Summary

The potential difference, E at any instant of a R-C circuit is

$$E_o = i_o\sqrt{R^2 + \frac{1}{C^2\omega^2}} \qquad \text{and} \qquad \tan\phi = \frac{1}{RC\omega}$$

The potential difference, E at any instant of a L-R circuit is

$$E_O = i_o\sqrt{R^2 + L^2\omega^2} \qquad \text{and} \qquad \tan\phi = \frac{L\omega}{R}$$

The potential difference E at any instant of a L-C-R series circuit is

$$E_0 = i_o\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} \quad \text{and} \quad \tan\phi = \frac{L\omega - \frac{1}{C\omega}}{R} = \frac{X_L - X_C}{R}$$

Here, $L\omega = X_L$ and $\frac{1}{C\omega} = X_C$

Therefore, $E_0 = i_o\sqrt{R^2 + (X_L - X_C)^2}$

The current, therefore lags behind the potential if $X_L > X_C$ and leads if $X_C > X_L$

Then the current I at any instant of a L-C-R parallel circuit is

$$I_0 = E_0\sqrt{\frac{1}{R^2}+\left(C\omega-\frac{1}{L\omega}\right)^2} \quad \text{and} \quad \tan\phi = \frac{C\omega-\frac{1}{L\omega}}{\frac{1}{R}} = \frac{R\left(X_L - X_C\right)}{X_L X_C}$$

Here $L\omega = X_L$ and $\frac{1}{C\omega} = X_C$

The current, therefore leads the potential if $X_L > X_C$ and lags behind if $X_C > X_L$

The resonance frequency of L-C-R series or parallel circuit is $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

The quality factor of L-C-R series circuit is $Q = \frac{\omega_0}{\Delta\omega}$

Questions

1. A coil of resistance 50 Ω and inductance of 4 H is connected in series with a condenser of 4 μF and an AC supply of 200 v and 50 Hz. Find out,
   (a) The impedance in the circuit
   (b) The phase difference between the current and the voltage.
   (c) The potential difference across the inductor coil
   (d) The potential difference across the capacitor.

2. A resistance of 15 Ω is joined in series with an inductance of 0.6 H. What capacitance should be put in series with the combination to obtain maximum current? What will be the potential difference across the resistor, inductor and the capacitor? The current is being supplied by a 200 V and 50 Hz AC mains.

3. The AC supply of frequency 1000Hz and 110 V is joined across a circuit containing a resistor of 20 Ω, an inductor of 2 mH and a capacitor of 2 μF. Find the value of the current. What must be the value of the capacitor in order that the current would be a maximum?

Answers

(1) (a) The impedance of the series L-C-R. Circuit is given by

$$Z=\left[R^2+\left(L\omega-\frac{1}{C\omega}\right)^2\right]^{1/2}$$

given that $R=50\Omega \quad L=4H \quad \omega=2\pi\times f=2\pi\times 50 \quad C=4\times 10^{-6}F$

$$\therefore Z=\left[50^2+\left(4\times 2\pi\times 50-\frac{1}{2\pi\times 50\times 4\times 10^{-6}}\right)^2\right]^{1/2}$$

$$=\left[2500+\left(1256.6-7.95\times 10^2\right)^2\right]^{1/2}$$

$$Z=464.33\Omega$$

(b) The current lags behind the applied voltage by θ

$$\text{Then} \quad \tan\theta = \frac{X_L - X_C}{R} = \frac{L\omega - 1/\omega C}{R}$$

$$\tan\theta = \frac{4\times 2\pi\times 50 - \dfrac{1}{2\pi\times 50\times 4\times 10^{-6}}}{50}$$

$$= 9.232$$

$$= \tan 83^{\circ}.8183$$

$$\theta = 83^{\circ}49'$$

(c) The potential difference across the inductance is

$$V_L = i_o L\omega = \frac{V_o}{Z} L\omega$$

$$= \frac{200}{464.33} \times 4 \times 2\pi \times 50$$

$$\underline{\underline{V = 541.26V}}$$

(d) The potential difference across the capacitor is

$$V_C = \frac{i_o}{C\omega} = \frac{V_o}{Z\omega C}$$

$$V_C = 200V \qquad Z = 464.33\Omega$$

$$\omega = 2\pi \times 50 \qquad C = 4 \times 10^{-6} F$$

Then, $V_C = \dfrac{200}{464.33 \times 2\pi \times 50 \times 4 \times 10^{-6}}$

$$= \underline{\underline{342.76V}}$$

Note: $V_L - V_C = 198.5V$

(2) For a L-C-R series circuit

$$E_o = i_o \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega} \right)^2}$$

When $L\omega = \frac{1}{C\omega}$, The maximum current flows in the circuit

Therefore $C = \frac{1}{L\omega^2} = \frac{1}{L(2\pi f)^2}$

Given that L = 0.6 H   f = 50 Hz

$$\therefore C = \frac{1}{0.6 \times (2\pi \times 50)^2} = \underline{\underline{16.8 \times 10^{-6}\,F}}$$

The maximum current is $i_o = \frac{E_o}{R}$

$E_o = 200V \qquad R = 15\Omega$

$$i_o = \frac{200}{15} = 13.3A$$

Potential difference across R.

$$V_R = i_o \times R = 13.3 \times 15$$

$$= 119.5\ V$$

Potential difference across L

$$V_L = i_o X_L = i_o \omega L = i_o L.2\pi f$$

$$V = 13.3 \times 0.6 \times 2\pi \times 50$$

$$= 2506.9 \text{ V}$$

Potential difference across C is

$$V_C = \frac{i_o}{C\omega} = \frac{13.3}{2\pi f C}$$

$$= \frac{13.3}{2\pi \times 50 \times 1.68 \times 10^{-5}}$$

$$V = 2510 \text{ V}$$

The slight difference between V_L and V_C is due to approximations in the calculations.

(3) $\text{Current} = \dfrac{\text{e.m.f}}{\text{impedance}}$

Here, e.m.f = 110 V

The impedance of this series circuit is given by $Z = \sqrt{R^2 + \left(L\omega - \dfrac{1}{C\omega}\right)^2}$

$R = 20\Omega \quad L = 2\text{mH} = 2\times10^{-3}\,\text{H}$

$C = 2\mu\text{F} = 2\times10^{-6}\,\text{F}$

$\omega = 2\pi f = 2\pi\times1000 = 6283.18 \text{ rad s}^{-1}$

$$\therefore Z = \sqrt{20^2 + \left(2\times10^{-3}\times6283.18 - \frac{1}{6283.18\times2\times10^{-6}}\right)^2}$$

$$= \sqrt{400 + \left(12566.36\times10^{-3} - 79.5\right)^2}$$

$$Z = 69.8\ \Omega$$

$$\therefore \text{ Current } = \frac{110}{69.8} = 1.57\text{A}$$

Current is maximum when $\omega L = \frac{1}{C\omega}$

$$\therefore C = \frac{1}{L\omega^2} = \frac{1}{L\times(2\pi f)^2}$$

$$= \frac{1}{2\times10^{-3}\times(6283.18)^2}$$

$$C = 1.26\times10^{-5}\,\text{F}$$