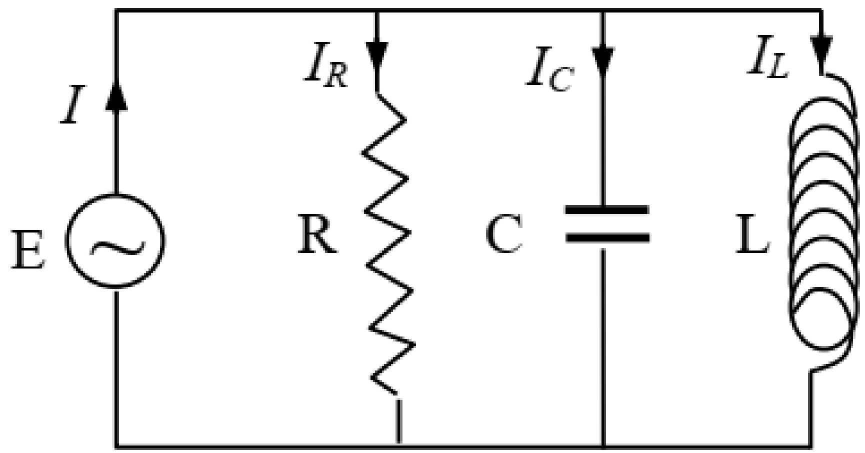
L-C-R Parallel Circuits



Let the branch currents through resistor, capacitor and inductor be I_R, I_C and I_L.

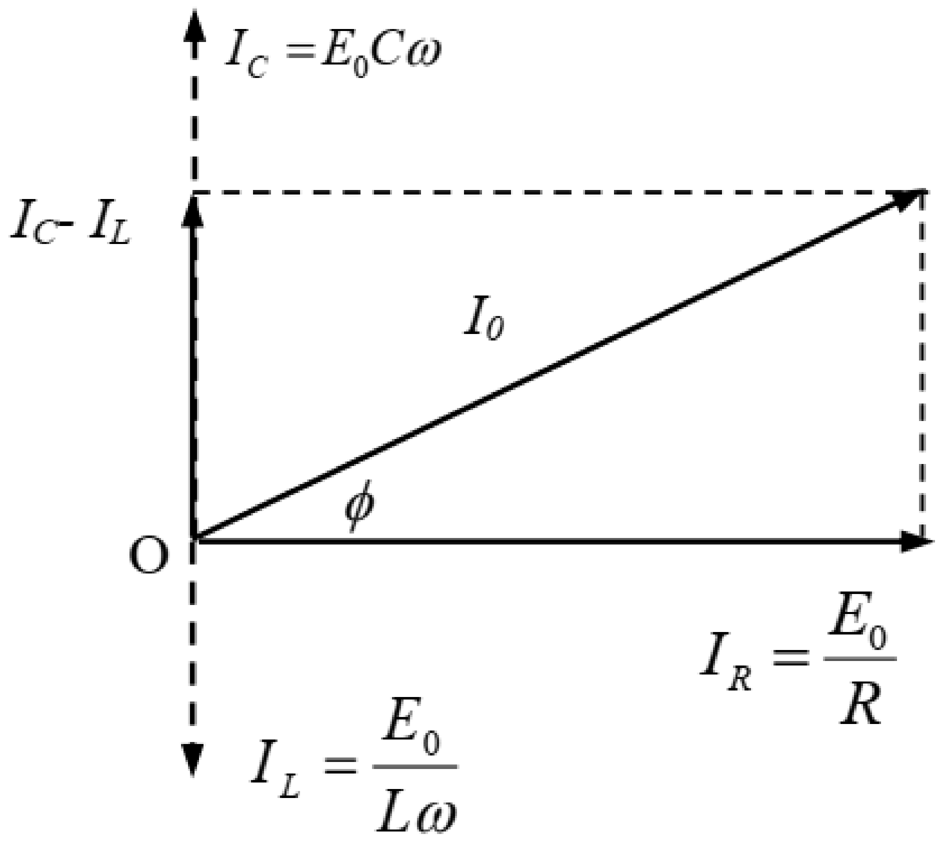
Let $E = E_o \sin \omega t$ be the voltage.

Then, $I = I_R + I_C + I_L$

$$I_R = \frac{E_0 \sin \omega t}{R},$$

$$I_C = C\frac{dv_C}{dt} = C\omega E_0 \sin\left(\omega t + \frac{\pi}{2}\right),$$

$$I_L = \frac{\int -v_L \, dt}{L} = \frac{1}{L\omega} E_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$



$$I = \frac{E_0 \sin \omega t}{R} + C\omega E_0 \sin\left(\omega t + \frac{\pi}{2}\right) + \frac{1}{L\omega} E_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Then, $I_0{}^2 = \left(\frac{E_0}{R}\right)^2 + E_0{}^2\left(C\omega - \frac{1}{L\omega}\right)^2$

$$I_0 = E_0\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}$$ and $$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}}$$

But $L\omega = X_L$, inductive reactance and $\frac{1}{C\omega} = X_C$, capacitive reactance

Therefore,

$$I_0 = E_0 \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L} \right)^2}$$

If I makes the angle ϕ with ωt

$$\tan \phi = \frac{C\omega - \frac{1}{L\omega}}{\frac{1}{R}} = \frac{1/X_C - 1/X_L}{1/R} = \frac{R(X_L - X_C)}{X_L X_C}$$

Then the current I at any instant can be given by,

$$I = I_o \sin(\omega t + \phi)$$

The current, therefore leads the potential if $X_L > X_C$ and lags behind if $X_C > X_L$

Resonance

The impedance of a circuit containing inductance resistance, and capacitance connected in parallel is

$$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}}$$

In this circuit when $\left(C\omega - \frac{1}{L\omega}\right) = 0$ the impedance become it is maximum.

Then $\quad L\omega = \frac{1}{\omega C} \quad$ or $\quad X_L = X_C$

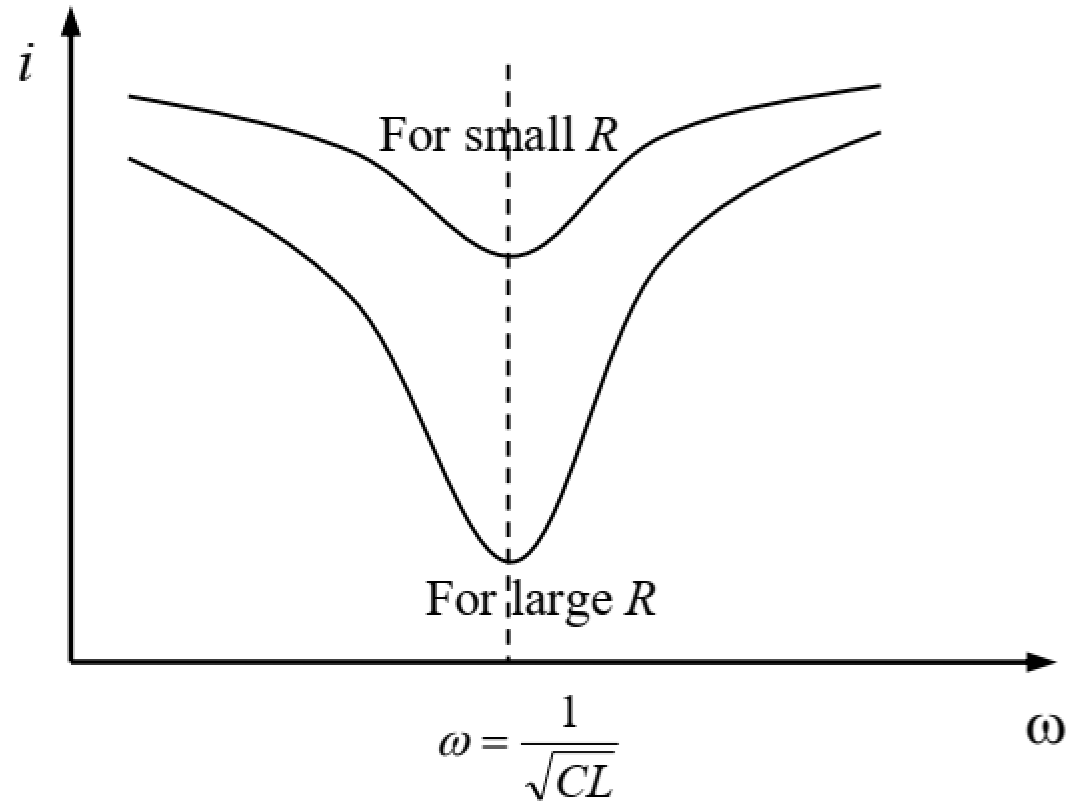
$$\omega^2 = \frac{1}{LC} \qquad \omega = \frac{1}{\sqrt{LC}}$$

Since $\tan\phi = \frac{X_L - X_C}{R}$, phase angle $\phi = 0$, when $X_L = X_C$

Then the current of this circuit is in phase with the voltage which we call the *resonance* condition.

The resonance frequency of the parallel LCR circuit is also $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

But when the frequency of the impressed voltage equals this resonance frequency the current in the LCR parallel circuit reaches its minimum.


i
For small R
For large R
ω
ω = 1/√CL