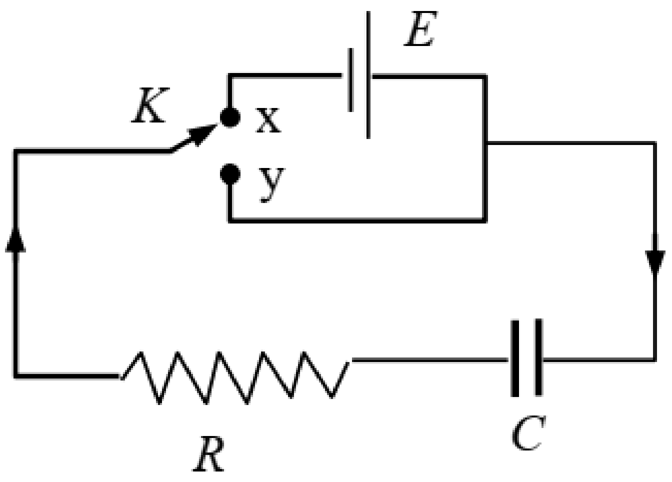
C-R Series Circuit

A capacitor C and a resistor R in series are connected to cell E through a two way switch. When the switch is in the upper position at point X is connected to the battery which charges the capacitor through resistor R.



If the capacitor is initially uncharged then the initial potential difference across it is zero, and entire battery potential appears across the resistor.

Then the initial current $I = E/R$ where E is the e.m.f. of the battery.

As the capacitor charges its voltage increases and the potential difference across the resistor decreases, corresponding to decrease in current.

After a long time the capacitor has become fully charged and the entire battery voltage appears across the capacitor.

Now there is no potential difference across the resistor.

Let q be the charge on the capacitor and i the charging current at some instant after switch is thrown up.

Then the instantaneous potential differences across C and R are

$$V_C = q/c \qquad V_R = iR$$

Therefore $$E = q/c + iR$$

But $$i = \frac{dq}{dt}$$

Therefore $$E = \frac{q}{c} + R\frac{dq}{dt}$$

$$Ec - q = Rc.\frac{dq}{dt}$$

$$\frac{dq}{Ec-q} = \frac{1}{Rc}.dt$$

Let at t = 0, q = 0 and at t = t, q = q. By integrating both sides.

$$\int_0^q \frac{dq}{Ec-q} = \frac{1}{Rc}\int_0^t dt$$

$$-Log\left|\frac{Ec-q}{Ec}\right| = \frac{1}{Rc}t$$

$$\frac{Ec-q}{Ec} = e^{-t/cR}$$

$$q = Ec\left(1-e^{-t/cR}\right)$$

When $t \rightarrow \propto$ $\quad q = Ec$

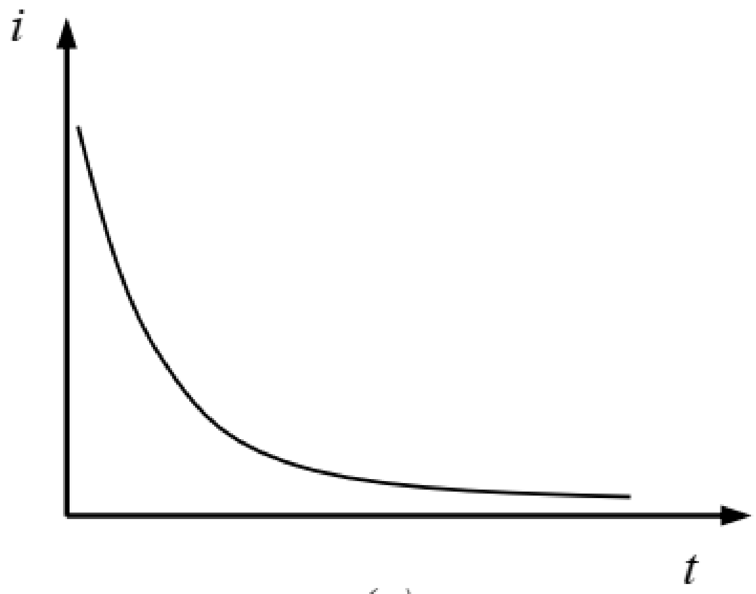
Let this be q_o

Then time derivative of this expression is

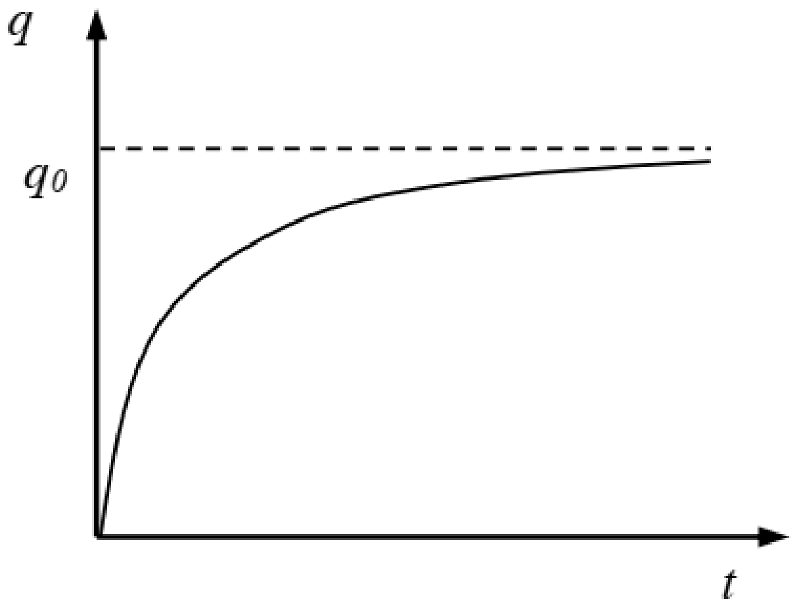
$$\frac{dq}{dt} = \frac{E}{R} e^{-t/Rc}$$

$$i = i_o e^{-t/Rc} \quad \text{where} \quad i_o = E/R$$

The charge and the current are therefore both exponential functions of time. These are shown in fig. (a) and (b).



(a)



(b)

At time t = RC the current has decreased to 1/e of its initial value and the charge has increased to within 1/e of its final value.

The product RC is called the time constant or the relaxation time, of the circuit.
It is the time in which the current would decrease to zero, if it continued decrease at its initial rate.

The half-life of the current is the time for the current to decrease to half its initial value.

Setting $i = i_o/2$

We find $t_{1/2} = RC \ln 2 = 0.693\, RC$

Suppose next that the capacitor has acquired a charge q_o and that the switch is thrown to the down position.

The capacitor then discharges through the resistor and its charge eventually reduces to zero.

$$V_R + V_C = 0$$

then $iR + \frac{q}{c} = 0$

$$\frac{dq}{dt}R + \frac{q}{c} = 0 \quad \rightarrow \quad \frac{dq}{q} = -\frac{1}{Rc}dt$$

$$\int_{q_o}^{q} \frac{dq}{q} = -\int_0^t \frac{1}{Rc}dt$$

$$\ln\frac{q}{q_o} = -\frac{t}{Rc} \qquad\qquad q = q_o e^{-t/Rc}$$

Figure shows the buildup of charge on closed circuit and the discharge on open circuit.