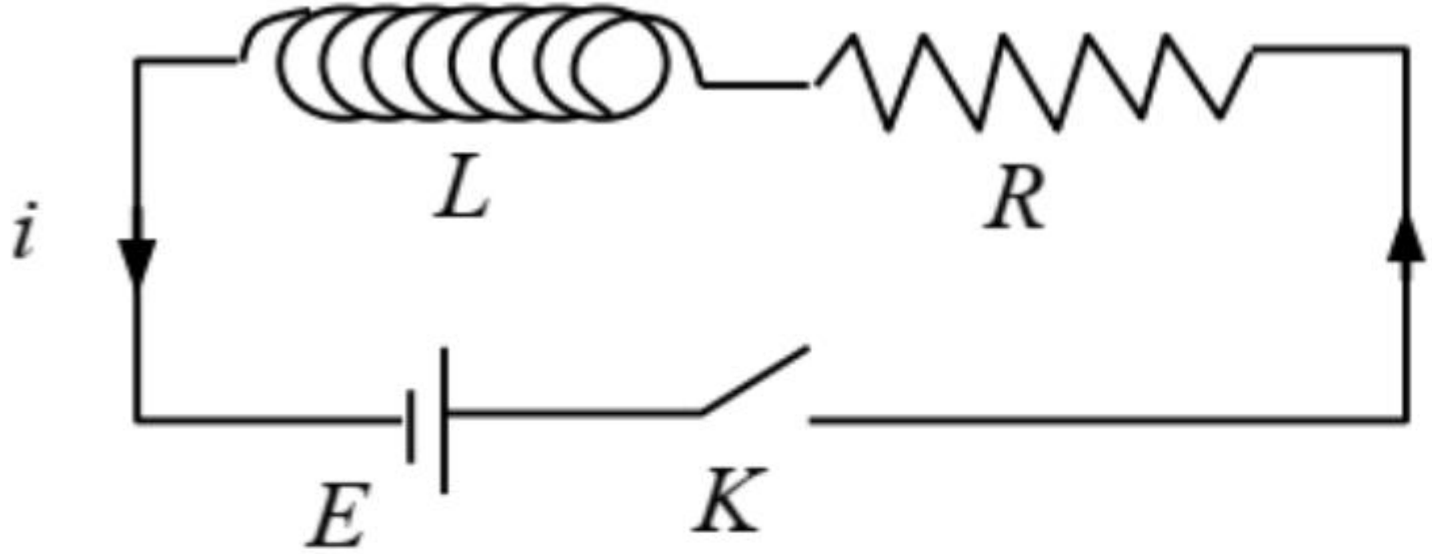
L-R Series Circuit

Consider a non-resistive inductor in series with a non-inductive resistor as shown in the figure below.



When the switch is suddenly closed, the current does not instantly rise to its final value because of the self-induced e.m.f., but grows at a rate which depends on the inductance and the resistance of the circuit.

At some instant, let *i* be the current in the circuit. Then, the instantaneous e.m.f induced across the inductor is $V_L = -L\left(\frac{di}{dt}\right)$.

From Ohm's law,

$$E - L\frac{di}{dt} = i\,R$$

$$E = L\frac{di}{dt} + iR$$

The rate of increase of current

$$\frac{di}{dt} = \frac{E - iR}{L} = \frac{E}{L} - \frac{iR}{L}$$

At the instant the circuit is closed, I = 0 and the current grows at the rate,

$$\left(\frac{di}{dt}\right)_{initial} = \frac{E}{L}$$

The greater the self-inductance L, the more slowly does the current increase. After time t the current reaches its final-steady value I_o. Then, the rate of increase is zero and

$$0 = \frac{E}{L_1} - \frac{R}{L} I_o \qquad I_o = \frac{E}{R}$$

To obtain an expression for the current as a function of time, rearrange the expression as

$$\frac{Ldi}{E - iR} = dt$$

At t = 0, i = 0 and at t = t, $i = i$.

Therefore, integrating both sides we get

$$\int_0^i \frac{Ldi}{E - iR} = \int_0^t dt$$

$$-\frac{L}{R}\log\left|\frac{E - iR}{E}\right| = t$$

$$\log\left|\frac{E - iR}{E}\right| = -\frac{RT}{L}$$

$$\frac{E - iR}{E} = e^{-Rt/L}$$

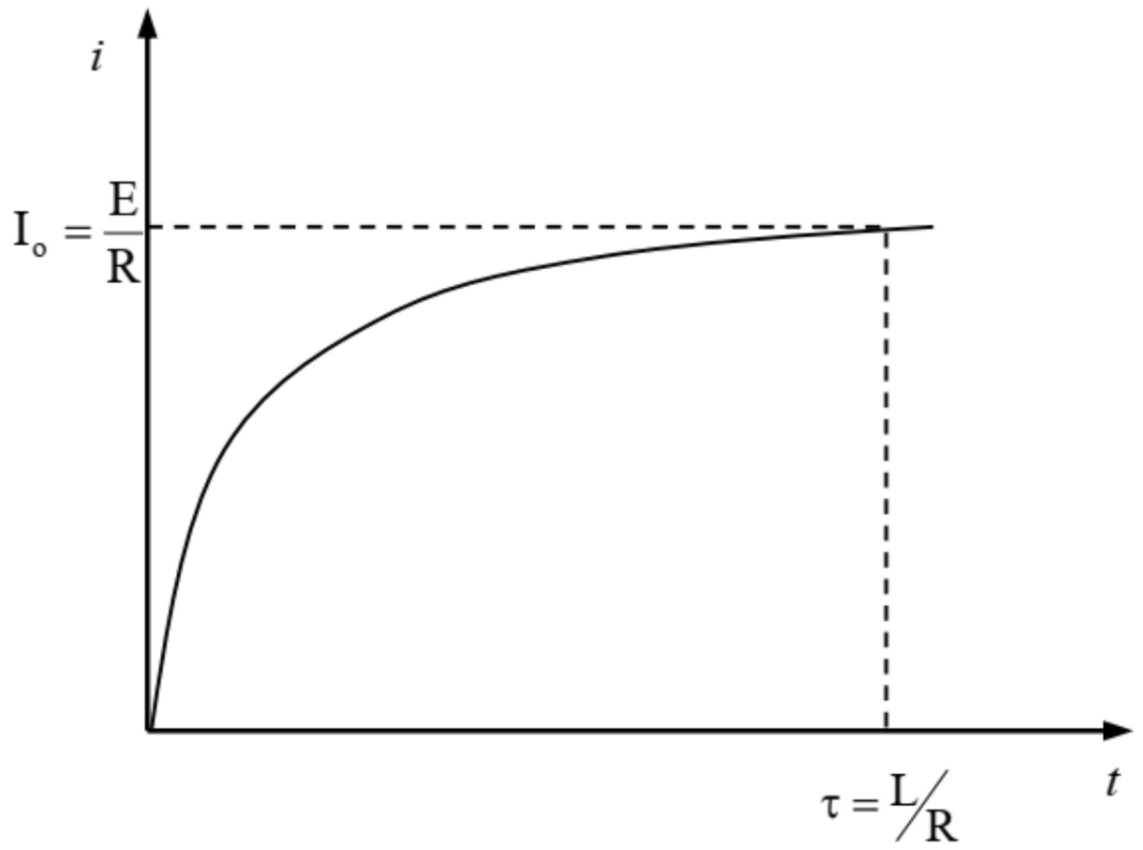
$$E - E\,e^{-Rt/L} = iR$$

$$i = \frac{E}{R}\left(1 - e^{-Rt/L}\right) or$$

$$i = I_0\left(1 - e^{-Rt/L}\right)$$

At time t = 0, I = 0 and $\frac{di}{dt} = \frac{E}{L}$ and

as $t \to \infty$, $I \to \frac{E}{R}$ and $\frac{di}{dt} = 0$



Time equal to L/R the current has risen to $\left(1-\frac{1}{e}\right)$ or about 0.63 of its final value.

This time is called the **time constant**, or the decay constant, for this circuit.

It is denoted by τ (tau). Then $\tau = L/R$

Decay of Current

Suppose that the battery is removed from the circuit by opening the switch.

Now the e.m.f in the circuit is zero.

The resistance of the circuit remains unchanged and the current in the circuit decays from maximum value $i_o = \left(E/R\right)$ to zero.

Hence from Ohm's law

$$-L\frac{di}{dt} = Ri$$

$$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating we get,

$$\log_e i = -\frac{Rt}{L} + A$$

Where A is a constant of integration. Since at $t = 0,\ i = i_o$ we have $\log_e i_o = A$.

Substituting this value for A in equation we get

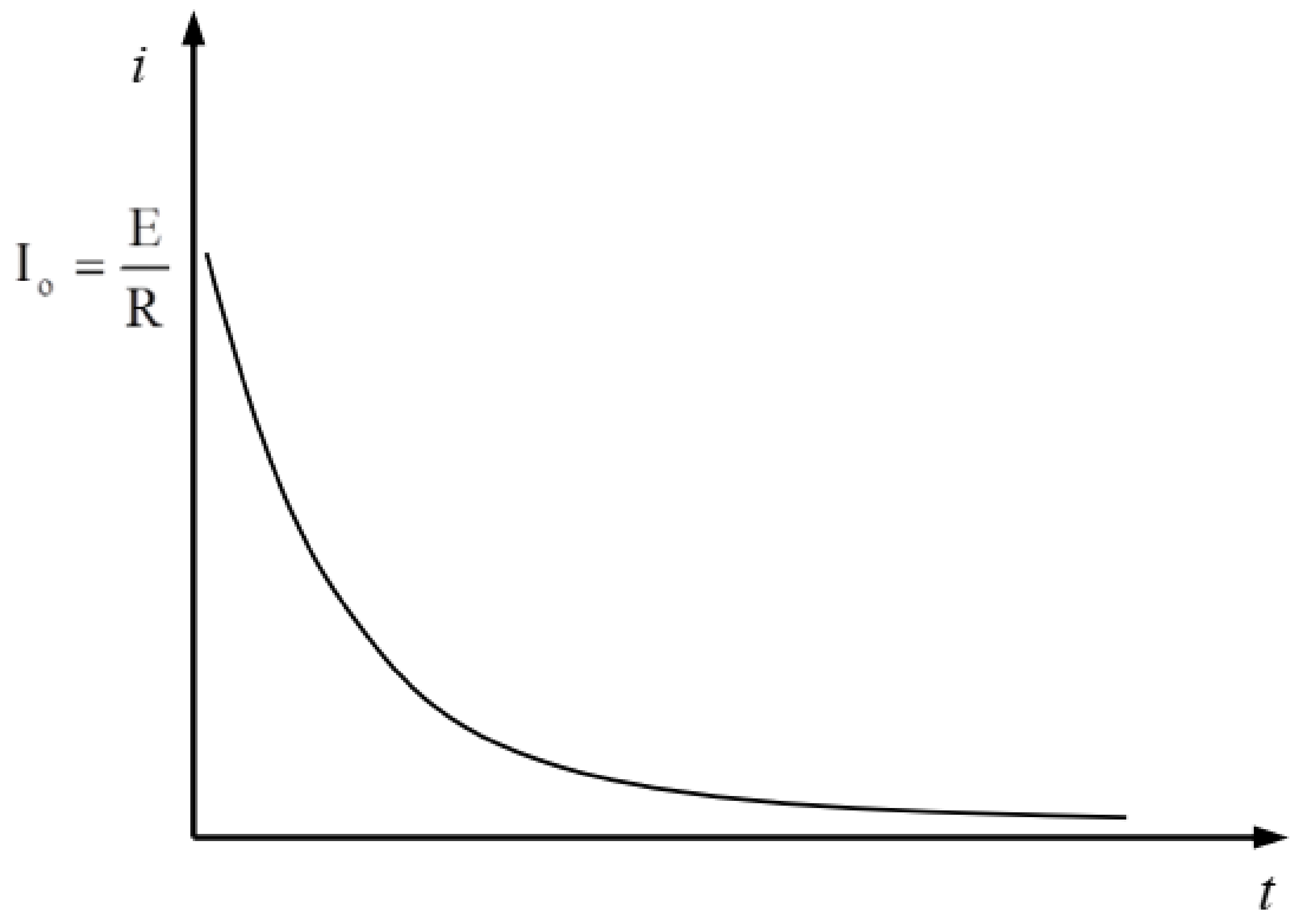
$$\log_e i = -\frac{Rt}{L} + \log_e i_o$$

$$\log_e \frac{i}{i_o} = -\frac{Rt}{L}$$

$$\frac{i}{i_o} = e^{-\frac{R}{L}t}$$

$$i = i_0 \, e^{-\frac{R}{L}t}$$

From this it is clear that the current in the circuit decays exponentially.


i
$I_o = \frac{E}{R}$
t

After a time $t = L/R$, the current in the circuit is given by

$$i = i_o e^{-\frac{R}{L}\times\frac{L}{R}}$$

$$= i_o e^{-1}$$

$$= \frac{1}{e} i_o$$

This time L/R is called the time constant of decay current.

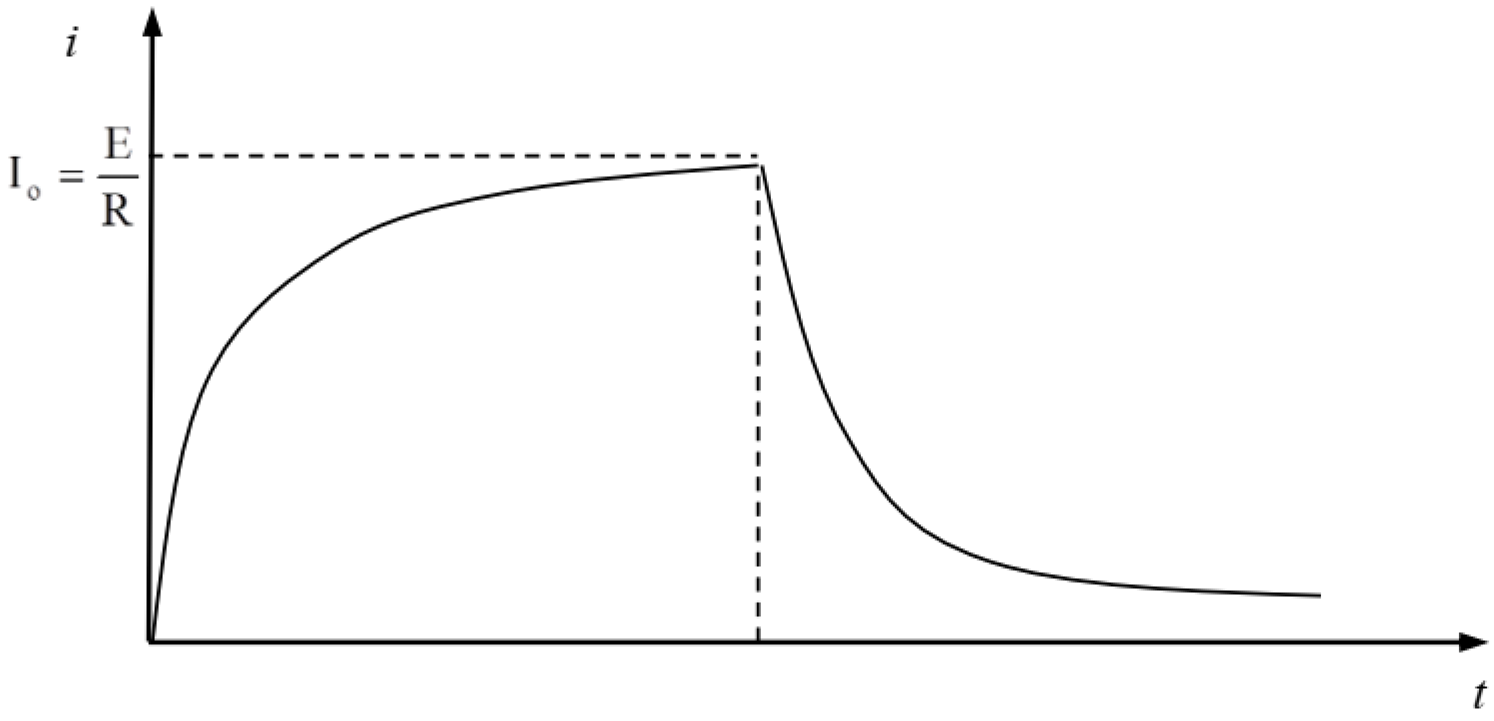
Therefore the time constant L/R of a R –L circuit may be defined as the time in which the current in the circuit falls to 1/e of its maximum value when external source of e.m.f is removed.

The energy required to maintain the current during this decay is provided by the energy stored in the magnetic field of the inductor.

From equation $i = i_o e^{-\frac{R}{L}.t}$ the rate of decay of current in the circuit is given by

$$\frac{di}{dt} = -i_o \frac{R}{L} e^{-\frac{R}{L}t} = -i_o \frac{R}{L} \frac{i}{i_o} = -\frac{R}{L} i$$

When closed and open circuits are combined together, variation of i with time t is shown in the figure.



Question 01)

An inductor of self-inductance 300 mH and resistance 5Ω is connected to a battery of negligible Internal resistance. Calculate the time in which the current will attain half its final steady value.