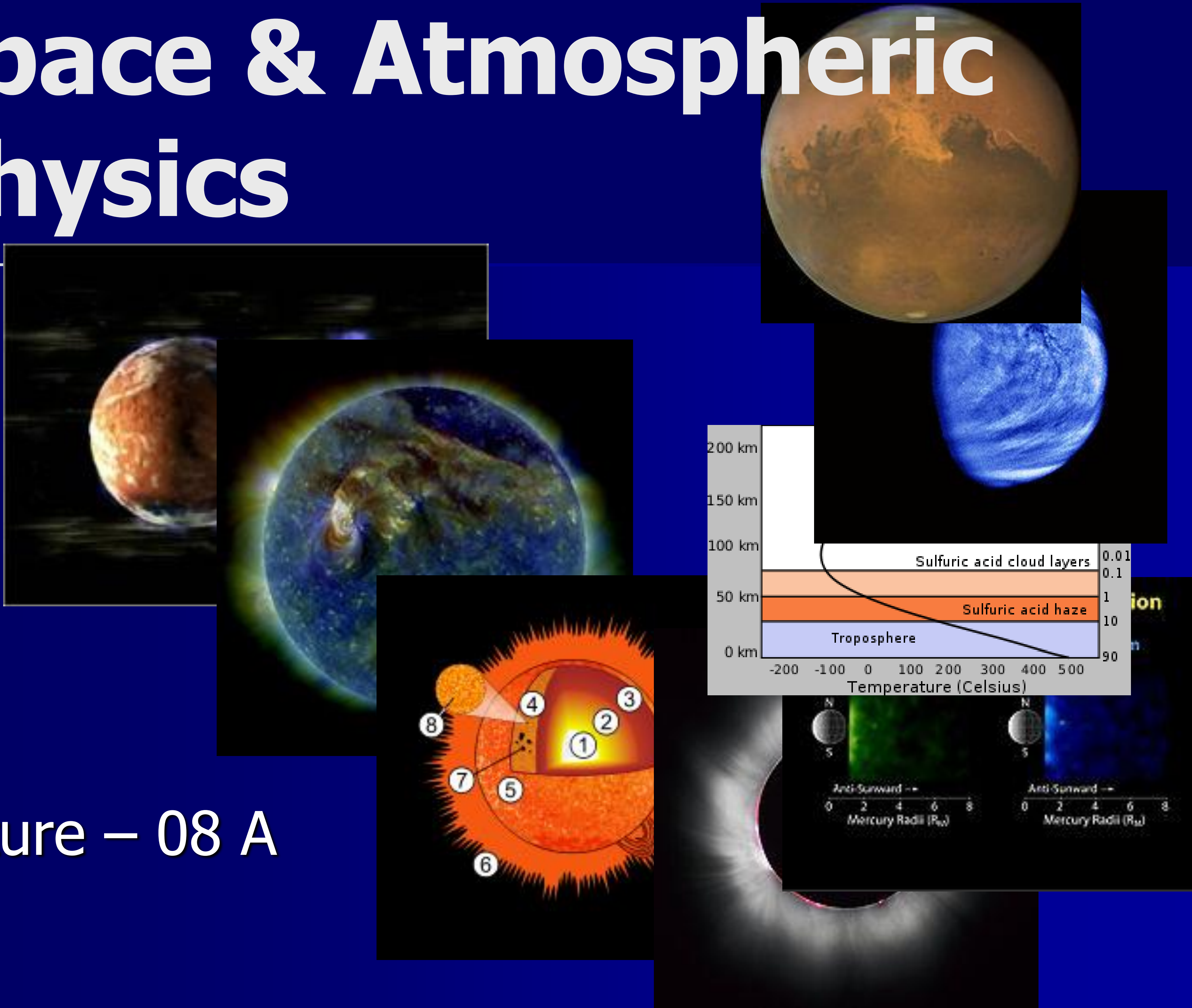
Space Physics

Space & Atmospheric Physics



Lecture – 08 A

Continuous Assignments 01 – 07 Marks

4th Year Special B.Sc. (Honors) in Physics - 2024

Assignment #

No.	Index No.	Name	01	02	03	04	05	06	07	
1	AS2020048	Ms.H.N. Edirisinghe (Helani)	100	100	50	ab	20	00	70	
2	AS2020250	Mr.T.I.K Peiris (Isuru)	100	100	50	100	90	20	90	
3	AS2020275	Mr.H.P.S De Silva (Pathum)	ab	ab	50	00~~70~~ *	ab	00	90	
4	AS2020368	Ms.D.A.D.D.Dissanayake (Dinithi)	100	100	50	100	80	15	100	
5	AS2020378	Mr.H.V.H.C.Dayananda (Hasitha)	ab	100	50	100	90	15	80	
6	AS2020459	Ms. D.D.M. Manurangi (Madushika)	100	100	50	80	40	20	60	
7	AS2020462	Mr.C.S. Wijesinghe (Chamika)	100	100	50	70	100	70	80	
8	AS2020500	Mr.D.N.U.Keashan (Nisal)	100	100	50	100	80	15	ab	
9	AS2020609	Mr.H.V.D.Hettiarachchi (Vimuth)	100	100	50	00~~70~~ *	80	ab	80	
10	AS2020636	Mr.A.M. Musaraf	ab *	100	50	100	80	20	100	
11	AS2020658	Mr.L. S. D. Lekamge (Sathira)	100	100	50	100	90	10	90	
12	AS2020907	Ms.K. Thahir (Khadeeja)	100	100	50	80	00	20	90	
13	AS2020916	Mr.H.A.C.M.Dissanayake (Chathura)	100	100	50	100	ab	00	80	
14	AS2020920	Ms.T. Paramananthan (Thanuja)	100	100	50	100	00	10	90	
15	AS2020930	Ms.G.A.A.Perera (Amasha)	100	100	50	80	ab	15	50	
16	AS2020948	Mr.C.C. Liyanage (Chinthaka)	100	100	50	100	90	00	90	

PHY 497 2.0 – Space & Atmospheric Physics

Continuous Assignment – 08

What is the **Chapmen Layer Theory**?

What is the importance of this theory to Radio Transmission?

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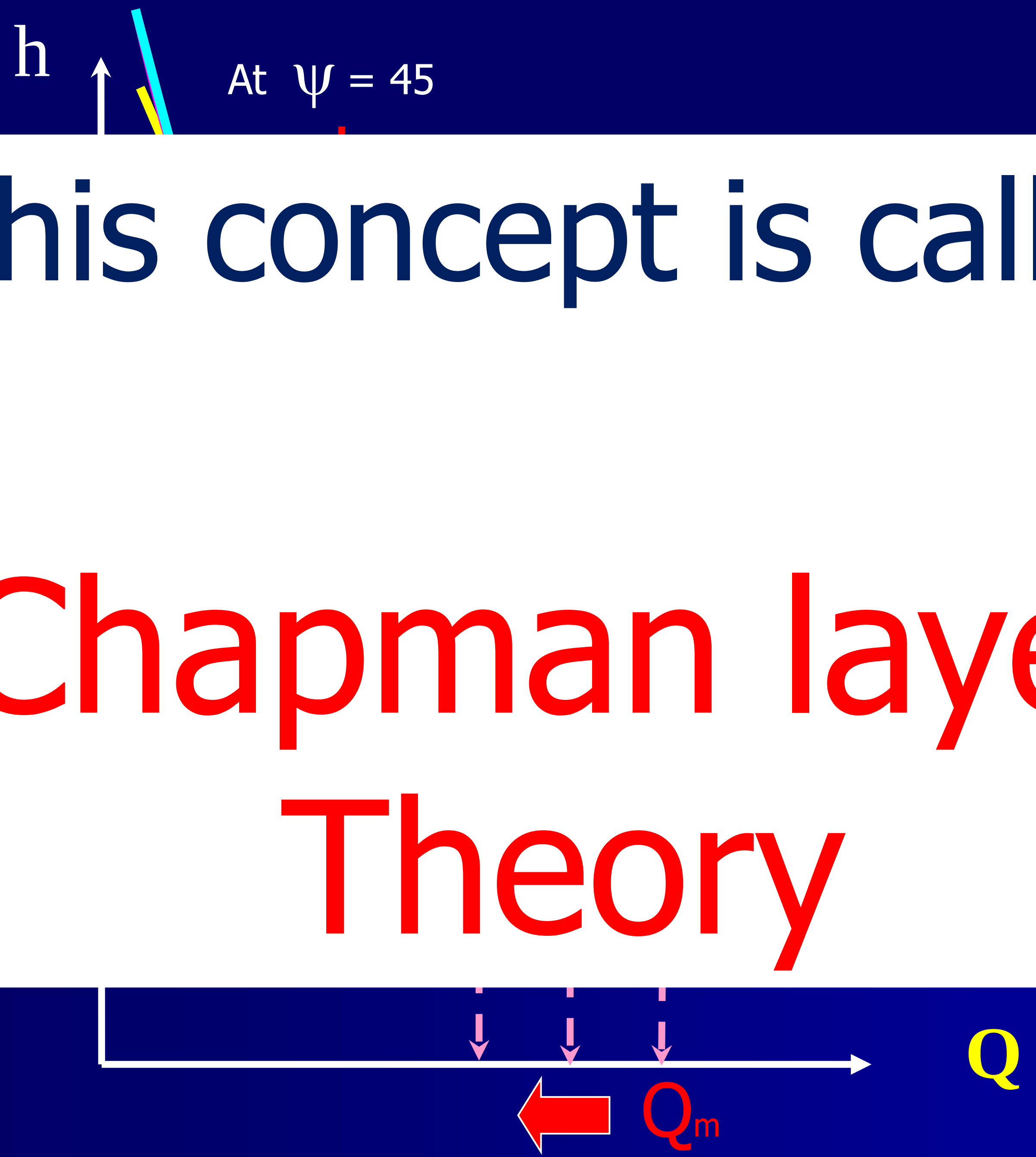
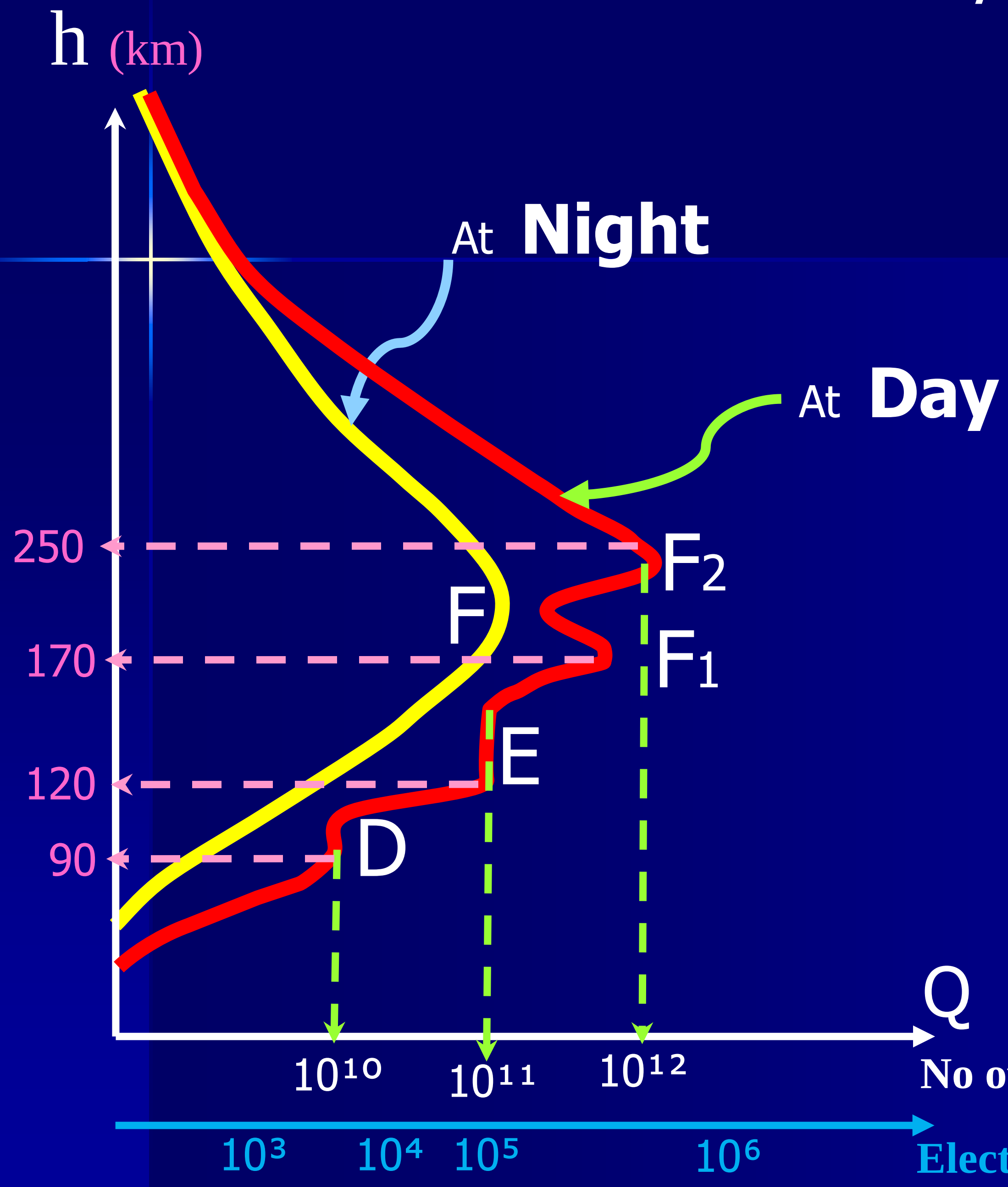
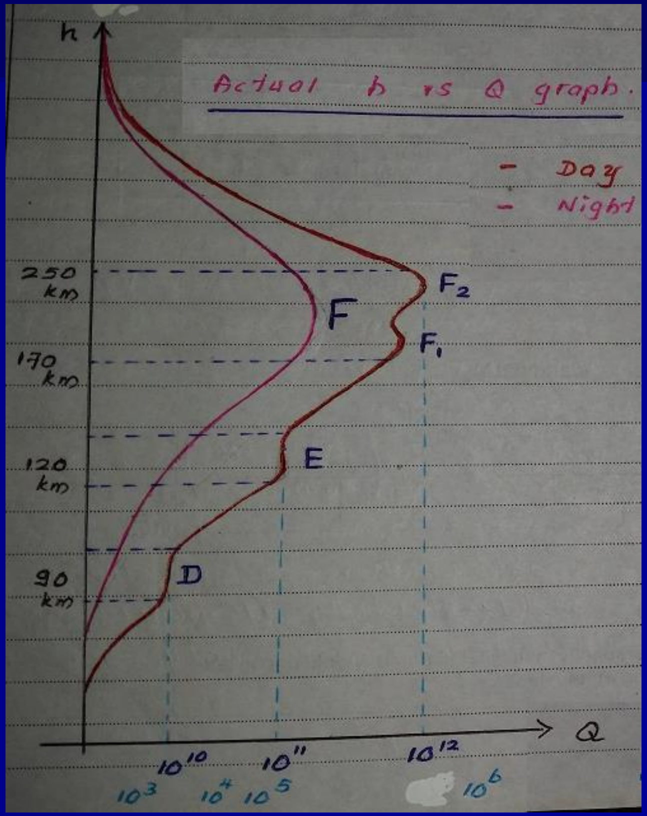
This concept is called

Chapman layer Theory



Actual *h* vs *Q* graph





Plasma Frequency

Let us consider an ionized layer with an uniform electron density N and radio waves of frequency f incident normally (at right angles) upon the layer. If the frequency is above a limiting frequency f_p the waves will pass through the layer, whereas if **$f < f_p$**, the waves will be reflected back. This critical frequency is called the **Plasma Frequency, f_p** and is proportional to the square root of the **electron density, N** of the Layer

$$f_p = \frac{e}{2\pi(\varepsilon_o m)^{1/2}} N^{1/2}$$

$$f_p = 9 N^{1/2}$$

$$f_p \,\alpha\, N^{1/2}$$

Eg : If electron density at some height is 10^{12} e^n/m^3, Find the plasma frequency of the medium at that height.

$$f_p = 9N^{1/2} \Rightarrow f_p = 9\times\left(10^{12}\right)^{1/2} \Rightarrow f_p = 9\times10^6$$

$$\Rightarrow f_p = 9\ MHz$$

That means, if we send a Radio Wave of frequency 9 MHz , it is reflected from the region of the atmosphere when the electron density is 10^{12} e^n/m^3.
That height is situated at **F** (actually **F_2** region)

But if we send **UHF (300 MHz)** or **VHF (30 MHz)** signal (Radio Wave); the wave **goes through the ionosphere** without any reflection !

Ionospheric regions

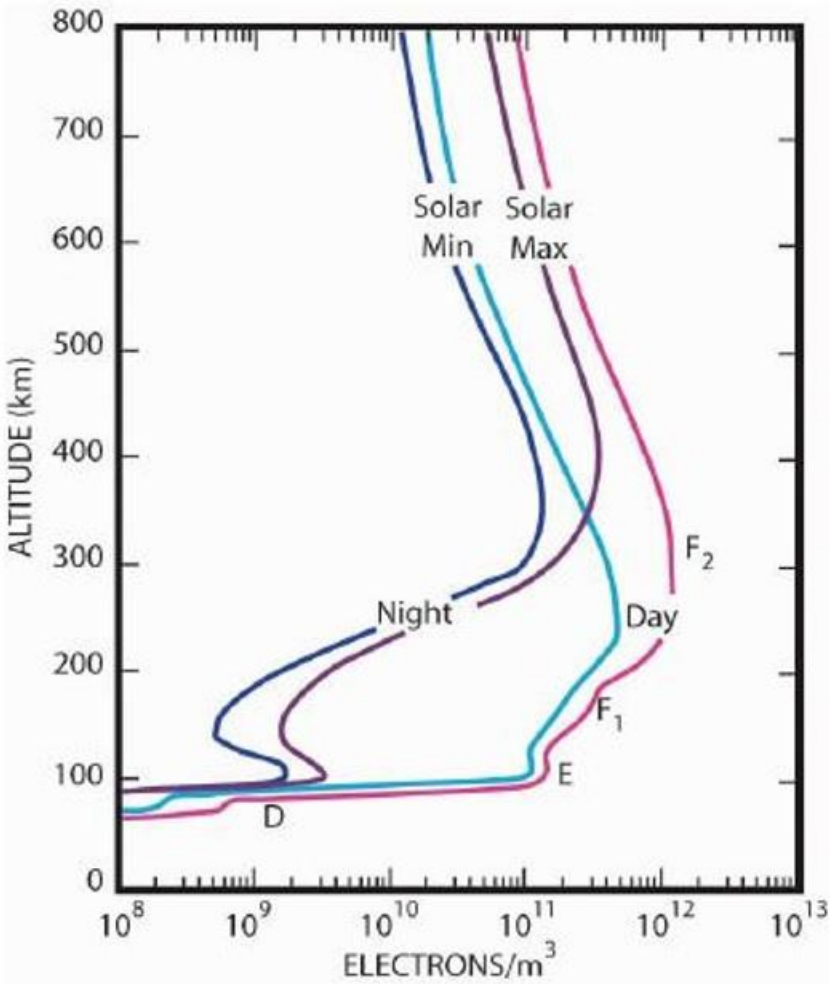


Figure: Typical ionospheric electron density profiles.

Ionospheric regions and typical daytime electron densities:

- D region: 60–90 km, $n_e = 10^8$–10^{10} m^{-3}
- E region: 90–150 km, $n_e = 10^{10}$–10^{11} m^{-3}
- F region: 150–1000 km, $n_e = 10^{11}$–10^{12} m^{-3} .

Ionosphere has great variability:

- Solar cycle variations (in specific upper F region)
- Day-night variation in lower F, E and D regions
- Space weather effects based on short-term solar variability (lower F, E and D regions)

For D region :

- D region: 60–90 km, $n_e = 10^8 – 10^{10}\ \text{m}^{-3}$

$$f_p = 9\,N^{1/2} \Rightarrow f_p = 9 \times \left(10^8\right)^{1/2} \Rightarrow f_p = 9 \times 10^4 \Rightarrow f_p = 90\ kHz$$

$$\Rightarrow f_p = 9 \times \left(10^{10}\right)^{1/2} \Rightarrow f_p = 9 \times 10^5 \Rightarrow f_p = 900\ kHz$$

That means, if we send a Radio Wave of frequency 90 kHz to 900 kHz, it is reflected from the **D** region; when the electron density is 10^8 - 10^{10} e^n/m^3.

For E region :

- E region: 90–150 km, $n_e = 10^{10}$–10^{11} m^{-3}

$$f_p = 9N^{1/2} \Rightarrow f_p = 9\times\left(10^{10}\right)^{1/2} \Rightarrow f_p = 9\times10^5$$

$$\Rightarrow f_p = 900\ kHz$$

$$\Rightarrow f_p = 9\times\left(10^{11}\right)^{1/2} \Rightarrow f_p = 9\times10^5\times\sqrt{10}$$

$$\Rightarrow f_p = 2.85\ MHz$$

That means, if we send a Radio Wave of frequency 900 kHz to 2.85 MHz, it is reflected from the **E** region; when the electron density is 10^{10} - 10^{11} e^n/m^3.

For F region :

- F region: 150–1000 km, $n_e = 10^{11}$–10^{12} m^{-3} .

$$f_p = 9\,N^{1/2} \Rightarrow f_p = 9\times\left(10^{11}\right)^{1/2} \Rightarrow f_p = 9\times10^{5}\times\sqrt{10}$$

$$\Rightarrow f_p = 2.85\ MHz$$

$$\Rightarrow f_p = 9\times\left(10^{12}\right)^{1/2} \Rightarrow f_p = 9\times10^{6}$$

$$\Rightarrow f_p = 9\ MHz$$

That means, if we send a Radio Wave of frequency 2.85 MHz to 9 MHz, it is reflected from the **F** region; when the electron density is 10^{11} - 10^{12} e^n/m^3.

In conclusion, the plasma frequency of an ionized region is the natural frequency at which the electrons of the region would oscillate about their position of equilibrium if their original condition was disturbed.

The disturbance in this case is caused by the **electric field** of the wave which also varies in a harmonic fashion with the frequency *f* of the **Radio Wave**. i.e.;

$$E_x = E_o \cos \omega t$$

$$\Rightarrow \quad E_x = E_o \cos 2\pi f t$$

As a result, the electrons become forced harmonic oscillators because they are forced to oscillate in the frequency of the radio wave rather than in their own natural plasma frequency.

The equation of the forced harmonic oscillator with an external force F is;

$$\vec{F} = m\vec{a}$$

$$\Rightarrow \quad F_o \cos\omega t - \frac{e^2 N x}{\varepsilon_o} = m\frac{d^2x}{dt^2}$$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} = -m\frac{e^2 N}{\varepsilon_o m}x + F_o \cos\omega t$$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} = -m\omega_p{}^2 x + F_o \cos\omega t$$

And its solution is : $x = x_o \cos\omega t$

Where, $x_o = \dfrac{F_o}{m\left(\omega^2 - \omega_p^2\right)}$

$$x = x_o \cos \omega t$$

Where,

$$x_o = \frac{F_o}{m\left(\omega^2 - \omega_p^2\right)}$$

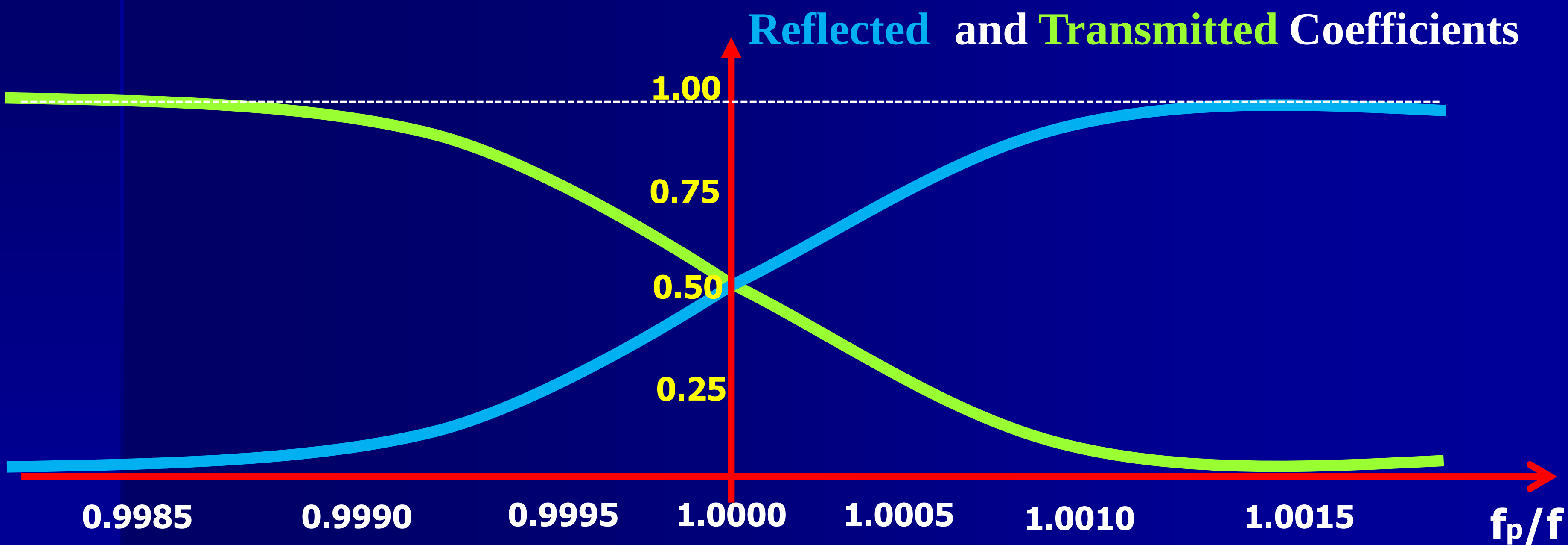
When the two frequencies are far apart, the **amplitude x_o is small** and **tends to zero** for **large values of ω**.

When on the other hand **ω approaches ω_p**, the **amplitude of the oscillation becomes very large**. It is very much like pushing a child on a swing. One gets the best results when the periodic pushes are coordinated with the natural period of the swing.

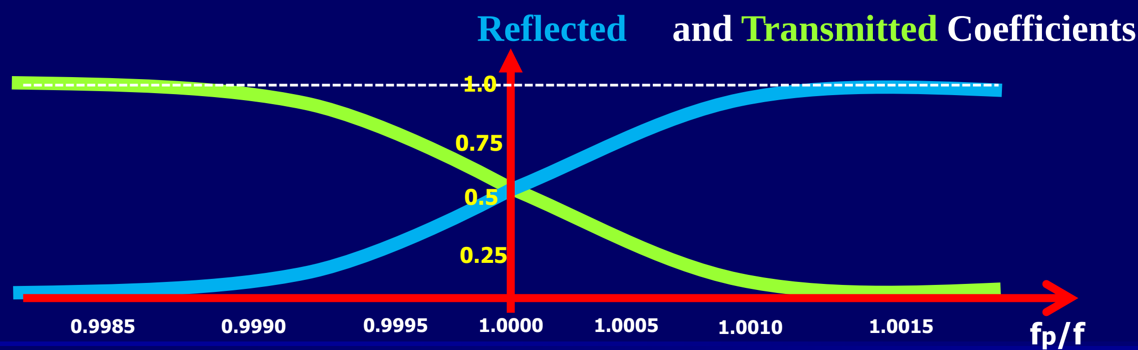
At **ω = ω_p**, the **amplitude appears to become infinite**, but **this not actually happen** because frictional and other forces that are normally negligible become important near the resonance frequency.

At frequencies below the plasma frequency of the medium the **transmitted** (forward) **wave tends to zero**, while the **reflected** (backward) **wave tends to reach the full intensity** of the incoming wave !

In the full wave solution of the problem, liked in quantum mechanics the **transmission** and **reflection coefficients** very smoothly with frequency **0** to **1**.



The change of the reflection |R| and the transmission |T| coefficients with frequency, for a parabolic electron density profile.



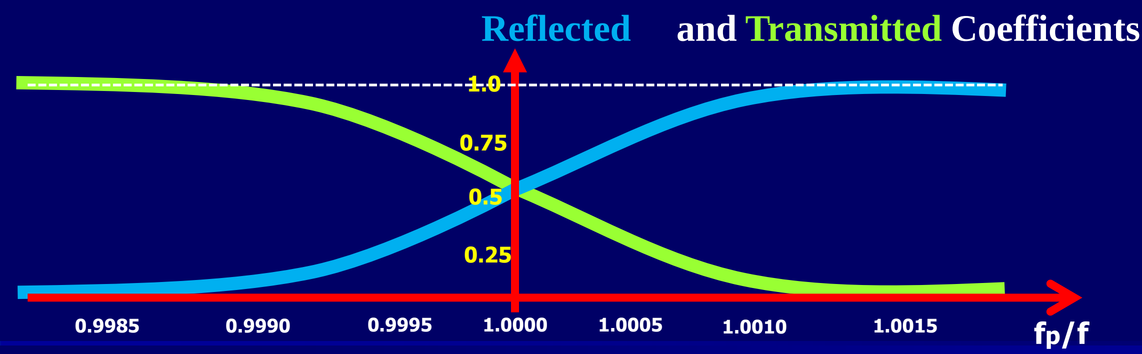
As seen from the above diagram, which describes the passage of radio waves through a parabolic layer of maximum plasma frequency f_p, most of this variation takes place very near f_p and for this reason we can adopt the classical step function formulation and simply state that radio waves with $f > f_p$ will be able to pass through this medium, whereas radio waves with $f < f_p$ will be reflected from this medium.

The group velocity of radio waves, i.e.: the velocity with which a group of radio waves (a radio signal) propagates through a plasma, is given by the following relation :

$$V_{gr} = \frac{c}{\mu_{gr}} \qquad (01)$$

Where μ_{gr} is the group index of reflection, which is related to the index of reflection μ of the plasma through the expression,

(P.T.O)



$$\frac{1}{\mu_{gr}} = \mu = \left(1 - \frac{f_p}{f^2}\right)^{1/2} \quad \rightarrow 02$$

From 01 and 02 it follows that, $V_{gr} = \mu\, c$

which says that the group velocity becomes zero when , $\mu = 0$

$$\Rightarrow \quad f = f_p = \left(\frac{e^2}{\varepsilon_o m} N\right)^{1/2} \quad \rightarrow 03$$

Which occurs, as we have seen, when the waves are about to be reflected. It should be made clear that this is the case only for **normal incidence**.

When the radio waves approach a plasma layer at an angle θ to the normal, then the **critical frequency** (the heights frequency reflected by the layer) **fc** is;

$$f_c = f_p \sec\theta$$

In accordance with that we have discussed up now, radio waves transmitted vertically from the ground will be reflected in the ionosphere at a height where the plasma frequency of the ionosphere becomes equal to the frequency of the wave.

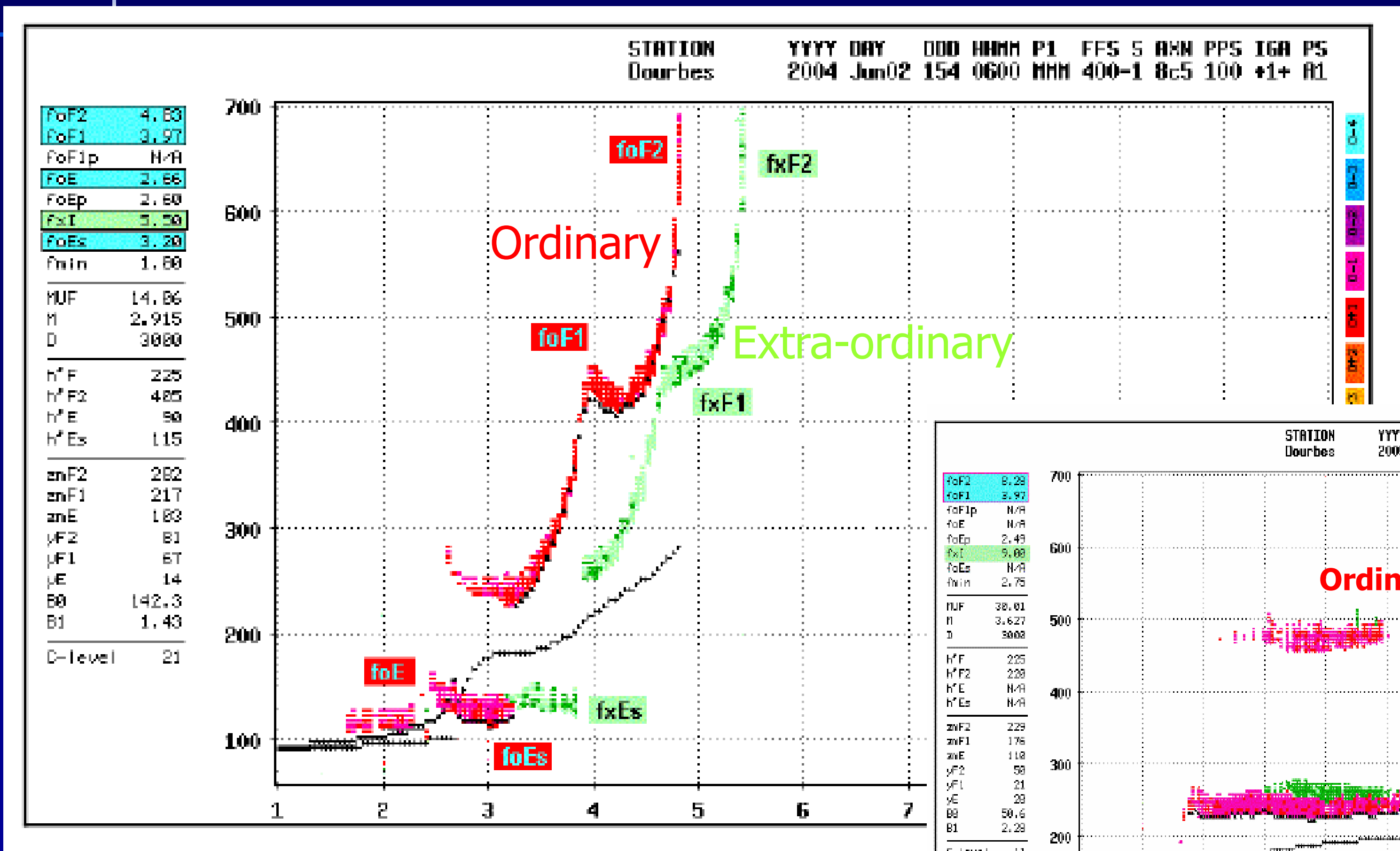
As seen from the above equation, for oblique transmission the same layer will be able to reflect considerably higher frequencies.

The presence of the **Earth Magnetic Field** makes the ionosphere a magneto-active plasma. (i.e.; A plasma with an embedded magnetic field) Radio waves in magneto-active plasma split into two modes of propagation called the **Ordinary** and the **Extra-ordinary**.

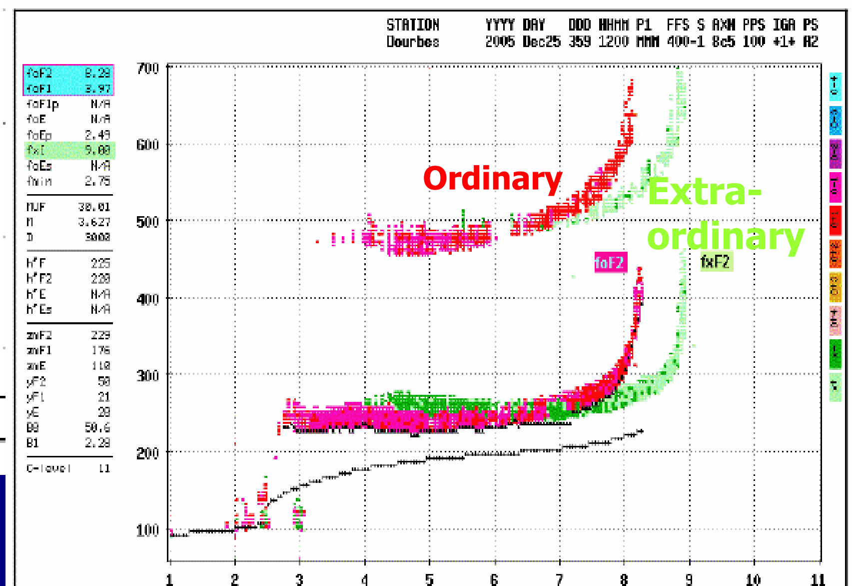
Each mode has its own index of reflection which is much more completed than equation 03.

$$f = f_p = \left(\frac{e^2}{\varepsilon_o m} N\right)^{1/2} \qquad (03)$$

As a result the two modes propagate with different group velocities and are reflected at different heights in the ionosphere. Thus for each transmitted radio pulse we receive back to separate echoes. This is clearly seen in the **ionogram** of the following figure.



A typical ionogram showing the ordinary and extra-ordinary traces from the different ionospheric layers



A typical wintertime ionogram from the same ionosonde.

The horizontal axis of the ionogram gives the **Transmission Frequency of the ionospheric sounder,** and the vertical axis is the **equivalent height**. To a first approximation, the ordinary f_o and the extra-ordinary f_x frequencies reflected from the same ionospheric layer are related through the expression.

$$f_x - f_o = \frac{1}{2} f_H \qquad \text{(04)}$$

Where f_H is the **cyclotron frequency** of the **Earth's Magnetic Field H**.

$$f_H = \frac{1}{2\pi} \times \frac{eH}{mc}$$

If H is expressed in Gauss and f_H in MHz, then, $f_H = 2.8H$.

In the terrestrial ionosphere fH ≈ 1.0 – 1.5 MHz. As seen from the equation 04 the highest frequency that will be reflected by the ionosphere will be the frequency of the extra-ordinary mode reflected at N_{max}. This is called the **Maximum Usable Frequency** (MUF) and its values and variations around the globe are of great importance to all radio telecommunications.

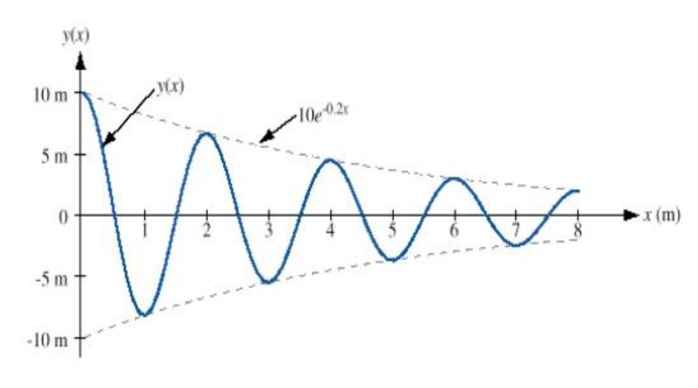
The Ionosphere

Collision Frequency & Absorption

The rate at which electrons collide with neutral particles and ions is called **Collision Frequency**.

If a passing radio wave had set the electrons in harmonic motion, these collisions would disrupt it and the ordered (harmonic) **energy of the electrons** will be converted into **random (thermal) kinetic energy**. As a result, the radio wave will have to spent some more of its energy to start again the harmonic motion of the electrons and in this manner collisions cause the **attenuation of radio waves**.

Collisions of electrons with electrons, because both particles have the same mass, **contribute much less to the thermalization** of their energy than do collisions with the much heavier ions and neutral particles.



The Ionosphere

Collision Frequency & Absorption

For this reason **electron-electron collisions can be neglected** in most cases in computing the collision frequency which cause the attenuation of the passing radio waves.

The collision frequency of electrons with the neutral particles f_n is proportional to **the physical cross section** of the neutral particles $\sigma_n \approx 10^{-15}$ cm^2, **their concentration** (particle number density) N_n and the **thermal velocity of the electrons** V_e.

$$f_n \; \alpha \; f_n(\sigma_n, N_n, V_e)$$

Thus we have, $$f_n = \sigma_n . N_n . V_e$$

σ_n cross area, $$\sigma_n = \pi r_n^2$$

The Ionosphere

Collision Frequency & Absorption

Thermal velocity of the electrons, Ve

$$K.E \ \alpha \ T$$

$$K.E = \frac{3}{2}kT$$

$$T.E = K.E$$

$$\Rightarrow \frac{1}{2}mV_e^2 = \frac{3}{2}kT \Rightarrow V_e = \left(\frac{3kT}{m}\right)^{1/2}$$

The collision frequency of electrons with the neutral particles,

$$f_n = \sigma_n.N_n.V_e \Rightarrow f_n = \left(\pi r_n^2\right).N_n.\left(\frac{3kT}{m}\right)^{1/2}$$

$$\Rightarrow f_n = \frac{\pi r_n^2 \sqrt{3k}}{\sqrt{m}}.N_n.T^{1/2}$$

C_n

The Ionosphere

Collision Frequency & Absorption

Numerical const ~ 10^{-10} in CGS sys

Collision frequency of electrons with the neutral particles

$$f_n = C_n \cdot N_n \cdot T^{1/2}$$

Temperature in K

The collision frequency of electrons with the ions

The collision of electrons with ions are actually **Coulomb Collisions** in which the **electrons are scattered by the ions through the interaction of their electric fields** rather than through physical contact. The collision cross-section, therefore, is much larger than the physical cross-section of the ions.

The Ionosphere

Collision Frequency & Absorption

As a first approximation one can say that the **maximum distance for an effective interaction is the distance r** at which the kinetic energy of the electrons is equal to the Coulomb Potential of the two particles.

$$C.P.E \ \alpha \ F \times d \quad \rightarrow \quad K.E = \left(\frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} \right) \times r \quad \rightarrow \quad K.E = \left(k \frac{e^2}{r^2} \right) \times r$$

For CGS units k=1

$$\therefore \ K.E = \frac{e^2}{r}$$

The collision frequency of electrons with the ions f_i is proportional to **the physical cross section** of the ions σ_i, **their concentration** (particle number density) N_i and the **thermal velocity of the electrons** V_e.

$$f_i \ \alpha \ f_i(\sigma_i, N_i, V_e)$$

Thus we have,

$$f_i = \sigma_i . N_i . V_e$$

σ_n cross area,

$$\sigma_i = \pi r^2$$

The Ionosphere

Collision Frequency & Absorption

Thermal velocity of the electrons, V_e

Coulomb Potential Energy, $$C.P.E = T.E$$

$$\frac{e^2}{r} = \frac{1}{2} m V_e^2 \quad \rightarrow 01$$

K.E. of the e^n = Thermal E. of the e^n $$T.E = K.E$$

$$\frac{1}{2} m V_e^2 = \frac{3}{2} kT \quad \rightarrow 02$$

Using 01 and 02 ;

$$\therefore \; r = \frac{2e^2}{3kT}$$ and, $$V_e = \left(\frac{3kT}{m}\right)^{1/2}$$

The Ionosphere

Collision Frequency & Absorption

$$f_i = \sigma_i . N_i . V_e$$

$$f_i = \left(\pi r^2\right) . N_i . V_e$$

$$f_i = \pi \left(\frac{2e^2}{3kT}\right)^2 . N_i . \left(\frac{3kT}{m}\right)^{1/2}$$

$$f_i = \frac{4\sqrt{3}\ \pi\ e^4 k^{-3/2}}{9\sqrt{m}} . N_i . T^{-3/2}$$

C_i

Numerical const ~10

Collision frequency of electrons with irons

$$f_i = C_i \cdot N_i \cdot T^{-3/2}$$

Temperature in K

The Ionosphere

Collision Frequency & Absorption

Numerical const ~ 10^{-10} *in CGS sys*

Collision frequency of electrons with the neutral particles

$$f_n = C_n \cdot N_n \cdot T^{1/2}$$

Temperature in K

Numerical const ~10

Collision frequency of electrons with irons

$$f_i = C_i \cdot N_i \cdot T^{-3/2}$$

Temperature in K

Electrons Number Density

Ions Number Density

$$N_i = N_e = N$$

Total Collision frequency

$$f = f_n + f_i$$

The Ionosphere

Collision Frequency & Absorption

Total Collision frequency

$$f = f_n + f_i$$

$$f = C_n \cdot N_n \cdot T^{1/2} + C_i \cdot N_i \cdot T^{-3/2}$$

$$f = C_n \cdot N_n \cdot T^{1/2} + C_i \cdot N \cdot T^{-3/2}$$

$$\because \; N_i = N_e = N$$

From the collision frequency we can now compute the absorption coefficient. It can be shown that the damping force due to the collision is to a first approximation proportional to the velocity $V=\dot{r}$ of the electrons. Also that the constant of proportionality is equal to mf, where m is the mass of the electrons and f is the collision frequency.

The Ionosphere

Collision Frequency & Absorption

Hence, the equation of motion of an electron under the oscillating force of a field E and in the presence of collision is,

$$\vec{F} = m\,\vec{a}$$

$$\Rightarrow\quad -e\,E = m\,\ddot{r} + m\,f\,\dot{r} \quad \rightarrow 03$$

Since, $E = E_o e^{i\omega t}$ and, $r = r_o e^{i\omega t}$, we can write the above equation in the following form

$$-e\,E = -m\,\omega^2 r + i\,m\,f\,\omega\,r$$

Because, $r = r_o e^{i\omega t}$ $\Rightarrow$ $\dot{r} = \frac{dr}{dt} = \frac{d\left(r_o e^{i\omega t}\right)}{dt}$ $\Rightarrow$ $\dot{r} = i\omega r$

$$\Rightarrow\quad \ddot{r} = \frac{d\dot{r}}{dt} = \frac{d\left(i\omega r\right)}{dt} = \frac{d\left(i\omega r_o e^{i\omega t}\right)}{dt} \quad\Rightarrow\quad \ddot{r} = -\omega^2 r$$

The Ionosphere

Collision Frequency & Absorption

Now, since the **dipole moment** between an electron and an ion separated by a distance r is equal to -er, and since we have N electron ion pairs per unit volume, the polarizibility P, i.e.; The induced dipole moment per unit volume is **P = - N e r** .

By introducing P in the equation 03 we obtain :

$$-e\,E = m\,\omega^2\,\frac{P}{eN} - i\,m\,f\,\omega\,\frac{P}{eN}$$

because,

$$r = -\frac{P}{eN}$$

$$P = -\left[\frac{e^2 N}{m\omega^2\left(1 - i\dfrac{f}{\omega}\right)}\right]E$$

But we know, Plasma frequency

$$\omega_p^2 = \frac{e^2 N}{\varepsilon_o m}$$

$$\varepsilon_o \omega_p^2 = \frac{e^2 N}{m}$$

$$\left\{ \frac{1}{4\pi\varepsilon_o} = k = 1 \quad \text{For CGS units k=1} \quad \therefore \frac{1}{4\pi} = \varepsilon_o \right\}$$

The Ionosphere

Collision Frequency & Absorption

$$P = -\left[\frac{e^2 N}{m\omega^2\left(1 - i\dfrac{f}{\omega}\right)}\right]E$$

and

$$\varepsilon_o \omega_p^2 = \frac{e^2 N}{m}$$

also

$$\frac{1}{4\pi} = \varepsilon_o$$

$$P = -\left[\frac{\varepsilon_o \omega_p^2}{\omega^2\left(1 - i\dfrac{f}{\omega}\right)}\right]E$$

$$P = -\left[\frac{\omega_p^2}{4\pi\ \omega^2\left(1 - i\dfrac{f}{\omega}\right)}\right]E$$

$$P = -\left[\frac{\omega_p^2 / \omega^2}{4\pi\left(1 - i\dfrac{f}{\omega}\right)}\right]E$$

The Ionosphere

Collision Frequency & Absorption

From the Electro-Magnetic Theory on the other hand we have : $D = \varepsilon E$

$$D = E + \frac{P}{\varepsilon_o}$$

OR

$$D = E + 4\pi P$$

$$D = E + 4\pi\left(-\left[\frac{\omega_p^2 / \omega^2}{4\pi\left(1 - i\frac{f}{\omega}\right)} \right] E \right)$$

$$D = \left(1 - \frac{\omega_p^2 / \omega^2}{\left(1 - i\frac{f}{\omega}\right)} \right) E$$

The Ionosphere

Collision Frequency & Absorption

Then, $D = \varepsilon E$ and,

$$D = \left(1 - \frac{\omega_p^2 / \omega^2}{\left(1 - i\dfrac{f}{\omega} \right)} \right) E$$

$$\Rightarrow \quad \varepsilon = \left(1 - \frac{\omega_p^2 / \omega^2}{\left(1 - i\dfrac{f}{\omega} \right)} \right)$$

From which we conclude that the square of the **complex index of reflection** **n^2**, which is equal to the **complex di-electric constant** **ε**, is given by the expression,

$$n^2 = \varepsilon = 1 - \frac{\omega_p^2 / \omega^2}{\left(1 - i\dfrac{f}{\omega} \right)}$$

The Ionosphere

Collision Frequency & Absorption

From which we conclude that the square of the **complex index of reflection** **n^2**, which is equal to the **complex di-electric constant** **ε**, is given by the expression,

$$n^2 = \varepsilon = 1 - \frac{\omega_p^2 / \omega^2}{\left(1 - i\frac{f}{\omega}\right)} \quad \ldots\ldots \Rightarrow \quad n^2 = \left[1 - \frac{\left(\omega_p^2/\omega^2\right)}{\left(1 + \left(\frac{f}{\omega}\right)^2\right)}\right] - i\left[\left(\frac{f}{\omega}\right)\frac{\left(\omega_p^2/\omega^2\right)}{\left(1 + \left(\frac{f}{\omega}\right)^2\right)}\right]$$

Let **μ** then be the **Real Part** and **ϰ** the **Imaginary Part** of the complex index of reflection, so that,

$$\mathbf{n = \mu - i\,\varkappa}$$

$$\mathbf{n^2 = (\mu - i\,\varkappa)^2} = 1 - \frac{\left(\omega_p^2/\omega^2\right)}{1 + \left(\frac{f}{\omega}\right)^2} - i\left(\frac{f}{\omega}\right)\frac{\left(\omega_p^2/\omega^2\right)}{1 + \left(\frac{f}{\omega}\right)^2}$$

The Ionosphere

Collision Frequency & Absorption

When the collision frequency f, is much smaller than the operating frequency ω; i.e.:

When f « ω then **ϰ** « 1 and equations of **μ** & **ϰ** become,

$$\mu^2 \approx 1 - \frac{\omega_p^2}{\omega^2}$$

and

$$\chi \approx \frac{f\,\omega_p^2}{2\mu\omega^3}$$

The electric field of a wave in a medium with a complex index of refraction n is given by the expression,

$$E = A\,e^{-i(kr-\omega t)}$$

$$\Rightarrow E = A\,e^{-i(nk_o r-\omega t)}$$

$$\Rightarrow E = A\,e^{-i((\mu - i\chi)k_o r-\omega t)}$$

$$\Rightarrow E = A\,e^{-i(\mu k_o r-\omega t)}e^{-k_o \chi r}$$

$$\Rightarrow E = E_o e^{-k_o \chi r}$$

The Ionosphere

Collision Frequency & Absorption

And since the intensity of the radiation I is proportional to the square of the electric field E; we have,

$$I \;\alpha\; E^2$$

$$\Rightarrow \quad I = a\,E^2$$

$$\Rightarrow \quad I = a\left(E_o e^{-k_o \chi r}\right)^2$$

$$\Rightarrow \quad I = a\,E_o^2\,e^{-2k_o \chi r}$$

$$\Rightarrow \quad I = I_o\,e^{-\kappa\, r}$$ and $$\kappa = 2k_o \chi$$

$$\Rightarrow \quad I = I_o\,e^{-\tau}$$

The Ionosphere

Collision Frequency & Absorption

$I = I_o e^{-\tau}$ where $\tau = \kappa r$ and $\kappa = 2k_o \chi$

Where κ is the **absorption coefficient** of the medium

Absorption coefficient of the medium : $\kappa = 2k_o \chi$

$$\kappa = 2\left(\frac{\omega}{c}\right)\chi$$

$$\kappa = 2\left(\frac{\omega}{c}\right)\left(\frac{f\,\omega_p^2}{2\mu\omega^3}\right)$$

$$\kappa = \left(\frac{f}{\mu c}\right)\left(\frac{\omega_p^2}{\omega^2}\right)$$

$\omega = 2\pi f_o$

$$\kappa = \left(\frac{f}{\mu c}\right)\left(\frac{f_p^2}{f_o^2}\right)$$

And τ is the **Optical Thickness** or **Opacity** of the medium, $\tau = \int \kappa\, dr$

$\Rightarrow \tau = \kappa r$

The Ionosphere

Collision Frequency & Absorption

From equations of κ and τ we see that the absorption coefficient κ and the opacity τ of the medium are directly proportional to the collision frequency f.

For a very weakly ionized plasma, like the lower ionosphere (D-Region) where $N_n \gg N_i$, we can set $f=f_n$ and we get,

$$\kappa_n \alpha \ \frac{N.N_n.T^{1/2}}{f_o^2}$$

The electron density N in the D-Region is usually low and therefore the κ_n of the D-Region is usually low !

The absorption of the D-layer is usually measured with radio receivers which monitor continuously the radio noise from our galaxy. These receivers are called **Riometers**, where the prefix **rio** stands for the initials of the words **relative ionospheric opacity**.

The Ionosphere

Collision Frequency & Absorption

During the span of a day both the **ionospheric absorption** and the **galactic radio background change**.
The first because the **electron density of the D-layer varies with the Zenith Angle of the Sun**, and
The second because the galactic radio noise is concentrated in the plane of the galaxy and especially towards the galactic center and as the earth rotates our antenna focuses an different regions of the galaxy.

Through long observations, we can take into account these variations, which have also a seasonal component and we can establish the normal levels of ionospheric absorption and the expected, under normal conditions, intensity I of the galactic radio emission. Any decreases of the signal strength to a new level I′ represents an increase in the D-region absorption and is usually expressed in dB units.

$$dB = 10 Log_{10}\left(\frac{I'}{I}\right)$$

The Ionosphere

Collision Frequency & Absorption

$$dB = 10 \log_{10}\left(\frac{I'}{I}\right)$$ and $$\log_{10} x = 2.3 \ln x$$

$$dB = 23 \ln\left(\frac{I'}{I}\right)$$

$$dB = 23 \ln\left(\frac{I_o e^{-\tau'}}{I_o e^{-\tau}}\right)$$

$$dB = 23 \ln\left(e^{\tau-\tau'}\right)$$

$$dB = 23 \left(\tau - \tau'\right)$$

$$dB = 23 \tau'\left(\frac{\tau}{\tau'} - 1\right)$$

$$dB = -23 \tau'\left(1 - \frac{\tau}{\tau'}\right)$$

$$dB \approx -23 \tau'$$

Because, $\tau \ll \tau'$.

Where τ and τ' are the normal and the enhanced opacity of the ionosphere, which are essentially proportional to the normal and enhanced electron density of the D-Region.

The Ionosphere

Collision Frequency & Absorption

Riometer observations are usually conducted in the frequency range between 15 MHz and 60 MHz. **The reason is that lower frequencies can hardly penetrate through the ionosphere** while higher frequencies as seen from,

$$\chi \approx \frac{f}{2\mu}\left(\frac{\omega_p^2}{\omega^3}\right),$$

suffer very little attenuation which is difficult to measure.

For a fully ionized plasma, like the solar corona, we can set $f = f_i$ and we get,

$$\tau_i = \kappa_i \; r \; \alpha \left(\frac{N^2}{f_o^2 \; T^{3/2}}\right) r$$

Here it is important to note that in certain cases through κ_i might be very small, the total attenuation represented by the opacity τ_i might be quite large simply because the radio waves have travelled a very long distance r in the medium. This is quite often the case in **astronomical observations**.


Thank You !
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