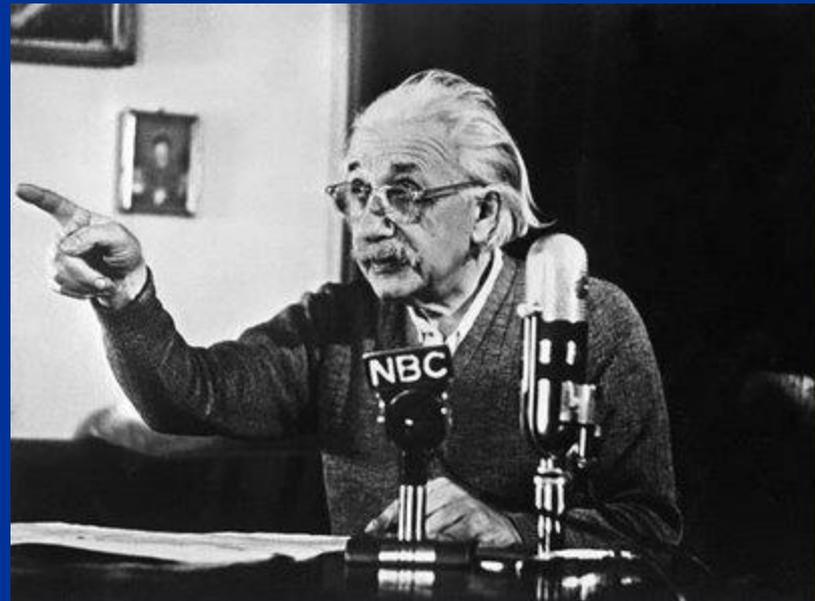


# Special Theory of **Relativity**



11<sup>th</sup> Lecture

# General Theory of Relativity

# Brief Introduction to General Theory of Relativity

Soon after the Special Theory was published in 1905, Einstein turned his attention to phenomena that occurred when observers were not restricted to movement with constant relative velocities (i.e. with zero acceleration) but with varying velocities (i.e. with accelerations not zero). The results of his reasoning embody **The General Theory of Relativity**, which was presented in 1916.

Everyone has ridden in an elevator at one time or another and has perhaps noticed that when the elevator is accelerating upwards, he is pushed towards the floor of the elevator. Also, if the passenger is carrying something, it too, is pushed towards the floor. He feels heavier and, furthermore, the faster the elevator starts up (the greater the acceleration), the heavier everything becomes.



# Brief Introduction to General Theory of Relativity

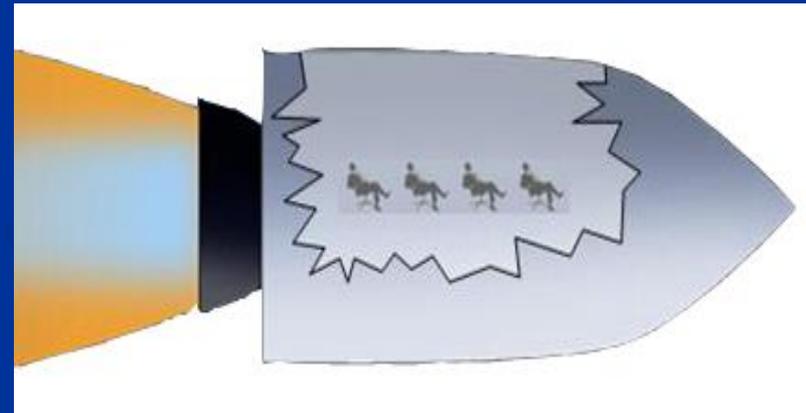
Conversely, when the elevator is accelerating downward, everything becomes lighter and the greater the acceleration downward, the lighter everything becomes. In particular, if the elevator should accelerate downward as fast as falling objects accelerate towards the Earth, then objects in the elevator would have no weight at all, and everybody and everything would tend to float around in the elevator like soap bubbles! And if the elevator plummeted earthward with an even higher acceleration, everybody and everything would be pressed to the ceiling of the elevator. When the elevator stops accelerating, and moves with a constant velocity up or down, these effects will not occur.

Although the elevator passengers might be flustered momentarily by what is happening, they are not completely in the dark as to the cause of their discomfort. They know that the Earth's gravitational force on them has something to do with the peculiar effects they experience.

# Brief Introduction to General Theory of Relativity

But now suppose these people are in a rocket travelling out in interstellar space, say on a star seeing trip. They have no weight, since weight is the force with which a large mass pulls on the object, and they are out beyond the pull, or gravitational field, of the earth. Hence, they must be tied down in some way to keep from floating around.

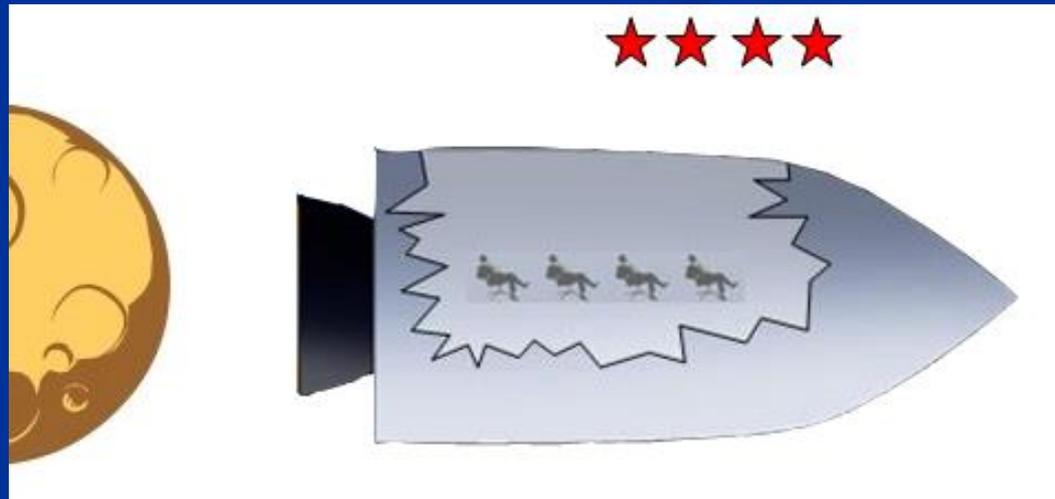
Now; when the rocket accelerates in the forward direction relative to the distant stars, they are pushed back against their seats as the following figure, and when the rocket decelerates, they are thrust forward in the same way that people are in any conveyance which is speeding up slowing down on the Earth.



# Brief Introduction to General Theory of Relativity

Thus, the people in the rocket would automatically associate a pressure backward with an acceleration of the rocket, and a thrust forward with a deceleration of the rocket. At other times, when the rocket is neither accelerating nor decelerating, they do not experience any effect.

But now suppose that while they are journeying along in space with a constant velocity relative to the distant stars, a stray planet comes along. It is not seen by anyone in the rocket and narrowly misses hitting the rocket, passing by its tail, as in the following figure.



# Brief Introduction to General Theory of Relativity

If we assume the rocket motors and controls are such as to keep the rocket moving with constant velocity relative to the distant stars, the question is what will the passengers feel? With the planet in their vicinity they are again given weight, and they feel this by being pulled towards the planet as it passes, i.e.; pushed to the back of their seats. But since they don't know of the presence of the planet and since the effect is the same as if the rocket accelerated, they will erroneously conclude that, that is what happened and not give it a second thought.

The larger question is whether there is any way they can tell (without looking outside) if the forces they feel are due to acceleration or the gravitational pull of a nearby mass. The answer is that there is no way of distinguishing the two. Einstein was stuck by this equivalence of acceleration and gravitational forces and stated this observation in what is known as the **principle of equivalence at a single point in space effects of gravitation and accelerated motion are equivalent and cannot be distinguished from each other.**

# Brief Introduction to General Theory of Relativity

Going back to the accelerator, we wonder if the increased weight of the passengers, caused by the elevator being accelerated upward, could not also be caused by additional gravitational forces. If certainly could, suppose that the elevator and its passengers were suddenly 'transplanted' to Jupiter without their knowing it. They would all feel heavier because Jupiter has a mass over 300 times greater than that of the Earth and so exerts a greater gravitational pull on objects on its surface, causing objects to weigh 2.5 times more than they do on Earth. Hence a 50kg man would weigh 125kg on Jupiter, and would probably sag to the floor as a result. Furthermore, he would attribute his increased weight to an upward acceleration of the elevator, not knowing that an increased gravitational mass caused it.

Or, if the elevator were transplanted to Mercury, which has a mass of  $1/25$  that of the Earth, everything would weigh only a third as much and the 50kg man would weigh only 16.75kg. He and the other occupants would interpret their lighter weight as due to the elevator accelerating downward. Again we see that the effects of accelerated motion and gravitation are the same.

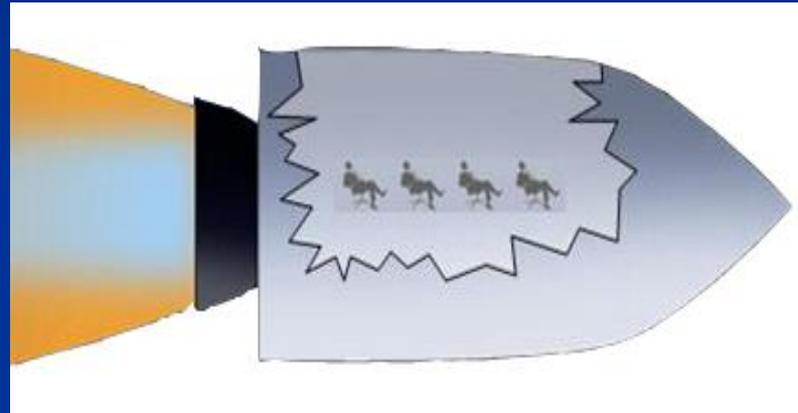
# Brief Introduction to General Theory of Relativity

It would appear that the principle of equivalence is a simple, or even trivial, observation. It would seem so to one not versed in the history of scientific accomplishment, but Einstein was the first to draw attention to this conclusion. If it had no further ramifications, the principle would be considered interesting and then promptly forgotten. With this principle of equivalence as the basic postulate of the General Theory, **Einstein applied a branch of mathematics previously developed by Riemann and others, i.e.; tensor calculus, and obtained three important conclusions, each of which was tested experimentally.** These will now be discussed in detail.

# Brief Introduction to General Theory of Relativity



The Concept of Gravity : Space & Time Curves

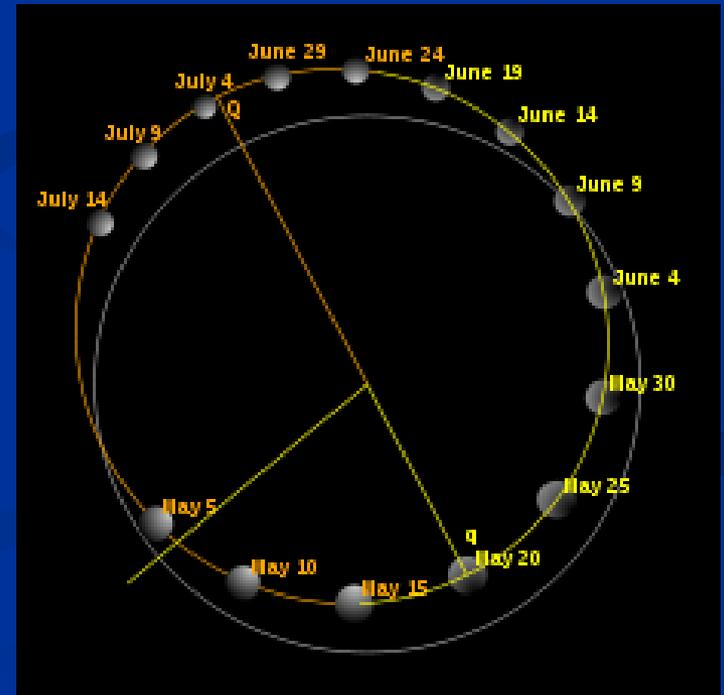


The principle of equivalence at a single point in space effects of gravitation and accelerated motion are equivalent and cannot be distinguished from each other.

# Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

Gravitation was a topic which had intrigued people for many years because of the mysterious way in which it acts. A freely falling object always falls towards the Earth. But how is it possible for the Earth to pull the object towards it without literally reaching up and grabbing the object? The air does not help, because objects are also pulled by the Earth even when they are in a vacuum.

Another mystery was the strange force which the sun seemed to exert on the planets to keep them continually revolving about the Sun. Kepler had deduced, after considerable observation, that the paths travelled by the planets were ellipses, but he did not know why they were ellipses.



# Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

A satisfactory answer to both of these unknowns was given by Newton in 1687 when he published what is now known as Newton's Law of Universal Gravitation. This says that every object in the universe attracts every other object with a gravitational force given by the following Equation:

$$F = \frac{Gm_1m_2}{d^2}$$

Where,

$m_1$  - the mass of one object,

$m_2$  - that of the other,

$d$  - the distance between them, and

$G$  - a universal constant (Newton's Gravitational constant)

# Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

When an object is falling freely, the force of attraction of the Earth on the object is obtained from the above equation by inserting the mass of the object for  $m_1$  and the mass of the Earth for  $m_2$ .

It should be noted that Newton's equation was entirely the result of observation. He observed falling objects and the movements of the planets about the Sun and evolved his formula as the one which best fitted the facts- thus obtaining what is called an empirical equation. With this equation for the force of attraction between two masses, he then arrived at the equations for the paths the planet make around the Sun. He found that these paths were ellipses, which were stationary with respect to the Sun as in the following figure.

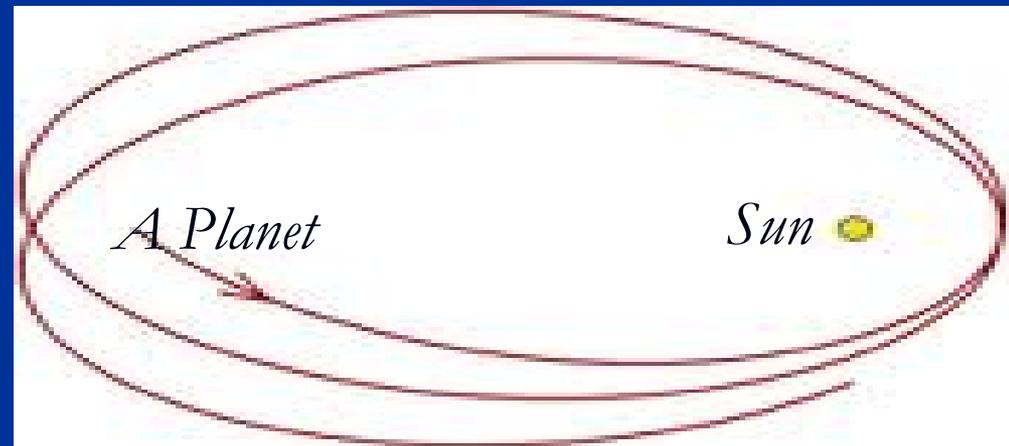
*Under Newton's Law a planet's orbit is a stationary ellipse*



# Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

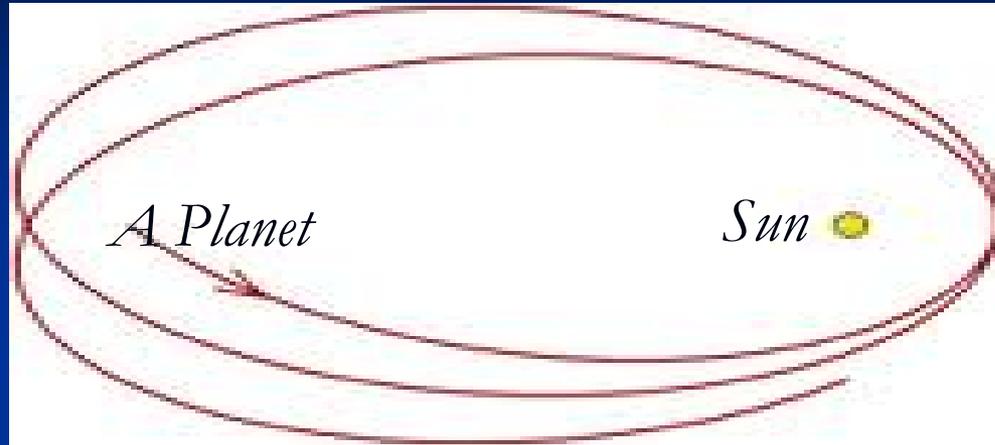
The planets travel over and over again in the same elliptical paths in space. Since observations through years verified this, Newton's Law of gravitation was hailed as a great accomplishment, as indeed it was.

In his development of General Theory, Einstein concerned himself with the development of a theory of gravitation. For this reason the **General Theory of Relativity** is also called the Einstein theory of gravitation. As a result of this theory; Einstein also determined the Equation of the paths the planets make in their journey about the Sun. The end result was approximately the same as Newton's but there was a slight difference. **Although Einstein also found that these ellipses were not stationary but were slowly rotating in space, as in the following figure.**



*Under Einstein's Theory a planet's orbit is a rotating ellipse.*

# Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit



But this predicted rotation is so light as to be scarcely detectable for most of the planets. The orbit of the Earth, for example, rotates at a rate of only 3.8 seconds of arc every century. When we remember that there are 324,000 seconds of arc in a right angle, it can be realized how small a value 3.8 seconds of arc really is **about one one-hundred thousandth of a right angle**. And, further, it takes 100 years for the Earth's orbit to rotate by this amount. At this rate, it would take about 34 million years for the Earth's orbit to rotate one revolution!

## Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

Strictly speaking, since the elliptical orbits of the planets are rotating, the orbits **rosette patterns** similar to the paths of the electrons about the nucleus in the Sommerfeld theory. But since this rotation rate is so small, it would take too long for a rosette to be complicated. Hence, we think of the planetary orbit as rotating ellipses, and not as rosette patterns.

Obviously, since the Einstein theory of gravitation produces different results from the Newton law of gravitation, one of these is incorrect, however **slightly**.

As we shall see, the Einstein theory is correct one, but we first would like to see how these two theories differ mathematically. If the Newton law of gravitation is modified so that the results agree with Einstein's, i.e., so that the orbits of the planets turn out to be slightly rotating ellipses instead of stationary ones, then the corrected formula should be in the following equation;

## Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

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As we shall see, the Einstein theory is **correct**, but we first would like to see how these two theories differ mathematically. If the Newton law of gravitation is  $F = G \frac{M_1 M_2}{d^2}$  and it agrees with Einstein's, i.e., so that the orbits of the planets turn out to be slightly rotating ellipses instead of stationary ones, then the corrected formula should be in the following equation;

## Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

Which is only slightly different from Newton's law is correct to a high degree of approximation. It is for this reason that **Newton's law of gravitation has given such good results for so many years.**

One proof of the General Theory of Relativity consisted in looking for a planet whose orbit rotated the most over a given period of time. The theory showed that the amount of rotation would be greatest for the planet with the highest orbital velocity. But it was also necessary to use a planet whose orbit was as elliptical as possible, since some of the planetary orbits, like the Earth's, are so nearly circular that is difficult to tell whether or not they have rotated. With an orbit that is very flat, or higher elliptical, it is easy to see in which direction it points, and hence its rotation can be detected.

# Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

It so happens that the **planet Mercury** has one of the flattest orbits and the greatest orbital velocity. There had been a peculiar and puzzling behavior of this planet's orbit for many years: it had a rotation of about 43 seconds of arc per century which could not be accounted for. (Although the total rotation of Mercury's orbit is about 574 seconds of arc per century, it was known that 531 seconds of this was due to the gravitational effect of the other planets)

In 1845 the French mathematician Laverrier showed that this excess amount of rotation would be produced if Mercury were being influenced by another planet between it and the Sun. The anticipated planet was eagerly looked for by astronomers but has never been found. (The planet Neptune was also predicted by Laverrier as a result of variations in Uranus' orbit, and was thus subsequently discovered in 1930 as a result of still-remaining variations in Uranus' orbit)

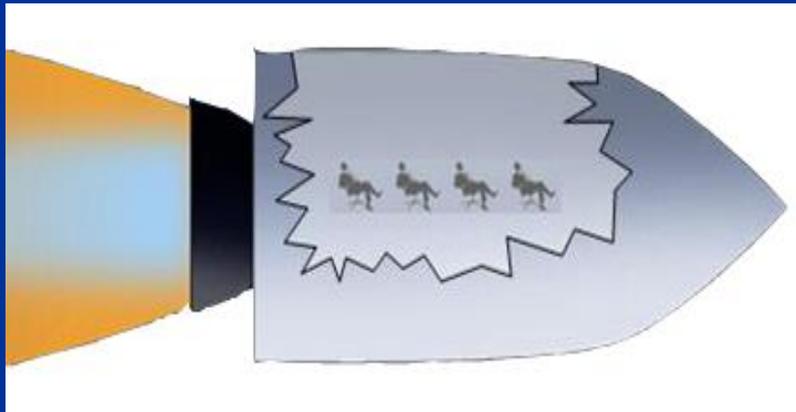
# Newton's Law of Universal Gravitation and Einstein's Theory of Gravitation- Rotation of Mercury Orbit

The cause of the excess rotation of Mercury's orbit remained a mystery until the introduction of the General Theory of Relativity. When the General Theory was used to compute the amount of the excess rotation of Mercury's orbit for the period of a century, the result was 43 seconds of arc-the exact amount of rotation which previously could not be explained. This constituted the first proof of the General Theory. It should be noted that this particular proof is the most convincing of the three so far known, since the effect is a relatively large one compared to the other two be discussed.

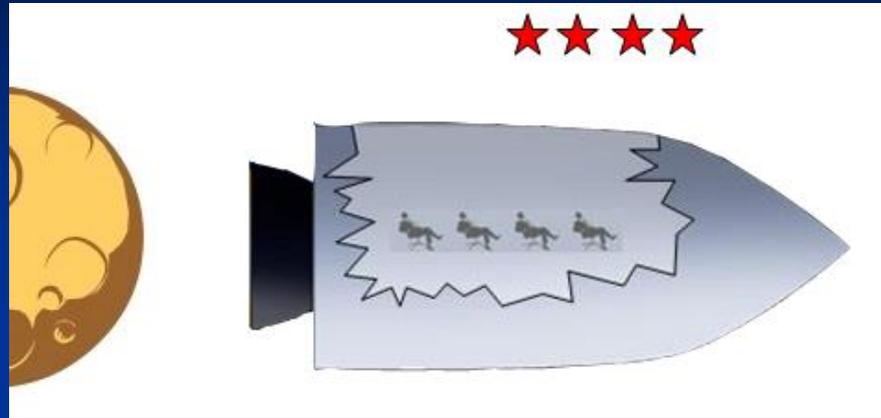
We found in the discussion of the Sommerfeld theory of atomic orbits that the rotation of the orbits there was caused by the variation in mass of the electron as predicted by the special Theory. You may wonder if the excess rotation of Mercury's orbit could be due to the same effect. Since Mercury travels in an elliptical orbit, its orbital velocity varies and, hence, its mass would vary, with its orbit being caused to rotate as a result. However, it can be shown mathematically that the amount of this rotation predicted by the Special Theory will be only one-sixth that predicted by the general theory. In this case it would account for only about 7 of the excess 43 seconds of arc per century that Mercury's orbit is known to rotate. The General Theory, then, satisfactorily accounts for the excess rotation.

# Effect of a Gravitational Mass on a light beam - weighing a Beam of light

In connection with the General Theory, Einstein also investigated the behavior of a beam of light under the influence of gravitational field due to a large mass. His results are best presented by again referring to the following figures, where now we will assume that the rocket passes a row of stars in its flight. Since there is only a single skylight in the ceiling of the rocket, a single beam of light from each star will enter the rocket via the skylight as the rocket passes each star.

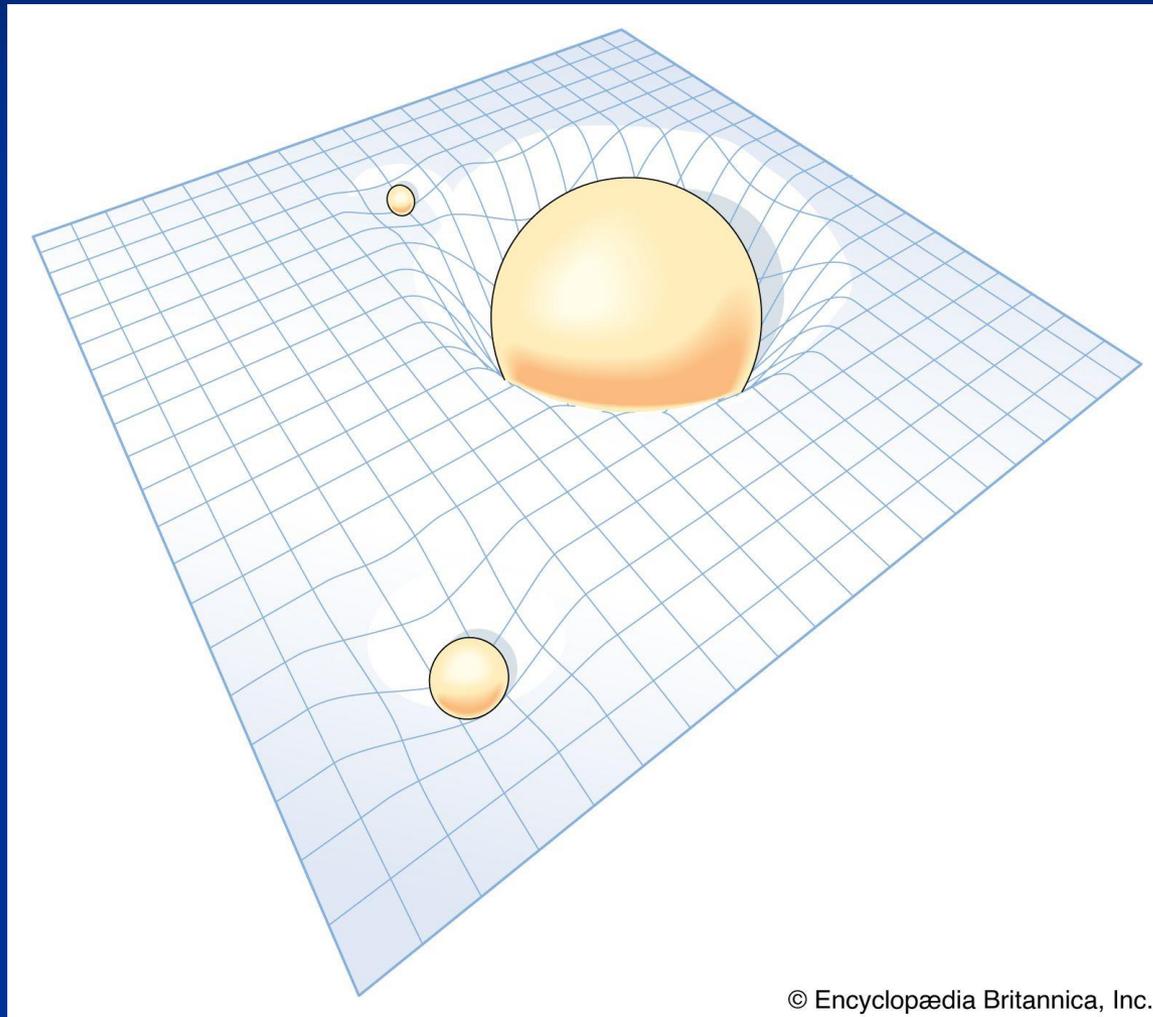


# Effect of a Gravitational Mass on a light beam - weighing a Beam of light



In the above diagram, it is seen that while the rocket is passing the row of stars, the stray planet is passing by the rear of the rocket. What effect does this have on the resultant light beam in the rocket? The general Theory states that the gravitational field due to the planet's mass will attract the light beam towards it in the same way that the Earth will pull a flying bullet or arrow towards it. This causes the light beam inside the rocket to be curved.

# What is curve space-time?



# What is curve space-time?

## Newton's Gravity Concept

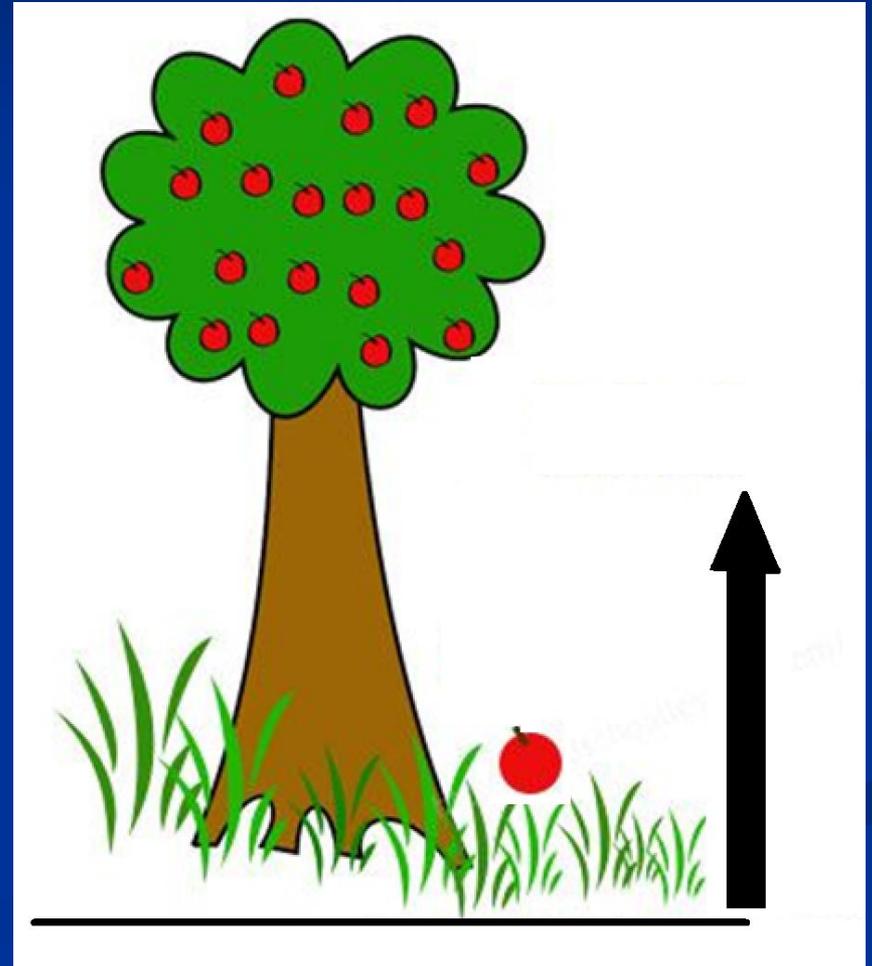
Newton's law of universal gravitation is usually stated as that every particle attracts every other particle in the universe



The key difference between Einstein and Newton gravity is that **Einstein gravity describes that gravity is a curvature in a 4-dimensional space-time fabric proportional to object masses**, whereas Newton gravity describes gravity as a force expressed mutually between two objects in relation to their masses. Moreover, Einstein considered gravity as a push while Newton considered gravity as a pull.

**What is curve space-time?**

## **Einstein's Gravity Concept**



# Einstein vs Newton Gravity

More Information Online [WWW.DIFFERENCEBETWEEN.COM](http://WWW.DIFFERENCEBETWEEN.COM)

## Einstein Gravity

## Newton Gravity

### DEFINITION

Einstein gravity is described by the general relativity or general theory of relativity, which is a geometric theory of gravitation published by Albert Einstein in 1915

Newton gravity states that every particle tends to attract every other particle in the universe with force

### NATURE

Considered gravity as a push

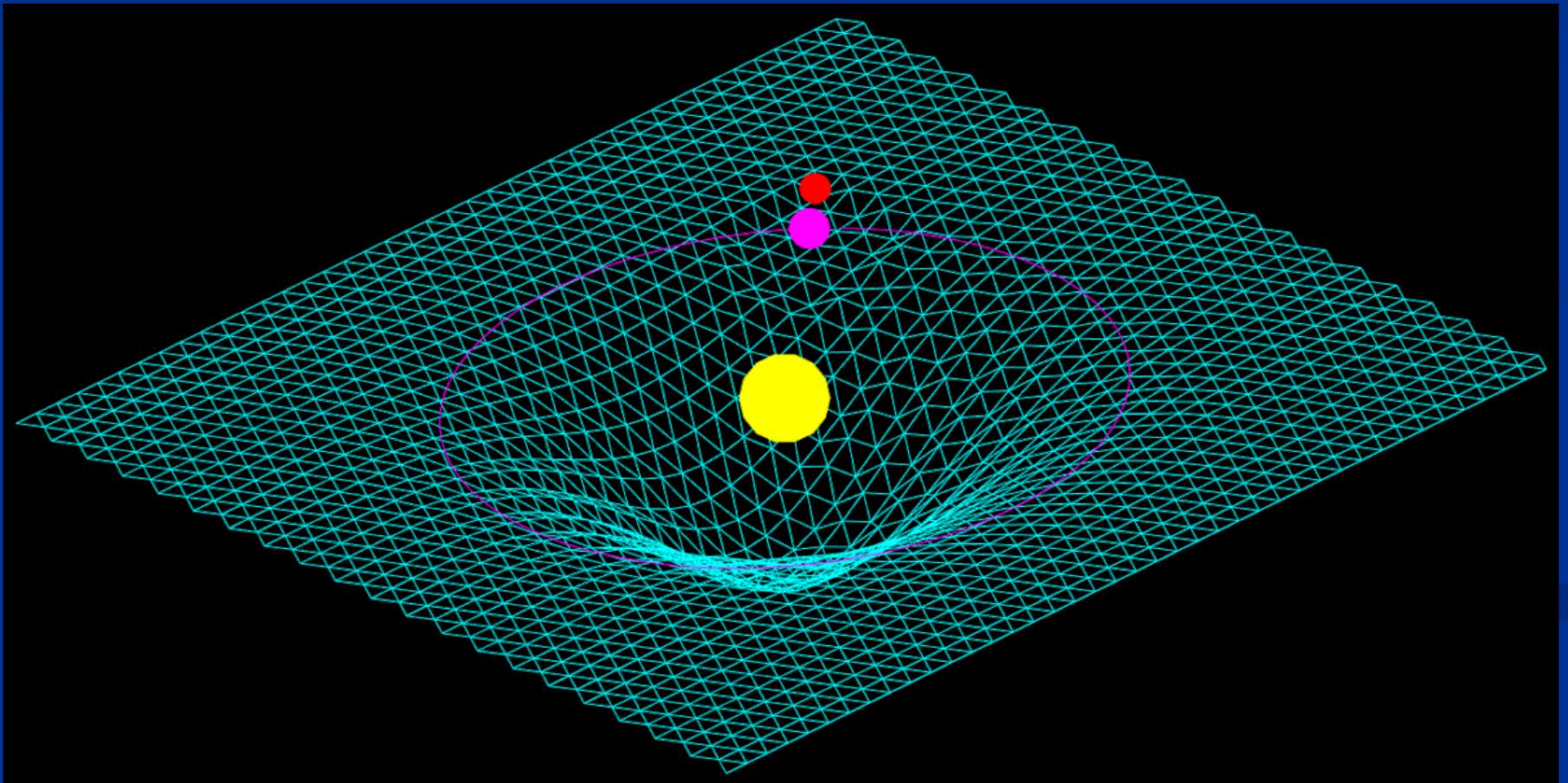
Considered gravity as a pull

### DESCRIPTION OF GRAVITY

Describes that gravity is a curvature in a 4-dimensional space-time fabric proportional to object masses

Describes gravity as a force expressed mutually between two objects in relation to their masses

# The motion of Earth & Moon around the Sun according to the Einstein's GTR



# Effect of a Gravitational Mass on Time (Slowing Down of atomic clocks on the Sun and stars)

Another result of the General Theory is the **effect of a gravitational mass on time**. The prediction is that all time processes will be slower on a large mass than on a small mass, or that

**Time** will move more slowly on a relatively larger planet, such as Jupiter, than on Earth.

Although a clock which runs at a certain rate on the Earth will run slower on Jupiter, it will run even slower on the Sun.

Indeed, Einstein found that a second of time on the Sun should correspond to 1.000002 Earth seconds.

## Effect of a Gravitational Mass on a light beam - weighing a Beam of light

It is not surprising to find that a flying bullet or arrow is pulled towards the Earth. These have weight (even while in the flight), but most people are surprised to find that a light beam also has a weight. This did not surprise the scientists of the day, however, because one of the theories of the light says that the light is made up of small particles are called photons which travel at a velocity of  $3 \times 10^8$  m/s. These were considered to have mass, and it was reasoned that if they did, when light falls on a surface these photons would exert a pressure on the surface similar to the pattern of falling rain-drops on a roof. This effect has been observed and is known as

**radiation pressure.** The pressure is very small, and for the Sun's rays on the Earth it is only about a tenth of an ounce per acre ( $7 \times 10^{-6}$  N/m<sup>2</sup>), giving a total of but 160 tons ( $1.62 \times 10^5$  kg) for the entire surface of the Earth. Fortunately, gravitational attraction of the Sun on the Earth is many times stronger, so that we are not pushed off into space by the radiation pressure of the Sun's rays.

# Radiation Pressure in General Theory of Relativity

Radiation pressure :

$$P_r = \frac{\text{Force}}{\text{Area}}$$



$$\frac{\frac{\partial p}{\partial t}}{A}$$

Where momentum :

$$p = \frac{\mathcal{E}}{c}$$

And Plank Law :

$$\mathcal{E} = h f$$

or

$$\mathcal{E} = \hbar \omega$$

Radiation pressure :

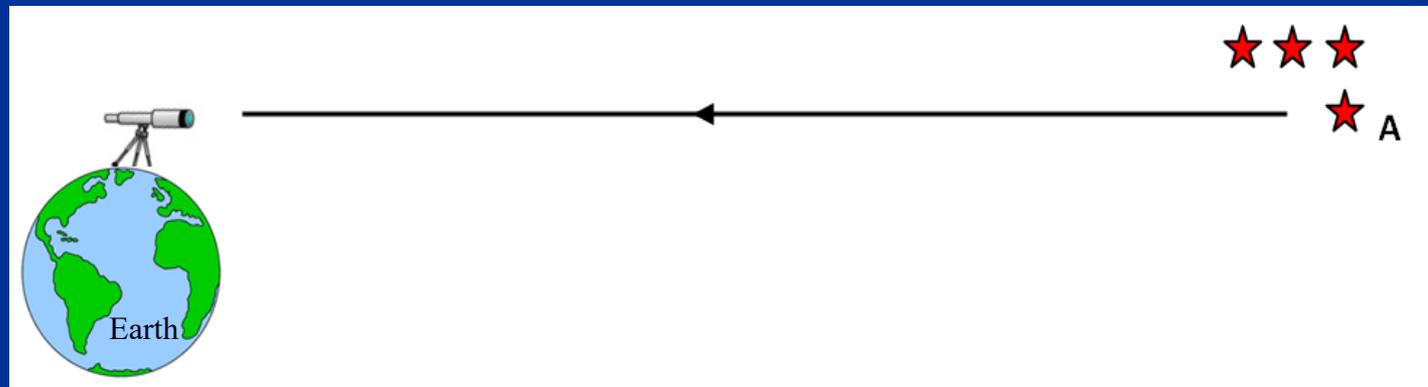
$$\frac{1}{Ac} \frac{\partial \mathcal{E}}{\partial t}$$

# Effect of a Gravitational Mass on a light beam - weighing a Beam of light

To test the prediction of the General Theory that light is deflected by a gravitational field, actually we would have to 'weigh a beam of light'. It isn't possible to catch a lot of photons and pile them on a scale, as we could with bullets, since no one has yet succeed in building a photon trap. **(Moreover, scientists today believe that the rest mass of the photon is zero!)** So the photons must be weighted while in flight. This is not difficult to do in theory, since if a beam of light is affected by a gravitational field, its path will be curved, and this is easy to determine, providing the curvature is sufficient. But if a beam of light is not affected by a gravitational field, its path through it will be a straight line, which is also easy to determine.

# Effect of a Gravitational Mass on a light beam - weighing a Beam of light

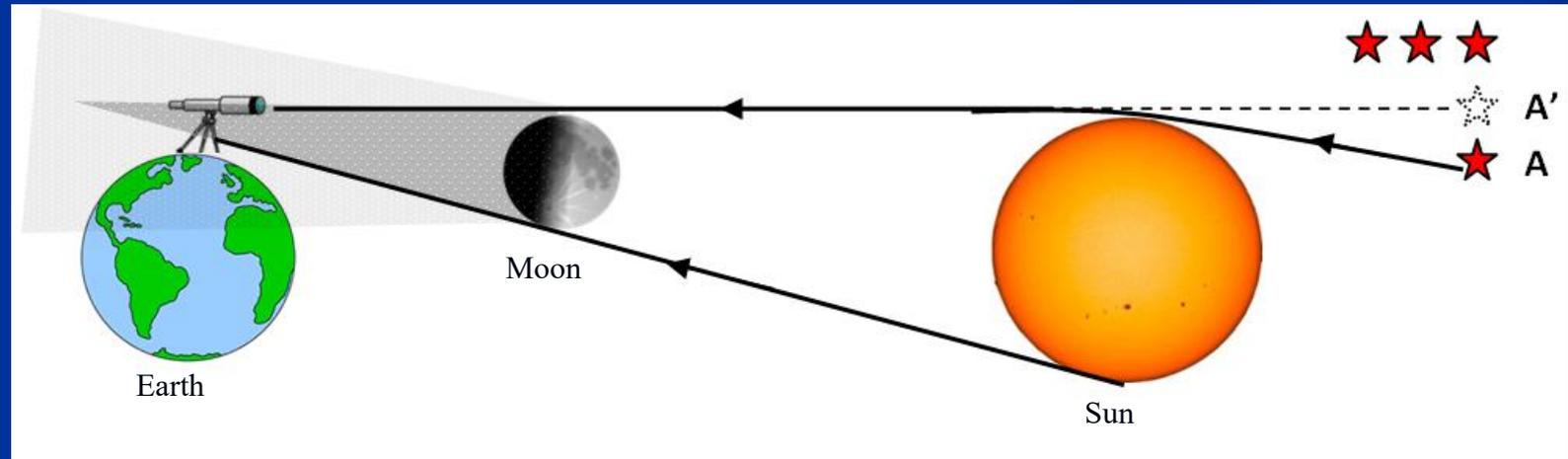
The beam of light must, of course, come from a star. The procedure for weighing is illustrated by the following figure. The initial position of the star A, whose light is to be weighted, is shown in figure.



(a) Incoming light beam is straight.

# Effect of a Gravitational Mass on a light beam - weighing a Beam of light

There are no intervening gravitational masses, so the star's light travels in a straight line from the star to the observer on the Earth. Later on, the earth has travelled sufficiently far in its orbit so that the sun comes between the earth and the star in such a position that the light from the star just grazes the sun's surface on its way to the observer on the earth as in the following figure.



(b) Incoming light beam is bend sun's mass

# Effect of a Gravitational Mass on a light beam - weighing a Beam of light

But here a difficulty is encountered, for if the star's light is just grazing the sun's surface the observer will not be able to see the star, since the sun's light will be too bright. The only solution is to observe the star's light grazing the sun during a total eclipse of the sun during a total eclipse of the sun when the moon blots out the sun's light completely, as illustrated. For this reason Einstein suggested that this effect be looked for during a total solar eclipse.

Since the deflection of the star's light, while grazing the sun is so slight, precise photographic techniques are necessary. The actual procedure consists of photographing the star while in position (*a*), showing its position relative to its neighboring stars, and then again at a later time during a total solar eclipse, as in (*b*). In this later position, it will appear to the observer (and his camera) that the star is at the apparent position *A'*. A picture taken at this position is compared with that of position (*a*). The comparison should show that the star appears to have moved, provided that the Sun's mass has deflected the star's light as predicted by the General Theory. In particular, Einstein predicted that the deflection would be 1.74 seconds of arc for a light beam grazing the sun's surface.

# Effect of a Gravitational Mass on a light beam - weighing a Beam of light

The most favorable total eclipse of the Sun after the General Theory was presented in 1916 occurred on 29th May 1919. This eclipse was particularly favorable for the test because the earth and the Sun happen to be lined up with a patch of bright stars at the end of May every year, and so were a number of stars to choose from during this eclipse. Accordingly, two British astronomical expeditions were fitted out. **One, under A. C. Crommelin, went to Sobral in Northern Brazil, while the other, under A. S. Eddington, went to the Gulf of Guinea.** Both groups photographed a number of stars, and upon their return to England the photographic plates were developed and compared with pictures taken when the Sun was not in the same stars' vicinity.

The Sobral group found that their stars had moved on average of 1.98 seconds of arc, and the principle group that theirs had moved 1.6 seconds of arc. This nearness to the 1.74 seconds of arc predicted by Einstein was sufficient to verify the effect. Since then, more than ten different results have been reported which also confirm the prediction.

# Effect of a Gravitational Mass on a light beam - weighing a Beam of light

It is interesting to speculate how massive a star would have to be so that its gravitational attraction is strong enough to prevent any of the star's light from leaving the star. It can be shown that for a star of the same radius as the Sun this would occur if its mass were approximately 400,000 times the Sun's mass. If such stars existed, we would never be able to see them, regardless of how close they were or how brilliantly they shone.

Using the Newton law of gravitation, a value can also be obtained for the bending of a light ray. This turns out to be exactly one half of the value given by the General Theory of Relativity; and so far the Sun the value would be 0.87 seconds of arc. None of the experimental determination to date has been of this order of magnitude all being larger and within a reasonable range of the Einstein value. This again points up the slight difference between the Newton law and the Einstein Theory.



*Thank You !*