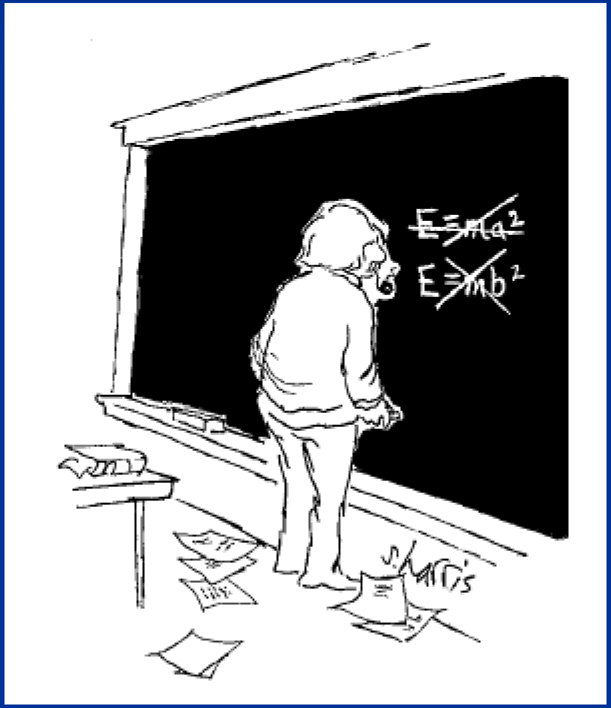
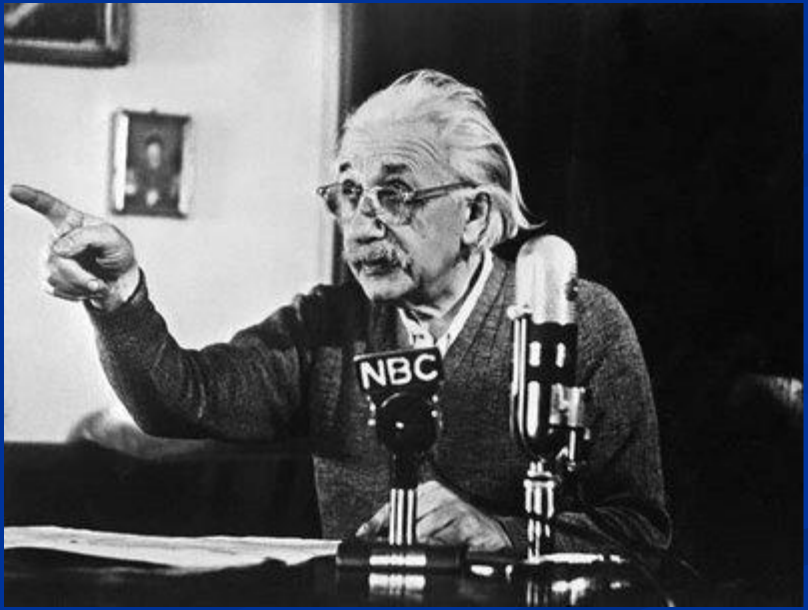
Special Theory of Relativity





3rd Lecture

Special Theory of Relativity

Einstein's Two Postulates in STR

Postulate 01 : The Principle of Relativity:

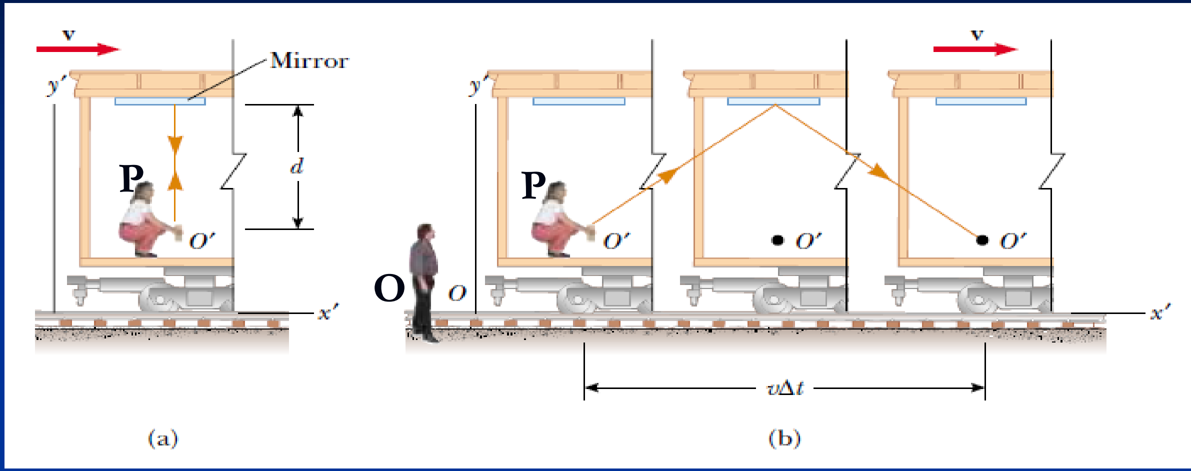
The laws of physics must be the same in all inertial reference frames.

The laws of Physics are the same for all observers in uniform motion relative to one another.

Postulate 02 : The constancy of the speed of light :

The speed of light in vacuum has the same value, $c = 3 \times 10^8$ m/s in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Measurement of Time in STR



$$\Delta t_O = \Delta t_P \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

This equation express the fact that for the moving observer the period of the clock is shorter than in the frame of the ground observer itself !

Where, v is the Relative Speed of the Two Frames

$$\Delta t_O \rangle \Delta t_P$$

Time interval w. r. t the stationary frame

Time interval w. r. t the moving frame

Time Dilation

Suppose,

$$t_2 = t_1 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

This equation is called relativistic time equation!

If v > 0 $\Rightarrow$ $\frac{v}{c} < 1$ $\Rightarrow$ $\frac{v^2}{c^2} < 1$ $\Rightarrow$ $1-\frac{v^2}{c^2} < 1$

$\Rightarrow$ $\sqrt{1-\frac{v^2}{c^2}} < 1$ $\Rightarrow$ $t_2 = t_1 \frac{1}{(<1)}$

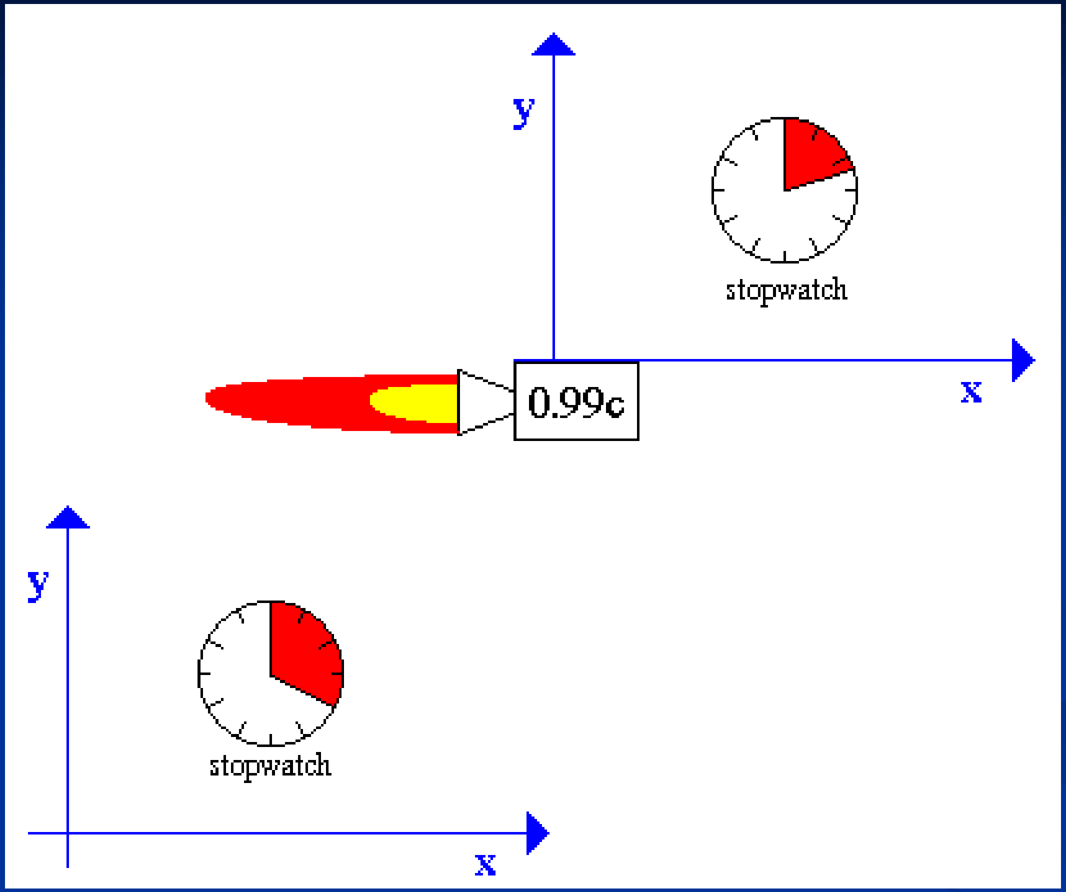
$$t_2 > t_1$$

Time interval w. r. t the stationary frame

Time interval w. r. t the moving frame

This is called Time Dilation !

Time Dilation

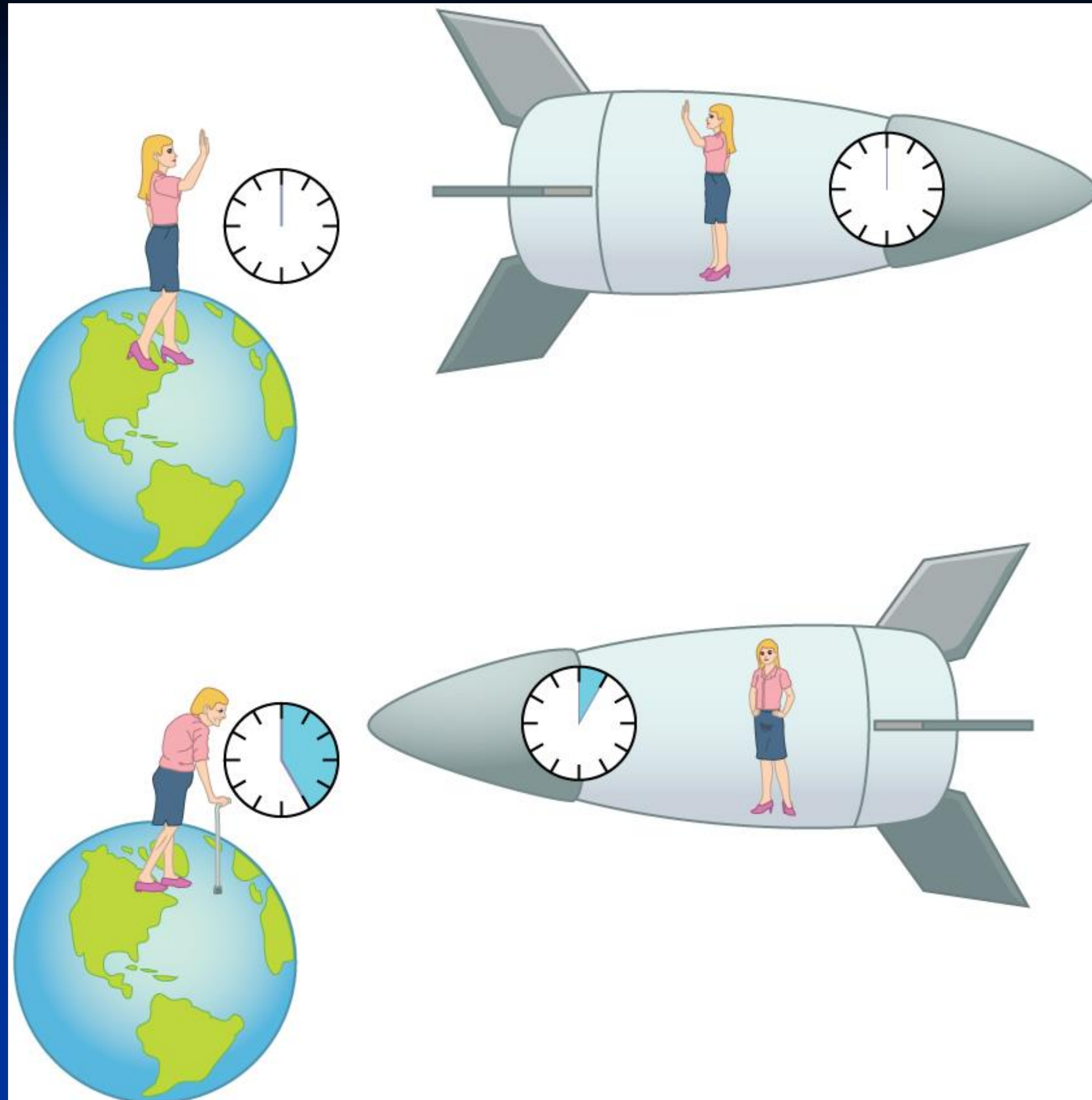


$$t_2 \rangle t_1$$

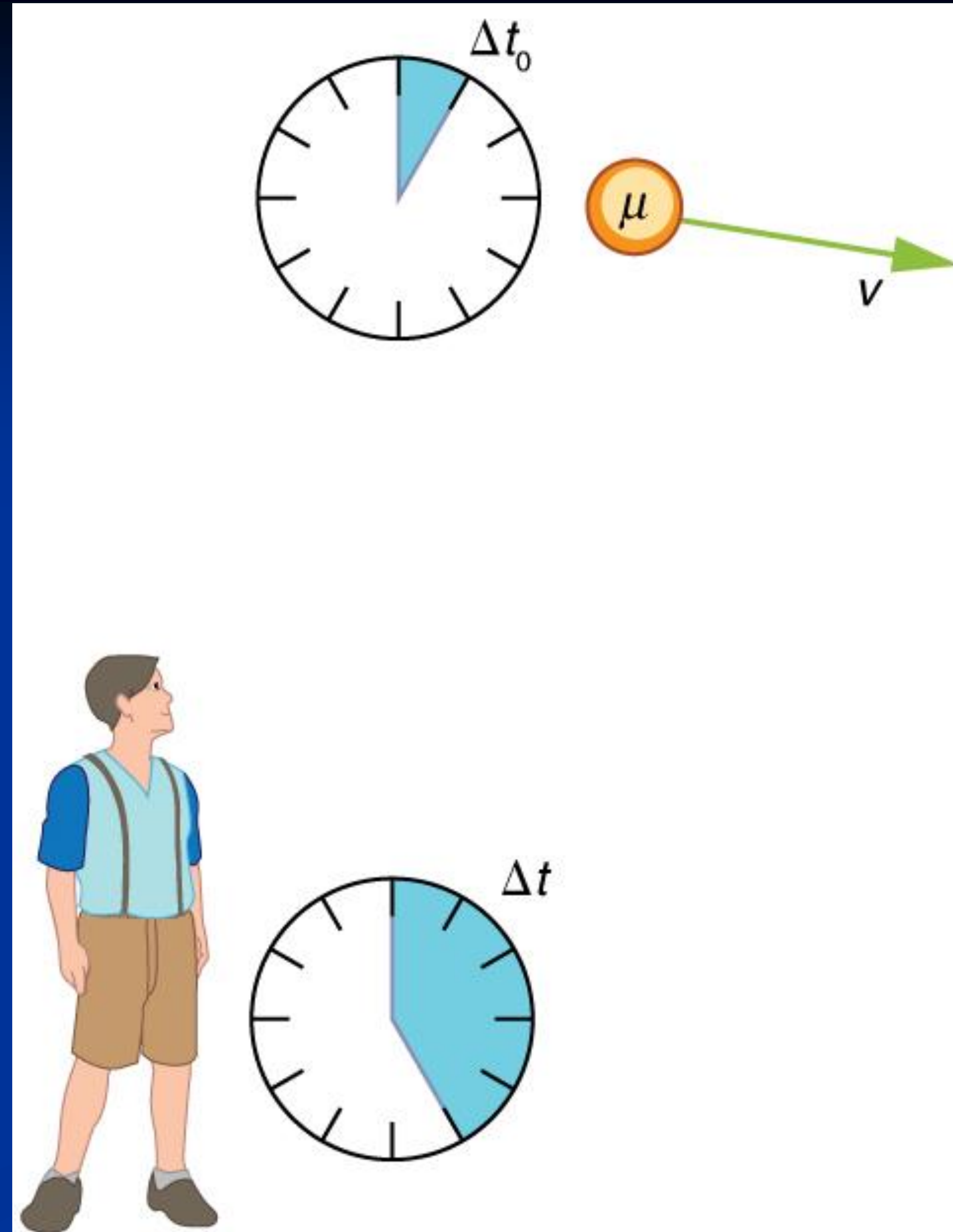
Time interval w. r. t the stationary frame

Time interval w. r. t the moving frame

Time Dilation



Time Dilation



Proper Time & Improper Time

Proper Time :

In relativity, proper time is time measured by a single clock between events that occur at the same place as the clock. It depends not only on the events but also on the motion of the clock between the events!

Improper Time :

Time measured with two clocks or a single moving clock!

Proper Time & Improper Time

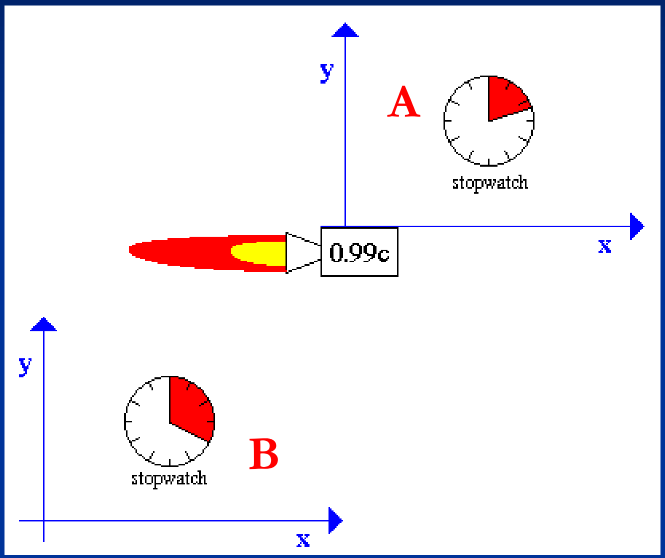
Improper Time $=$ Proper Time x $\sqrt{1-\frac{v^2}{c^2}}$

This means, that time is dilated (extended) in motion. This gives rise to two questions,

(01) Do moving clocks really run slow ???

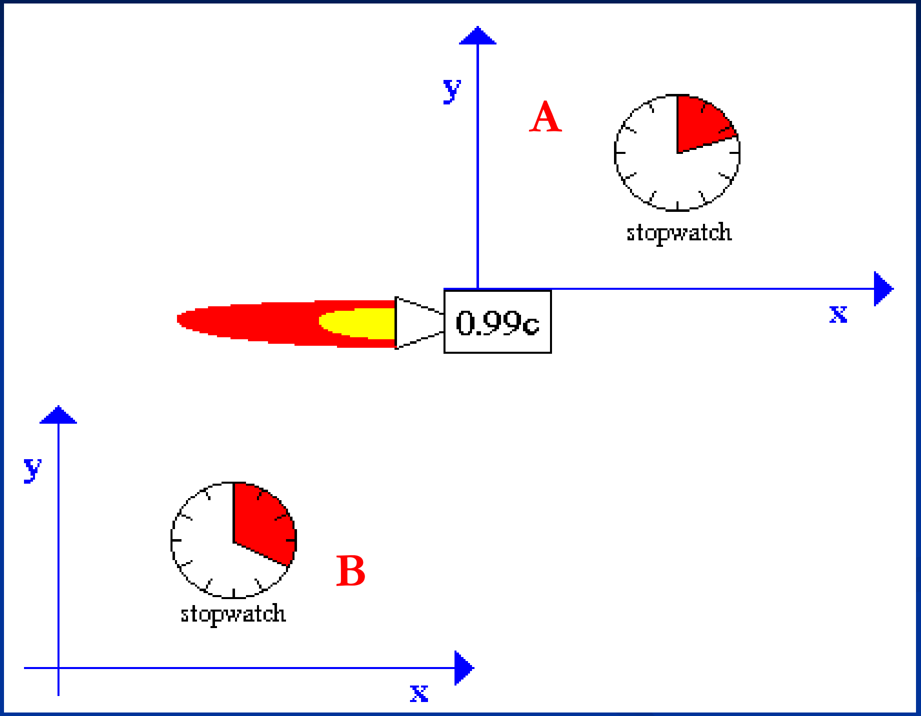
(02) The clock mechanism affected by motion ???

Proper Time & Improper Time



The above results tells us that " *moving clocks run slow* ". When the clock is moving one is measuring improper time. A stationary clock measures proper time. It seems as through there is a Clock Paradox in question **01** started above. For instance, suppose an observer A is moving with a uniform velocity and another observer B is "stationary". Then according to A his watch runs slower than that of B, as a consequence of measurement of proper and improper time !

Proper Time & Improper Time



Question **02** started above, regarding whether the clock mechanism is affected during motion is simply non-sense. As our frames are inertial all physical laws, including mechanics remain unaltered.

Example 01 :

A particle X, which is created in a particle accelerator, travels a total distance of $100.0\ m$ between two detectors in $410\ ns$ as measured in the laboratory frame before decaying into other particles. What is the lifetime of the particle X as measured in its own frame ???

Velocity of the particle X w.r.t lab frame :

$$v = \frac{distance}{time}$$

$$v = \frac{100\,m}{410\,ns}$$

$$v = 2.44 \times 10^{8}\ ms^{-1}$$

Using relativistic time equation :

$$t_2 = t_1 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t_1 = ?$

$t_2 = 410 \times 10^{-9}\ s$

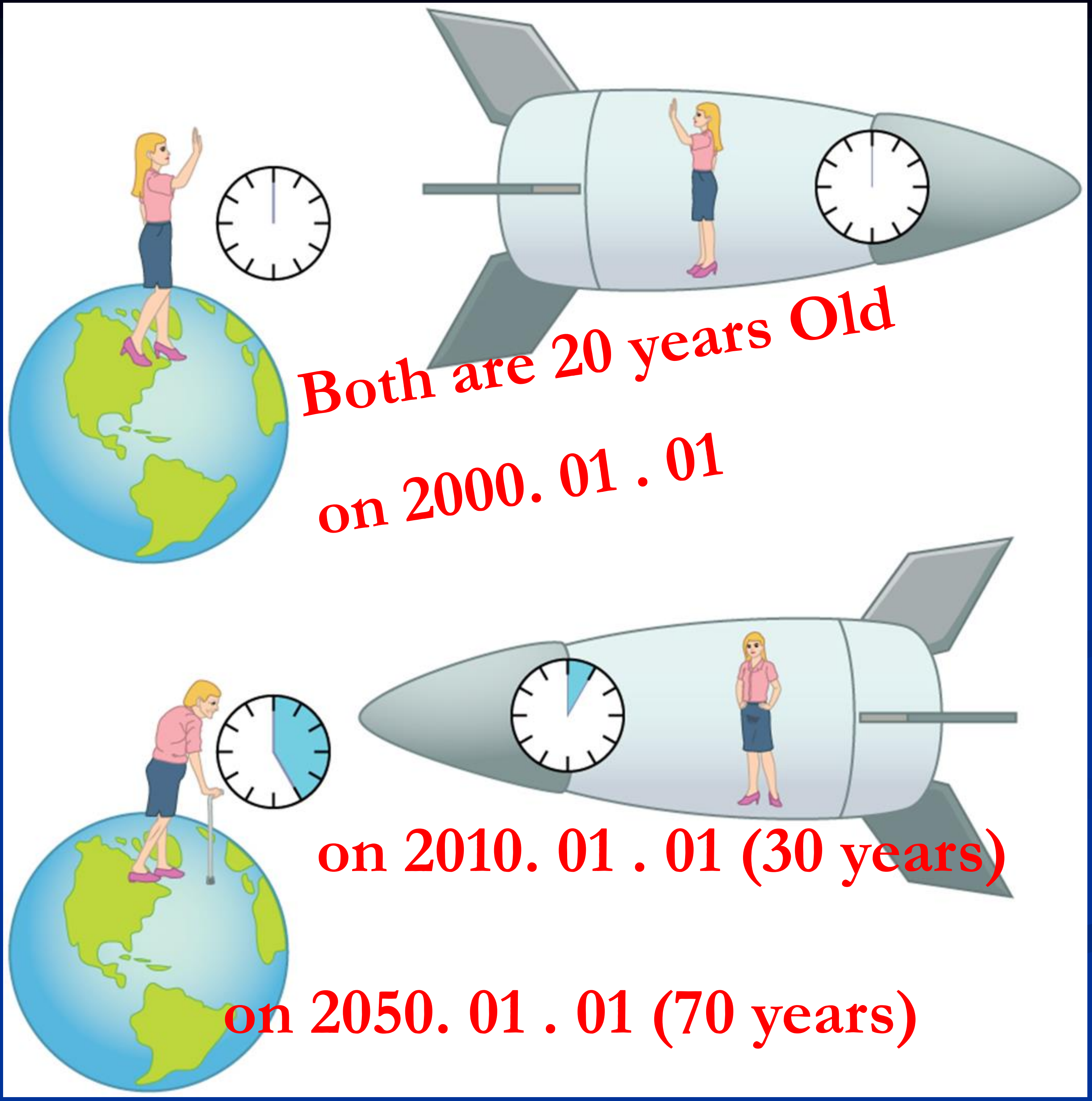
$v = 2.44 \times 10^{8}\ ms^{-1}$

$$t_1 = t_2 \sqrt{1 - \frac{v^2}{c^2}}$$

$$t_1 = 410 \sqrt{1 - \frac{(2.44 \times 10^{8})^2}{(3.0 \times 10^{8})^2}}$$

$$t_1 = 238\ ns$$

Example 02 :



Both are 20 years Old

on 2000. 01 . 01

on 2010. 01 . 01 (30 years)

on 2050. 01 . 01 (70 years)

Find the Velocity of the space-ship ?

Experiment on Time Dilation :

The experimental evidence for time dilation was provided by Ives and Stilwel (1938) who measured the change in frequency of spectral lines emitted by fast moving atoms. The effect observed was small as the velocities of atoms was only about c/2; but it was convincing.

The real, conclusive evidence came from the experiment of Rossi & Hall (1941) involving **muons**, and we discuss this below :

The muon is a charged particle with mass about 200 times that of an electron. It decays to an electron plus a neutrino – antineutrino pair.

$$\mu \rightarrow e + \gamma_{\mu} + \overline{\gamma_{\mu}}$$

Antineutrino

Neutrino

Experiment on Time Dilation :

$$\mu \rightarrow e + \gamma_{\mu} + \overline{\gamma_{\mu}}$$

Antineutrino

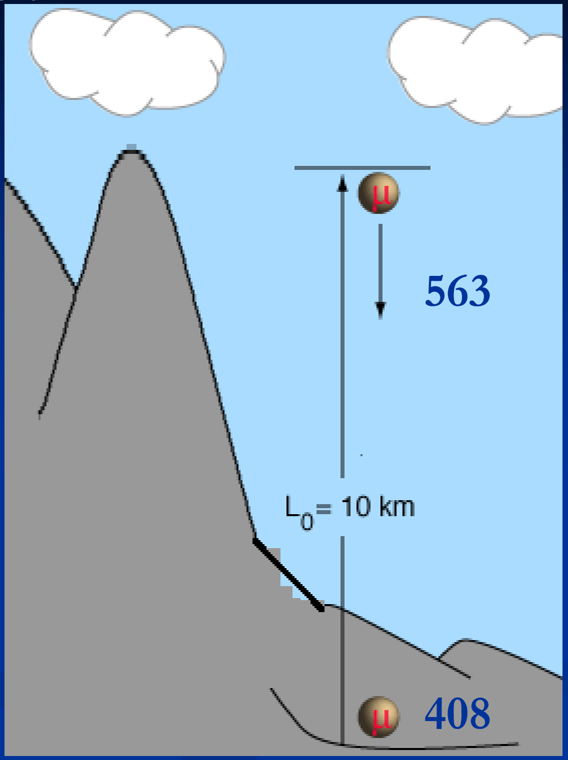
Neutrino

and the half life for the muon decay is *1.53 x 10^(-6) s.* (This means that if there are No number of muons at time t = 0, then after *1.53 x 10^(-6) s*, there will be No/2 muons left. The rest No/2 would have decayed.)

Cosmic rays contain large number of muons at high altitudes and they mostly travel vertically downward at speeds comparable with that of light. The experiment of Rossi & Hall consisted in measuring the number of muons and their time of flight at the top of **Mountain Washington** (height 6265 ft) in New Hampshire, and at 10 ft above sea level.

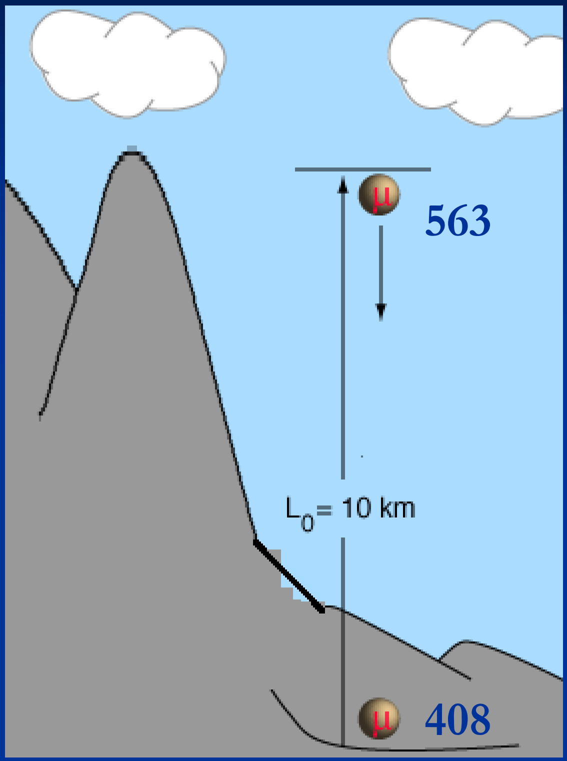
Experiment on Time Dilation…

Only those muons with speed between 0.9950 c and 0.9954 c were counted. This number at the mountain top was found to be 563±10 per hour.



However, at sea level the number of muons was found to be 408±9 per hour. **It must be said that in the experiment Rossi & Hall did not detect the 408±9 muons remaining out of the 563±10 per hour**. The ground experiment was done at some outer place which was close to the mountain.

Experiment on Time Dilation…



Thus, the measured time of the muon flight, which is the improper time is,

$$t_{im} = \frac{(6265 - 10)\,ft}{0.9952 \times 3 \times 10^8 \times 3.28\ (ft/m)}$$

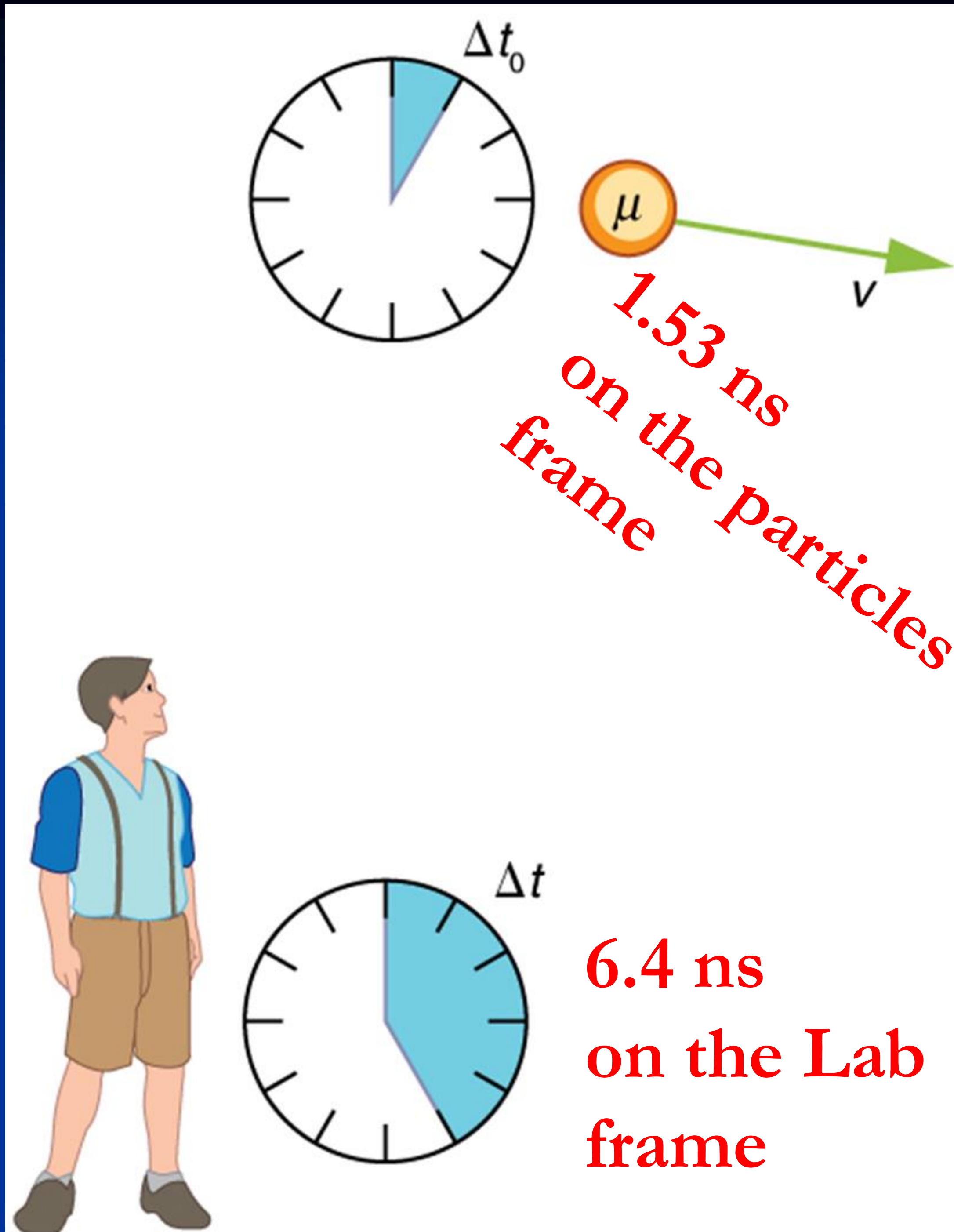
Avarage Speed   Meter → Feet

$$t_{im} = 6.4 \times 10^{-6}\,s$$

This experiment verifies that, $t_{improper} \rangle\ t_{proper}$

This experiment clearly shows that time dilation can be a very significant effect for clocks that are in high speed relative motion!

Summary



$$\Delta t_0 < \Delta t$$

Assuming a car is moving with a constant velocity in 10 s (w.r.t the car's frame) from,

(a) 60 m/s

(b) c/10 m/s

(c) c/2 m/s

Where c is the velocity of light.

$$t_A = t_B \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

```
In[1]:= tb = 10; (* Time in Seconds *)
        c = 3*10^(8); (* Velocity of Light *)
        v = 60; (* Speed of the vehicle *)
        ta = N[tb*(1/Sqrt[1 - v^2/c^2]), 50](* Time in Seconds *)

Out[3]= 10.000000000000200000000000006000000000000200000000
```

13^{th} Decimal Point

$$t_A = t_B \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

```
In[1]:= tb = 10; (* Time in Seconds *)
        c = 3 * 10 ^ (8); (* Velocity of Light *)
        v = 60; (* Speed of the vehicle *)
        ta = N[tb * (1 / Sqrt[1 - v ^ 2 / c ^ 2]), 50] (* Time in Seconds *)

Out[3]= 10.000000000000200000000000000060000000000000200000000
```

13th Decimal Point

```
In[4]:= tb = 10; (* Time in Seconds *)
        c = 3 * 10 ^ (8); (* Velocity of Light *)
        v = c / 10; (* Speed of the particle *)
        ta = N[tb * (1 / Sqrt[1 - v ^ 2 / c ^ 2]), 50] (* Time in Seconds *)

Out[5]= 10.050378152592120754893735565668747527051783471483
```

2nd Decimal Point

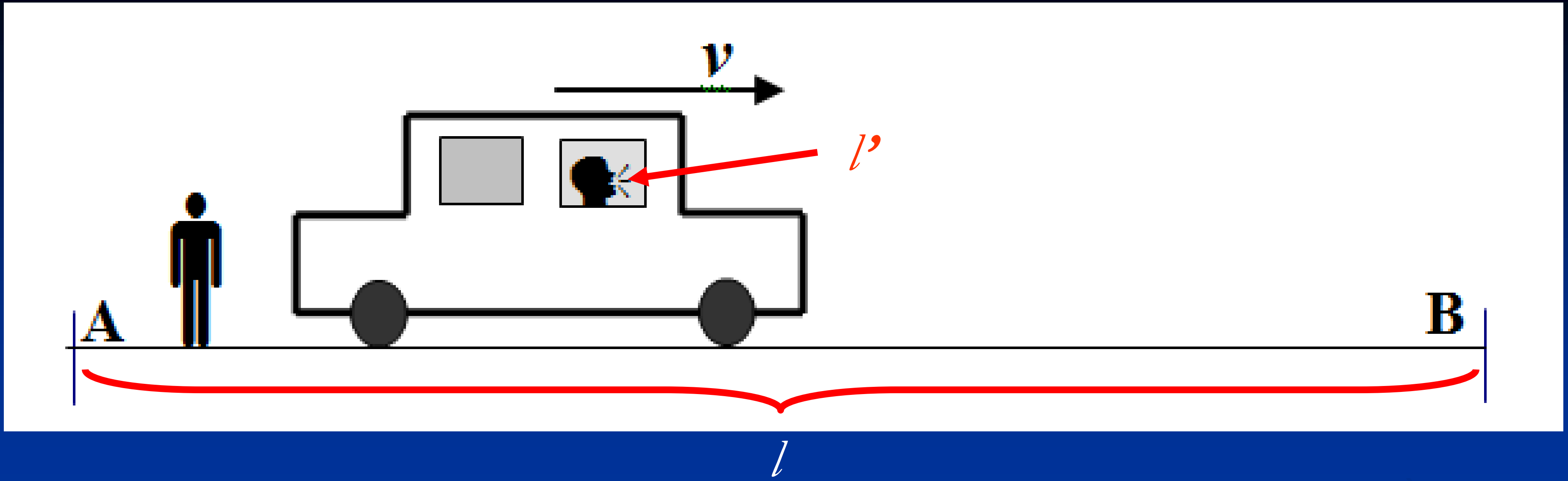
```
In[6]:= tb = 10; (* Time in Seconds *)
        c = 3 * 10 ^ (8); (* Velocity of Light *)
        v = c / 2; (* Speed of the particle *)
        ta = N[tb * (1 / Sqrt[1 - v ^ 2 / c ^ 2]), 50] (* Time in Seconds *)

Out[7]= 11.547005383792515290182975610039149112952035025403
```

Measurement of Length in STR

So, time is relative ! What about distance ???

In order to think about this note that when we say that the distance between two objects is l we imagine measuring the position of these objects simultaneously… but simultaneously is relative, so **we can expect distance to be a relative concept too**!

Length contraction is the observation that a moving object appears shorter than a stationary object! Like time dilation, length contraction is a consequence of the postulates of relativity. Length contraction and time dilation are related, and introduce a relation between length and time (Or space and time, if you prefer!)

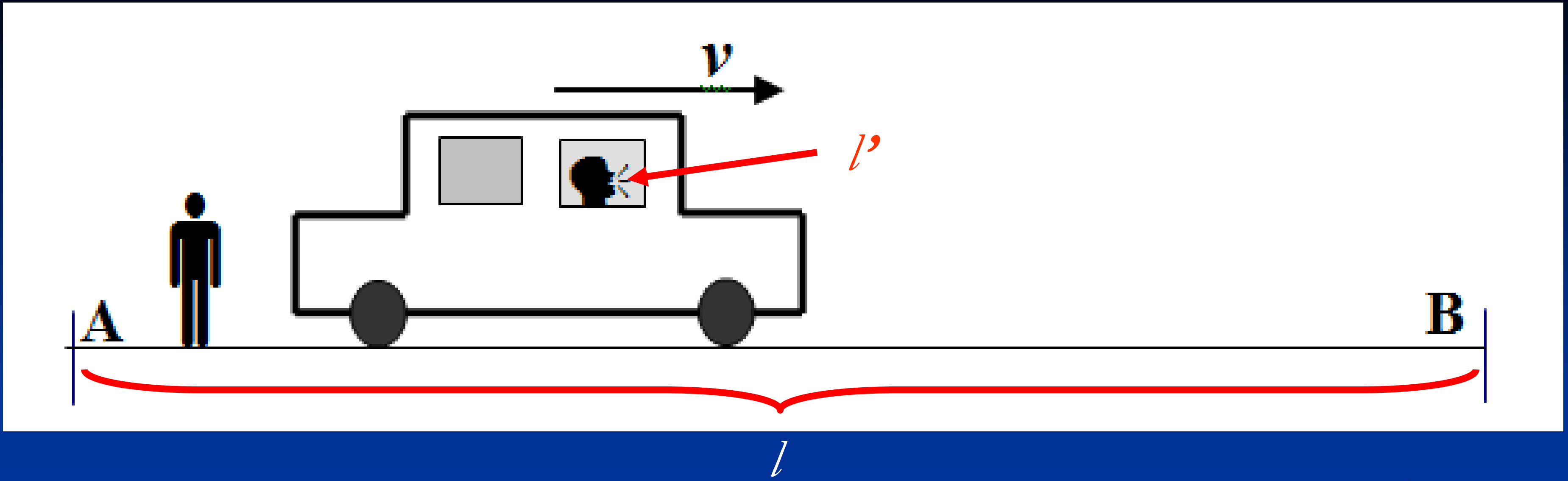


Suppose a car is moving with a uniform speed v relative to an observer in a stationary frame. This observer notes that it covers a distance l between two check posts (A & B) in time t. As two different clocks (one at each post) are involved in the measurement of this time, this time is improper time.

$$t_{im} = \frac{l}{v}$$

The Observer in the car covers the distance between the check-posts in proper time, such that:

$$t_{im} = t_{pro}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

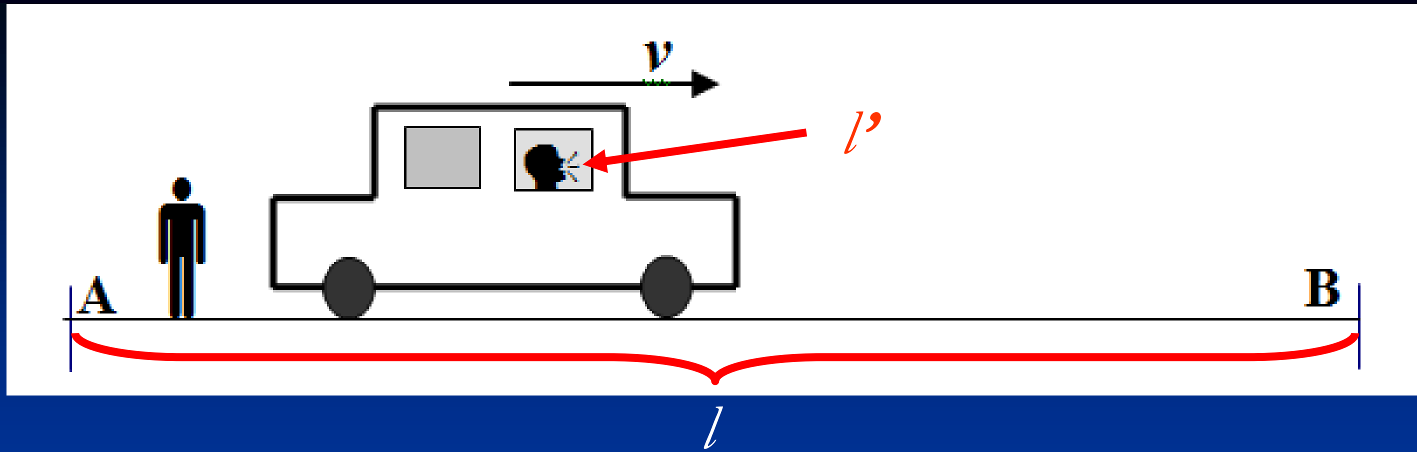


Thus, the distance l' that the observer in the car measures is given by,

$$t_{pro} = \frac{l^1}{v}$$

(l' is measured by using **speedometer in the car** !)

$$t_{im} = t_{pro}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{l}{v} = \frac{l^1}{v} \times \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow l^1 = l\sqrt{1-\frac{v^2}{c^2}}$$



$$l^1 = l\sqrt{1-\frac{v^2}{c^2}}$$

This equation is called relativistic length equation!

If v > 0 $\Rightarrow$ $\frac{v}{c} < 1$ $\Rightarrow$ $\frac{v^2}{c^2} < 1$ $\Rightarrow$ $1-\frac{v^2}{c^2} < 1$

$\Rightarrow$ $\sqrt{1-\frac{v^2}{c^2}} < 1$ $\Rightarrow$ $l^1 = l\ (<1\ )$

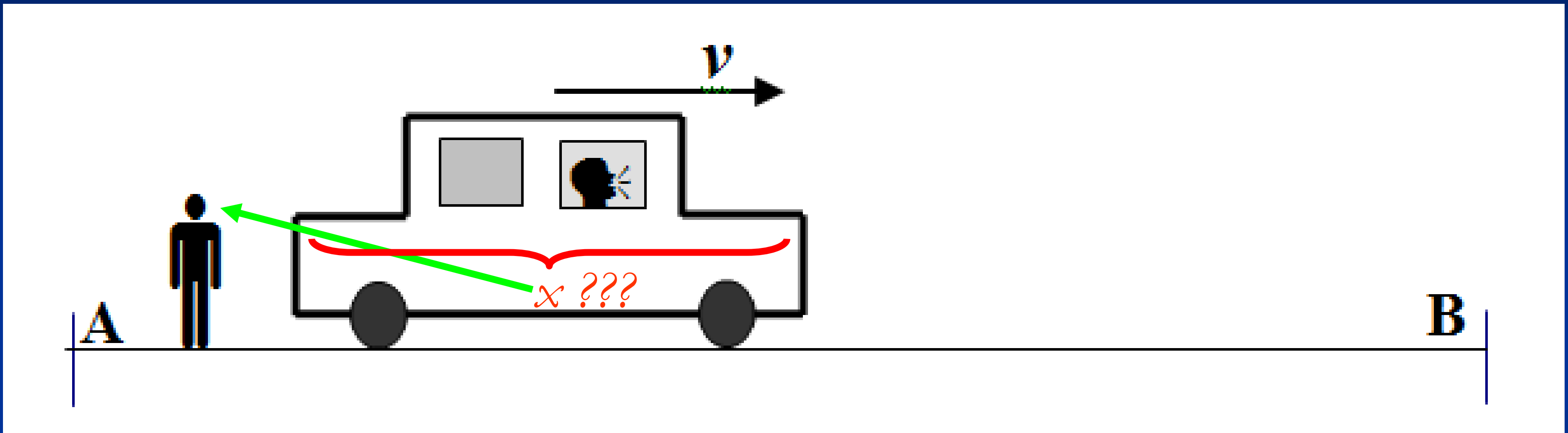
$\Rightarrow$ $l^1 < l$

Length measured by an observer in the car

Length measured by observer on the Earth

This is called Length Contraction !

What is the length of the car as seen by an observer on the Earth ???



Is it less than the original length ???

Is it greater than the original length ???

Is it equal to the original length ???

Thank You !