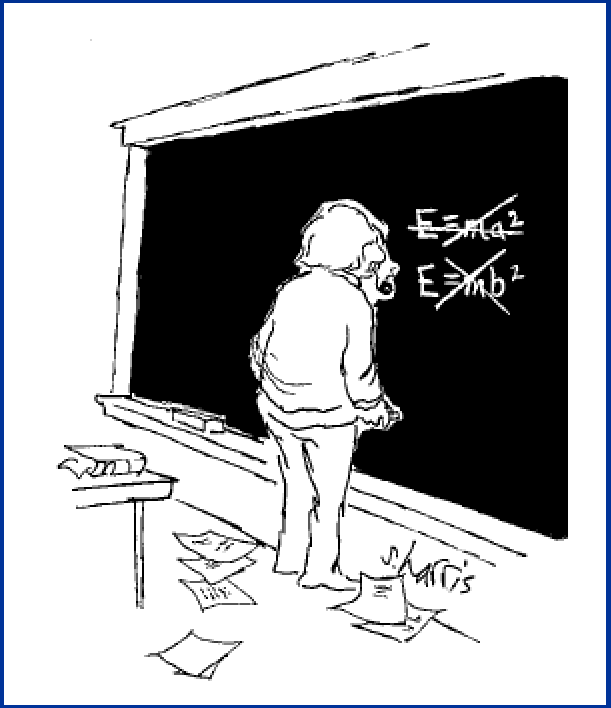
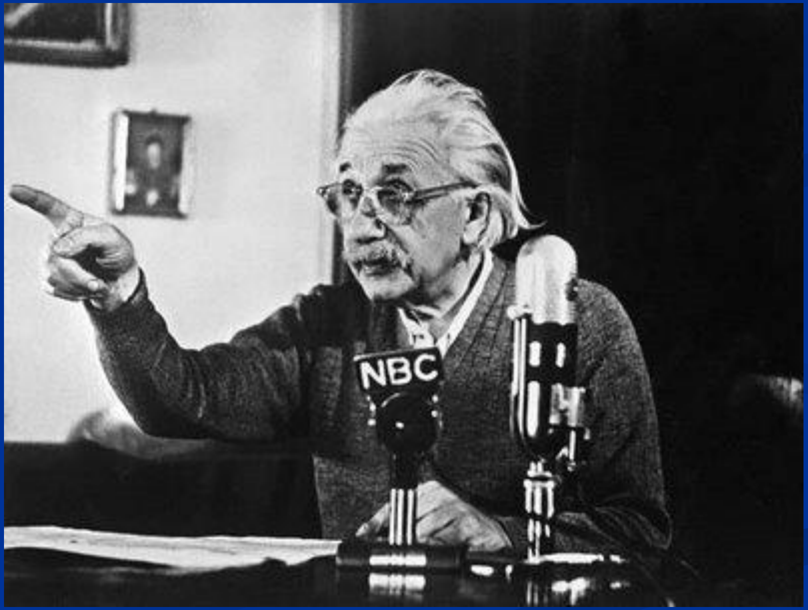
Special Theory of Relativity





13th Lecture

2017

UNIVERSITY OF SRI JAYEWARDANEPURA

B.Sc. General/Special Degree Third Year Course Unit Examination – October, 2017.

PHY 207 1.0 / PHY 257 1.0 / PHY 302 1.0 / PHY 327 1.0

- Special Theory of Relativity

Time : One hour

Answer all questions

Assume, velocity of Light (c) = $3 \times 10^8 \text{ ms}^{-1}$

<u>01.</u> Write down the **two** main Einstein's Postulates in Special Theory of Relativity (STR).

Obtain the following relativistic time equation, starting from the above Postulates in STR.

$$t_2 = t_1 \frac{1}{\sqrt{1 - v^2/c^2}}$$ (Symbols have their usual meanings)

How many times will the half-life of an unstable particle increase, compared its stationary value, if it moves with a velocity of *0.99 c* ?

01. Write down the **two** main Einstein's Postulates in Special Theory of Relativity (STR).

Obtain the following relativistic time equation, starting from the above Postulates in STR.

$$t_2 = t_1 \frac{1}{\sqrt{1 - v^2/c^2}}$$ (Symbols have their usual meanings)

How many times will the half-life of an unstable particle increase, compared its stationary value, if it moves with a velocity of *0.99 c* ?

$$t_2 = t_1 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{t_2-t_1}{t_1} = \frac{t_1 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - t_1}{t_1}$$

02. Derive an expression for the length contraction ($l^1 = l\sqrt{1 - v^2/c^2}$) starting from the relativistic time equation (Symbols have their usual meanings).

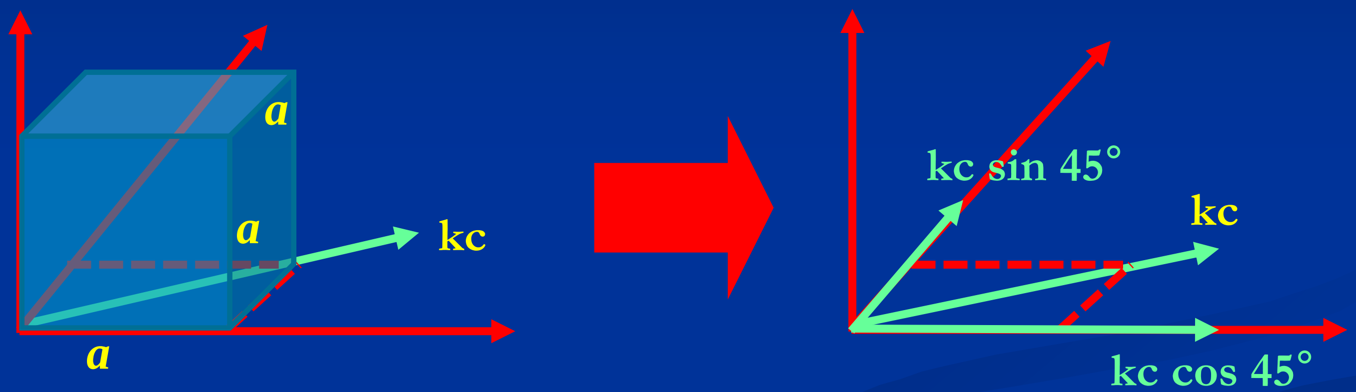
A solid cube has each side a while at rest. It is now set into motion with velocity kc where k is a constant and c is the velocity of light, along the direction of the base diagonal.

What is its volume as seen by a stationary observer ?

02. Derive an expression for the length contraction ($l^1 = l\sqrt{1 - {}^{v^2}\!/_{c^2}}$) starting from the relativistic time equation (Symbols have their usual meanings).

A solid cube has each side a while at rest. It is now set into motion with velocity kc where k is a constant and c is the velocity of light, along the direction of the base diagonal.

What is its volume as seen by a stationary observer ?



$$l^1 = l\sqrt{1-\frac{v^2}{c^2}}$$

$$l_x^{\,1} = a\sqrt{1-\frac{(kc\cos 45^\circ)^2}{c^2}}$$

$$l_y^{\,1} = a\sqrt{1-\frac{(kc\sin 45^\circ)^2}{c^2}}$$

Volume as seen by Sta; Obs; = $V = l_x^1 \times l_y^1 \times l_z^1 = l_x^1 \times l_y^1 \times a$

03. Derive the expression, $E^2 = m_o^{\,2} c^4 + p^2 c^2$, starting from the equation, $E = m c^2$ (symbols have their usual meanings).

Hence, obtain the equation, $m = {m_o}\Big/{\sqrt{1-\frac{v^2}{c^2}}}$ for **mass variation** in relativistic dynamics. (Symbols have their usual meanings)

If $M = {m}/{m_o}$ and $\beta = \frac{v}{c}$, sketch the variation of M vs β.

For a particle of the rest mass m_0, relativistic mass m, rest energy E_0 and total relativistic energy E, prove the following relations;

$$\textbf{(a)} \quad v = \frac{c\sqrt{(E+E_0)(E-E_0)}}{E}$$

$$\textbf{(b)} \quad v = \frac{c\sqrt{(m+m_0)(m-m_0)}}{m}$$

03. Derive the expression, $E^2 = m_o^2 c^4 + p^2 c^2$, starting from the equation, $E = m c^2$ (symbols have their usual meanings).

Hence, obtain the equation, $m = {m_o}\Big/{\sqrt{1-\frac{v^2}{c^2}}}$ for **mass variation** in relativistic dynamics. (Symbols have their usual meanings)

If $M = {}^{m}\!/\!{}_{m_o}$ and $\beta = \dfrac{v}{c}$, sketch the variation of M vs β.

$$m = \frac{m_o}{\sqrt{1 - \dfrac{v^2}{c^2}}} = m_o \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

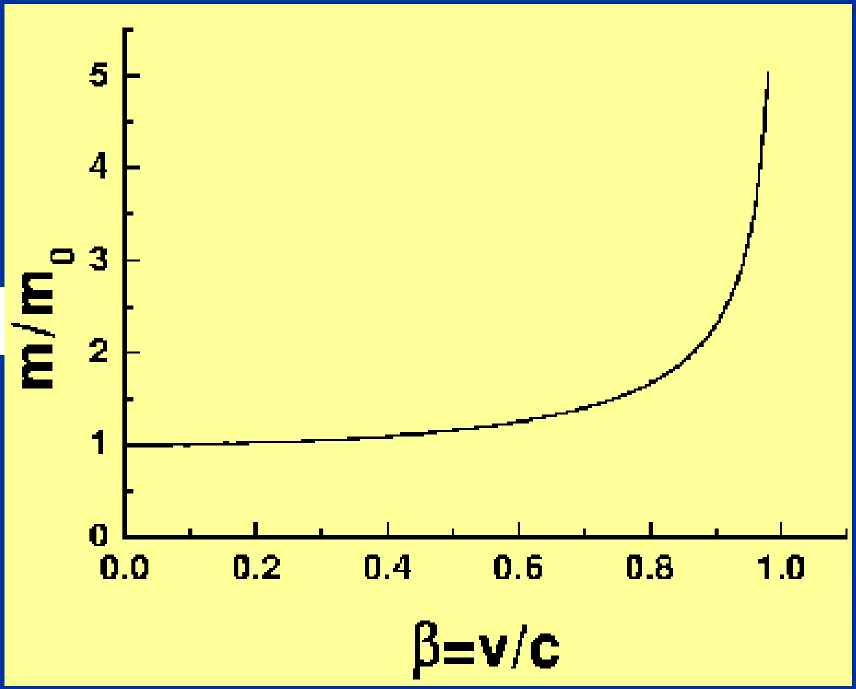
$$\Rightarrow \frac{m}{m_o} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$\Rightarrow M = \left(1 - \beta^2\right)^{-1/2}$$

Where, $\beta = \dfrac{v}{c}$

and $\dfrac{m}{m_o} = M$

$M =$



For a particle of the rest mass m_0, relativistic mass m, rest energy E_0 and total relativistic energy E, prove the following relations;

$$(a)\quad v = \frac{c\sqrt{(E+E_0)(E-E_0)}}{E}$$

$$(b)\quad v = \frac{c\sqrt{(m+m_0)(m-m_0)}}{m}$$

$$E^2 = p^2c^2 + m_o^2c^4$$

$$E^2 = (mv)^2c^2 + \left(m_oc^2\right)^2$$

$$E^2 = (mc)^2v^2 + E_o^2$$

$$E^2 = \frac{E^2}{c^2}v^2 + E_o^2$$

$$\frac{c^2\left(E^2 - E_o^2\right)}{E^2} = v^2$$

$$\frac{c\sqrt{(E+E_o)(E-E_o)}}{E} = v$$

For a particle of the rest mass m_0, relativistic mass m, rest energy E_0 and total relativistic energy E, prove the following relations;

$$(a)\quad v=\frac{c\sqrt{(E+E_0)(E-E_0)}}{E}$$

$$(b)\quad v=\frac{c\sqrt{(m+m_0)(m-m_0)}}{m}$$

$$E^2=p^2c^2+m_o^2c^4$$

$$\left(mc^2\right)^2=(mv)^2c^2+\left(m_oc^2\right)^2$$

$$m^2c^4=m^2v^2c^2+m_o^2c^4$$

$$m^2c^4-m_o^2c^4=m^2v^2c^2$$

$$\frac{\left(m^2-m_o^2\right)c^4}{m^2c^2}=v^2$$

$$\frac{c\sqrt{(m+m_o)(m-m_o)}}{m}=v$$

04. What is meant by the **Doppler Effect** in Relativity for a moving light source?

You are given the following mathematical equation for the Doppler effect,

$$f_o = \frac{f_s}{\gamma\left(1-\beta\ \cos\theta\right)}.$$

Where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = \frac{v}{c}$ and other symbols have their usual meanings.

In Astronomy, Astronomical Charts display "**actual**" colour and **speed of stars** in the Universe. Such charts indicate the colours of the stars observed on the Earth. However, indicating their "actual" colours and speeds seems beyond common sense.

Write down your views on the basis on which above information are included in Astronomical Charts.

04. What is meant by the **Doppler Effect** in Relativity for a moving light source?

You are given the following mathematical equation for the Doppler effect,

$$f_o = \frac{f_s}{\gamma\left(1-\beta\cos\theta\right)}.$$ Where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = \frac{v}{c}$ and other symbols have their usual meanings.

In Astronomy, Astronomical Charts display "**actual**" colour and **speed of stars** in the Universe. Such charts indicate the colours of the stars observed on the Earth. However, indicating their "actual" colours and speeds seems beyond common sense.

Write down your views on the basis on which above information are included in Astronomical Charts.

For the stars are situated very far, we can observed their frequency with respect to the Earth frame. Also we san observed their frequency shift with respect to the Earth frame.
Then we get a such colour lamp (Na Lamp) and move it a very fast in the laboratory frame that we can observed the same frequency shift with respect to the laboratory frame. Then we assume that is the velocity of the star.
Then we calculated the original colour using Doppler's Effect equation in STR

Assignments

Special Theory of Relativity

Assignment No 02

Assume, velocity of Light (c) = 3×10^8 ms^{-1}

Identify the symbols E, p and m_o in the following equation,

$$E^2 - p^2 c^2 = m_o^2 c^4 .$$

Obtain the equation,

$$m = m_o \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

for **mass variation** in relativistic dynamics. (Symbols have their usual meanings)

The rest mass of the electron is, $m_e = 9.1 \times 10^{-31}\ kg$. **What** will be its mass if it was moving with velocity $0.8c$?

Identify the symbols E, p and m_o in the following equation,

$$E^2 - p^2 c^2 = m_o{}^2 c^4 .$$

Obtain the equation,

$$m = {m_o}\Big/{\sqrt{1-\frac{v^2}{c^2}}}$$

for **mass variation** in relativistic dynamics. (Symbols have their usual meanings)

The rest mass of the electron is, $m_e = 9.1\times10^{-31}\ kg$. **What** will be its mass if it was moving with velocity *0.8c* ?

Special Theory of Relativity

Assignment No 01

Assume, velocity of Light (c) = 3×10^8 ms^{-1}

An **alpha** particle and a **beta** particle, which are created in a **particle accelerator**, travel a total distance of 10.0 m between two detectors in 50 ns and 40 ns respectively, as measured in the **laboratory frame**.

(*a*) What is the lifetime of the alpha particle as measured in its own frame?

(*b*) What is the lifetime of the alpha particle as measured in the frame of the beta particle?

An **alpha** particle and a **beta** particle, which are created in a **particle accelerator**, travel a total distance of 10.0 m between two detectors in 50 ns and 40 ns respectively, as measured in the **laboratory frame**.

(*a*) What is the lifetime of the alpha particle as measured in its own frame?

An **alpha** particle and a **beta** particle, which are created in a **particle accelerator**, travel a total distance of 10.0 m between two detectors in 50 ns and 40 ns respectively, as measured in the **laboratory frame**.

(*a*) What is the lifetime of the alpha particle as measured in its own frame?

(*b*) What is the lifetime of the alpha particle as measured in the frame of the beta particle?

Thank You !