Practice on probability distributions

1. X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

a) $P(x < 40) = P(x-30/4<40-30/4) = P(z<2.5) = 1-P(z>2.5) = 1-P(z<-2.5) = 1-0.0062$

b) $P(x > 21) = P(z>21-30/4) = P(z>-2.25) = 1-P(z<-2.25) = 1-0.0122$

c) $P(30 < x < 35) = P(30-30/4<z<35-30/4) = P(0<z<1.25) = P(z<1.25)-P(z<0) = 1-P(z>1.25) – 0.5$

$= 0.5-P(z<-1.25) = 0.5-0.1056$

2. For a certain type of computers, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. John owns one of these computers and wants to know the probability that the length of time will be between 50 and 70 hours.

X=battery life

$P(50<z<70) = P(50-50/15<z<70-50/15) = P(0<z<1.33) = P(z<1.33) – P(z<0) = 1-P(z>1.33)-0.5$

$=0.5-P(z<-1.33) =0.5- 0.0918$

3. Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

4. The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant in a period of time

a) less than 19.5 hours?

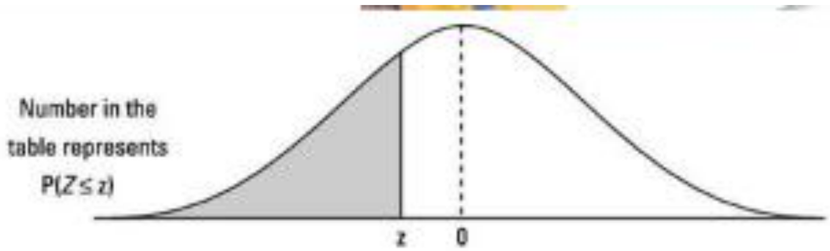
b) between 20 and 22 hours?

5. A large group of students took a test in Physics and the final grades have a mean of 70 and a standard deviation of 10. If we can approximate the distribution of these grades by a normal distribution, what percent of the students

a) scored higher than 80?

b) should pass the test (grades$\geq$60)?

c) should fail the test (grades$<$60)?



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
–3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

6.

In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

(a) Find the probability that

(i) exactly 6 ask for water with their meal,

(ii) less than 9 ask for water with their meal.

(5)

A second random sample of 50 customers is selected.

(b) Find the smallest value of n such that

$$P(X < n) \geqslant 0.9$$

where the random variable X represents the number of these customers who ask for water.

(3)

X=number of customers ask for water out of 10 customers

X follows a binomial distribution where success event is that a customer asks for a water

P(success)=3/5   thus X~Bin(10,3/5)

a) I) P(X=6) = 10C6*(3/5)^6*(2/5)^4
   ii) P(X<9) = 1 - P(X≥9) = 1- P(X=9) - P(X=10) = 0.95
b) P(X<8) = 1 - P(X≥8) = 1 - P(X=8) - P(X=9) - P(X=10) = 0.82
   Since P(X<8) < 0.9, n = 9 is the smallest number where P(X<n) ≤ 0.9.

7.

In a game, players select sticks at random from a box containing a large number of sticks of different lengths. The length, in cm, of a randomly chosen stick has a continuous uniform distribution over the interval [7, 10].

A stick is selected at random from the box.

(a) Find the probability that the stick is shorter than 9.5 cm.

(2)

To win a bag of sweets, a player must select 3 sticks and wins if the length of the longest stick is more than 9.5 cm.

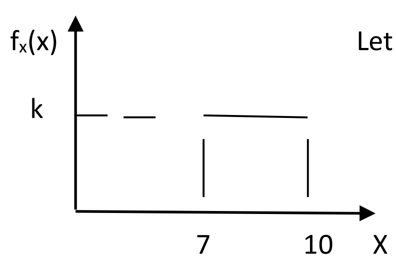
(b) Find the probability of winning a bag of sweets.

(2)

To win a soft toy, a player must select 6 sticks and wins the toy if more than four of the sticks are shorter than 7.6 cm.

(c) Find the probability of winning a soft toy.

(4)



Let X be the random variable of the length of a randomly chosen stick

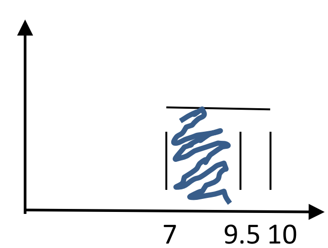
PDF of X can be drawn as this.

K is constant that we can calculate using definition of pdf

area under the function should be 1.

K*(10-7) = 1 $\therefore$k = 1/3

(a)



Here we are asked to find the darken area

P(X<9.5) = P(7<X<9.5) = (1/3)*(9.5 – 7) = 2.5/3

For (b), we can define a random variable (Y = number of sticks, out of 3, longer than 9.5) which follows a binomial distribution with success probability is P(X>9.5) and n=3. You have to calculate P(Y>0) = p(Y=0)

For (c), calculate P(X<7.6) and take it as the success probability for a new random variable (W = number of stick, out of 6, shorter than 7.6) which follows a binomial distribution with n = 6.Find P(W>4)

8.

A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.
(2)

(b) Find the probability that there are at least 2 defective components in the box.
(3)

(c) Using a suitable approximation, find the probability that a batch of 250 components contains between 1 and 4 (inclusive) defective components.
(4)

9.

Patients arrive at a hospital accident and emergency department at random at a rate of 6 per hour.

(a) Find the probability that, during any 90 minute period, the number of patients arriving at the hospital accident and emergency department is

(i) exactly 7

(ii) at least 10
(5)

A patient arrives at 11.30 a.m.

(b) Find the probability that the next patient arrives before 11.45 a.m.
(3)

(a) (i) X = number of patients arrive per hour. X ~ Poisson(6)
P(X=7) = $e^{-6}.6^7/6!$ =
(ii) P(10≤X) = 1 – P(X<10) = 1 – P(X=0) – P(X=1) …. – P(X=9) =
(b) Now the time frame is 15 min. So Y = number of patients arrive per 15 min. Y ~ Poisson(6/4)
P(Y=1) =

In a village, power cuts occur randomly at a rate of 3 per year.

(a) Find the probability that in any given year there will be

(i) exactly 7 power cuts,

(ii) at least 4 power cuts.

(5)

(b) Use a suitable approximation to find the probability that in the next 10 years the number of power cuts will be less than 20

(6)