# Practice PMF, CDF and PDF

1.  The random variable X has p.m.f as given by:

$$f(X) = \begin{cases} 0 & \text{if } x < 4 \\ 0.1 & \text{if } x = 4 \\ 0.3 & \text{if } x = 5 \\ 0.3 & \text{if } x = 6 \\ 0.2 & \text{if } x = 8 \\ 0.1 & \text{if } x = 9 \\ 0 & \text{if } x > 9 \end{cases}.$$

1.  Calculate the probabilities P(X≤6.5), P(X>8.1), P(5<X<8).
2.  Calculate the c.d.f F(x)
3.  Calculate E[X} and Var[X]      (additional work)
4.  Draw the p.m.f and c.d.f

2.  For discrete random variable x, p.m.f is given by;

$$P(X=x) = \begin{cases} kx & x = 1,2,3,4,5 \\ k(10-x) & x = 6,7,8,9 \end{cases}$$

1.  Find the value of the constant k.
2.  Find the probabilities P(X=5), P(X=7), and P(4<X≤7).
3.  Find the c.d.f of X
4.  Find the probabilities P(X<3), P(X≥7) and P(3≤X<7).
5.  Draw the pmf and cdf.

3.  A fair six-sided die has '1' on new face, '2'on two of its faces and '3' on the remaining three faces. The die is thrown twice, and X is the random variable 'total score thrown'. Find
    (a)  The probability distribution of X
    (b)  The probability that total score is more than 4
    (c)  Find the c.d.f of X.

4.  Let X be a continuous random variable whose probability density function is:

$$f_X(x) = \begin{cases} 0 & x < -1 \\ x+1 & -1 \le x \le 0 \\ 1-x & 0 < x \le 1 \\ 0 & x > 1 \end{cases}$$

    (a)  Graph the probability density function $f_X$ of X.
    (b)  Verify that fX is a valid probability density function.
    (c)  Find P(X< -0.5) and P(-0.7<X≤0.3).

Answers

1.

1. $P(X \leq 6.5) = 0.7, P(X > 8.1) = 0.1, P(5 < X < 8) = 0.3$.

2.

$$F(X) = \begin{cases} 0 & \text{if } x < 4 \\ 0.1 & \text{if } 4 \leq X < 5 \\ 0.4 & \text{if } 5 \leq X < 6 \\ 0.7 & \text{if } 6 \leq X < 8 \\ 0.9 & \text{if } 8 \leq X < 9 \\ 1.0 & \text{if } x \geq 9 \end{cases}.$$

3. $E[X] = 0.1 * 4 + 0.3 * 5 + 0.3 * 6 + 0.2 * 8 + 0.1 * 9 = 6.2$

4. $E[X^2] = 0.1 * 16 + 0.3 * 25 + 0.3 * 36 + 0.2 * 64 + 0.1 * 81 = 40.8, Var[X] = 40.8 - 6.2^2 = 2.36$