

University of Sri Jayewardenepura.

B. Sc. General/Honours Degree Third Year

Course Unit – 2025

PHY 308 1.0

PHYSICS PRACTICALS

[Computational Physics]



This handout is intended to be a brief introduction to *Mathematica*. It describes essentially some capabilities of *Mathematica*, and assumes no prior knowledge of the system.

WOLFRAMRESEARCH

PHY 308 1.0

PHYSICS PRACTICAL

Attendance for the Practical Classes (2025)

According to the UGC regulations, the students <u>should have at</u> <u>least 80% attendance</u> for the lectures or practical classes to be eligible to sit for the final examination.

Method of Evaluation for PHY 308 1.0 Physics Practical:

Continuous Assessments	- 40 marks
Attendance	- 20 marks
Final Examination	- 60 marks
Total	- 100 marks

Continuous Assessments:

A number of assignments given for a year would vary between 04 to 06 depending on the time factor. All these assignments are compulsory

Final Examination:

Practical Paper – Duration 2 or 3 hours (Open Book Examination)

> Dr. Madhuranga Fernando Department of Physics.

Mathematica 11 - 13 Software

Mathematica is a powerful mathematical software in the world. The working environment of this software is totally different from other mathematical software. **Mathematica** is built on the powerful unifying idea that everything can be represented as a symbolic expression.

Executing Method

Any given input / command can be run by pressing both *Shift* and *Enter* keys simultaneously.

Basic Operations

Numerical Form	Mathematica Form
a+b	a + b
a-b	<i>u</i> + <i>b</i>
$a \times b$	a-b
u × U	a * b
$\frac{a}{b}$	a/b
υ	

Defining a variable

When defining a variable it is a common practice to use simple letters.

E.g.:a = 5b = -1c = 2

In-built functions / Keywords

The first letter of any given in-built function must be capitalized. Some in-built functions are as follows.

Print Plot Do If Table Solve N - Numerical Function There are three kinds of brackets used in Mathematica.

(1) Parenthesis – ()

We can use parenthesis in usual way.

Numerical Form	Mathematica Form
(2x-5)(3x-6)	(2 * x - 5) * (3 * x - 6)

(2) Box bracket - []

We can use box bracket to type the argument of any in-built function. E.g.:-

N[1/7] {Here N gives the numerical value of its argument}

We can also do the above calculation in following way.

N[1/7,10] {This will give the value of 1/7 up to ten decimal points}

(3) Curly bracket – { }

Curly brackets are used to define a Range and an Array,

(1) <u>Defining a range</u>
E.g.:- i = 1 to 100 → {i, 1,100}
(2) <u>Defining an array</u>
E.g.:- marks = {50, 60, 70, 80, 90}

Exercise:

Find the numerical values of (i) $\frac{1}{13}$, (ii) $\frac{1}{17}$ and (iii) $\frac{1}{\pi}$. **Hint:**

Numerical Form	Mathematica Form
π	Pi

Exercise:

If $a = \frac{1}{11}$, find the numerical values of (i) a^2 , (ii) $\frac{1}{a}$, (iii) $\frac{1}{a^2}$, (iv) a^{10} , (v) $a + \frac{1}{a}$, (vi) $a^2 + 1$, (vii) $a - \frac{1}{a}$, (viii) $a \left(a + \frac{1}{a} \right)$, (ix) $\frac{a - \frac{1}{a}}{a + \frac{1}{a}}$ and (x) $a + \frac{1}{a} + \frac{1}{a^2}$

Exercise:

Find the numerical values of (i) π , (ii) $\frac{22}{7}$, (iii) $\pi - \frac{22}{7}$, (iv) π^2 and (v) $\pi^2 - 10$

Mathematica can be used in three major ways.

- (1) Calculator
- (2) Software package
- (3) **Programming language**

Mathematica as a Calculator

We can use **Mathematica** just like a calculator.

Numerical Form	Mathematica Form
3+4	3+4
2^{20}	2^20
$\sin(60^{\circ})$	Sin[60Degree]
$(1.765)^2(2.8)^{\frac{1}{3}}$	$(((1.765)^2)^*((2.8)^(1/3)))/(Sqrt[1.9125831])$
$\sqrt{1.9125831}$	

When we type questions like above, Mathematica prints back their answers.

Numerical Form	Mathematica Form
sin 60°	Sin[60 Degree]
$\cos 60^{\circ}$	Cos[60 Degree]
tan 60°	Tan[60 Degree]
cosec 60°	Csc[60 Degree]
sec 60°	Sec[60 Degree]
$cot \ 60^{o}$	Cot[60 Degree]

Exercises:

E.g.:-

Find the numerical values of the following expressions,

- (i) $\sin 85^{\circ} + \cos 85^{\circ}$
- (ii) $\sin^2 85^o + \cos^2 85^o$
- (iii) $\sec^2 35^\circ \tan^2 35^\circ$
- (iv) $\csc^2 55^\circ \cot^2 55^\circ$

Mathematica as a Software Package

We can use **Mathematica** like a software package. Here we can't use it in our own way. That means they are built to do some specific tasks. Some common packages are given below.

E.g.:- MSWORD, MSEXCEL, MSPOWERPOINT, WINAMP, VLC Player, Windows Media Player, Nero, U Tube Player, ...

Exercises:

(1) Finding the numerical value of π up to 20 decimal places. N[Pi, 20]

- (2) Plotting the function $y = \sin x$ within the range $-\pi$ to π . $Plot[Sin[x], \{x, -Pi, Pi\}]$
- (3) Plotting the function $y = e^x$ within the range 2 to 6. $Plot[Exp[x], \{x, 2, 6\}]$
- (4) Solving the equation $3x^3 + 2x^2 + 1 = 0$ Solve $[3 * x^3 + 2 * x^2 + 1 = 0, x]$

(5) If $Z_1 = 1 + 2j$ and $Z_2 = 4 - 3j$ (where, $j = \sqrt{-1}$) find the following expressions.

(i) $Z_1 + Z_2$ (ii) $Z_1 - Z_2$ (iii) $\frac{1}{Z_1}$ (iv) $\frac{1}{Z_1} + \frac{1}{Z_2}$ (v) $\frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}}$

Hint:

Mathematical Form $Z_1 = 1 + 2j$

Mathematica Form z1 = 1 + 2I(where, $I = \sqrt{-1}$)

Mathematica as a Programming language

Mathematica is a flexible and intuitive programming language.

(1) Finding the area of a rectangular shaped land whose length is 5m and width is 3m.

$$a = 5$$

 $b = 3$
 $area = a * b$
Print["Area is", area, "m²"]

Exercises:

Lengths of the two sides of a rectangular triangle are 6 cm and 8 cm. Find the length of the diagonal.

Some important guidelines on writing programs

(1) Comments

It is a good practice to comment our program in order to understand the inclusion later.

E.g.:- $ugc = 6.67 * (10^{(-11)})$ (* universal gravity constant *)

(2) Semicolon (;)

If we put a semicolon after any input, it will prevent the output being displayed.

E.g.:-	inputs	outputs
	a = 5	5
	b = 10;	

Additional to above tasks, there are some add-on packages that come with **Mathematica** to do some specific tasks. Here are some examples for standard packages embedded in **Mathematica**.

- (1) Miscellaneous
- (2) Statistics
- (3) DiscreteMath
- (4) Geometry

Under the package **Miscellaneous** there are several sub packages as *WorldPlot*, *ChemicalElements*, *Geodesy*, *StandardAtmosphere* and etc. Let's now consider the add-on package **WorldPlot**. This loads the **WorldPlot** package.

<<Miscellaneous `WorldPlot`

This gives a map of Asia with all countries drawn as white with black outlines.

WorldPlot[World]



WorldPlot[{World, RandomGrays}]



WorldPlot[Asia]



Exercise:

Plot the Other Continents i:e; NorthAmerica, SouthAmerica, Europe, Africa,, MiddleEast and Oceania.

Now consider the standard add-on package called *ChemicalElements*. This loads the *ChemicalElements* package.

Other Packages:

• <<Miscellaneous ` Dictionary`

SpellCheck["She is an actress"]	{She is an actress, {}}
SpellCheck["She is a actress"]	{She is a actress, {}}
SpellCheck["Sha is an actress"]	{ <mark>Sha</mark> is an actress,{Sha}}
SpellCheck["Sha is an actres"]	{Sha is an actres, {Sha, actres}}
SpellCheck["Sha is a actres"]	{Sha is an actres,{Sha,actres}}
	SpellCheck["She is an actress"] SpellCheck["She is a actress"] SpellCheck["Sha is an actress"] SpellCheck["Sha is an actres"] SpellCheck["Sha is a actres"]

• <<Miscellaneous ` Calendar`

Eg:	DayOfWeek[{2010,09,15}] DayOfWeek[{1973,11,26}]	Wednesday Monday
Eg:	DaysPlus[{2010,09,15},10] DaysPlus[{2010,09,15},50]	{2010,9,25} {2010,11,4}
Eg:	DaysBetween[{1973,11,26},{1985,11,03}] DaysBetween[{1973,11,26},{1985,11,03}]/365 DaysBetween[{1973,11,26},{1985,11,03}]/365//N	$ 4360 \\ 872 \\ 73 \\ 11.9452 $

DaysBetween[{your birthday}, {date of today}]/365//N = your age (in years)

<u>Tute 01</u>

 ${\bf Q1.}$ Find the solutions for the following problems.

Mathematical Form	Mathematica Form
2.3 + 5.63	2.3 + 5.63
$2.4 + 8.9^{2}$	2.4 + 8.9^2
$4.4+7.2^2$	
$(3+4)^2 - 2(3+1)$	$(3+4)^{2-2*(3+1)}$
2 ⁴⁵	2^45
2 ⁴⁵	2^45 //N
$2.^{45}$	2.^45
452 ÷ 62	452/62
Find the value for π	
Find the above for 40 decimal places	
77 ²	77^2
77 ² +1	% + 1
(4 + 3 i)/(2 - i)	(4+3 I)/(2-I)
3 ¹⁰⁰	
$2^3 + 6$	
3x - x + 2	3 * x - x + 2
-l+2x+7x	
35.8 ⁵⁰	(35.8)^50
59. ÷ 4	59./4
2 ×3× 6	2*3*6 or 2 3 6
54 ×6 ¹⁰	54 * 6^10 or 54 6^10

Q2.

Mathematical Form	Mathematica Form
Type, $x^2 + x - 4x^2$	$x^{2} + x - 4^{*}(x^{2})$
Type, $xy + 2x^2y + y^2x^2 - 2yx$	$x*y+2*(x^2)*y+(y^2)*(x^2)-2*y*x$
Type, $(x + 2y + 1)(x - 2)^2$	$(x + 2*y + 1)*(x - 2)^2$
Expand the above	Expand[%]
expressions	
Factor the above	Factor[%]
expressions	
Find, $\sqrt{\left(1+0.1\right)^4}$	$Sqrt[(1 + 0.1)^{4}]$
Type, $log(1+Cos(x))$	Log[1+Cos[x]]
Find the value of	1 + 2*x /. x->3
(1+2x), when x=3	
Type, $t=1+x^2$	
Find t when $x= 2$	
Find t when x=5a	
Find the value of t when	
$x = \pi$	
Find , $\sum_{x=1}^{5} x^2$	Sum[x^2,{x,1,5}]
Find , $\sum_{i=1}^{10} rac{x^i}{i}$	
Find , $\sum \frac{x^i}{i}$, for i=1,3,5	Sum[x^i/i,{i,1,5,2}]
Find , $\sum_{i=1}^n i^2$	Sum[i^2,{i,1,n}]

Find , $\sum_{k=1}^{\infty} \frac{1}{k}$	Sum[1/1^4, {1,1, Infinity}]
$\sum_{i=1}^{n} i^4$	
\sim 1	<pre>Sum[1/(i!+(2i)!), {i,1,Infinity}]</pre>
Find , $\sum \frac{1}{(1+(2i)!)}$	
$\frac{1}{i=1} (i: \tau(2i):)$	NT [0,]
for the above	IN [0]
	$[2^{1}]$
Find, $\sum_{i=1}^{N} x^i y^i$	$\operatorname{Sum}[X \perp Y], \{j, 1, 1\}, \{1, 1, 5\}]$
<i>i,j=</i> 1	
Find the value of $l+x+x^2$,	$1 + x + x^2 / . x \rightarrow (2-y)$
when x→2-y	
Туре х→3+у	х→3+у
Find value of x -9 using	x^2-9/.%
the above	
Generate this product	<pre>Product[x+i, {i, 1, 4}]</pre>
(x+1)(x+2)(x+3)(x+4)	
$\lim_{t \to 0} (1)$	
$ n > 0 \left \frac{1}{n+1} \right $	Limit[(1/(1+x)),x->0]
$x \to 0 (x+1)$	
$\lim (x-1)$	
$r \rightarrow 1$ $\left(\frac{r^2-1}{r^2-1}\right)$	
$\lim_{n \to \infty} (1)$	
$x \rightarrow 1$ $\left(\overline{x-1} \right)$	
$\lim_{x \to -2x+1} (\frac{x^2 - 2x + 1}{x^2 - 2x + 1})$	
$x \rightarrow 1$ $(x - 1)$	
$\lim_{x \to \infty} \left(\frac{2}{x} \right)$	
$\lim_{x \to 1} \left(\frac{x - 1}{x} \right)$	
$x \rightarrow 1 (x - 1)$	
lim (sin r)	
$\frac{1}{3}$	
$x \rightarrow 0$ (x)	
$\lim_{x \to \infty} (\tan x)$	
$\begin{vmatrix} x - > 0 \end{vmatrix} \left(\frac{x}{r} \right)$	
$\lim (\sin x + \cos x)$	
$x \to 0$ $(x \to 0)$	

Q3.

Mathematical Form	Mathematica Form
If $f = x^n$, find f' or $\frac{df}{dx}$	D[x^n,x]
If $f = x^n$, find f''' or $\frac{d^3f}{dx^3}$	D[x^n, {x, 3}]
If $f = Sinx$, find f^1	
If $f = x Sinx$, find f^1	
If $f = \frac{x^2 + 5x + 1}{x^3 - 1}$, find f^1	
If $f = Cos^2 x Sinx$, find f^1	
If $f = \frac{5}{x^2 + 1}$, find f^1	
If $f = \frac{5}{\sqrt{x^2-1}}$, find f^1	

If $f = \frac{5}{x^2 - 1}$, find f^1	
If $f = \frac{5}{\sqrt{x^2+1}}$, find f^1	
If $f = Sinx$, find f^2	
If $f = Sinx$, find f^3	
If $f = Sinx$, find f^{10}	
If $f = Sinx$, find f^{51}	
• 1	Integrate $\left[\frac{1}{x^4-1}\right]$
Find , $\int \frac{1}{(x^4-1)} dx$	
Find , $\int_{0}^{1} x^2 dx$	<pre>Integrate[x^2, {x, 0, 1}]</pre>
Find , $\int_{1}^{1} x^2 dx$	<pre>NIntegrate[x^2, {x, 0, 1}]</pre>
0 0	Integrate $[x^2, \{x, 0, 1\}]//N$
Find , $\int_0^1 \frac{1}{\sqrt{x(1-x)}} dx$	
Find , $\int \frac{1}{(x^4 - a^4)} dx$	
Find , $\int_{-a}^{a} \frac{1}{(x^4 - a^4)} dx$	
Find , $\int_{0}^{\pi} e^{(1-x^2)} dx$	
Find , $\int_{0}^{\pi} Sin(2x) dx$	
Find , $\int \ln x dx$	<pre>Integrate[Log[x], x]</pre>
Find , $\int \log x dx$	<pre>Integrate[Log[10,x], x]</pre>
Find , $\int_{a}^{b} \ln x dx$	
Find , $\int \sqrt{(x+1) \times \sqrt{(x-1)}} dx$	
Find , $\int \sqrt{\left(\frac{x+1}{\sqrt{x-1}}\right)} dx$	
Find , $\int \sqrt{\frac{x+1}{x-1}} dx$	
Find , $\int \sqrt{\frac{x-1}{x+1}} dx$	
Find , $\int_{0}^{\pi} (5 \sin x + 8 \cos x) dx$	
Find , $\int_{0}^{\pi} (x \sin^2 x)^{\frac{1}{2}} dx$	

Quit Kernel

If you encounter with any of the following situations when you are working with Mathematica, you are advised to follow the Quit Kernel procedure.

- 1. Mathematica stucks while you are working.
- 2. Mathematica produces an erroneous result even if your input statement is correct.
- 3. Mathematica produces an incorrect result even if your input statement is correct.
- 4. If the machine becomes slow due to the processing of the particular calculation.
- 5. If any loop will run continuously without control.

Quit Kernel Procedure:

Move the cursor to the kernel that appears on the Mathematica toolbar and click on it. Select,

Quit kernel ------ Local

A dialog box will appear & choose Quit.

Write a complete Mathematica program segment to do the following exercise.



The cube was transformed in to a sphere by melting it. Assuming,

- *a.* No material is loss in the process
- **b.** 20% of material is loss in the process

Calculate the radius (r) of the Sphere.

Define a Function

Mathematical Form

$$f(x) = \sin(x^2)$$
$$f(\frac{\pi}{2}) = \sin\left(\left(\frac{\pi}{2}\right)^2\right) = \sin\left(\frac{\pi^2}{4}\right)$$

iv. Solve, x³+2x-1=0
v. Solve, x-3x²=0

Mathematica Form

$$f[x_{-}] = Sin [x^{2}]$$

$$f[\pi/2] = f\{Pi/2\}$$

$$f[\pi/2] // N = 0.624266$$

Tute 02

Q1. Given $f(x) = x^6 - 21x^5 + 175x^4 + 735x^3 + 1624x^2 - 1764x + 720$

f[x]=x^6-21*x^5+175*x^4+735*x^3+1624*x^2-1764*x+720

i. If x=0.5, find f(x)f[0.5] When $x = \sqrt{2}$, find f(x) ii. f[Sqrt[2]] Find $f(y) = y^6 - 21y^5 + 175y^4 + 735y^3 + 1624y^2 - 1764y + 720$ iii. f[y] **Q2.** Given $f(x) = (1+x)^2$ Expand, $(1+x)^2$ i. ii. Factor the above result iii. If x=0.25, find f(x)iv. Find, f(z) **Q3.** Given $f(x) = (1+x+3y)^2$ where y is a constant i. Expand, $(1+x+3y)^2$ ii. Find f(x) for x = 0.5, 1, 1.5, 2, 2.5, 3iii. When $x = \sqrt{3}$, find f(x) iv. Find f(z) Factor the above result in part(i) v. **Q4.** Given $f(x) = x^{10} - 1$ i. Factor above f(x) ii. Expand above f(x) iii. When $x=\sqrt{3}$, find f(x) iv. Find f(z) Q5. Find the answers for the following equations. Solve, 8x+3=0 Solve[8*x + 3 == 0, x]i. ii. Solve, 8x+3.0=0
iii. Solve, x²-5x-3=0 Solve[8*x + 3.0 == 0, x]

vi	. Solve,	1+5.0x=0			
vii	. Solve,	$x^2 - 9 = 0$			
viii	. Solve,	$2x+x^2=0$			
ix	. Solve,	$x^{2}+2x-7=0$			
X	. Solve,	$x^{4}-5x^{2}-3=0$			
xi	. Solve,	$x^{6}-1=0$			
xii	. Solve,	$7x^4 - 5x^2 - 6.0 = 0$)		
xiii	. Solve,	$ax^2+bx+c=0$,	(here a, b	o & c are Co	onstants)
xiv	. Solve,	$7x^5 - 3x^3 + 8 = 0$	· ·		
XV	. Solve,	$x^{4}-1=0$			
xvi	. Solve,	x-1.0=0			
xvii	. Solve,	$x^{3}-3x+5=0$			
xviii	. Solve,	$7x^{5}+6x^{2}+3=0$			
	(* Do	the same u	sing Tabl	e command	*)
	f={x^2 Table[%[1]	-2*x+1,x^3-1, Solve[f[[i]]	x^4-1,2*x- ,x], {I,1,	7} Length[f]}]	
Q6. Gi	ven e = ($(x-1)^2 + (2+x))$	/((a+x)(x-3	3) ²), a is a	a constant
e	$= ((x-1)^{2})$	2+(2+x))/((a+	x)*(x-3)^2)	
i.	Expand th	e above (e)		Expan	d[e]
ii.	Expand ou	t everything,	, including	g the denomi Expan	.nator, dAll[e]
iii.	Collect denominat	all the te or	erms Toget	her over Toget	a common her[%]
iv.	Break the	expression a	apart into	terms with	simple
v.	Factor ev	erything(rep	roducing th	ne original	form),
vi.	Simplify(e)		Simpl	r[%] ify[e]
Q7.	Given v=(v=((3+ 2	3+2x) ² (x+2y) ² 2x) ² (x+2y) ²)		
i	Expand th	le above v			
ii	Groups to	ogether term	s in v th	at involve	the same
	power of	Х		Collect[v,	x]
iii	Groups to	gether power	of y	Collect[v,	y]
iv	Separate	factors out	the piece	that does n	not depend
	on y			FactorTerm	s[v,y]
00 at		2 + 4 + 2 + 2		(1.0.)	
Qo. GI	Lven e=(1+	3x+4y-)-		e = (1+3x+)	4y^2)^2
i	Expand ab	ove e		f = Expand	[e]
ii	Find the	coefficient d	of x in f	Coefficien	t[f,x]
iii	Find the	coefficient d	of x^2 in f	Coefficien	t[f,x^2]
iv	Find the	highest power	r of y that	appears ir	n Exponent
v	Find the	fourth term :	in f	<pre>Exponent[f Part[f,4]</pre>	,y]

14

- i. Make a list of values (a table) of r^2 , with r running from 1 to 10. i.e. $\left\{ r^2 \right\}_{r=1}^{10} = \{1^2, 2^2, 3^2, 4^2, \dots, 10^2\}$ Hint: Table [expression, {x, x min, x max}]
- ii. Make a list of values of Sin (n/5), with n running from 0 to 25.
- iii. Make a list of values of $x^i + y^j$ with i running from 1 to 3 & j running from 1 to 2. Put them in table format.
- iv. Create a list of values containing four copies of the symbol x using "Table" command. i.e. {x, x, x, x}
- **Q10.** Plot the following functions in given regions.

(a) f(x) = tan(x)i Plot, f(x) for x = -3 to 3 range. ii Plot, f(x) for $x = -\pi$ to $+\pi$ range. iii Plot, f(x) for $x = -2\pi$ to $+2\pi$ range. iv Plot, f(x) for $x = -10\pi$ to $+10\pi$ range.

(b) f(x,y) = sin(xy)
i Plot, f(x,y) for x, 0 to 4 and y, 0 to 4
in three dimensional space.(3D)

Hint:

Mathematical Form

Mathematica Form

 $f(x, y) = \sin(xy) \qquad \qquad f[x_{-}, y_{-}] = Sin[x^*y]$

- - Put,

AxesLabel→{"Length","Width","Height"}

(c) f(x,y)=Sin(x)Sin(y)

- i ContourPlot f(x,y) for x = -2, 2 range and y = -2, 2 range.
 - ii DensityPlot f(x, y) for above range.

Q11. $f(x, y) = 10 \sin x + 5 \cos y$

- i Plot f(x, y) for x = -10, 10, y = -10, 10, in 3D
- ii Put, PlotPoints -> 40
- iii See the difference in the figure by varying no of plotpoints. In the same plot show axes labels as "time", "depth" and "value".

Q12. f(x) = Sin x, g(x) = Sin 2x, h(x) = Sin 3x

i Plot the above functions in same axis for x goes from 0 to 2 \sim range.

f={Sin[x], Sin[2*x], Sin[3*x]}
Table[Plot[f[[i]], {x,0,2*Pi}], {i,1,3}]
Show[%]

ii Draw f(x) in red color

- iii Draw g(x) in green color
- iv Draw h(x) in blue color
 - v Plot the above (ii), (iii) and (iv) functions in same axis

- **Q13.** Write the following statement by using conditionals, Loops & control structures in Mathematica.
 - i If 7 > 8 the answer should be x, otherwise y.
 ii If 7 > 8 the answer should be print "x",
 otherwise print "y".
 Hints:
 If[Condition, True, False,Otherwise]
 Eq: If[2>3, Print["True"], Print["False"]]

False
If[3>2, Print["True"], Print["False"]]
True
If[Mala > Kamala, Print["T"], Print["F"],
Print["Otherwise"]]
Otherwise
iii Print i² for i = 1 to 4 increment by 1

V	Print i^2 for $i = 1$ to 40 increment by 2
vi	Print i^3 for $i = 1$ to 40 increment by 5
vii	Print i^3 + (i+1) ³ for i = 1 to 40 increment by 5
viii	Make a array (table) i for i = 1 to 100 increment by 1
	Table[i,{i,1,100}]
ix	Make a array (table) i^2 for $i = 1$ to 100 increment by 5
	Table[i^2,{i,1,100,5}]
х	Print i , starting from i = 1 for i < 14 increment by 1
	<pre>For[i=1, i < 14, Print[i]; i++]</pre>
xi	Print i , starting from i = 1 for i < 14 increment by 2
	For[i=1, i < 14, Print[i]; i + = 2]
xii	Print i , starting from i = 15 for i $>$ 5 decrement by 1
	For[$i=15$, $i > 5$, Print[i]; i]
xiii	Print i , starting from i = 15 for i > 5 decrement by 0.5
	<pre>For[i=15, i > 5, Print[i]; i-=0.5]</pre>
xiv	Print n for n - 1 > 5 starting from n = 20 in decreasing order.
XV	Print n for n - 1 < 15 starting from n = 2 in increasing order.

Exercise:

(I) Make a data table as follows.

 $X = \{1, 2, 3, 4, \dots, 200\}$

Hence make the table A. Such that $A = \{1^2, 2^2, ..., 200^2\}$

(II) Make a data table as follows.

$$X = \{1, 2, 3, 4, 5, 6, \dots, 100\}$$

Hence make the table A. Such that $A = \left\{1, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}, \frac{1}{5^2}, \dots, \frac{1}{99^2}, \frac{1}{100^2}\right\}$

(III)



a) Find the values of \mathbf{r} in the above diagram for x = 0, 1, 2, ...

b) Also find the angle α as depicted in the diagram for each point.

List and Table Manipulation

Max[x]

Min[x]

MatrixForm[*a*]

Eigenvalues[a]

Inverse[*a*]

Det[*a*]

a+ba-b $a \cdot b$

- 01. Let $x = \{8, 10, 1, -3, 5, 7, -2, 0, -13, 15\}$
 - *a*. Find the maximum value of x.
 - *b*. Find the minimum value of x.

02. Let $t = \{-10, 5, 10, 15, 25, -25, 75, -50, 9, 12, 78, 19, 20\}$. Do the following list manipulations and try to understand their operation.

- a) First[*t*]
- b) Last[*t*]
- c) Reverse[*t*]
- d) Take[t, 3]
- e) Take[*t*, -2]
- f) Drop[*t*, 2]
- g) Drop[*t*, -2]
- h) Length[*t*]
- i) Dimensions[*t*]
- 03. Let $a = \{1, 2, 3, 4\}$, $b = \{5, 6, 7\}$ and $c = \{\{-3, 5\}, \{7, 9\}\}$. We can perform the following operations on lists.

a)	Join[a,b]	{This concatenates list a and b together}
b)	Position[b , 6]	{This gives the position of the number 6 in b }
c)	Reverse[<i>a</i>]	{This reverse the order of the elements in list a }
d)	Flatten[C]	{This flattens out the list C }

04. Let
$$a = \{\{10, 20\}, \{30, 40\}\}$$
 and $b = \{\{-10, 8\}, \{40, -84\}\}$

- a) Find the dimension of a and b.
- b) Represent a and b in matrix form.
- c) Find the inverse of a and b.
- d) Find the Eigen values of a and b.
- e) Find the determinant of a and b.
- f) Find a+b, a-b and $a \times b$

$$(* a/b \text{ is not defined } *)$$

05. a. We can solve the m x = b equation in following way

Solve [m . x = = b, x]

If $m = \{\{1, 2, 3\}, \{2, 0, -3\}, \{1, 1, -1\}\}, x = \{x_1, x_2, x_3\}$ and $b = \{1, 0, -1\}$ and solve the equation m x = b

Exercise:

Solve the following simultaneous equation set

$$x + 2y + 3z = 6$$
$$2x - 3y + z = 11$$
$$-x + y - 2z = -9$$

b. We can also solve the above equation using predetermined case

if $\mathbf{m} \cdot \mathbf{x} = \mathbf{b}$ $\mathbf{x} = [\mathbf{m}^{T} \cdot \mathbf{m}]^{-1} \cdot \mathbf{m}^{T} \cdot \mathbf{b}$ This gives the solution of m x = b for non square matrix.

(Here m^T is the transpose of m and $[\mathbf{m}^T.\mathbf{m}]^{-1}$ is the inverse of $\mathbf{m}^T.\mathbf{m}$ metrics.)

By using the predetermined case solve the following equation

 $m = \{\{2, -1, 6\}, \{5, 4, 3\}, \{9, 10, 7\}, \{11, 13, 16\}, \{5, 7, 9\} \}$ $x = \{x_1, x_2, x_3\} \text{ and } b = \{1, 0, -1, -10, -19\}$

Exercise:

Solve the following simultaneous equation set

```
x + 2y + 3z = 6

2x - 3y + z = 11

-x + y - 2z = -9

5x - 3y - 8z = -13

6x + 2y + 5z = 17
```

- 6. Consider two complex numbers $Z_1 = 2 + 3i$ and $Z_2 = 3 2i$.
 - a) Find $Z_1 + Z_2$
 - b) Find $Z_1 Z_2$
 - c) Find $Z_1 * Z_2$ and Z_1 / Z_2
 - d) Find the Conjugates of Z_1 and Z_2
 - e) Find the Absolute values of Z_1 and Z_2 {Hint: $Z = |Z_0| e^{i\theta}$ and $|Z_0| = Abs[Z]$ }
 - f) Find the Argument of Z_1 and Z_2 {Hint: $Z = |Z_0| e^{i\theta}$ and $\theta = Arg[Z]$ }

Practical Example (1D Case)



If the gravity anomaly is given by the following Equation, $g=a x^2 + b x + c$ Where g is the gravity (Measured by gravimeter), x is the distance and a, b and c are unknown constants.

[Accuracy of "g" is depend on the gravimeter used and the accuracy of "x" is depend on the meter ruler or GPS System used]

We have find these a, b and c constants.

Gravity is usually measured using the gravimeter (gravity-meter) and therefore the accuracy depends on the gravimeter. Specifically, expensive gravimeters have high

accuracy than the cheaper gravimeter. By measuring the gravity at different points, constants a, b & c can be accurately determined.

In order to determine the constants a, b & c, a student measured the gravity at five points. These readings are given in the table below.

x (m)	Gravity (ms ⁻²)
1	1.39
2	2.21
3	3.21
4	4.39
5	5.81

By selecting a suitable method and using the data given in the table, find the values of a, b & c accurately.

(Values obtained for a, b & c can be confirmed with the actual values of a = 0.1, b = 0.5 and c = 0.8).

By using the values calculated for *a*, *b* & *c*, determine the gravity at x = 2.5 m and x = 4.5 m.



If the gravity anomaly is given by the following Equation, $g(x, y) = a x^2 + b y^2 + c y + d$, Where g is the gravity (Measured by gravimeter), x and y is the distance and a, b, c and d are unknown constants.

[Accuracy of "g" is depend on the **gravimeter** used and the accuracy of "x" and "y" is depend on the **meter ruler** or **GPS System** used] We have find these a, b, c and d constants.

By measuring the gravity at different points, constants a, b, c & d can be accurately determined. In order to determine the constants a, b, c & d, a student measured the gravity at eight points. These readings are given in the table below.

x (m)	y (m)	Gravity (ms ⁻²)
0	0	0.95
0	1	1.19
1	0	1.09
1	1	1.21
2	0	1.39
0	2	1.81
2	1	1.40
1	2	1.72

By selecting a suitable method and using the data given in the table, find the values of a, b, c & d accurately. (Values obtained for a, b, c & d can be confirmed with the actual values of a = 0.1, b = 0.2, c = -0.1 and d = 1.0). By using the values calculated for a, b, c & d determine the gravity at x = 1.5 m, y = 1.25 m and x = 0.25 m, y = 1.75 m.

Mathematical Modeling

7. Variation of Petrol Prices with Time is represented in the following table.

Time (Year)	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Petrol Price (Rs:)	48	50	55	55	59	69	82	91	102	110	115	120

The First row in the above table represent Time (in years) and the second row represent Average Prices of Petrol (in Rupees).

(a) Enter the above data into a table form as Petrol Price vs Time.

(b)

```
y = {1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007}
Or
y = Table[i, {i, 1998, 2007}]
p = {48, 50, 55, 55, 59, 69, 82, 91, 102, 110}
yp = Transpose[{y, p}]
TableForm[yp]
Plot the graph of Petrol Prices vs Time.
```

g1 = ListPlot[yp]
g1 = ListPlot[yp, PlotStyle -> PointSize[0.02]]
g1 = ListPlot[yp, PlotStyle -> {PointSize[0.02], Hue[0.7]}]

(c) Find the suitable equation passing through the points of the above graph.

Method I:

 $\begin{aligned} & \textbf{fyp} = \textbf{Fit[yp, {t, 1}, t]} - \textit{This is for a Linear (Simple) Function} \\ & \textbf{fyp} = \textbf{Fit[yp, {t^2, t, 1}, t]} - \textit{This is for a Quadratic Function} \\ & \textbf{fyp} = \textbf{Fit[yp, {t^3, t^2, t, 1}, t]} - \textit{This is for a Cubic Function} \\ & \dots \\ & \textbf{fyp} = \textbf{Fit[yp, {t^{100, t^{99, t^{98, \dots t^{2}, t, 1}, t]} - \textit{This is for a 100^{th}} \\ & \textit{order Polynomial Function.} \\ & \textbf{If you want to plot the corresponding function then,} \\ & \textbf{g2} = \textbf{Plot[fyp, {t, 1998, 2007}, \textbf{PlotStyle -> Hue[0.9]]} \\ & \textbf{Or} \\ & \textbf{g2} = \textbf{Plot[fyp, {t, y[[1]], y[[10]]}, \textbf{PlotStyle -> Hue[0.9]]} \end{aligned}$

Method II (Out of our control):

fyp = InterpolatingPolynomial[yp, t] – *This is for finding Interpolating function in Mathematica (we can not control this function like in Method I)*

Additional Method: (Out of our control):

fyp = Interpolation[yp] - This is for Interpolation and the function is not displayed in Mathematica. If you want to find some value using the above Interpolation then, fyp[2001] fyp[2001.5]

fyp[2010] – If the value in the argument is not in the defined data range (not included in the domain) Mathematica will fit an extrapolation function to get the corresponding function value.

(d) Plot the above graph in part (a) and the equation obtained in part (c) in the same graph by using different two colours.

Show[g1, g2]

(e) Find the Average Price of Petrol in 2008, 2009 and 2050 using the equation in part (c).

fyp /. t -> {2008, 2009, 2050}

(f) Find the corresponding year when Average Price of Petrol is Rs: 500.00 using the equation in part (c).

Solve[fyp == 500, t]

Exercise 01

There is a future threat of melting the glaciers in the polar regions of the Earth as a consequence of global warming. Ecologists have predicted that before the end of year 2050, the glaciers in the Polar Regions will melt entirely. The following table depicts the variation of the temperature and the prevailed carbon dioxide percentage of the environment with time for the last 27 years in the Arctic region of the earth. Here given values are the annual averages of daily records.

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991
Average									
Temperature (°C)	-40.0	-38.1	-38.0	-37.0	-36.4	-35.7	-35.2	-34.3	-33.9
Environment									
CO ₂	0.5	1.0	1.9	2.0	3.0	3.9	4.2	5.0	5.2
percentage									

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Average									
Temperature	-33.0	-32.2	-31.0	-29.7	-30.5	-30.1	-28.6	-28.0	-27.8
(°C)									
Environment									
CO ₂	6.0	7.1	7.9	8.9	9.1	10.1	11.3	11.1	12.0
percentage									

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Average Temperature (°C)	-27.0	-26.1	-25.6	-24.9	-24.0	-23.6	-22.9	-22.0	-21.5
Environment CO ₂ percentage	12.9	13.1	14.1	15.2	15.3	16.5	16.9	18.0	18.9

- (1) Make 2D arrays for the average temperature and the corresponding year and CO_2 percentage of the environment and the corresponding year using above data table.
- (2) Plot the graph of average temperature vs time (year).
- (3) Plot the graph of CO_2 percentage vs time (year).
- (4) Plot the above graphs on a same page using two different colours.
- (5) Find a reasonable linear equation (y = mx + c) considering the variation of the average temperature with time.
- (6) Find a reasonable linear equation (y = mx + c) considering the variation of the CO₂ percentage with time.
- (7) Find a suitable cubic equation ($y = ax^3 + bx^2 + cx + d$) considering the variation of the average temperature with time.
 - P.T.O
- (8) Find a suitable cubic equation ($y = ax^3 + bx^2 + cx + d$) considering the variation of the CO₂ percentage with time.
- (9) Using the equations in part (5) and (7) determine the year in which the melting process of ice begins in the Arctic region in each situation.(melting process of ice begins when the temperature of the environment is 0°C)
- (10) Find the amount of increase in CO_2 percentage by using part (6) and (8) in each year the melting process begins.

Exercise 02

Variation of Average Petrol Prices (92-octane) per year in Sri Lanka with Time is represented in the following table.

Time (Year)	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Petrol Price (Rs)	48	50	55	55	59	69	82	91	102	110
Time (Year)	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Petrol Price (Rs)	115	120	125	133	145	159	150	177	117	117
Time (Year)	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
Petrol Price (Rs)	142	133	137	157	303	355	355	309	_	_
								(?)		

The First row in the above table represent Time (in years) and the second row represent Average Prices of 92-octane Petrol (in Rupees).

(i) Draw the graph of Petrol Price vs. time (in years).

- (ii) Determine a **suitable model** which fits the above data set.
- (iii) Find the average Petrol Price in 2026, 2027, 2028, 2029 and 2030 using your model.
- By using the above model, estimate the average Petrol Price in year 2050.
 Is it reasonable to estimate the average Petrol Price in 2050 ? Explain your answer.

Exercise 03 (Home Work)

The variation of the monthly average temperature at the University premises for the last 13 months is given in the following table.

Г	Month	Temperature (°C)
1-	May 2019	33
2 –	June 2019	30
3 –	July 2019	31
4 –	Aug 2019	32
5 –	Sept 2019	33
6 –	Oct 2019	33
7 –	Nov 2019	32
8 –	Dec 2019	32
9 –	Jan 2020	32
10 -	Feb 2020	31
11 –	Mar 2020	32
12 –	Apr 2020	33
13 –	May 2020	33

- 1) Draw the graph of temperature vs. time (in months).
- 2) Determine a suitable model which fits the above data set.
- 3) Find the average temperature in the months, June 2020 and July 2020.
- 4) By using the above model, is it reasonable to estimate the average temperature for the month January 2025? Explain your answer.

Exercise 04 (Home Work)

Following table represents the Production and the corresponding Income for the past twelve years of a factory started in year 1995. Here the production and the income are given by the sum of the daily production and daily income. Here Income is given in billion rupees.

Time(Year)	1995	1996	1997	1998	1999	2000	2001	2002
Income	15.0	15.1	17.8	-	20.7	21.1	23.0	23.0
Production	746	800	819	-	890	926	1005	1022
	-			-				
Time(Year)	2003	2004	2005	2006	2007	2008	2009	2050
Income	23.4	26.8	28.9	29.0	33.1	29.5	?	?
Production	1081	1087	1147	1185	1240	1230	?	?

- **1.** Enter the above data for the time period 1995 to 2008 with the exception of data corresponding to year 1998 in 2D arrays.
 - i.e. Time (Year) and the corresponding income

Time (Year) and the corresponding production

If 12.5% of the income can be considered as the net profit, calculate the net profit for each year.

- **2.** (A) Plot the graph of Income vs. Time (Year)
 - (B) Plot the graph of Production vs. Time (Year)
 - (C) Plot the above two graphs in a same page using two colours.
- **3.** Find appropriate Linear (y = m t + c), Quadratic ($y = a t^2 + b t + c$) and Cubic ($y = a t^3 + b t^2 + c t + d$) equations considering the variation of the income with the time. If the income in year 1998 is 19.2 billion rupees, determine the suitable equation out of the above three equations. Hence find the expected income for the years 2009 and 2050.
- 4. Find a reasonable Linear equation (y = m t + c) considering the variation of the production with the time. Goods produced in year 1998 were 870. Find the Production in 1998 using above linear equation. Then calculate the percentage of the error of your answer.
- 5. Plot the graph of Production vs. Income. Assuming that a linear equation is suitable to represent the above data. Calculate the production when the income is 40 billion rupees. In which year the income is equal to 40 billion rupees?

Reading and Writing Files

Mathematica has in-built functions for file handling. i.e.; reading and writing files.

(1.) Reading a file

Consider the following text file at "S:\\t02\\testdata.txt" which contains three data columns.

📕 tes	tdata - No		
File E	dit Format	View Help	
79.69 79.70 79.71 79.72 79.73 79.74 79.75 79.76 79.77 79.78	7.33 7.30 7.26 7.19 7.16 7.12 7.05 6.98 6.91 6.84	10 12 15 28 30 35 38 42	
<			× •

We can read the above file using the following command.

a = ReadList["S:\\t02\\testdata.txt", Number, RecordLists -> True]

This reads the "testdata.txt" file of the **Number** format. Here the option setting "**RecordLists -> True**" makes ReadList create separate sub lists for objects that appear in separate records.

(2.) Writing a file



f = OpenWrite["S:\\phy_06_850 \\writedata.txt"]

- This opens a file "writedata.txt" to write data into it.

Write [f, data1, data2,..] - This writes a sequence of data to the specified file "f".

Close[f] - This indicates the end of the writing.

Eg:- Write the above read list "a" to a file writedata.txt as

f = OpenWrite["S:\\AS_08_850 \\writedata.txt"] Table[Write[f, {a[[i, 1]], a[[i, 2]], a[[i, 3]] }], {i, 1, Length[a]}] Close[f]

Or f = OpenWrite["S:\\AS_08_850 \\writedata.txt"]

Do[Write[f, {a[[i , 1]], a[[i , 2]], a[[i , 3]] }], {i, 1 ,Length[a]}] Close[f]

Reading Graphic and Sound files

Mathematica allows us to export and import graphics and sound files also. Built-in command **Import** enables us to read graphics and sound files while using the command **Export** we can write graphics and sound files in to a given location.

Reading a Graphic file

a = Import ["S:\\ Graphics\\ picture. jpg"] This imports an image stored in **jpg** format Show [a] This shows the imported graphics

Reading a Sound file

b = Import ["Z:\\ Sound\\ sound1. wav"] This imports an sound stored in **wav** format Show [b] This is for playing the wave file

Exporting a Graphic file

Export ["Z:\\ mypicture.jpg", a]	This exports the imported graphic "a" in to the given location with the ing format.
Export ["Z:\\ mypicture.bmp", a]	This is for convert jpg format to bmp . (* Check the size of the files *)
Exporting a Sound file	
Export ["Z:\\ mysound.wav", b]	This exports the imported sound file "b" in to the given location with the wav format.
Export ["Z:\\ mysound.mp3", b]	It can not convert way to mp3.

Writing and Reading Data files using Export and Import commands

In addition to conventional method for writing data files using "*Write*" command, **Mathematica** allows us to write data files using "Export" command. Here we use special data format types namely **tsv** and **csv**. Any file with "tsv" format within the Export command writes data separated with space into a text file and any file with "csv" format writes data into an Excel sheet.

d = Table [$\{i, i^2, i^3\}, \{i, 1, 2, 0.01\}$	}] This is our data table to be written
d2 = Table [$\{i, i^2, i^3\}, \{i, 1, 200\}$]	This is our data table to be written
EXPORT	
x = Export ["S:\\my\\write.tsv", d]	This writes above data file into a Text file
y = Export ["S:\\my\\write.csv", d]	This writes above data file into a Excel file
IMPORT	
Import ["S:\\my\\write.tsv"]	This reads the file within the inverted commas
Import ["S:\\my\\write.csv"]	This reads the file within the inverted commas

Solving Differential Equations using Mathematica

We can solve ordinary differential equations using **Mathematica**. It allows us to solve first order, second order differential equations.

DSolve [differential_equation, y[x], x]	This solves a differential equation for the
	function y with independent variable x.

Examples:-

Solve the following first order differential equation.

• $\frac{dy(x)}{dx} = k \ y(x)$; where k is a constant

DSolve[y'[x] == k * y[x], y[x], x] {Mathematica form for solving above equation}

• $\frac{dy(x)}{dx} = -k \ y(x)$; where k is a constant

Solve the following second order differential equation.

•
$$\frac{d^2 y(x)}{dx^2} = -\omega^2 y(x)$$
; where ω is a constant
 $DSolve[y''[x] = -\omega^2 x y[x], y[x], x]$ {Mathematica form for solving above equation}
• $\frac{d^2 y(x)}{dx^2} = \omega^2 y(x)$ = t where ω is a constant

• $\frac{a^2 y(x)}{dx^2} = \omega^2 y(x)$; where ω is a constant

We can also solve differential equations having boundary conditions.

Solve the following second order differential equation with boundary conditions y(0) = 1and y'(0) = 0.

$$\frac{d^2 y(x)}{dx^2} = a \frac{dy(x)}{dx} + y(x)$$

boundary condition
DSolve[{y''[x] == a * y'[x] + y[x], y[0] == 1, y'[0] == 0}, y[x], x]
equation

Using above methods we can solve some famous problems in Physics. Some of them are as follows,

- (1) Wave Equation
- (2) Heat Conduction Equation



1-D Wave Equation







Boundary conditions are, z[x, 0,t] = 0 $z[x, L_2,t] = 0$ z[0, y,t] = 0 $z[L_1, y,t] = 0$

Initial conditions are,

z[x, y, 0] = f(x, y) and z[x, y, 0] = 0

<u>1-D Heat Conduction Equation</u>



Boundary conditions are, $\theta[0, t] = 0$ $\theta[L, t] = 0$

And the initial condition is, $\theta[x, 0] = f(x)$

2-D Heat Conduction Equation

$$\frac{\partial \theta}{\partial t} = k \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)$$

Laplace Equation

$$\nabla^2 u = 0$$

Here $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$ and *u* may be gravity potential, magnetic potential or electrostatic potential which satisfies the Laplace equation.

Tute 03

- Q1. Lengths of the two sides of a rectangular triangle are 5cm & 8 cm. Find the length of the diagonal & the corresponding angles.
- Q2. Do the followings.
 - i Write an expression for the sum of the square values

 $\sum^{n} i^2$

of 1 to n. i:e;

- ii Using the above, find the value for the summation of square value of 1 to 10 and 1 to 100.
- iii Find the values of these expressions,
 - (a) $\sum_{i=1}^{10} i$ (b) $\sum_{i=1}^{100} i$ (c) $\sum_{i=1}^{1000} i$ (d) $\sum_{i=1}^{10000} i$ iv Find the summation, $\sum_{i=1}^{3}\sum_{j=1}^{i}x^{i}y^{j}$
- Q3. Find the solutions or numerical approximations of the following polynomial equations. (Solutions for 25 digit precision)

 - i $x^{5} + 7x + 1 = 0$ ii $x^{3} + x + 1 = 0$ iii $x^{5} 6x^{3} + 8x + 1 = 0$

 - $\sqrt{x} + \sqrt{x+1} = a$; where a' is a constant. iv
- Q4. Find solutions or numerical solutions of the following polynomial equations
 - i x + y = 2, x - 3y + z = 3, x - y + z = 0
 - ii a x + y = 0, x + (1 a) y = 0, where 'a' is a constant
 - iii $x^2 + y^2 = 1$, x + y = a, where 'a' is a constant

Q5. Draw the following graphics.

- (i) g1 = Show [Graphics [Circle[{0, 0}, 1] , AspectRatio ->Automatic , Axes -> Automatic]]
- (ii) g2 = Show [Graphics [Circle[{0, 0}, {2, 1}] , AspectRatio >Automatic , Axes -> Automatic]]
- (iii) g3 = Show [Graphics [Circle[{0, 0}, 2, {2, 3}] , AspectRatio >Automatic , Axes -> Automatic]]
- (iv) Show[g1, g2, g3]

Draw a circle centered whose center is (0,0) and radius is 5 cm. Find whether the following points are inside the circle or outside the circle. Points are :- (1, 3), (5, 6) and (10, 4)

Q6. (A) Following are the marks obtained by 10 students in their A/L examination, Find their z-score values.

Subject_1 = 67,78,69,80,56,88,75,66,72,78
Subject_2 = 83,79,75,88,86,92,80,78,82,87
Subject_3 = 56,67,60,70,74,68,60,77,63,65

Hints:



(B) Using above hints, find the z-scores for the Advanced Level Results given in the following location, "S\\data\\ALMarks.txt".

100001	7	42	33
100002	9	47	16
100003	17	35	24
100600	67	94	96

First column of this file displays the Index Numbers while other columns display marks of Physics, Chemistry and Combined Mathematics respectively.

(C) Find the z-scores for the Advanced Level Results given in the following location, "S\\data\\ALMarks2.txt".



Q8.



Two charges are apart as shown above. Find the electrical force between the charges. Given $q_1 = 5C$, $q_2 = 8C$, $x_1 = 20m$, $x_2 = 45m$, $y_1 = 10m$, $y_2 = 12m$ and $\frac{1}{4\pi\varepsilon_o} = 9 \times 10^{-9} Nm^2 C^{-2}$.



(500,0)





Q10. Consider a homogeneous cubic mass distributation having its centre at a depth of 100 m directly below the AB line. The line AB lies on the surface of the Earth.



We are trying to calculate the gravity anomaly caused by a body which takes the shape of a cube. In doing this calculation, we divided the body into a large number of small cubical masses and calculate gravity anomaly due to each and add them together.

In view of the complexity of this method, if we illustrate it by calculating gravity anomaly using a crude method first and then refine it in several steps.

Step I:

Consider a cubic volume of $50 \times 50 \times 50 \text{ m}^3$ whose centre is situated 100 m directly below the line AB on the surface of the Earth.

Step II:

Calculate the gravity anomaly due to the mass distributation at equal intervals of 1.0 m at 200 points on the line AB.

[Density of the cube is 3300 kgm⁻³ and Universal Gravity Constant, $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$]



Step III:

Now divide the mass into four equal parts by a vertical plane and by a horizontal plane.



Step IV:

Calculate gravity anomaly due to each cube and add them together.

Step V:

Extend the above method by dividing cross section of the cube into $\mathbf{n} \times \mathbf{n}$ equal parts.

Step VI:

Calculate the gravity due to this body at fields points situated at the nodes of the rectangular grid of 200 m x 200 m.



Step VII:

Divided the body into **n** x **n** x **n** small and equal cubes. And calculate gravity anomaly due to each cube at the nodes of the above grid and hence, determine the gravity anomaly due to the large cube on the same grid.

Step VIII:

Calculate the vertical component of gravity anomaly due to the mass distributation.





25 V electrical power supply whose internal resistance is 8 ohms is connected to an external resistor R. When its value equal to the values given in the following table,

 Find, Current (I) flowing through the resistor R, Voltage (V) across the resistor R, Electrical power (P) of resistor R.

r			
R (ohms)	I (A)	V (Volts)	P (W)
2			
4			
6			
8			
10			
12			
15			
20			
25			
30			
35			
40			
45			
50			

2. Complete the following table using Mathematica.

3. Draw the graphs of I vs R, V vs R and P vs R.

And draw the above graphs in the same page using three colours.

4. Find the suitable polynomial equations for the graphs I vs R, V vs R and P vs R.

Hint :
$$I = \frac{25}{(8+R)}$$
, $V = IR$ and $P = VI$.

Q12. A human skull excavated from an archeological site, when brought near a counter **R** records, 16 counts per minute. Whereas, the skull of human died recently records 21 counts per minute on the same counter **R**. (In both instances the counts on **R** are due to the radioisotope ${}^{14}_{6}C$ in the human skulls). However, when **R** is exposed to the environment away from any specimen, it records 06 counts per minute.

- (i). Estimate the age of the human skull excavated from the archaeological site.
- (ii). Find the corresponding time in years when the skull of human records 06 counts per minute on the same counter \mathbf{R} ?

{Hints: $C(t) = C_o e^{-\lambda t}$, $T_{\frac{1}{2}} = \frac{\ln[2]}{\lambda}$ and the half-life of ${}_{6}^{14}C$ isotope is 5730 years }

Input Box

• Find the volume of the rectangular body of sides *a*, *b* and *c*.

```
a=5;
b=10;
c=12;
vol=a*b*c;
Print["Volume of the Body : ",vol, " m^3"]
```

OR

(* Input Part *) a = Input["Enter the length of the body (in meters) : "] b = Input["Enter the width of the body (in meters) : "] c = Input["Enter the height of the body (in meters) : "] (* Calculation Part *) vol = a*b*c; (* Output Part *) Print["Volume of the Body : ",vol, " m^3"]

Manipulation

The single function **Manipulate** gives immediate access to a huge range of powerful interactive capabilities. For any expression with symbolic parameters, **Manipulate** function automatically creates an interface for manipulating the parameters. **Manipulate** function supports not only mouse and keyboard manipulation, but also game pads and other devices.

1) Manipulate frequency

Manipulate [Plot[Sin[freq * x], {x, -2Pi, 2Pi}], {freq, 1,5}]



2) Manipulation amplitude

 $Manipulate [Plot[amp * Sin[freq * x], \{x, -2Pi, 2Pi\}], \{freq, 1, 5\}, \{amp, 1, 5\}]$



3) Trigger (Many graphs can be plotted using a single command, "trig")

 $\label{eq:main_string} Manipulate [Plot[amp * trig[freq * x], \{x, -2Pi, 2Pi\}], \{freq, 1, 5\}, \{amp, 1, 5\}, \{trig, \{Sin, Cos, Tan\}\}]$



4) Manipulate a parameter in discrete steps

```
Manipulate[Factor[x^n - 1], {n, 10, 100, 1}]
```



THE NEW FEATURES IN Mathematica 8 ARE GIVEN BELOW:

1) Integration with Wolfram|Alpha

Mathematica 8 provides free-form linguistic input, allowing users to enter plain English and get immediate results and the *Mathematica* input for further exploration (without the need for syntax).

A notebook in Mathematica should be opened, *a box with a plus sign* will appear at the top left side corner. Then click over the box and chose the **"Wolfram Alpha query"** option and an equal sign inside a star will appear. Next you can type the necessary tasks in English which you have to do. eg. 1) plot sinx (plots a graph of sinx)



eg. 2) weather at panadura (weather of panadura, weather in panadura are also accepted)

This gives the present situation and the future forecast of weather at Panadura

ln[2]:=	Weather at pana + Latest recorded	dura I weather for Panadura, Sri Lan
	temperature	29 °C (heat index: 33 °C)
	conditions	few clouds
Out[2]=	relative humidity	70% (dew point: 23 °C)
	wind speed	3 m/s

eg. 3) calories toast bread + omelette

This gives the Chart of Calories, Cholesterol, Protein,... etc. obtained by eating a toast bread and omelette

summig ar	ly type of brea	su l'oscore	au, egg of preau, rye	or m		cou		
t interpret	ation:							
1	amount	50 grams	and and and a low		to and	- automates	50	asset estados
oread	type	toasted	total calories	Į d	bread	amount	50 grams	total calories
	amount	1 egg						
egg	type	omelet	total calories					

In[12]:= 🗯 calories of toast bread + omelette

The location of Piduruthalagala mountain will be displayed in a Sri Lankan map

ut interpretation:	
Piduruthalagala location	
sult:	Show coordinates

eg. 5) model ((1,3),(2,4),(3,5),(4,6),(5,7))

The suitable equation (fitted polynomial) which passes through the points (1, 3), (2, 4), (3, 5), (4, 6) and (5, 7) will be displayed.



eg. 6) solve $x^2 - 2x - 6 = 0$

This gives the roots of this equation with the root plot where the curve

intersects



eg. 7) integrate $x^3/3 + \sin x$

This gives the answer of $\int \left(\frac{x^3}{3} + \sin x\right) dx$ integration. If we need the steps how the final answer was obtained, click the "show steps" hyperlink command which is given in the output box.



The intermediate steps of the integration of x^4 :





2) Computable Document Format (CDF)

By using a CDF, can download the entire source code for our required applications. It provides the source code for that application in which we can make any changes in the variables according to our requirements.



3) Wolfram CDF Player

It interacts with applications from the Wolfram Demonstrations Project and views Mathematica examples, reports and files even whenever Mathematica software is not installed in that computer.

<u>Tute 04</u>

01. Consider a simple pendulum which oscillates with very small angles.



We can write the periodic time of the pendulum as,

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Where l = length of the string, H = Height of the room and g = gravitational acceleration

The file "S:\\T4Files\\simplem.txt" contains the values of x and T^2 .

- a) Read the above file.
- b) Draw the graphs of T^2 vs x.
- c) Find the gravitational acceleration using part (b).

02. Consider the function f(x) = x, where $x = \{-10, -9, -8, ..., 8, 9, 10\}$.

- a) Find the discrete Fourier transform of f(x).
- b) Find the real part of the f(x) and plot it.
- c) What are the characteristics of the above graph?
- d) Find the imaginary part of the f(x) and plot it.
- e) Find the inverse Fourier transform of the result of part (a).
- f) What are the characteristics of the above graph?
- g) Find the [(real part)² + (imaginary part)²]^($\frac{1}{2}$) of the f(x) and plot it.
- h) Find the inverse Fourier transform of the part (a)
- i) compare that Inverse Fourier Transform (in part h) with Original Data Set (f(x)).

03. Consider the following data sets

- (i) $\{0., 0.58, 0.95, 0.95, 0.58, 0., -0.58, -0.95, -0.95, -0.58\}$
- (ii) $\{1.,0.80,0.30,-0.30,-0.80,-1.,-0.80,-0.30,0.30,0.80\}$
- (iii) $\{0., 1.53, 0.36, 0.36, 1.53, 0., -1.53, -0.36, -0.36, -1.53\}$
- (iv) $\{0., 1.53, 1.53, 0.36, -0.36, 0.36, -0.36, -1.53, -1.53\}$
- (v) $\{2.,1.11,-0.5,-1.11,-0.5,0.,-0.5,-1.11,-0.5,1.11\}$
- (vi) {1.,1.39,1.26,0.64,-0.22,-1.,-1.39,-1.26,-0.64,0.22} (vii) {1.,-0.41,1.95,-0.04,1.58,-1.,0.41,-1.95,0.04,-1.58}
- (vii) {1.,-0.41,1.95,-0.04,1.58,-1.,0.41,-1.95,0.04,-1.58} (viii) {0.,-0.36,1.53,1.53,-0.36,0.,0.36,-1.53,-1.53,0.36}

Plot the above data sets as Lists. And do the following for all data sets.

- (a) Find the discrete Fourier transform of f(x).
- (b) Find the real part of the f(x) and plot it.
- (c) Find the imaginary part of the f(x) and plot it.

(d) Find the inverse Fourier transform of the result of part (a). And compare that Inverse Fourier Transform with Original Data Set in Part (i) to Part (viii).

Animations

04. A string with length L is tied at x = 0 and x = L. At t = 0, the string is slowly released giving initial shape f(x). Animate the wave function in the time interval t=0 to t =100 s.





The solutions of the above 1-D wave equation can be written in the following form

$$y(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi vt}{L}\right) \text{ where}$$
$$a_n = \left(\frac{2}{L}\right) \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

Hints:

You are given the following values, L=1 m, $E = 130 \times 10^8$ Nm⁻², $\rho = 8960$ kgm⁻³ for a Copper string and consider *n* goes from 1 to10. And also f(x, 0) is given below:





The 1-D wave equation is given in the following form,

 $y = A\sin(kx + \omega t)$, where A = Amplitude of the wave

 $k = \frac{2\pi}{\lambda}$ (This is called the wave number and λ is the wave length)

 $\omega = 2\pi f$ (Angular frequency of the wave and f is the frequency of the wave)

If A = 5, f = 50 Hz and k = 10 m⁻¹, find the displacement y after time t = 100 s and animate the wave function in the time interval t = 0 to t = 100 s.

06. Animate the following function in the time interval t = 0s to t = 50 s using appropriate ranges for x and y.

$$f(x, y, t) = \sin(x \times y + t),$$

where $x \in [0,3]$, $y \in [0,3]$ and time $t \in [0,50]$ with time intervals 0.1 s.

07. Animate the following function in the time interval t = 0 to t = 1 s using below ranges for x and y.

$$f(x, y, t) = \sin(x \times y + t),$$

where $x \in [0,3]$, $y \in [0,3]$ and time $t \in [0,1]$ with time intervals 0.001s.

2nd YEAR – PHYSICS PRACTICAL GUESS PAPER Computational Physics Practical

Time : One and half hours

Mathematica Software must be used for following all activities

Variation of Bus Fares form Panadura to Nugegoda with Prices of Petrol, Prices of Diesel & Time are represented in document of given location, "S\\data\\oilprice.txt". Sketch of this document is given in a following form.

1987	14.00	9.00	8.00
1988	17.00	9.00	8.00
1989	19.00	9.00	8.00
2006	102.00	70.00	21.00
2007	110.00	80.00	22.00
2008	115.00	90.00	25.00
2009	120.00	100.00	29.00

First column of this document is displayed time & other 2nd, 3rd, 4th columns are displayed Average Prices of petrol, diesel & bus fares are respectively.

- 1. Read the document, as a data table in "S\\data\\oilprice.txt".
- 2. (a) Draw the graph of Petrol Prices vs Time (years).
 - (b) Find the suitable Cubic Equation ($p = a t^3 + b t^2 + c t + d$) passing through the points of above graph.
 - (c) Plot the above graph & obtained cubic equation in a same graph paper by using various two colours.
 - (d) Find the Average Price of Petrol in 2008 & 2009 using your cubic equation.
 - (e) Give your comments for Price of Petrol in part D.
- (a) Plot the graph of Diesel Prices & Bus Fares by using the above document.
 - (b) Decide the suitable & reasonable polynomial $(y = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n)$ passing through the data points of above graph. And also, find the polynomial.
 - (c) Find the Bus Fares when Diesel Prices are Rs. 100.00, 120.00, 140.00, 160.00, 180.00 & 200.00 by using above polynomial.

- 4. (a) Plot the graph of Petrol Prices vs. Bus fares by using read document.
 - (b) Can you conclude that Prices of Petrol are increasing when Diesel Prices are increasing by studying above graph? Explain your answer.
- (a) Draw the graphs of Bus Fares with Prices of Diesel and Prices of Petrol in the same page by using various two colours.
 - (b) We know the Bus Fares are not increasing although Prices of Petrol are increasing & Prices of Diesel are not. By studying above graphs, can you conclude the Bus Fares are not increasing when Prices of Petrol are increasing? Explain your answer.

2nd YEAR - PHYSICS PRACTICAL EXAMINATION Computational Physics Practical Paper

Time : one & half hours

The Use of Mathematica software for all the following tasks is requied

The following table display the sales of five (05) brands of biscuits sold by the Ceylon Biscuits (Pvt) Limited in the past five years. The figures are listed in the table in millions.

Year Biscuit (Brand)	2002	2003	2004	2005	2006	2007
Mari	14.0	15.0	16.5	?	16.2	16.7
Nice	13.0	14.1	15.2	?	15.0	14.9
Lemmon- Puff	20.0	21.0	23.0	?	23.5	25.0
Chocolate	15.0	14.0	13.0	?	11.0	11.5
Cream Cracker	25.0	28.0	30.0	?	39.0	42.0

Table - 01

By using the amount of sales for years 2002, 2003, 2004, 2006 and 2007, determine the sales of biscuits for 2005 by using the following approaches,

- (a) A cubic polynomial of the form, $y = ax^3 + bx^2 + cx + d$.
- (b) A quadratic equation of the form, $y = ax^2 + bx + c$.
- (c) A linear equation of the form, y = mx + c.

If the actual biscuit sales for 2005 is shown in the following table (Table-02) using that data, determine which approach mentioned above {ie: part (a), (b) and (c)} is more suitable to represent the sales for each brand of biscuits.

Furthermore, determine the amount of biscuits expected to be sold in 2008 for Mari, Chocolate and Cream-Cracker.

Mari	16.0
Nice	14.9
Lemmon-Puff	23.0
Chocolate	12.5
Cream Cracker	35.0



Are the expected amounts of biscuits sold for 2008 absolutely correct or incorrect? Write down your views.

2nd YEAR - PHYSICS PRACTICAL EXAMINATION Computational Physics Practical Paper

Time : one & half hours

The Use of Mathematica software for all the following tasks is required

The following table shows the monthly averaged share price (book value of each share is Rs:10.00) of two companies, "SEYLAN MERCHANT LEASING LIMITED" and "SRI LANKA TELECOM LIMITED" for the past two years.

		SEYLAN	TELECOM
Month by Number	Month	LIMITED	LIMITED
01	2006 January	50.00	19.75
02	2006 February	50.00	19.50
03	2006 March	52.75	19.25
04	2006 April	51.75	19.50
05	2006 May	32.75	18.50
06	2006 June	31.75	18.75
07	2006 July	27.50	19.50
08	2006 August	26.75	20.00
09	2006 September	??.??	??.??
10	2006 October	28.00	24.25
11	2006 November	23.50	29.75
12	2006 December	16.75	29.75
13	2007 January	16.50	33.75
14	2007 February	16.50	43.25
15	2007 March	14.50	42.75
16	2007 April	13.25	39.25
17	2007 May	12.50	38.75
18	2007 June	13.25	39.25
19	2007 July	16.00	36.75
20	2007 August	16.50	36.00
21	2007 September	17.50	36.25
22	2007 October	16.50	35.00
23	2007 November	16.00	35.50
24	2007 December	17.00	33.00

Table - 01

In the above table the averaged share price in rupees for 23 months is displayed excluding the share price of the 09^{th} month.

1. Input the share price of "SEYLAN MERCHANT LEASING LIMITED" company for the 23 months displayed on the table into a 2-D data array.

Plot the corresponding data array.

2. Input the share price of "SRI LANKA TELECOM LIMITED" company for the 23 months displayed on the table into a 2-D data array.

Plot the corresponding data array.

3. Determine the linear equation that correctly represents the data points relevant to "SEYLAN MERCHANT LEASING LIMITED" company. Hence, estimate the share price for the 09th month of "SEYLAN MERCHANT LEASING LIMITED" company.

Plot the linear equation and the data points in part-1 on the same graph.

4. Determine the linear equation that correctly represents the data points relevant to "SRI LANKA TELECOM LIMITED" company.

Hence, estimate the share price for the 09th month of "SRI LANKA TELECOM LIMITED" company.

Plot the linear equation and the data points in part-2 on the same graph.

5. Determine the suitable polynomial that correctly represents the data points relevant to "SEYLAN MERCHANT LEASING LIMITED" company.

Hence, estimate the share price for the 09th month of "SEYLAN MERCHANT LEASING LIMITED" company.

Plot the suitable polynomial on the graph that includes the linear equation and the data points in part-3. Use three (03) different colours to distinguish the curves.

6. Determine the suitable polynomial that correctly represents the data points relevant to "SRI LANKA TELECOM LIMITED" company.

Hence, estimate the share price for the 09th month of "SRI LANKA TELECOM LIMITED" company.

Plot the suitable polynomial on the graph that includes the linear equation and the data points in part-4. Use three (03) different colours to distinguish the curves.

7. The following table shows the actual average share price for the 09th month of the two companies "SEYLAN MERCHANT LEASING LIMITED" and "SRI LANKA TELECOM LIMITED".

SEYLAN LIMITED	28.50
TELECOM LIMITED	21.00

Table	- 02
-------	------

By comparing the data in Table-02 with the values obtained from the linear equation and suitable polynomial equation deduct which is the more suitable representation for the two companies separately.

- 8. Hence estimate the average share price for the 25th month (ie; January, 2008), separately for each company.
- 9. By using the above method is it reasonable to estimate the average value of the share price for the 48th month (ie; December, 2009)? Explain your answer.

Total Marks	For Lecture's Use Only		
	Part I		
	Part II		
	Part III		
	Part IV		
	Part V		
	Total		
	Total		

2nd year - Physics Practical [**Computational Physics**] Assignment No. 03 (Time : 30 min)

Name : Index No :

Mathematica software should be used for the following exercises

The variation of the average temperature in °C of the atmosphere of the Earth with respect to the height in km from the surface of the Earth is given in the following table.

Height (km)	Temperature
	(°C)
0	29.5
1	28.0
2	24.1
3	20.9
4	17.7
5	17.8
6	16.1
7	13.3
8	11.5
9	10.6
10	9.29
11	7.17
12	7.10
13	5.34
14	6.73
15	4.33

- 1) Draw the graph of temperature vs. height.
- 2) Determine a suitable model which fits the above data set.
- 3) Find the average temperature at the heights 16 km and 17 km from the surface of the Earth.
- 4) By using the above model, is it reasonable to estimate the average temperature at 70 km? Explain your answer.

Total Marks	For Lecture's Use Only		
	Program Structure		
	Comments		
	Marks of Q1 – Q10		

2nd year - Physics Practical [Computational Physics] Assignment No. 01 (Time : 30 min)

Name :

Mathematica 7.1/8.0 software should be used for the following exercises

- (01). Obtain the numerical value of, $\frac{2^{23}}{10^8} \times \sqrt{2.7156}$.
- (02). Solve the following equation. $x^{7} + x^{2} + 2x - 7 = 0$
- (03). "Tanzania" belongs to which continent? Plot the map of the above continent.
- (04). Find the atomic masses of Cu and Ra.
- (05). Differentiate the following function; $f(x) = x \times \sin(x^2)$
- (06). Determine the following integrals:

(a).
$$\int \sin(e^x) dx$$

(b). $\int_{-1}^{+1} e^{-x^2} dx$

(07). Solve the following simultaneous equations.

$$x + y = 0$$
$$x - y = 20$$

(08). Write a complete Mathematica computer program to determine the area of the following sketch :



Total Marks	For Lecture's Use Only		
	Part I		
	Part II		
	Part III		
	Part IV		
	Part V		
	Total		

2nd year - Physics Practical [Computational Physics] Assignment No. 02 (Time : 15 min)

..... Name Index No :

Write a complete Mathematica program segment to do the following exercise.



The cylinder was transformed into a sphere by melting it. Assuming 15% of material is lost in the process, calculate the radius of the sphere.

(You are given the volume of a cylinder = $\pi r^2 h$ and volume of a sphere = $\frac{4}{3}\pi r^3$)

Department of Physics, USJP. General Degree 2nd Year **Computational Physics - Assignment No I**

Time : Half an hour

Mathematica Software must be used to do the following exercises

Using complex numbers, calculate the equivalence impedance of the following electrical (network) circuit.

Hence, obtain the equation of the resultant current i(t).

Plot the graphs of,

- (i) Voltage, V(t) vs. Time (t),
- (*ii*) Current, i(t) vs. Time (t).

Also plot the above two graphs in a same page.

<u>Hints:</u>

A) First, find the complex numbers Z_1 , Z_2 , Z_3 , Z_4 , Z_5 , and Z_6 for the above circuit.

After that, find the equivalence impedance, Z.

B) <u>Phasor Diagrams for the L, R and C circuit components, which may be used to find the current</u>

C) Plot Commands which may be used to draw more than one graph on the same page

Department of Physics, USJP. General Degree 2nd Year Computational Physics - Assignment No III

Time : 45 minutes

Mathematica Software must be used to do the following exercises

(a) Obtain first 10 terms of the arithmetic progression whose first term is 1 and common difference is 0.5 using "Tables" in "Mathematica".

- I. Find the values of **r** in the above diagram for x = 0, 1, 2, 3, 4, ..., 10.
- II. Also find the angle $\,\alpha\,$ and $\,\theta\,$ depicted in the diagram for each point.

- (c) Consider a rectangular grid of 21 x 21 points of spacing 100 m. There is a homogenous spherical mass distribution of radius 50 m, centered at a depth of 75 m directly below the center of the grid.
 - (i) Determine the gravity field (vertical component) at all grid points.
 - (ii) Plot the contour diagram of the gravity field.

[Density of the sphere 3000 kgm⁻³ & Universal gravitational constant, $G = 6.67 \times 10^{-11} N m^2 kg^{-2}$]
