

University of Sri Jayewardenepura.

B. Sc. General Degree Third Year

Course Unit – 2021

PHY 308 1.0 – PHYSICS PRACTICALS

Computational Physics

# Mathematica 11 / 12

This handout is intended to be a brief introduction to *Mathematica*. It describes essentially some capabilities of *Mathematica*, and assumes no prior knowledge of the system.

WOLFRAMRESEARCH

# PHY 308 1.0

# PHYSICS PRACTICAL

# **Method of Evaluation** for PHY 308 1.0 Physics Practical:

Continuous Assessments	- 40 marks
Attendance	- 20 marks
Final Examination	- 60 marks
Total	- 100 marks

# **Continuous Assessments:**

A number of assignments given for a year would vary between 04 to 06 depending on the time factor. All these assignments are compulsory

# **Final Examination:**

Practical Paper – Duration 2 or 3 hours (Open Book Examination)

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# Mathematica 11 / 12 Software

**Mathematica** is a powerful mathematical software in the world. The working environment of this software is totally different from other mathematical software. **Mathematica** is built on the powerful unifying idea that everything can be represented as a symbolic expression.

# **Executing Method**

Any given input / command can be run by pressing both *Shift* and *Enter* keys simultaneously.

# **Basic Operations**

Numerical Form	Mathematica Form
a+b	a+b
a-b	a-b
$a \times b$	a*b
$\frac{a}{1}$	a/b
h	

# Defining a variable

When defining a variable it is a common practice to use simple letters.

E.g.:-
$$a = 5$$

$$b = -1$$

$$c = 2$$

# <u>In-built functions / Keywords</u>

The first letter of any given in-built function must be capitalized. Some in-built functions are as follows.

Print
Plot
Do
If
Table
Solve
N - Numerical Function

There are three kinds of brackets used in Mathematica.

# (1) Parenthesis – ()

We can use parenthesis in usual way.

**Numerical Form** 

Mathematica Form

$$(2x-5)(3x-6)$$

$$(2*x-5)*(3*x-6)$$

# (2) Box bracket - []

We can use box bracket to type the argument of any in-built function.

N[1/7] {Here N gives the numerical value of its argument}

We can also do the above calculation in following way.

N[1/7,10] {This will give the value of 1/7 up to ten decimal points}

# (3) Curly bracket – { }

Curly brackets are used to define a Range and an Array,

E.g.:- 
$$i = 1 \text{ to } 100 \rightarrow \{i, 1, 100\}$$

(2) Defining an array

E.g.:-  $marks = \{50, 60, 70, 80, 90\}$ 

# **Exercise:**

Find the numerical values of (i)  $\frac{1}{13}$ , (ii)  $\frac{1}{17}$  and (iii)  $\frac{1}{\pi}$ .

**Hint:** 

Mathematica Form

#### **Exercise:**

If  $a = \frac{1}{11}$ , find the numerical values of (i)  $a^2$ , (ii)  $\frac{1}{a}$ , (iii)  $\frac{1}{a^2}$ , (iv)  $a^{10}$ ,

(v) 
$$a + \frac{1}{a}$$
, (vi)  $a^2 + 1$ , (vii)  $a - \frac{1}{a}$ , (viii)  $a \left( a + \frac{1}{a} \right)$ , (ix)  $\frac{a - \frac{1}{a}}{a + \frac{1}{a}}$  and

(x) 
$$a + \frac{1}{a} + \frac{1}{a^2}$$

#### **Exercise:**

Find the numerical values of (i)  $\pi$ , (ii)  $\frac{22}{7}$ , (iii)  $\pi - \frac{22}{7}$ , (iv)  $\pi^2$  and

(v) 
$$\pi^2 - 10$$

Mathematica can be used in three major ways.

- (1) Calculator
- (2) Software package
- (3) **Programming language**

# Mathematica as a Calculator

We can use **Mathematica** just like a calculator.

E.g.:-

Numerical Form Mathematica Form 3 + 43 + 4 $2^{20}$ 2^20  $\sin(60^{\circ})$ Sin[60Degree]  $(((1.765)^2)*((2.8)^(1/3)))/(Sqrt[1.9125831])$  $(1.765)^2(2.8)^{\frac{1}{3}}$ 

When we type questions like above, **Mathematica** prints back their answers.

Numerical Form	Mathematica Form
$sin~60^{o}$	Sin[60 Degree]
$\cos 60^{\circ}$	Cos[60 Degree]
$tan 60^{\circ}$	Tan[60 Degree]
cosec 60°	Csc[60 Degree]
sec 60°	Sec[60 Degree]
$cot 60^{\circ}$	Cot[60 Degree]
<i>coi</i> 00	Cottoo Degree

# **Exercises:**

Find the numerical values of the following expressions,

- $\sin 85^o + \cos 85^o$
- (ii)  $\sin^2 85^o + \cos^2 85^o$ (iii)  $\sec^2 35^o \tan^2 35^o$
- (iv)  $\csc^2 55^\circ \cot^2 55^\circ$

# Mathematica as a Package

We can use Mathematica like a software package. Here we can't use it in our own way. That means they are built to do some specific tasks. Some common packages are given

E.g.:- MSWORD, MSEXCEL, MSPOWERPOINT, WINAMP, VLC Player, Windows Media Player, Nero, U Tube Player, ...

#### **Exercises:**

(1) Finding the numerical value of  $\pi$  up to 20 decimal places. N[Pi, 20]

(2) Plotting the function  $y = \sin x$  within the range  $-\pi$  to  $\pi$ .

$$Plot[Sin[x], \{x, -Pi, Pi\}]$$

(3) Plotting the function  $y = e^x$  within the range 2 to 6.

$$Plot[Exp[x], \{x, 2, 6\}]$$

(4) Solving the equation  $3x^3 + 2x^2 + 1 = 0$ 

$$Solve[3*x^3+2*x^2+1=0, x]$$

(5) If  $Z_1 = 1 + 2j$  and  $Z_2 = 4 - 3j$  (where,  $j = \sqrt{-1}$ ) find the following expressions.

(i) 
$$Z_1 + Z_2$$
 (ii)  $Z_1 - Z_2$  (iii)

(iii) 
$$\frac{1}{Z_1}$$

$$(iv) \qquad \frac{1}{Z_1} + \frac{1}{Z_2}$$

(iv) 
$$\frac{1}{Z_1} + \frac{1}{Z_2}$$
 (v)  $\frac{1}{\sqrt{Z_1} + \sqrt{Z_2}}$ 

#### Hint:

Mathematical Form

$$Z_1 = 1 + 2j$$

Mathematica Form

$$z1 = 1 + 2I$$

(where, 
$$I = \sqrt{-1}$$
)

# Mathematica as a Programming language

Mathematica is a flexible and intuitive programming language.

(1) Finding the area of a rectangular shaped land whose length is 5m and width is 3m.

$$a = 5$$

$$b = 3$$

$$area = a * b$$

#### **Exercises:**

Lengths of the two sides of a rectangular triangle are 6 cm and 8 cm. Find the length of the diagonal.

# Some important guidelines on writing programs

(1) Comments

It is a good practice to comment our program in order to understand the inclusion later.

E.g.:- 
$$ugc = 6.67 * (10^{(-11)}) (*universal gravity constant*)$$

(2) Semicolon (;)

If we put a semicolon after any input, it will prevent the output being displayed.

E.g.:- inputs outputs 
$$a = 5$$
  $b = 10$ ;

Additional to above tasks, there are some add-on packages that come with **Mathematica** to do some specific tasks. Here are some examples for standard packages embedded in **Mathematica**.

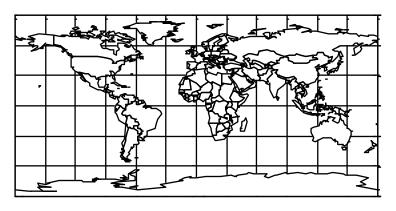
- (1) Miscellaneous
- (2) Statistics
- (3) DiscreteMath
- (4) Geometry

Under the package **Miscellaneous** there are several sub packages as *WorldPlot*, *ChemicalElements*, *Geodesy*, *StandardAtmosphere* and etc. Let's now consider the add-on package *WorldPlot*. This loads the *WorldPlot* package.

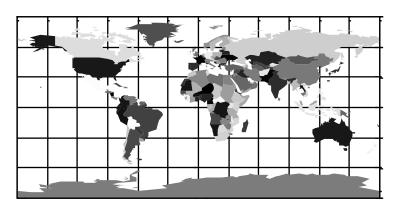
#### <<WorldPlot`

This gives a map of Asia with all countries drawn as white with black outlines.

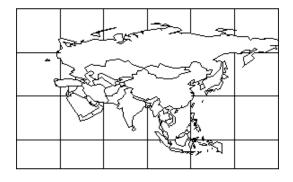
# WorldPlot[World]



#### WorldPlot[{World, RandomGrays}]



# WorldPlot[Asia]



# **Exercise:**

Plot the Other Continents i:e; NorthAmerica, SouthAmerica, Europe, Africa, ....., MiddleEast and Oceania.

Now consider the standard add-on package called *ChemicalElements*. This loads the *ChemicalElements* package.

#### << Chemical Elements `

This gives the list of the names of the chemical elements from Hydrogen to Ununbium.

#### **Elements**

{Hydrogen, Helium, Lithium, Beryllium, Boron, Carbon,..., Unununium, Ununbium}

This gives the atomic weight of **Oxygen** using the data in the package.

#### AtomicWeight[Oxygen]

15.9994

or

#### AtomicWeight[O]

15.9994

or

AtomicWeight[Elements[[8]]]

(\* Elements[[8]] is Oxygen \*)

15.9994

The first command gives the electronic configuration of **Oxygen** as a list in the standard format while the second gives the orbital labels in the list.

# ${\bf Electron Configuration [Oxygen]}$

**ElectronConfigurationFormat[Oxygen]** 

$$\{\{2\}, \{2, 4\}\}\$$
 $1s^2 2s^2 2p^4$ 

#### Other Packages:

#### <<Dictionary`</p>

Eg:	SpellCheck["She is an actress"]	{She is an actress,{}}
	SpellCheck["She is a actress"]	{She is a actress, {}}
	SpellCheck["Sha is an actress"]	{Sha is an actress,{Sha}}
	SpellCheck["Sha is an actres"]	{Sha is an actres, {Sha, actres}}
	SpellCheck["Sha is a actres"]	{Sha is an actres, {Sha, actres}}

#### <<Calendar`</li>

Eg:	DayOfWeek[{2010,09,15}] DayOfWeek[{1973,11,26}]	Wednesday Monday
Eg:	DaysPlus[{2010,09,15},10] DaysPlus[{2010,09,15},50]	{2010,9,25} {2010,11,4}
Eg:	DaysBetween[{1973,11,26},{1985,11,03}] DaysBetween[{1973,11,26},{1985,11,03}]/365	$\frac{4360}{872}$
	DaysBetween[{1973,11,26},{1985,11,03}]/365//N	11.9452

DaysBetween[ $\{your\ birthday\}, \{date\ of\ today\}\}/365//N = your\ age\ (in\ years)$ 

# **Tute 01**

 ${f Q1.}$  Find the solutions for the following problems.

Mathematical Form	Mathematica Form
2.3 + 5.63	2.3 + 5.63
$2.4 + 8.9^2$	2.4 + 8.9^2
$4.4+7.2^2$	
$\frac{(3+4)^2 - 2(3+1)}{2^{45}}$ $2^{45}$	$(3+4)^2-2*(3+1)$
$2^{45}$	2^45
$2^{45}$	2^45 //N
2. 45	2.^45
452 ÷ 62	452/62
Find the value for $\pi$	
Find the above for 40 decimal places	
77 <sup>2</sup>	77^2
77 <sup>2</sup> +1	% + 1
(4+3i)/(2-i)	(4+3 I)/(2-I)
3 <sup>100</sup>	
$2^3 + 6$	
3x-x+2	3*x - x + 2
-1+2x+7x	
35.8 <sup>50</sup>	(35.8)^50
59. ÷ 4	59./4
2 ×3× 6	2*3*6 or 2 3 6
54 ×6 <sup>10</sup>	54 * 6^10 or 54 6^10

# **Q2.**

Mathematical Form	Mathematica Form
Type, $x^2 + x - 4x^2$	$x^2 + x - 4*(x^2)$
Type, $xy+2x^2y+y^2x^2-2yx$	$x*y+2*(x^2)*y+(y^2)*(x^2)-2*y*x$ (x + 2*y + 1) * (x - 2) ^2
Type, $(x + 2y + 1)(x - 2)^2$	$(x + 2*y + 1)*(x - 2)^2$
Expand the above	Expand[%]
expressions	
Factor the above	Factor[%]
expressions	
Find, $\sqrt{(1+0.1)^4}$	$Sqrt[(1+0.1)^4]$
Type, $log(1+Cos(x))$	Log[1+Cos[x]]
Find the value of	1 + 2*x /. x->3
(1+2x), when $x=3$	
Type, $t=1+x^2$	
Find t when $x=2$	
Find t when x=5a	
Find the value of t when	
$x=\pi$	
Find, $\sum_{x=1}^{5} x^2$	$Sum[x^2, \{x, 1, 5\}]$
Find , $\sum_{i=1}^{10} \frac{x^i}{i}$	
Find, $\sum \frac{x^i}{i}$ , for i=1,3,5	Sum[x^i/i,{i,1,5,2}]
Find , $\sum_{i=1}^n \dot{t}^2$	Sum[i^2,{i,1,n}]

° 1	Sum[1/i^4, {i,1,Infinity}]
Find , $\sum_{i=1}^{\infty} \frac{1}{i^4}$	Sum[1/1 4, (1,1,111111111111111111111111111111
i=1 $i$	
Find , $\sum_{i=1}^{\infty} \frac{1}{(i!+(2i)!)}$	Sum[1/(i!+(2i)!),{i,1,Infinity}]
$\sum_{i=1}^{n} (i!+(2i)!)$	
Find the numerical value	N[%]
for the above	
$\sum_{i=3,j=i}^{i=3,j=i} z_i^{i} \cdot z_j^{i}$	$Sum[x^i y^j, \{j, 1, i\}, \{i, 1, 3\}]$
Find, $\sum_{i,j=1}^{i=3,j=i} x^i y^j$	
Find the value of $1+x+x^2$ ,	$1 + x + x^2 / . x \rightarrow (2-y)$
when $x\rightarrow 2-y$	
	x→3+y
Type $x\rightarrow 3+y$ Find value of $x-9$ using	x^2-9/.%
the above	
Generate this product	Product[x+i, {i, 1, 4}]
(x+1)(x+2)(x+3)(x+4)	
lim ( 1 )	
$ \lim_{x \to 0} \left( \frac{1}{x+1} \right) $	Limit[ $(1/(1+x)), x->0$ ]
` ´ ´	
$\lim_{x \to 1} \left( \frac{x-1}{x^2 - 1} \right)$	
$x \rightarrow 1 \left(x^2 - 1\right)$	
lim ( 1 )	
$ \lim_{x \to 1} \left( \frac{1}{x-1} \right) $	
x->1 $(x-1)$	
$\lim (x^2 - 2x + 1)$	
$\begin{vmatrix} \lim_{x \to 1} \left( \frac{x^2 - 2x + 1}{x - 1} \right) \end{vmatrix}$	
<u> </u>	
$\lim \left(x^2-1\right)$	
$ \lim_{x \to 1} \left( \frac{x^2 - 1}{x - 1} \right) $	
$\lim \left(\frac{\sin x}{x}\right)$	
$x \rightarrow 0 (x)$	
$\lim (\tan x)$	
$x \rightarrow 0$ ( $x$ )	
$\lim \left(\sin x + \cos x\right)$	
$x \rightarrow 0$ $\left(\frac{x}{x}\right)$	
- ( /	

# **Q3**.

Mathematical Form	Mathematica Form
If $f = x^n$ , find $f'$ or $\frac{df}{dx}$	D[x^n,x]
If $f = \mathbf{x}^n$ , find $f'''$ or $\frac{d^3 f}{dx^3}$	D[x^n, {x, 3}]
If $f = Sinx$ , find $f^1$	
If $f = x Sinx$ , find $f^1$	
If $f = \frac{x^2 + 5x + 1}{x^3 - 1}$ , find $f^1$	
If $f = Cos^2x Sinx$ , find $f^1$	
If $f = \frac{5}{x^2 + 1}$ , find $f^1$	
If $f = \frac{5}{\sqrt{x^2 - 1}}$ , find $f^1$	

If $f = \frac{5}{x^2 - 1}$ , find $f^1$	
If $f = \frac{5}{\sqrt{x^2 + 1}}$ , find $f^1$	
If $f = Sinx$ , find $f^2$	
If $f = Sinx$ , find $f^3$	
If $f = Sinx$ , find $f^{10}$	
	T., + + - [1/(-0.4.1)1
Find , $\int \frac{1}{(x^4 - 1)} dx$	Integrate[1/(x^4-1), x]
Find, $\int_{0}^{1} x^{2} dx$	<pre>Integrate[x^2, {x, 0, 1}]</pre>
Find, $\int_{1}^{1} x^{2} dx$	NIntegrate[x^2, {x, 0, 1}] or
0	<pre>Integrate[x^2, {x, 0, 1}]//N</pre>
Find, $\int_0^1 \frac{1}{\sqrt{x(1-x)}} dx$	
Find , $\int \frac{1}{(x^4 - a^4)} dx$	
Find , $\int_{-a}^{a} \frac{1}{(x^4 - a^4)} dx$	
Find , $\int\limits_0^\pi e^{(1-x^2)}dx$	
Find , $\int_{0}^{\pi} Sin(2x) dx$	
Find , $\int \ln x  dx$	<pre>Integrate[Log[x], x]</pre>
Find , $\int \log x  dx$	<pre>Integrate[Log[10,x], x]</pre>
Find , $\int_{a}^{b} \ln x  dx$	
Find , $\int \sqrt{(x+1)} \times \sqrt{(x-1)} dx$	
Find , $\int \sqrt{\left(\frac{x+1}{\sqrt{x-1}}\right)} dx$	
Find , $\int \sqrt{\frac{x+1}{x-1}} dx$	
Find , $\int \sqrt{\frac{x-1}{x+1}} dx$	
Find, $\int_{0}^{\pi} \left(5 \sin x + 8 \cos x\right) dx$	
Find, $\int_{0}^{\pi} \left(x \sin^2 x\right)^{\frac{1}{2}} dx$	

# **Quit Kernel**

If you encounter with any of the following situations when you are working with Mathematica, you are advised to follow the Quit Kernel procedure.

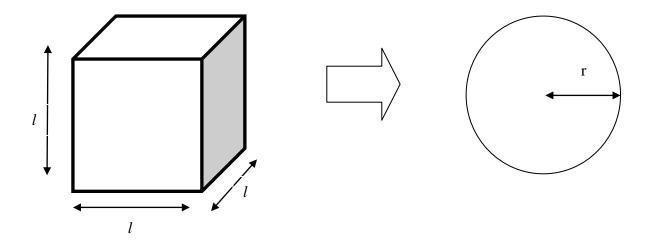
- 1. Mathematica stucks while you are working.
- 2. Mathematica produces an erroneous result even if your input statement is correct.
- 3. Mathematica produces an incorrect result even if your input statement is correct.
- 4. If the machine becomes slow due to the processing of the particular calculation.
- 5. If any loop will run continuously without control.

#### **Quit Kernel Procedure:**

Move the cursor to the kernel that appears on the Mathematica toolbar and click on it. Select,

A dialog box will appear & choose Quit.

Write a complete Mathematica program segment to do the following exercise.



The cube was transformed in to a sphere by melting it. Assuming,

- a. No material is loss in the process
- b. 20% of material is loss in the process

Calculate the radius (r) of the Sphere.

# Define a Function

#### **Mathematical Form**

#### **Mathematica Form**

$$f(x) = \sin(x^2)$$
$$f(\frac{\pi}{2}) = \sin((\frac{\pi}{2})^2) = \sin(\frac{\pi^2}{4})$$

$$f\left[x_{-}\right] = Sin\left[x^{2}\right]$$

$$f[Pi/2] = Sin[Pi^2/4]$$

$$f[Pi/2]//N = 0.624266$$

# Tute 02

**Q1.** Given 
$$f(x) = x^6 - 21x^5 + 175x^4 + 735x^3 + 1624x^2 - 1764x + 720$$

# $f[x]=x^6-21*x^5+175*x^4+735*x^3+1624*x^2-1764*x+720$

If x=0.5, find f(x)

- f[0.5]
- When  $x = \sqrt{2}$ , find f(x)
- f[Sqrt[2]]
- iii. Find  $f(y) = y^6 21y^5 + 175y^4 + 735y^3 + 1624y^2 1764y + 720$

- **Q2.** Given  $f(x) = (1+x)^2$ 
  - i. Expand,  $(1+x)^2$
  - Factor the above result ii.
  - iii. If x=0.25, find f(x)
    - iv. Find, f(z)
- **Q3.** Given  $f(x) = (1+x+3y)^2$  where y is a constant
  - i. Expand,  $(1+x+3y)^2$
  - Find f(x) for x = 0.5, 1, 1.5, 2, 2.5, 3
  - iii. When  $x = \sqrt{3}$ , find f(x)
    - iv. Find f(z)
      - v. Factor the above result in part(i)
- **Q4.** Given  $f(x) = x^{10}-1$ 
  - i. Factor above f(x)
  - ii. Expand above f(x)
  - iii. When  $x=\sqrt{3}$ , find f(x)
    - iv. Find f(z)
- Q5. Find the answers for the following equations.
  - i. Solve, 8x+3=0
- Solve[8\*x + 3 == 0, x]Solve[8\*x + 3.0 == 0, x]
- ii. Solve, 8x+3.0=0
- iii. Solve,  $x^2-5x-3=0$
- iv. Solve,  $x^3 + 2x 1 = 0$
- v. Solve,  $x-3x^2=0$ vi. Solve, 1+5.0x=0
- vii. Solve,  $x^2-9=0$

```
ix. Solve, x^2 + 2x - 7 = 0
    x. Solve, x^4-5x^2-3=0
    xi. Solve, x^6-1=0
   xii. Solve, 7x^4-5x^2-6.0=0
  xiii. Solve, ax^2+bx+c=0, (here a, b & c are Constants)
   xiv. Solve, 7x^5 - 3x^3 + 8 = 0
   xv. Solve, x^4-1=0
   xvi. Solve, x-1.0=0
  xvii. Solve, x^3-3x+5=0
 xviii. Solve, 7x^5 + 6x^2 + 3 = 0
          (* Do the same using Table command *)
          f={x^2-2*x+1,x^3-1,x^4-1,2*x-7}
          Table[ Solve[f[[i]],x], {I,1,Length[f]}]
Q6. Given e = ((x-1)^2 + (2+x))/((a+x)(x-3)^2), a is a constant
    e = ((x-1)^2+(2+x))/((a+x)*(x-3)^2)
   i. Expand the above (e)
                                           Expand[e]
      Expand out everything, including the denominator,
                                            ExpandAll[e]
 iii. Collect all the terms Together over a common
       denominator
                                            Together[%]
  iv. Break the expression apart into terms with simple
       Denominator
                                            Apart[%]
  v. Factor everything (reproducing the original form),
                                            Factor[%]
  vi. Simplify(e)
                                            Simplify[e]
Q7. Given v = (3+2x)^2 (x+2y)^2 v = ((3+2x)^2 (x+2y)^2)
   i Expand the above v
  ii Groups together terms in v that involve the same
      power of x
                                      Collect[v,x]
  iii Groups together power of y Collect[v,y]
  iv Separate factors out the piece that does not depend
       on y
                                       FactorTerms[v,y]
Q8. Given e = (1+3x+4y^2)^2
                                      e = (1+3x+4y^2)^2
   i Expand above e
                                       f = Expand[e]
  ii Find the coefficient of x in f Coefficient[f,x]
  iii Find the coefficient of x^2 in f Coefficient[f,x^2]
  iv Find the highest power of y that appears in Exponent
                                       Exponent[f,y]
    v Find the fourth term in f
                                      Part[f,4]
```

viii. Solve,  $2x+x^2=0$ 

- Q09. Generate the following tables.
  - i. Make a list of values (a table) of  $r^2$ , with r running from 1 to 10. i.e.  $\left\{r^2\right\}_{r=1}^{10} = \{1^2, 2^2, 3^2, 4^2, \dots, 10^2\}$

Hint: Table [expression, {x, x\_min, x\_max}]

- ii. Make a list of values of Sin (n/5), with n running from 0 to 25.
- iii. Make a list of values of  $x^i + y^j$  with i running from 1 to 3 & j running from 1 to 2. Put them in table format.
- iv. Create a list of values containing four copies of the
   symbol x using "Table" command.
   i.e. {x, x, x, x}
- Q10. Plot the following functions in given regions.
  - (a) f(x) = tan(x)
    - i Plot, f(x) for x = -3 to 3 range.
    - ii Plot, f(x) for  $x = -\pi$  to  $+\pi$  range.
    - iii Plot, f(x) for  $x = -2\pi$  to  $+2\pi$  range.
    - iv Plot, f(x) for  $x = -10\pi$  to  $+10\pi$  range.
  - **(b)**  $f(x,y) = \sin(x y)$ 
    - i Plot, f(x,y) for x, 0 to 4 and y, 0 to 4 in three dimensional space.(3D)

Hint:

#### Mathematical Form Mathematica Form

F 3

- $f(x, y) = \sin(xy) \qquad \qquad f[x_{-}, y_{-}] = Sin[x * y]$ 
  - observe the variations.

    ➤ Put, PlotPoints→40
  - Put, Mesh→False
  - ▶ Put, FaceGrids→All
  - Put, AxesLabel → {"Length (m)",
    "Width (m)", "Height (m)"}
  - Put, PlotLabel  $\rightarrow$  "The graph of h vs x"

Add the following to the above plot and

- (c) f(x,y) = Sin(x) Sin(y)
  - i ContourPlot f(x,y) for x = -2, 2 range and y = -2, 2 range.
  - ii **DensityPlot** f(x,y) for above range.

```
i Plot f(x,y) for x = -10, 10, y = -10, 10, in 3D
         Put, PlotPoints -> 40
        See the difference in the figure by varying no of
    iii
          plotpoints. In the same plot show axes labels as
          "time", "depth" and "value".
Q12. f(x) = \sin x, g(x) = \sin 2x, h(x) = \sin 3x
         Plot the above functions in same axis for x goes
          from 0 to 2 \pi range.
               f={Sin[x], Sin[2*x], Sin[3*x]}
               Table[ Plot[ f[[i]], {x,0,2*Pi}], {i,1,3}]
               Show[%]
     ii Draw f(x) in red color
          Plot[f[[1]], \{x,0,2*Pi\}, PlotStyle \rightarrow Red]
                               \mathtt{Or}
          Plot[f[[1]], \{x,0,2*Pi\}, PlotStyle \rightarrow Hue[0.9]]
                               or
          Plot[f[[1]], {x,0,2*Pi}, PlotStyle->
                                    RGBColor[1,0,0]]
          Draw g(x) in green color
    iii
          Draw h(x) in blue color
     iv
          Plot the above (ii), (iii) and (iv) functions in
          same axis
               f={Sin[x], Sin[2*x], Sin[3*x]}
               col={Red, Green, Blue}
               Table[ Plot[ f[[i]], {x,0,2*Pi}, PlotStyle->
                                         col[[i]]], {i,1,3}]
               Show[%]
Q13. Write the following statement by using conditionals,
      Loops & control structures in Mathematica.
          If 7 > 8 the answer should be x, otherwise y.
          If 7 > 8 the answer should be print "x",
     ii
          otherwise print "y".
          Hints:
          If[Condition, True, False, Otherwise]
               If[2>3, Print["True"], Print["False"]]
               False
               If[3>2, Print["True"], Print["False"]]
               If[Mala > Kamala, Print["True"],
               Print["False"], Print["Otherwise"]]
               Otherwise
    iii Print i^2 for i = 1 to 4 increment by 1
                                    Do[i^2,{i,1,4}]
          Print i^2 for i = 1 to 4 increment by 0.25
                                    Do[i^2,{i,1,4,0.25}]
```

Q11.

f(x,y)=10 Sin x + 5 Cos y

- v Print  $i^2$  for i = 1 to 40 increment by 2
- vi Print  $i^3$  for i = 1 to 40 increment by 5
- vii Print  $i^3 + (i+1)^3$  for i = 1 to 40 increment by 5
- viii Make a array (table) i for i = 1 to 100 increment by 1

#### Table[i, {i, 1, 100}]

ix Make a array (table)  $i^2$  for i = 1 to 100 increment by 5

#### Table[i^2,{i,1,100,5}]

x Print i , starting from i = 1 for i < 14 increment by 1

xi Print i , starting from i = 1 for i < 14 increment by 2

#### For[i=1, i < 14, Print[i]; i + = 2]

xii Print i , starting from i = 15 for i > 5 decrement by 1

xiii Print i , starting from i = 15 for i > 5 decrement by 0.5

- xiv Print n for n-1>5 starting from n=20 in decreasing order.
- xv Print n for n 1 < 15 starting from n = 2 in increasing order.

#### **Exercise:**

(I) Make a data table as follows.

$$X = \{1, 2, 3, 4, ..., 200\}$$

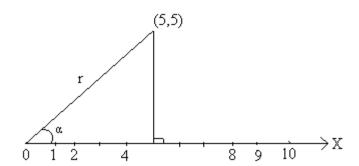
Hence make the table A. Such that  $A = \{1^2, 2^2, ..., 200^2\}$ 

(II) Make a data table as follows.

$$X = \{1,2,3,4,5,6,\dots,100\}$$

Hence make the table A. Such that  $A = \left\{1, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}, \frac{1}{5^2}, \dots, \frac{1}{99^2}, \frac{1}{100^2}\right\}$ 

**(III)** 



- a) Find the values of  $\mathbf{r}$  in the above diagram for x = 0, 1, 2, ...
- b) Also find the angle  $\alpha$  as depicted in the diagram for each point.

# **List and Table Manipulation**

- **01.** Let  $x = \{8, 10, 1, -3, 5, 7, -2, 0, -13, 15\}$ 
  - a. Find the maximum value of x.

Max[x]

**b.** Find the minimum value of x.

Min[x]

- **02.** Let  $t = \{-10, 5, 10, 15, 25, -25, 75, -50, 9, 12, 78, 19, 20\}$ . Do the following list manipulations and try to understand their operation.
  - *a*) First[*t*]
  - *b*) Last[*t*]
  - *c*) Reverse[*t*]
  - d) Take[t, 3]
  - e) Take[t, -2]
  - f) Drop[t, 2]
  - g) Drop[t, -2]
  - h) Length[t]
  - i) Dimensions[t]
- 03. Let  $a = \{1, 2, 3, 4\}$ ,  $b = \{5, 6, 7\}$  and  $c = \{\{-3, 5\}, \{7, 9\}\}$ . We can perform the following operations on lists.
  - a) Join[a,b] {This concatenates list a and b together}
  - b) **Position**[b, 6] {This gives the position of the number 6 in b}
  - c) **Reverse**[a] {This reverse the order of the elements in list a}
  - d) **Flatten**[C] {This flattens out the list C}
- **04.** Let  $a = \{\{10, 20\}, \{30, 40\}\}\$ and  $b = \{\{-10, 8\}, \{40, -84\}\}\$ 
  - a) Find the dimension of a and b.
  - b) Represent a and b in matrix form.

MatrixForm[a]

c) Find the inverse of a and b.

f) Find a+b, a-b and  $a \times b$ 

- Inverse[a]
- d) Find the Eigen values of a and b.
- Eigenvalues[a]
- e) Find the determinant of a and b.
- $\mathbf{Det}[a]$  a+b
- a-b  $a \cdot b$

$$\begin{pmatrix} * & a/h \\ h & \text{is not defined} \end{pmatrix}$$

**05.** a. We can solve the mx = b equation in following ways

$$x + 2y + 3z = 1$$

$$2x - 3z = 0$$

$$x + y - z = -1$$

Solve  $[m \cdot x = = b, x]$ 

Solve  $[\{x_1 + 2*x_2 + 3*x_3 == 0, 2*x_1 - 3*x_3 == 0, x_1 + x_2 - x_3 == 0\}, \{x_1, x_2, x_3\}]$ 

OR

If  $m = \{\{1, 2, 3\}, \{2, 0, -3\}, \{1, 1, -1\}\}, x = \{x_1, x_2, x_3\}$  and  $b = \{1, 0, -1\}$  and solve the equation m x = b

If  $\mathbf{m} \cdot \mathbf{x} = \mathbf{b}$ 

 $x = m^{-1} \cdot b$  (\* This is called <u>Even Determine Case</u> \*)

This gives the solution of  $mx = \overline{b}$  for square matrix.

#### **Exercise:**

Solve the following simultaneous equation set

$$x + 2y + 3z = 6$$
  
 $2x - 3y + z = 11$   
 $-x + y - 2z = -9$ 

b. We can also solve the above equation using **Predetermined Case** 

If 
$$\mathbf{m} \cdot \mathbf{x} = \mathbf{b}$$
  
 $\mathbf{x} = [\mathbf{m}^{\mathrm{T}} \cdot \mathbf{m}]^{-1} \cdot \mathbf{m}^{\mathrm{T}} \cdot \mathbf{b}$ 

 $\mathbf{x} = [\mathbf{m}^{T}.\mathbf{m}]^{-1}.\mathbf{m}^{T}.\mathbf{b}$ This gives the solution of m x = b for non square matrix.

(Here  $m^T$  is the transpose of m and  $[\mathbf{m}^T.\mathbf{m}]^{-1}$  is the inverse of  $\mathbf{m}^T.\mathbf{m}$  metrics.)

By using the predetermined case solve the following equation

$$\begin{aligned} m &= \{\{2, -1, 6\}, \{5, 4, 3\}, \{9, 10, 7\}, \{11, 13, 16\}, \{5, 7, 9\}\} \\ x &= \{x_1, x_2, x_3\} \quad \text{and} \quad b &= \{1, 0, -1, -10, -19\} \end{aligned}$$

#### **Exercise:**

Solve the following simultaneous equation set

$$x+2y+3z = 6$$

$$2x-3y+z = 11$$

$$-x+y-2z = -9$$

$$5x-3y-8z = -13$$

$$6x+2y+5z = 17$$

We can also solve the above equation using **Underdetermined Case** c.

If 
$$m \cdot x = b$$
  
  $x = m^{T} \cdot [m \cdot m^{T}]^{-1} \cdot b$ 

This gives the solution of m x = b for non square matrix in Underdetermined Case [Backus & Gilbert Method].

By using the underdetermined case solve the following equation

$$\begin{aligned} m &= \{\{2, -1, 6\}, \{5, 4, 3\}\} \\ x &= \{x_1, x_2, x_3\} \quad \text{and} \quad b = \{1, 0\} \end{aligned}$$

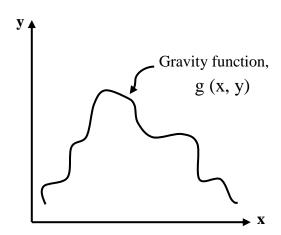
# **Exercise:**

Solve the following simultaneous equation set

$$x+2y+3z=6$$
$$2x-3y+z=11$$

#### 06. Gravity Problem

# 1- D Case :



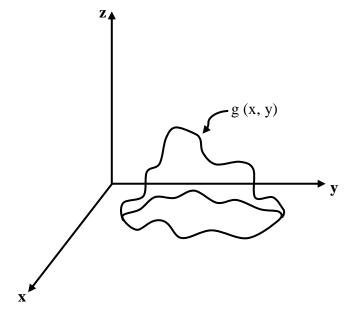
The above figure shows the variation of the gravity (anomaly) with the field coordinates (x) which can be expressed by the equation,  $g(x) = a x^2 + b x + c$ , where a, b & c are unknown constants. Gravity is usually measured using the gravimeter (gravity-meter) and therefore the accuracy depends on the gravimeter. Specifically, expensive gravimeters have high accuracy than the cheaper gravimeter. By measuring the gravity at different points, constants a, b & c can be accurately determined.

In order to determine the constants a, b & c, a student measured the gravity at five points. These readings are given in the table below.

x(m)	Gravity (ms <sup>-2</sup> )
1	1.39
2	2.21
3	3.21
4	4.39
5	5.81

- 1. By selecting a suitable method and using the data given in the table, find the values of a, b & c accurately. (Values obtained for a, b & c can be confirmed with the actual values of a = 0.1, b = 0.5 and c = 0.8).
- 2. By using the values calculated for a, b & c, determine the gravity at x = 2.5 m and x = 4.5 m.





The above figure shows the variation of the gravity (anomaly) with the field coordinates (x & y) which can be expressed by the equation  $g(x, y) = a x^2 + b y^2 + c y + d$ , where a, b, c & d are unknown constants.

Gravity values with the field coordinates (x & y) are shown in the table below.

<i>x</i> ( <i>m</i> )	y (m)	Gravity (ms <sup>-2</sup> )
0	0	0.95
0	1	1.19
1	0	1.09
1	1	1.21
2	0	1.39

0	2	1.81
2	1	1.40
1	2	1.72

- 1. By selecting a suitable method and using the data given in the table, find the values of a, b, c & d accurately. (Values obtained for a, b, c & d can be confirmed with the actual values of a = 0.1, b = 0.2, c = -0.1 and d = 1.0).
- 2. By using the values calculated for a, b, c & d determine the gravity at x = 1.5 m, y = 1.25 m and x = 0.25 m, y = 1.75 m.
- **07.** Consider two complex numbers  $Z_1 = 2 + 3i$  and  $Z_2 = 3 2i$ .
  - a) Find  $Z_1 + Z_2$
  - b) Find  $Z_1$   $Z_2$
  - c) Find  $Z_1 * Z_2$  and  $Z_1/Z_2$
  - d) Find the Conjugates of  $Z_1$  and  $Z_2$
  - e) Find the Absolute values of  $Z_1$  and  $Z_2$  {Hint:  $Z = |Z_0| e^{i\theta}$  and  $|Z_0| = Abs[Z]$ }
  - f) Find the Argument of  $Z_1$  and  $Z_2$  {Hint:  $Z = |Z_0| e^{i\theta}$  and  $\theta = Arg[Z]$ }

# **Mathematical Modeling**

**8.** Variation of Petrol Prices with Time is represented in the following table.

Time (Year)	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Petrol Price (Rs:)	48	50	55	55	59	69	82	91	102	110	115	120

Time (Year)	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Petrol Price (Rs:)	48	50	55	55	59	69	82	91	102	110		

The First row in the above table represent Time (in years) and the second row represent Average Prices of Petrol (in Rupees).

(a) Enter the above data into a table form as Petrol Price vs Time.

```
y = \{1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, \dots \} Or y = Table[i, \{i, 1998, 2019\}] p = \{48, 50, 55, 55, 59, 69, 82, 91, 102, 110, \dots \} yp = Transpose[\{y, p\}] TableForm[yp]
```

(b) Plot the graph of Petrol Prices vs Time.

g1 = ListPlot[yp]

```
g1 = ListPlot[yp, PlotStyle -> PointSize[0.02]]
g1 = ListPlot[yp, PlotStyle -> {PointSize[0.02], Hue[0.7]}]
```

(c) Find the suitable equation passing through the points of the above graph.

#### Method I:

```
fyp = Fit[yp, {t, 1}, t] - This is for a Linear (Simple) Function
fyp = Fit[yp, {t^2, t, 1}, t] - This is for a Quadratic Function
fyp = Fit[yp, {t^3, t^2, t, 1}, t] - This is for a Cubic Function
....
fyp = Fit[yp, {t^100, t^99, t^98, ... t^2, t, 1}, t] - This is for a 100<sup>th</sup>
order Polynomial Function.
If you want to plot the corresponding function then,
g2 = Plot[fyp, {t, 1998, 2019}, PlotStyle -> Hue[0.9]]
Or
g2 = Plot[fyp, {t, y[[1]], Last[y]}, PlotStyle -> Hue[0.9]]
```

#### Method II:

**fyp** = InterpolatingPolynomial[yp, t] - This is for finding Interpolating function in Mathematica (we can not control this function like in Method I)

#### **Additional Method:**

 $\mathbf{fyp} = \mathbf{Interpolation[yp]} - This is for Interpolation and the function is not displayed in Mathematica.$ 

If you want to find some value using the above Interpolation then,

fyp[2001] fyp[2001.5]

**fyp[2020]** – If the value in the argument is not in the defined data range (not included in the domain) Mathematica will fit an extrapolation function to get the corresponding function value.

(d) Plot the above graph in part (a) and the equation obtained in part (c) in the same graph by using different two colours.

Show[{g1, g2}]

(e) Find the Average Price of Petrol in 2018, 2019, 2020 and 2050 using the equation in part (c).

fyp /. 
$$t \rightarrow \{2018, 2019, 2020, 2050\}$$

(f) Find the corresponding year when Average Price of Petrol is Rs: 500.00 using the equation in part (c).

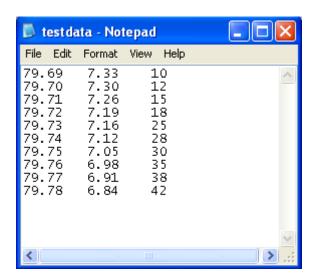
Solve[fyp == 500, t]

# **Reading and Writing Files**

Mathematica has in-built functions for file handling. i.e.; reading and writing files.

#### (1.) Reading a file

Consider the following text file at "D:\\t02\\testdata.txt" which contains three data columns.

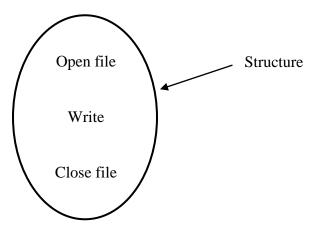


We can read the above file using the following command.

#### a = ReadList["D:\\t02\\testdata.txt", Number, RecordLists -> True]

This reads the "testdata.txt" file of the **Number** format. Here the option setting "**RecordLists -> True**" makes ReadList create separate sub lists for objects that appear in separate records.

# (2.) Writing a file



# f = OpenWrite["D:\\AS2018555 \\writedata.txt"]

- This opens a file "writedata.txt" to write data into it.

Write [f, data1, data2,...] - This writes a sequence of data to the specified file "f".

**Close[f]** - This indicates the end of the writing.

Eg:- Write the above read list "a" to a file writedata.txt as

# Reading Graphic and Sound files

**Mathematica** allows us to export and import graphics and sound files also. Built-in command **Import** enables us to read graphics and sound files while using the command **Export** we can write graphics and sound files in to a given location.

# Reading a Graphic file

a = Import ["D:\\ Graphics\\ picture. jpg"] This imports an image stored in jpg format

Show [a] This shows the imported graphics

# Reading a Sound file

 $b = Import \ [\text{``D:} \setminus Sound \setminus sound 1. \ wav"] \qquad This \ imports \ an \ sound \ stored \ in \ \textbf{wav} \ format$ 

Show [b] This is for playing the wave file

# **Exporting a Graphic file**

Export ["D:\\ mypicture.jpg", a] This exports the imported graphic "a" in to the

given location with the jpg format.

Export ["D:\\ mypicture.bmp", a] This is for convert **jpg** format to **bmp**.

(\* Check the size of the files \*)

# Exporting a Sound file

Export ["D:\\ mysound.wav", b] This exports the imported sound file "b" in to the

given location with the wav format.

Export ["D:\\ mysound.mp3", b] It can not convert way to mp3.

# Writing and Reading Data files using Export and Import commands

In addition to conventional method for writing data files using "Write" command, **Mathematica** allows us to write data files using "Export" command. Here we use special data format types namely **tsv** and **csv**. Any file with "tsv" format within the Export command writes data separated with space into a text file and any file with "csv" format writes data into an Excel sheet.

 $d = \text{Table } [\{i, i^2, i^3\}, \{i, 1, 2, 0.01\}]$  This is our data table to be written

 $d2 = \text{Table } [\{i, i^2, i^3\}, \{i, 1, 200\}]$  This is our data table to be written

# **EXPORT**

 $x = \text{Export ["D:\my}\write.tsv", d]$  This writes above data file into a Text file

 $y = \text{Export ["D:\my\write.csv", d]}$  This writes above data file into a Excel file

#### **IMPORT**

Import ["D:\\my\\write.tsv"] This reads the file within the inverted commas

Import ["D:\\my\\write.csv"] This reads the file within the inverted commas

#### Solving Differential Equations using Mathematica

We can solve ordinary differential equations using **Mathematica**. It allows us to solve first order, second order differential equations.

DSolve [differential\_equation, y[x], x] This solves a differential equation for the function y with independent variable x.

Examples:-

Solve the following first order differential equation.

• 
$$\frac{dy(x)}{dx} = k \ y(x)$$
 ; where k is a constant

DSolve[y'[x]] = k \* y[x], y[x], x] {Mathematica form for solving above equation}

• 
$$\frac{dy(x)}{dx} = -k \ y(x)$$
 ; where k is a constant

Solve the following second order differential equation.

• 
$$\frac{d^2y(x)}{dx^2} = -\omega^2y(x)$$
 ; where  $\omega$  is a constant   
  $DSolve[y''[x] = -\omega^2 y[x], y[x], x]$  {Mathematica form for solving above equation}

• 
$$\frac{d^2y(x)}{dx^2} = \omega^2y(x)$$
 ; where  $\omega$  is a constant

We can also solve differential equations having boundary conditions.

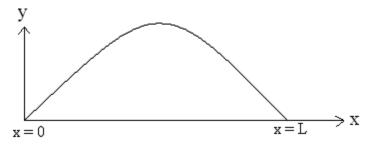
Solve the following second order differential equation with boundary conditions y(0) = 1 and y'(0) = 0.

$$\frac{d^2y(x)}{dx^2} = a\frac{dy(x)}{dx} + y(x)$$
boundary condition
$$DSolve[\{y''[x] = a * y'[x] + y[x], y[0] = 1, y'[0] = 0\}, y[x], x]$$
equation

Using above methods we can solve some famous problems in Physics. Some of them are as follows,

- (1) Wave Equation
- (2) Heat Conduction Equation
- (3) Laplace Equation

#### 1-D Wave Equation



$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

Boundary conditions are,

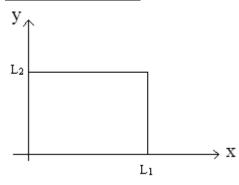
$$y[0, t] = 0$$

$$y[L, t] = 0$$

And the initial condition is,

$$y[x, 0] = f(x)$$

# **2-D Wave Equation**



$$\frac{\partial^2 z}{\partial t^2} = v^2 \left( \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right)$$

Boundary conditions are,

$$z[x, 0,t] = 0$$

$$z[x, L_2,t] = 0$$

$$z[0, y,t] = 0$$

$$z[L_1,\,y,\!t]=0$$

Initial conditions are,

$$z[x, y, 0] = f(x,y)$$
 and  $z[x, y, 0] = 0$ 

#### 1-D Heat Conduction Equation

$$\theta = 0 \qquad \qquad \theta = 0$$

$$\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}$$

Boundary conditions are,

$$\theta[0, t] = 0$$
  
$$\theta[L, t] = 0$$

And the initial condition is,

$$\theta[x, 0] = f(x)$$

#### 2-D Heat Conduction Equation

$$\frac{\partial \theta}{\partial t} = k \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)$$

#### **Laplace Equation**

$$\nabla^2 u = 0$$

Here  $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$  and u may be gravity potential, magnetic potential or electrostatic potential which satisfies the Laplace equation.

# Tute 03

- Q1. Lengths of the two sides of a rectangular triangle are 5cm & 8 cm. Find the length of the diagonal & the corresponding angles.
- Q2. Do the followings.
  - i Write an expression for the sum of the square values of 1 to n. i:e;  $\sum_{i=1}^{n}i^{2}$
  - ii Using the above, find the value for the summation of square value of 1 to 10 and 1 to 100.
  - iii Find the values of these expressions,

(a) 
$$\sum_{i=1}^{10} i$$
 (b)  $\sum_{i=1}^{1000} i$  (c)  $\sum_{i=1}^{10000} i$  (d)  $\sum_{i=1}^{100000} i$ 

iv Find the summation, 
$$\sum_{i=1}^{3} \sum_{j=1}^{i} x^{i} y^{j}$$

Q3. Find the solutions or numerical approximations of the following polynomial equations. (Solutions for 25 digit precision)

i 
$$x^5 + 7x + 1 = 0$$

ii 
$$x^3 + x + 1 = 0$$

iii 
$$x^5 - 6x^3 + 8x + 1 = 0$$

iv 
$$\sqrt{x} + \sqrt{x+1} = a$$
; where 'a' is a constant.

**Q4.** Find solutions or numerical solutions of the following polynomial equations

$$i$$
  $x + y = 2$ ,  $x - 3y + z = 3$ ,  $x - y + z = 0$ 

ii 
$$a \times y = 0$$
,  $x + (1 - a) y = 0$ , where 'a' is a constant

- iii  $x^2 + y^2 = 1$ , x + y = a, where 'a' is a constant
- Q5. Draw the following graphics.
  - (i)  $g1 = Show [ Graphics [ Circle[{0, 0}, 1] , AspectRatio -> Automatic , Axes -> Automatic ] ]$
  - (ii)  $g2 = Show [ Graphics [ Circle[{0, 0}, {2, 1}] , AspectRatio >Automatic , Axes -> Automatic ] ]$
  - (iii)  $g3 = Show [ Graphics [ Circle[{0, 0}, 2, {2, 3}] , AspectRatio >Automatic , Axes -> Automatic ] ]$
  - (iv) Show[g1, g2, g3]

Draw a circle centered whose center is (0,0) and radius is 5 cm. Find whether the following points are inside the circle or outside the circle.

Points are :- 
$$(1, 3)$$
 ,  $(5, 6)$  and  $(10, 4)$ 

 $\mathbf{Q6.}$  (A) Following are the marks obtained by 10 students in their A/L examination, Find their z-score values.

Subject 
$$1 = 67,78,69,80,56,88,75,66,72,78$$

$$Subject_2 = 83,79,75,88,86,92,80,78,82,87$$

Subject 
$$3 = 56,67,60,70,74,68,60,77,63,65$$

Hints:

Mean, 
$$\frac{1}{x} = \frac{\sum_{i=1}^{n} X_i}{N}$$

Standard Deviation, 
$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} x_i^2 - \left(\frac{\sum_{i=1}^{N} x_i}{N}\right)^2}{N-1}}$$

$$Z_{c1} = \frac{X_i - \overline{X}}{\sigma}$$

Z-score for one student  $Z_c = (Zc1+Z_{c2}+Z_{c3})/3$ 

If Zc < 0,  $Zc \rightarrow 0$  and If Zc > 0,  $Zc \rightarrow Zc$ 

(B) Using above hints, find the z-scores for the Advanced Level Results given in the following location, "D\\data\\ALMarks.txt".

100001	7	42	33
100002	9	47	16
100003	17	35	24
100600	67	94	96

First column of this file displays the Index Numbers while other columns display marks of Physics, Chemistry and Combined Mathematics respectively.

(C) Find the z-scores for the Advanced Level Results given in the following location, "D\\data\\ALMarks2.txt".

# Q7.

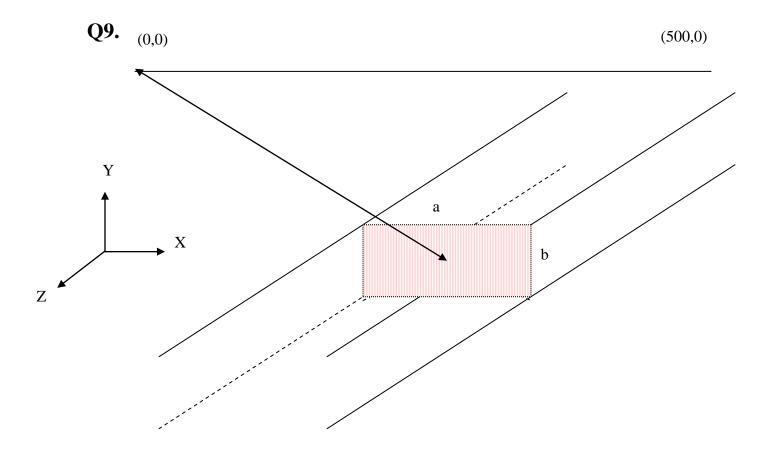


Two masses m1 & m<sub>2</sub> are apart as shown in the figure. Find the gravitational force between the two masses. Given m1 = 3kg , m<sub>2</sub> = 5kg, x<sub>1</sub> = 20m, x<sub>2</sub> = 45m, y<sub>1</sub> = 10m, y<sub>2</sub> = 12m and  $G = 6.67 \times 10^{-11}~Nm^2kg^{-2}$ .

# **Q8.**



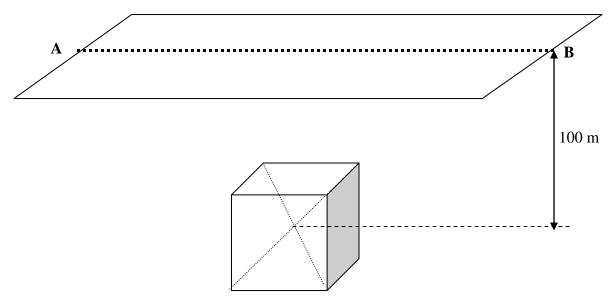
Two charges are apart as shown above. Find the electrical force between the charges. Given  $q_1=5C,\,q_2=8C$ ,  $x_1=20\text{m},\,x_2=45\text{m},\,y_1=10\text{m},\,y_2=12\text{m}$  and  $\frac{1}{4\pi\varepsilon_o}=9\times10^{-9}~Nm^2C^{-2}.$ 



Consider a 1000 km long cylinder whose cross section has a rectangular shape. Its centre is (100,100). Given a=100m, b=50m, density = 2900 kg  $\rm m^{-3}$ 

Find the Gravitational acceleration along X-axis for 100 points starting from (0,0).

**Q10.** Consider a homogeneous cubic mass distribution having its centre at a depth of 100 m directly below the AB line. The line AB lies on the surface of the Earth.



We are trying to calculate the gravity anomaly caused by a body which takes the shape of a cube. In doing this calculation, we divided the body into a large number of small cubical masses and calculate gravity anomaly due to each and add them together.

In view of the complexity of this method, if we illustrate it by calculating gravity anomaly using a crude method first and then refine it in several steps.

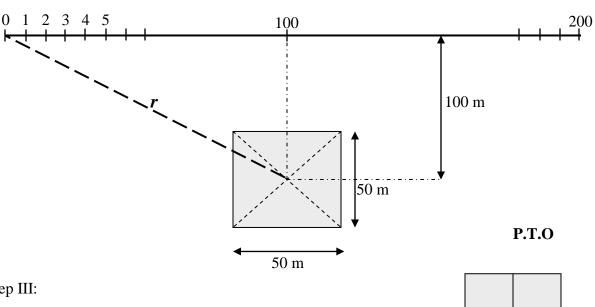
# Step I:

Consider a cubic volume of 50 x 50 x 50 m<sup>3</sup> whose centre is situated 100 m directly below the line AB on the surface of the Earth.

# Step II:

Calculate the gravity anomaly due to the mass distribution at equal intervals of 1.0 m at 200 points on the line AB.

[Density of the cube is 3300 kgm<sup>-3</sup> and Universal Gravity Constant,  $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$ 



# Step III:

Now divide the mass into four equal parts by a vertical plane and by a horizontal plane.



# Step IV:

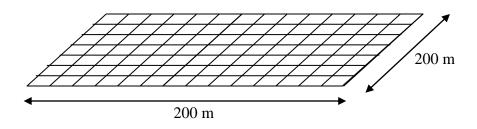
Calculate gravity anomaly due to each cube and add them together.

#### Step V:

Extend the above method by dividing cross section of the cube into  $\mathbf{n} \times \mathbf{n}$  equal parts.

#### Step VI:

Calculate the gravity due to this body at fields points situated at the nodes of the rectangular grid of 200 m x 200 m.



#### Step VII:

Divided the body into  $\mathbf{n} \times \mathbf{n} \times \mathbf{n}$  small and equal cubes. And calculate gravity anomaly due to each cube at the nodes of the above grid and hence, determine the gravity anomaly due to the large cube on the same grid.

# Step VIII:

Calculate the vertical component of gravity anomaly due to the mass distribution.

\*\*\*\*\*\*\*\*\*\*

**Q11.** There is a future threat of melting the glaciers in the polar regions of the Earth as a consequence of global warming. Ecologists have predicted that before the end of year 2050, the glaciers in the Polar Regions will melt entirely. The following table depicts the variation of the temperature and the prevailed carbon dioxide percentage of the environment with time for the last 27 years in the Arctic region of the earth. Here given values are the annual averages of daily records.

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991
Average									
Temperature	-40.0	-38.1	-38.0	-37.0	-36.4	-35.7	-35.2	-34.3	-33.9
(°C)									
Environment									
$CO_2$	0.5	1.0	1.9	2.0	3.0	3.9	4.2	5.0	5.2
percentage									

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Average Temperature (°C)	-33.0	-32.2	-31.0	-29.7	-30.5	-30.1	-28.6	-28.0	-27.8
Environment CO <sub>2</sub> percentage	6.0	7.1	7.9	8.9	9.1	10.1	11.3	11.1	12.0

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Average									
Temperature	-27.0	-26.1	-25.6	-24.9	-24.0	-23.6	-22.9	-22.0	-21.5
(°C)									
Environment									
$CO_2$	12.9	13.1	14.1	15.2	15.3	16.5	16.9	18.0	18.9
percentage									

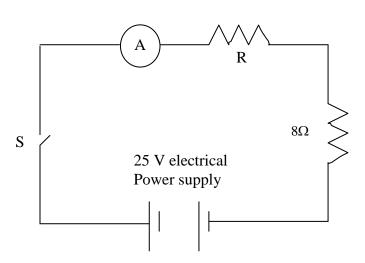
- (1) Make 2D arrays for the average temperature and the corresponding year and CO<sub>2</sub> percentage of the environment and the corresponding year using above data table.
- (2) Plot the graph of average temperature vs time (year).

- (3) Plot the graph of CO<sub>2</sub> percentage vs time (year).
- (4) Plot the above graphs on a same page using two different colours.
- (5) Find a reasonable linear equation (y = mx + c) considering the variation of the average temperature with time.
- (6) Find a reasonable linear equation (y = mx + c) considering the variation of the  $CO_2$  percentage with time.
- (7) Find a suitable cubic equation ( $y = ax^3 + bx^2 + cx + d$ ) considering the variation of the average temperature with time.

P.T.O

- (8) Find a suitable cubic equation ( $y = ax^3 + bx^2 + cx + d$ ) considering the variation of the CO<sub>2</sub> percentage with time.
- (9) Using the equations in part (5) and (7) determine the year in which the melting process of ice begins in the Arctic region in each situation. (melting process of ice begins when the temperature of the environment is  $0^{\circ}$ C)
- (10) Find the amount of increase in  $CO_2$  percentage by using part (6) and (8) in each year the melting process begins.

Q12.



25 V electrical power supply whose internal resistance is 8 ohms is connected to an external resistor R. When its value equal to the values given in the following table,

- Find, Current (I) flowing through the resistor R, Voltage (V) across the resistor R, Electrical power (P) of resistor R.
- 2. Complete the following table using Mathematica.

R (ohms)	I ( A )	V (Volts )	P ( W )
2			
4			
6			
8			
10			

•		
•		
•		
45		
50		

3. Draw the graphs of I vs R, V vs R and P vs R.

And draw the above graphs in the same page using three colours.

4. Find the suitable polynomial equations for the graphs I vs R, V vs R and P vs R.

$$\text{Hint} : I = \frac{25}{(8+R)}, \quad V = IR \quad and \quad P = VI.$$

- Q11. A human skull excavated from an archeological site, when brought near a counter  $\mathbf{R}$  records, 16 counts per minute. Whereas, the skull of human died recently records 21 counts per minute on the same counter  $\mathbf{R}$ . (In both instances the counts on  $\mathbf{R}$  are due to the radioisotope  ${}^{14}_{6}C$  in the human skulls). However, when  $\mathbf{R}$  is exposed to the environment away from any specimen, it records 06 counts per minute.
  - (i). Estimate the age of the human skull excavated from the archaeological site.
  - (ii). Find the corresponding time in years when the skull of human records 06 counts per minute on the same counter **R**?

{Hints: 
$$C(t) = C_o e^{-\lambda t}$$
,  $T_{\frac{1}{2}} = \frac{\ln[2]}{\lambda}$  and the half-life of  ${}_{6}^{14}C$  isotope is 5730 years }

# **Input Box**

• Find the volume of the rectangular body of sides a, b and c.

```
a=5;
b=10;
c=12;
vol=a*b*c;
Print["Volume of the Body: ",vol, " m^3"]

OR

(* Input Part *)
a = Input["Enter the length of the body (in meters): "]
b = Input["Enter the width of the body (in meters): "]
c = Input["Enter the height of the body (in meters): "]

(* Calculation Part *)
vol = a*b*c;

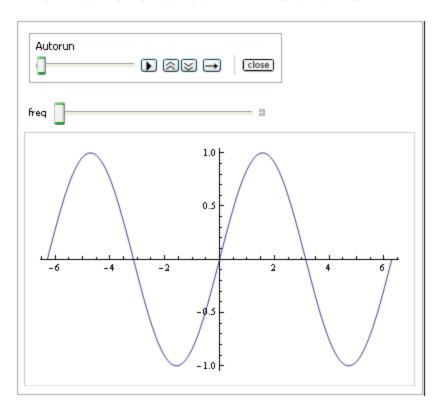
(* Output Part *)
Print["Volume of the Body: ",vol, " m^3"]
```

# Manipulation

The single function **Manipulate** gives immediate access to a huge range of powerful interactive capabilities. For any expression with symbolic parameters, **Manipulate** function automatically creates an interface for manipulating the parameters. **Manipulate** function supports not only mouse and keyboard manipulation, but also game pads and other devices.

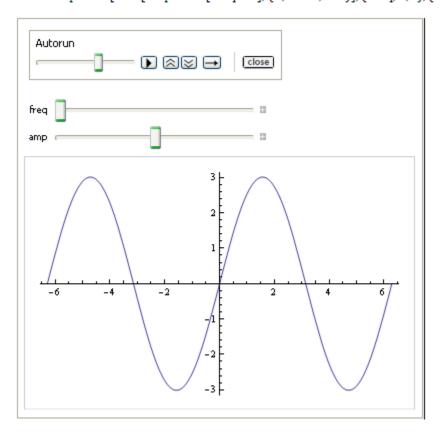
# 1) Manipulate frequency

Manipulate [Plot[Sin[freq  $\times x$ ], {x, -2Pi, 2Pi}], {freq, 1,5}]



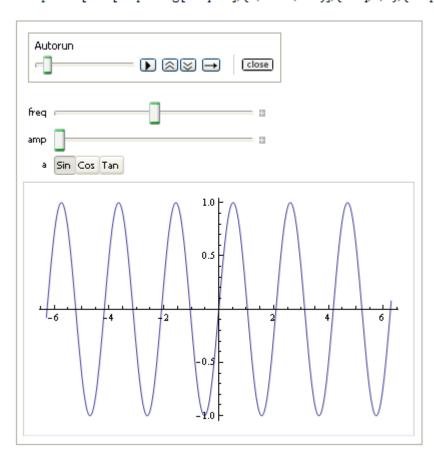
# 2) Manipulation amplitude

Manipulate [Plot[amp \* Sin[freq \* x], {x, -2Pi, 2Pi}], {freq, 1,5}, {amp, 1,5}]



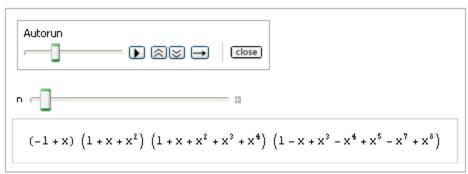
# 3) Trigger (Many graphs can be plotted using a single command, "trig")

 $Manipulate[Plot[amp*trig[freq*x], \{x, -2Pi, 2Pi\}], \{freq, 1, 5\}, \{amp, 1, 5\}, \{trig, \{Sin, Cos, Tan\}\}]$ 



#### 4) Manipulate a parameter in discrete steps

# Manipulate[Factor[ $x^n - 1$ ], {n, 10, 100, 1}]



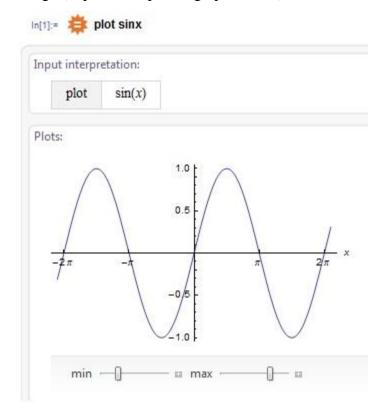
#### THE NEW FEATURES IN Mathematica 8 ARE GIVEN BELOW:

#### 1) Integration with Wolfram|Alpha

*Mathematica* 8 provides free-form linguistic input, allowing users to enter plain English and get immediate results and the *Mathematica* input for further exploration (without the need for syntax).

A notebook in Mathematica should be opened, *a box with a plus sign* will appear at the top left side corner. Then click over the box and chose the "Wolfram Alpha query" option and an equal sign inside a star will appear. Next you can type the necessary tasks in English which you have to do.

#### eg. 1) plot sinx (plots a graph of sinx)



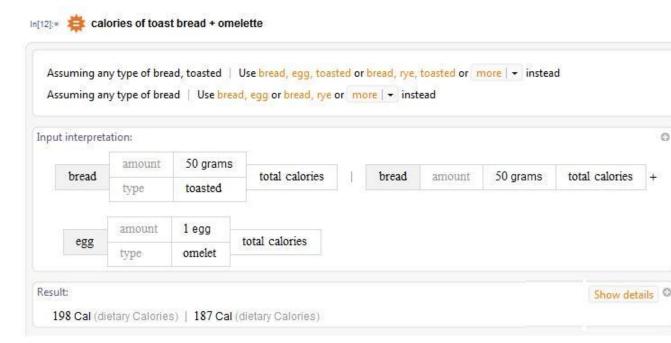
eg. 2) weather at panadura (weather of panadura, weather in panadura are also accepted)

This gives the present situation and the future forecast of weather at Panadura



eg. 3) calories toast bread + omelette

This gives the Chart of Calories, Cholesterol, Protein,... etc. obtained by eating a toast bread and omelette



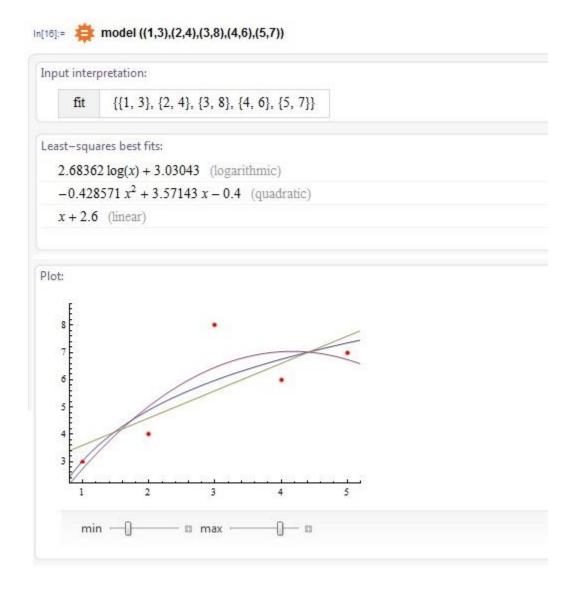
#### eg. 4) location of pidruthalagala

The location of Piduruthalagala mountain will be displayed in a Sri Lankan map



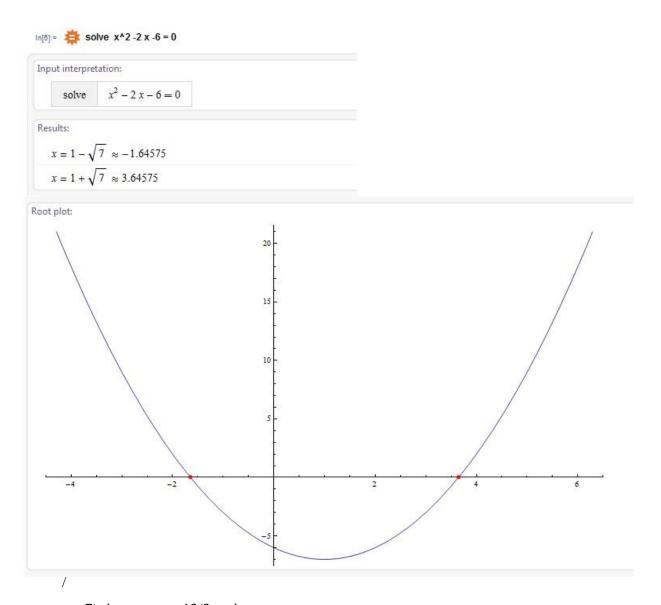
#### eg. 5) model ((1,3),(2,4),(3,5),(4,6),(5,7))

The suitable equation (fitted polynomial) which passes through the points (1, 3), (2, 4), (3, 5), (4, 6) and (5, 7) will be displayed.



#### eg. 6) solve $x^2 - 2x - 6 = 0$

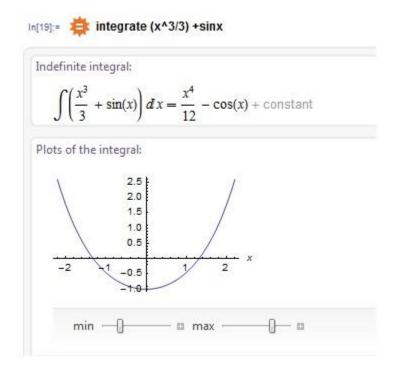
This gives the roots of this equation with the root plot where the curve intersects



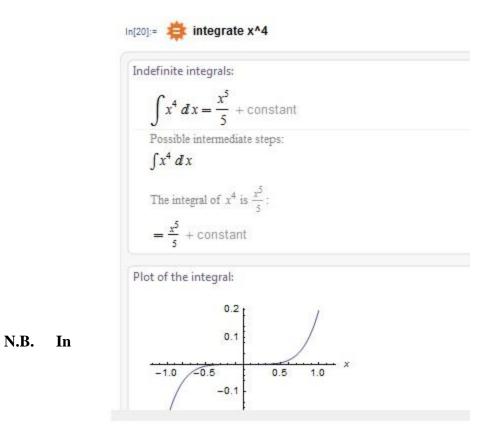
eg. 7) integrate  $x^3/3 + \sin x$ 

This gives the answer of  $\int \left(\frac{x^3}{3} + \sin x\right) dx$  integration. If we need

the steps how the final answer was obtained, click the "show steps" hyperlink command which is given in the output box.



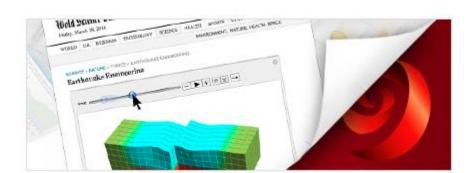
The intermediate steps of the integration of  $x^4$ :



addition to the above methods, the above examples can also be done using the website <a href="https://www.wolframalpha.com">www.wolframalpha.com</a>.

#### 2) Computable Document Format (CDF)

By using a CDF, can download the entire source code for our required applications. It provides the source code for that application in which we can make any changes in the variables according to our requirements.

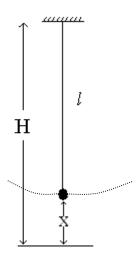


#### 3) Wolfram CDF Player

It interacts with applications from the Wolfram Demonstrations Project and views Mathematica examples, reports and files even whenever Mathematica software is not installed in that computer.

### Tute 04

01. Consider a simple pendulum which oscillates with very small angles.



We can write the periodic time of the pendulum as,

$$T=2\pi\sqrt{\frac{l}{g}}\;,$$

Where l = length of the string, H = Height of the room and g = gravitational acceleration

The file "D:\\T4Files\\simphm.txt" contains the values of x and  $T^2$ .

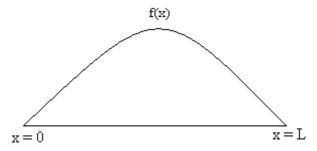
- a) Read the above file.
- b) Draw the graphs of  $T^2$  vs x.
- c) Find the gravitational acceleration using part (b).
- 02. Consider the function f(x) = x, where  $x = \{-10, -9, -8, ..., 8, 9, 10\}$ .
  - a) Find the discrete Fourier transform of f(x).
  - b) Find the real part of the f(x) and plot it.
  - c) Find the imaginary part of the f(x) and plot it.
  - d) Find the inverse Fourier transform of the result of part (a).
- 03. Consider the following data sets
  - (i)  $\{0.,0.58,0.95,0.95,0.58,0.,-0.58,-0.95,-0.95,-0.58\}$
  - (ii)  $\{1.,0.80,0.30,-0.30,-0.80,-1.,-0.80,-0.30,0.30,0.80\}$
  - (iii)  $\{0.,1.53,0.36,0.36,1.53,0.,-1.53,-0.36,-0.36,-1.53\}$
  - (iv)  $\{0.,1.53,1.53,0.36,-0.36,0.,0.36,-0.36,-1.53,-1.53\}$
  - (v)  $\{2.,1.11,-0.5,-1.11,-0.5,0.,-0.5,-1.11,-0.5,1.11\}$
  - (vi) {1.,1.39,1.26,0.64,-0.22,-1.,-1.39,-1.26,-0.64,0.22}
  - (vii) {1.,-0.41,1.95,-0.04,1.58,-1.,0.41,-1.95,0.04,-1.58}
  - (viii) {0.,-0.36,1.53,1.53,-0.36,0.,0.36,-1.53,-1.53,0.36}

Plot the above data sets as Lists. And do the following for all data sets.

- (a) Find the discrete Fourier transform of f(x).
- (b) Find the real part of the f(x) and plot it.
- (c) Find the imaginary part of the f(x) and plot it.
- (d) Find the inverse Fourier transform of the result of part (a). And compare that Inverse Fourier Transform with Original Data Set in Part(i) to Part(viii).

#### **Animations**

O1. A string with length L is tied at x = 0 and x = L. At t = 0, the string is slowly released giving initial shape f(x). Animate the wave function in the time interval t=0 to t=100 s.



1-D wave equation is as follows,  

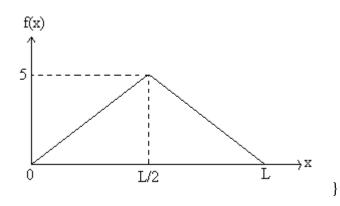
$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}, \quad \text{Here } v = \sqrt{\frac{E}{\rho}}$$

The solutions of the above 1-D wave equation can be written in the following form

$$y(x,t) = \sum_{n=1}^{\infty} a_n sin\left(\frac{n\pi x}{L}\right) cos\left(\frac{n\pi vt}{L}\right)$$
where

$$a_n = \left(\frac{2}{L}\right) \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

Hints: You are given the following values, L=1 m,  $E=130 \times 10^8$  Nm<sup>-2</sup>,  $\rho=8960$  kgm<sup>-3</sup> for a Copper string and consider n goes from 1 to 10. And also f(x, 0) is given below:



**02.** The 1-D wave equation is given in the following form,

 $y = A\sin(kx + \omega t)$ , where A = Amplitude of the wave

 $k = \frac{2\pi}{\lambda}$  (This is called the wave number and  $\lambda$  is the wave length)

 $\omega = 2\pi f$  (Angular frequency of the wave and f is the frequency of the wave)

If A = 5, f = 50 Hz and k = 10 m<sup>-1</sup>, find the displacement y after time t = 100 s and animate the wave function in the time interval t = 0 to t = 100 s.

**03.** Animate the following function in the time interval t = 0s to t = 50 s using appropriate ranges for x and y.

$$f(x, y, t) = \sin(x \times y + t),$$

where  $x \in [0,3]$ ,  $y \in [0,3]$  and time  $t \in [0,50]$  with time intervals 0.1 s.

**04.** Animate the following function in the time interval t = 0 to t = 1 s using below ranges for x and y.

$$f(x, y, t) = \sin(x \times y + t),$$

where  $x \in [0,3]$ ,  $y \in [0,3]$  and time  $t \in [0,1]$  with time intervals 0.001s.

\*\*\*\*\*\*\*\*\*\*\*\*

### 2<sup>nd</sup> YEAR – PHYSICS PRACTICAL GUESS PAPER Computational Physics Practical

Time: One and half hours

#### Mathematica Software must be used for following all activities

Variation of Bus Fares form Panadura to Nugegoda with Prices of Petrol, Prices of Diesel & Time are represented in document of given location, "D\\data\\oilprice.txt". Sketch of this document is given in a following form.

1987	14.00	9.00	8.00
1988	17.00	9.00	8.00
1989	19.00	9.00	8.00
2006	102.00	70.00	21.00
2007	110.00	80.00	22.00
2008	115.00	90.00	25.00
2009	120.00	100.00	29.00

First column of this document is displayed time & other 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup> columns are displayed Average Prices of petrol, diesel & bus fares are respectively.

- 1. Read the document, as a data table in "D\\data\\oilprice.txt".
- 2. (a) Draw the graph of Petrol Prices vs Time (years).
  - (b) Find the suitable Cubic Equation ( $p = a t^3 + b t^2 + c t + d$ ) passing through the points of above graph.
  - (c) Plot the above graph & obtained cubic equation in a same graph paper by using various two colours.
  - (d) Find the Average Price of Petrol in 2008 & 2009 using your cubic equation.
  - (e) Give your comments for Price of Petrol in part D.
- 3. (a) Plot the graph of Diesel Prices & Bus Fares by using the above document.

- (b) Decide the suitable & reasonable polynomial  $(y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n)$  passing through the data points of above graph. And also, find the polynomial.
- (c) Find the Bus Fares when Diesel Prices are Rs. 100.00, 120.00, 140.00, 160.00, 180.00 & 200.00 by using above polynomial.
- 4. (a) Plot the graph of Petrol Prices vs. Bus fares by using read document.
  - (b) Can you conclude that Prices of Petrol are increasing when Diesel Prices are increasing by studying above graph? Explain your answer.
- 5. (a) Draw the graphs of Bus Fares with Prices of Diesel and Prices of Petrol in the same page by using various two colours.
  - (b) We know the Bus Fares are not increasing although Prices of Petrol are increasing & Prices of Diesel are not. By studying above graphs, can you conclude the Bus Fares are not increasing when Prices of Petrol are increasing? Explain your answer.

\*\*\*\*\*\*\*

# 2<sup>nd</sup> YEAR - PHYSICS PRACTICAL EXAMINATION Computational Physics Practical Paper

Time: one & half hours

#### The Use of Mathematica software for all the following tasks is requied

The following table display the sales of five (05) brands of biscuits sold by the Ceylon Biscuits (Pvt) Limited in the past five years. The figures are listed in the table in millions.

Year Biscuit (Brand)	2002	2003	2004	2005	2006	2007
Mari	14.0	15.0	16.5	?	16.2	16.7
Nice	13.0	14.1	15.2	?	15.0	14.9
Lemmon- Puff	20.0	21.0	23.0	?	23.5	25.0
Chocolate	15.0	14.0	13.0	?	11.0	11.5
Cream Cracker	25.0	28.0	30.0	?	39.0	42.0

Table - 01

By using the amount of sales for years 2002, 2003, 2004, 2006 and 2007, determine the sales of biscuits for 2005 by using the following approaches,

- (a) A cubic polynomial of the form,  $y = ax^3 + bx^2 + cx + d$ .
- (b) A quadratic equation of the form,  $y = ax^2 + bx + c$ .
- (c) A linear equation of the form, y = mx + c.

If the actual biscuit sales for 2005 is shown in the following table (Table-02) using that data, determine which approach mentioned above {ie: part (a), (b) and (c)} is more suitable to represent the sales for each brand of biscuits.

Furthermore, determine the amount of biscuits expected to be sold in 2008 for Mari, Chocolate and Cream-Cracker.

Mari	16.0
Nice	14.9
Lemmon-Puff	23.0
Chocolate	12.5
Cream Cracker	35.0

Table - 02

Are the expected amounts of biscuits sold for 2008 absolutely correct or incorrect? Write down your views.

\*\*\*\*\*\*

## 2<sup>nd</sup> YEAR - PHYSICS PRACTICAL EXAMINATION Computational Physics Practical Paper

Time: one & half hours

#### The Use of Mathematica software for all the following tasks is required

The following table shows the monthly averaged share price (book value of each share is Rs:10.00) of two companies, "SEYLAN MERCHANT LEASING LIMITED" and "SRI LANKA TELECOM LIMITED" for the past two years.

		SEYLAN	TELECOM
Month by Number	Month	LIMITED	LIMITED
01	2006 January	50.00	19.75
02	2006 February	50.00	19.50
03	2006 March	52.75	19.25
04	2006 April	51.75	19.50
05	2006 May	32.75	18.50
06	2006 June	31.75	18.75
07	2006 July	27.50	19.50
08	2006 August	26.75	20.00
09	2006 September	??.??	??.??
10	2006 October	28.00	24.25
11	2006 November	23.50	29.75
12	2006 December	16.75	29.75
13	2007 January	16.50	33.75
14	2007 February	16.50	43.25
15	2007 March	14.50	42.75
16	2007 April	13.25	39.25
17	2007 May	12.50	38.75
18	2007 June	13.25	39.25
19	2007 July	16.00	36.75
20	2007 August	16.50	36.00
21	2007 September	17.50	36.25
22	2007 October	16.50	35.00
23	2007 November	16.00	35.50
24	2007 December	17.00	33.00

Table - 01

In the above table the averaged share price in rupees for 23 months is displayed excluding the share price of the 09<sup>th</sup> month.

- 1. Input the share price of "SEYLAN MERCHANT LEASING LIMITED" company for the 23 months displayed on the table into a 2-D data array.
  - Plot the corresponding data array.
- 2. Input the share price of "SRI LANKA TELECOM LIMITED" company for the 23 months displayed on the table into a 2-D data array.
  - Plot the corresponding data array.
- 3. Determine the linear equation that correctly represents the data points relevant to "SEYLAN MERCHANT LEASING LIMITED" company.

Hence, estimate the share price for the 09<sup>th</sup> month of "SEYLAN MERCHANT LEASING LIMITED" company.

Plot the linear equation and the data points in part-1 on the same graph.

4. Determine the linear equation that correctly represents the data points relevant to "SRI LANKA TELECOM LIMITED" company.

Hence, estimate the share price for the 09<sup>th</sup> month of "SRI LANKA TELECOM LIMITED" company.

Plot the linear equation and the data points in part-2 on the same graph.

5. Determine the suitable polynomial that correctly represents the data points relevant to "SEYLAN MERCHANT LEASING LIMITED" company.

Hence, estimate the share price for the 09<sup>th</sup> month of "SEYLAN MERCHANT LEASING LIMITED" company.

Plot the suitable polynomial on the graph that includes the linear equation and the data points in part-3. Use three (03) different colours to distinguish the curves.

6. Determine the suitable polynomial that correctly represents the data points relevant to "SRI LANKA TELECOM LIMITED" company.

Hence, estimate the share price for the 09<sup>th</sup> month of "SRI LANKA TELECOM LIMITED" company.

Plot the suitable polynomial on the graph that includes the linear equation and the data points in part-4. Use three (03) different colours to distinguish the curves.

7. The following table shows the actual average share price for the 09<sup>th</sup> month of the two companies "SEYLAN MERCHANT LEASING LIMITED" and "SRI LANKA TELECOM LIMITED".

SEYLAN LIMITED	28.50
TELECOM LIMITED	21.00

Table - 02

By comparing the data in Table-02 with the values obtained from the linear equation and suitable polynomial equation deduct which is the more suitable representation for the two companies separately.

- 8. Hence estimate the average share price for the 25<sup>th</sup> month (ie; January, 2008), separately for each company.
- 9. By using the above method is it reasonable to estimate the average value of the share price for the 48<sup>th</sup> month (ie; December, 2009)? Explain your answer.

\*\*\*\*\*\*\*\*

#### Mathematica software should be used for the following exercises

The variation of the birthrate of a particular country with time is given in the following table.

Time (Years)	Birthrate
1999	0.81
2000	1.41
2001	4.07
2002	5.04
2003	5.77
2004	6.09
2005	8.29
2006	7.41
2007	8.59
2008	9.94
2009	10.7
2010	12.6
2011	14.1

- 1) Draw the graph of birthrate vs. time.
- 2) Determine a suitable model which fits the above data set.
- 3) Find the average birthrate in the years 2012 and 2013.
- 4) By using the above model, is it reasonable to estimate the average birthrate for the year 2050? Explain your answer.

#### Mathematica software should be used for the following exercises

The variation of the death rate of a particular country with time is given in the following table.

Time (Years)	Death rate		
1999	28.1		
2000	25.5		
2001	25.7		
2002	21.3		
2003	21.1		
2004	19.6		
2005	18.3		
2006	17.4		
2007	15.1		
2008	12.0		
2009	7.38		
2010	9.06		
2011	5.14		

- 1) Draw the graph of death rate vs. time.
- 2) Determine a suitable model which fits the above data set.
- 3) Find the average death rate in the years 2012 and 2013.
- 4) By using the above model, is it reasonable to estimate the average death rate for the year 2050? Explain your answer.

#### Mathematica software should be used for the following exercises

The variation of the height in feet of a child with respect to his age is given in the following table.

Age (Years)	Height (feet)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	0.94 2.150 3.25 3.64 3.83 4.14 4.63 4.64 4.65 4.65 4.64 4.75 4.78 4.82

- 1) Draw the graph of height vs. age.
- 2) Determine a suitable model which fits the above data set.
- 3) Find the average heights of the child, at the ages of 17 and 18.
- 4) By using the above model, is it reasonable to estimate the height of the child at the age 50? Explain your answer.

#### Mathematica software should be used for the following exercises

The variation of the average temperature in °C of the atmosphere of the Earth with respect to the height in km from the surface of the Earth is given in the following table.

Height (km)	Temperature		
	(°C)		
0	29.5		
1	28.0		
2	24.1		
3	20.9		
4	17.7		
5	17.8		
6	16.1		
7	13.3		
8	11.5		
9	10.6		
10	9.29		
11	7.17		
12	7.10		
13	5.34		
14	6.73		
15	4.33		

- 1) Draw the graph of temperature vs. height.
- 2) Determine a suitable model which fits the above data set.
- 3) Find the average temperature at the heights 16 km and 17 km from the surface of the Earth.
- 4) By using the above model, is it reasonable to estimate the average temperature at 70 km? Explain your answer.

#### Mathematica software should be used for the following exercises

Assuming the Mean Molecular Distance at the surface of the Earth is 1 Å, the variation of the Mean Molecular Distance in Å with height in km from the surface of the Earth is given in the following table.

Mean Molecular Distance			
(Å)			
1.00			
1.30			
1.44			
1.43			
1.65			
1.89			
2.18			
2.10			
2.66			
2.81			
3.13			
3.76			
4.17			
4.94			
5.37			
6.25			

- 1) Draw the graph of mean molecular distance vs. height.
- 2) Determine a suitable model which fits the above data set.
- 3) Find the mean molecular distance at heights 16km and 17km.
- 4) By using the above model, is it reasonable to estimate the mean molecular distance at 80 km? Explain your answer.

#### Mathematica software should be used for the following exercises

The variation of the monthly average temperature at the University premises for the last 14 months is given in the following table.

temperature (°C)
010 11
010 12
)10 13
010 14
010 14
010 14
16
16
18
18
19
20
20
)11 21

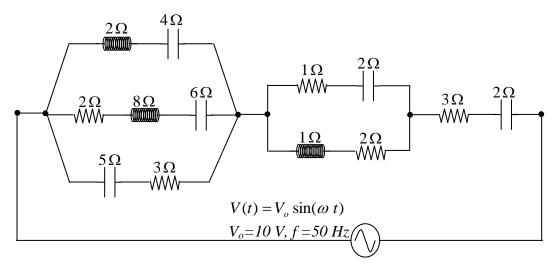
- 1) Draw the graph of temperature vs. month.
- 2) Determine a suitable model which fits the above data set.
- 3) Find the average temperature in the months, March 2011 and April 2011.
- 4) By using the above model, is it reasonable to estimate the average temperature for the month January 2014? Explain your answer.

# Department of Physics, USJP. General Degree 2<sup>nd</sup> Year Computational Physics - Assignment No I

Time: Half an hour

#### Mathematica Software must be used to do the following exercises

Using complex numbers, calculate the equivalence impedance of the following electrical (network) circuit.



Hence, obtain the equation of the resultant current i(t).

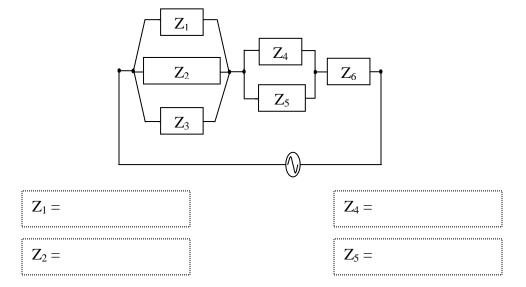
Plot the graphs of,

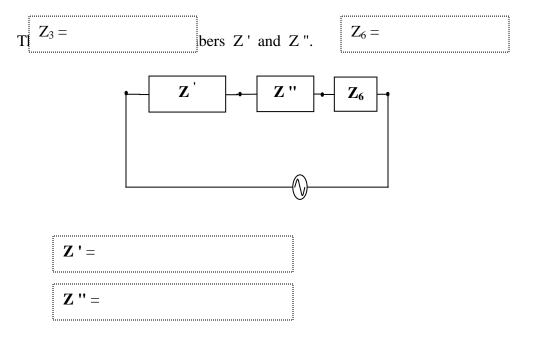
- (i) Voltage, V(t) vs. Time (t),
- (ii) Current, i(t) vs. Time (t).

Also plot the above two graphs in a same page.

#### **Hints:**

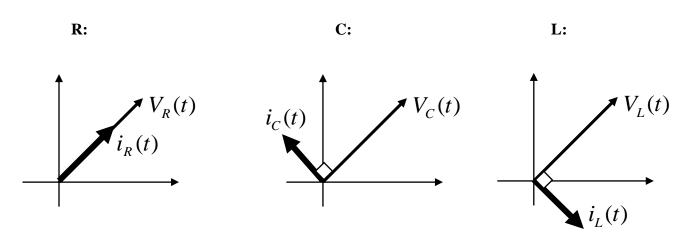
A) First, find the complex numbers  $Z_1$ ,  $Z_2$ ,  $Z_3$ ,  $Z_4$ ,  $Z_5$ , and  $Z_6$  for the above circuit.



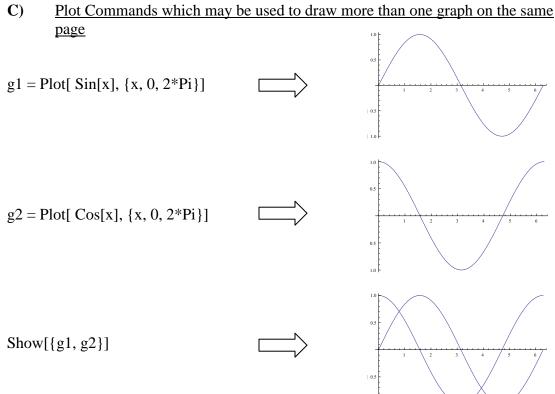


After that, find the equivalence impedance, **Z**.

#### B) Phasor Diagrams for the L, R and C circuit components, which may be used to find the current



Plot Commands which may be used to draw more than one graph on the same



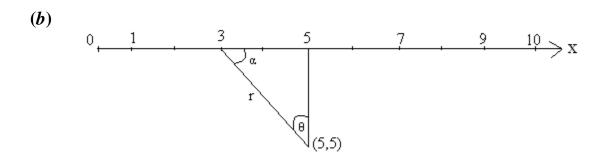
# Department of Physics, USJP. General Degree 2<sup>nd</sup> Year

#### **Computational Physics - Assignment No III**

#### Time: 45 minutes

#### Mathematica Software must be used to do the following exercises

(a) Obtain first 10 terms of the arithmetic progression whose first term is 1 and common difference is 0.5 using "Tables" in "Mathematica".



- I. Find the values of  $\mathbf{r}$  in the above diagram for  $x = 0, 1, 2, 3, 4, \dots, 10$ .
- II. Also find the angle  $\alpha$  and  $\theta$  depicted in the diagram for each point.
- (c) Consider a rectangular grid of 21 x 21 points of spacing 100 m. There is a homogenous spherical mass distribution of radius 50 m, centered at a depth of 75 m directly below the center of the grid.
  - (i) Determine the gravity field (vertical component) at all grid points.
  - (ii) Plot the contour diagram of the gravity field.
    - [ Density of the sphere 3000 kgm $^{-3}$  & Universal gravitational constant,  $G = 6.67 x 10^{-11} \, N \, m^2 \, kg^{-2}$  ]

# Department of Physics, USJP. Physics Special 3<sup>rd</sup> Year Computational Physics Practical No II

Time: One Hour

#### Mathematica Software must be used to do the following exercises

Following table represents the Production and the corresponding Income for the past twelve years of a factory started in year 1995. Here the production and the income are given by the sum of the daily production and daily income. Here Income is given in billion rupees.

Time(Year)	1995	1996	1997	1998	1999	2000	2001	2002
Income	15.0	15.1	17.8	-	20.7	21.1	23.0	23.0
Production	746	800	819	-	890	926	1005	1022

Time(Year)	2003	2004	2005	2006	2007	2008	2009	2050
Income	23.4	26.8	28.9	29.0	33.1	29.5	?	?
Production	1081	1087	1147	1185	1240	1230	?	?

- **1.** Enter the above data for the time period 1995 to 2008 with the exception of data corresponding to year 1998 in 2D arrays.
  - i.e. Time (Year) and the corresponding income

    Time (Year) and the corresponding production

If 12.5% of the income can be considered as the net profit, calculate the net profit for each year.

- **2.** (A) Plot the graph of Income vs. Time (Year)
  - (B) Plot the graph of Production vs. Time (Year)
  - (C) Plot the above two graphs in a same page using two colours.
- 3. Find appropriate Linear (y = m t + c), Quadratic ( $y = a t^2 + b t + c$ ) and Cubic ( $y = a t^3 + b t^2 + c t + d$ ) equations considering the variation of the income with the time. If the income in year 1998 is 19.2 billion rupees, determine

the suitable equation out of the above three equations. Hence find the expected income for the years 2009 and 2050.

- **4.** Find a reasonable Linear equation (y = m t + c) considering the variation of the production with the time. Goods produced in year 1998 were 870. Find the Production in 1998 using above linear equation. Then calculate the percentage of the error of your answer.
- **5.** Plot the graph of Production vs. Income. Assuming that a linear equation is suitable to represent the above data. Calculate the production when the income is 40 billion rupees. In which year the income is equal to 40 billion rupees?

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