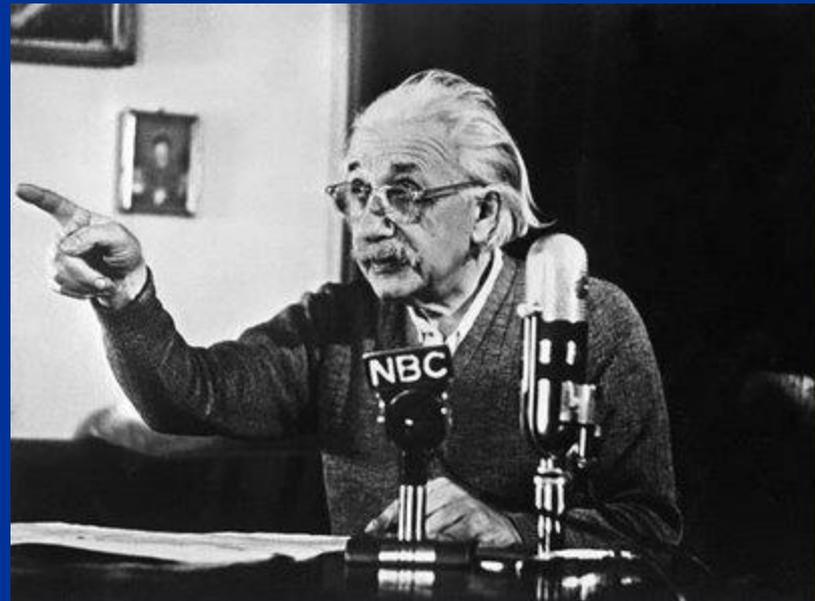


Special Theory of **Relativity**

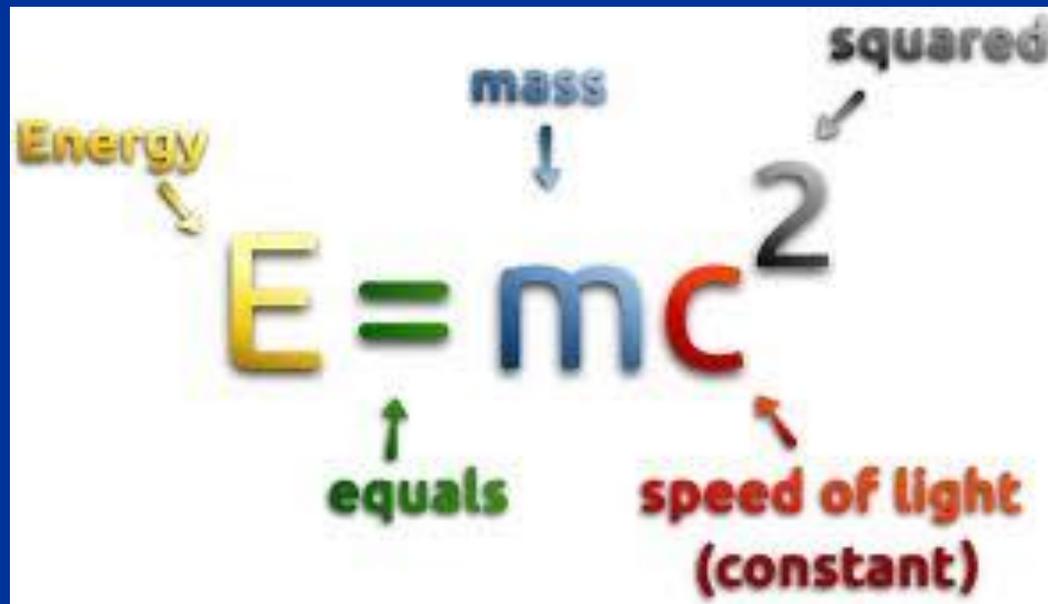


06th & 07th Lecture

The mass – energy equivalence

In Physics, **mass – energy equivalence** is the concept that the mass of a body is a measure of its energy content.

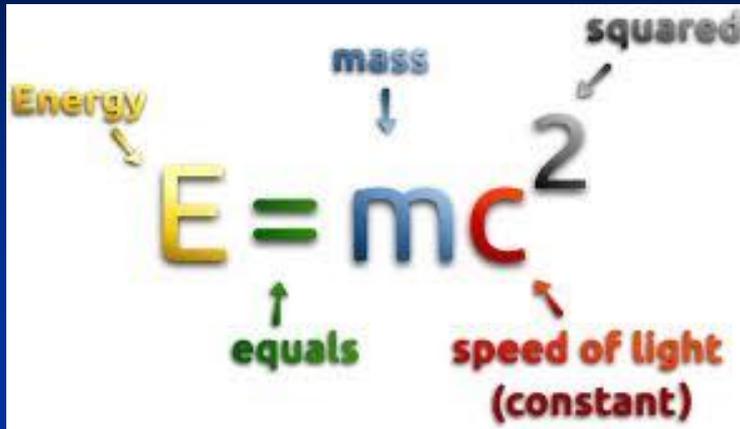
Albert Einstein proposed **mass – energy equivalence** in 1905. The equivalence is described by the famous equation,



The diagram shows the equation $E = mc^2$ with the following labels and arrows:

- Energy** (yellow text) with an arrow pointing to the letter **E**.
- mass** (blue text) with an arrow pointing to the letter **m**.
- squared** (grey text) with an arrow pointing to the superscript **2**.
- equals** (green text) with an arrow pointing to the equals sign **=**.
- speed of light (constant)** (red text) with an arrow pointing to the letter **c**.

The mass – energy equivalence



Proof of $E = mc^2$ [Einstein's Box]

This is a Hypothetical Experiment.

Find the mass-equivalence energy of a 1kg.

Using, $E = mc^2$

→ $E = (1) (3 \times 10^8)^2$

→ $E = 9 \times 10^{16} \text{ J}$

This is a very large energy. Using this energy, we can vaporize $\sim 10^{10} \text{ kg}$ of water at the room temperature (30°C)!

$$\therefore E = ms\theta + mL$$

Relativistic Energy - Momentum Equation

What is relativistic energy ???

The energy of a moving body as measured by an observer in the same frame of reference as the body.

What is relativistic momentum ???

The momentum of a moving body as measured by an observer in the same frame of reference as the body.

Derivation of the Relativistic Energy - Momentum equation :

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

$$p = m v$$



$$m = \frac{p}{v}$$

Using Einstein's equivalence Mass-Energy equation;

$$E = mc^2$$



$$E = \frac{P}{v} c^2$$



$$E = pc^2 \frac{1}{v}$$



Work done = Force x Distance



$$dE = F dx$$

Using Newton's second law,

$$F = ma$$



$$F = m \frac{dv}{dt}$$



$$F = \frac{d(mv)}{dt}$$



$$F = \frac{dp}{dt}$$

Then work done ,

$$dE = F dx$$



$$dE = \frac{dp}{dt} dx$$



$$\frac{1}{v} = \frac{dp}{dE}$$



$$dE = dp \cdot v$$



$$dE = dp \frac{dx}{dt}$$

Substitute this $1/v$ expression to equation 01;

$$E = p c^2 \frac{dp}{dE} \quad \text{because,} \quad \frac{1}{v} = \frac{dp}{dE} \quad \rightarrow \quad E = p c^2 \frac{1}{v}$$

$$\rightarrow E \cdot dE = p c^2 \cdot dp$$

Integrating the above equation;

$$\int E \cdot dE = \int p c^2 \cdot dp + k$$

$$\rightarrow \frac{E^2}{2} = c^2 \frac{p^2}{2} + k$$

To find the value k , apply a boundary condition:

If $t = 0$, v should be 0 . Therefore, $p = mv = 0$ and $E = E_0 = m_0 c^2$, Where, m_0 is the *Rest Mass* of the body.

$$\rightarrow \frac{E_0^2}{2} = k$$

$$\frac{E^2}{2} = c^2 \frac{p^2}{2} + k$$



$$\frac{E_o^2}{2} = k$$



$$\frac{E^2}{2} = c^2 \frac{p^2}{2} + \frac{E_o^2}{2}$$



$$E^2 = p^2 c^2 + E_o^2$$

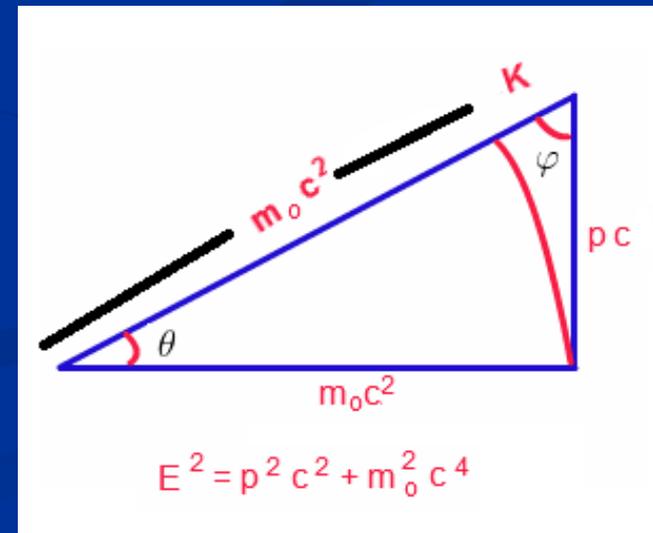


$$E^2 = p^2 c^2 + m_o^2 c^4$$

Finally we got the above relationship.

This is called,

“Relativistic Energy-Momentum equation”



“Relativistic Energy-Momentum equation”

$$E^2 = p^2 c^2 + m_0^2 c^4$$

Where,

E – Relativistic Energy

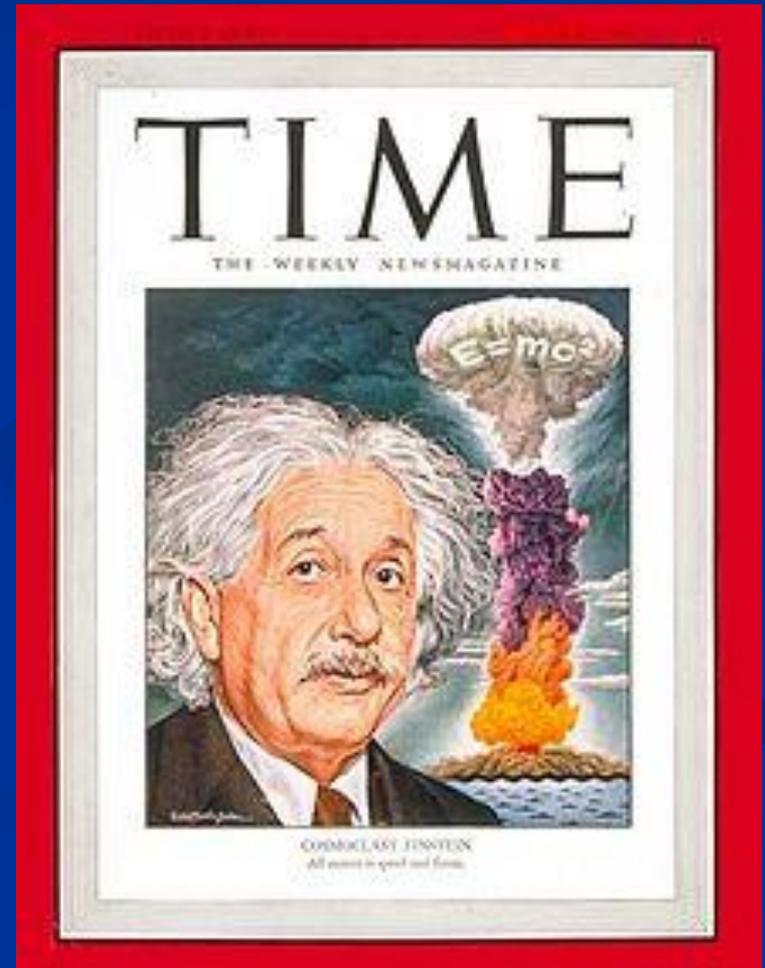
The energy of the body which is moving with velocity v .

p – Relativistic Momentum

The momentum of the body which is moving with velocity v .

m_0 – Rest Mass of the body

The mass of the body which is at rest.

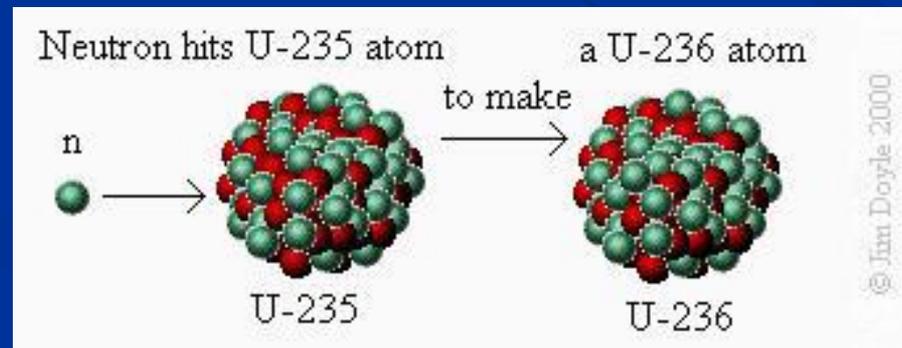


Practical Application of Equivalence of mass and energy

Nuclear fission takes place when a heavy atomic nucleus, such as uranium, breaks into two or more smaller pieces with the release of some energy. During this process some of the mass of the original atom is converted into energy in accordance with the equation $E = mc^2$.

First we fire a neutron (n) at the uranium-235 (U-235) atom so that it sticks to it. After a short while the uranium-235 splits into an atom of barium-141 (Ba-141), an atom of krypton-92 (Kr-92) and three neutrons. We can show this schematically.

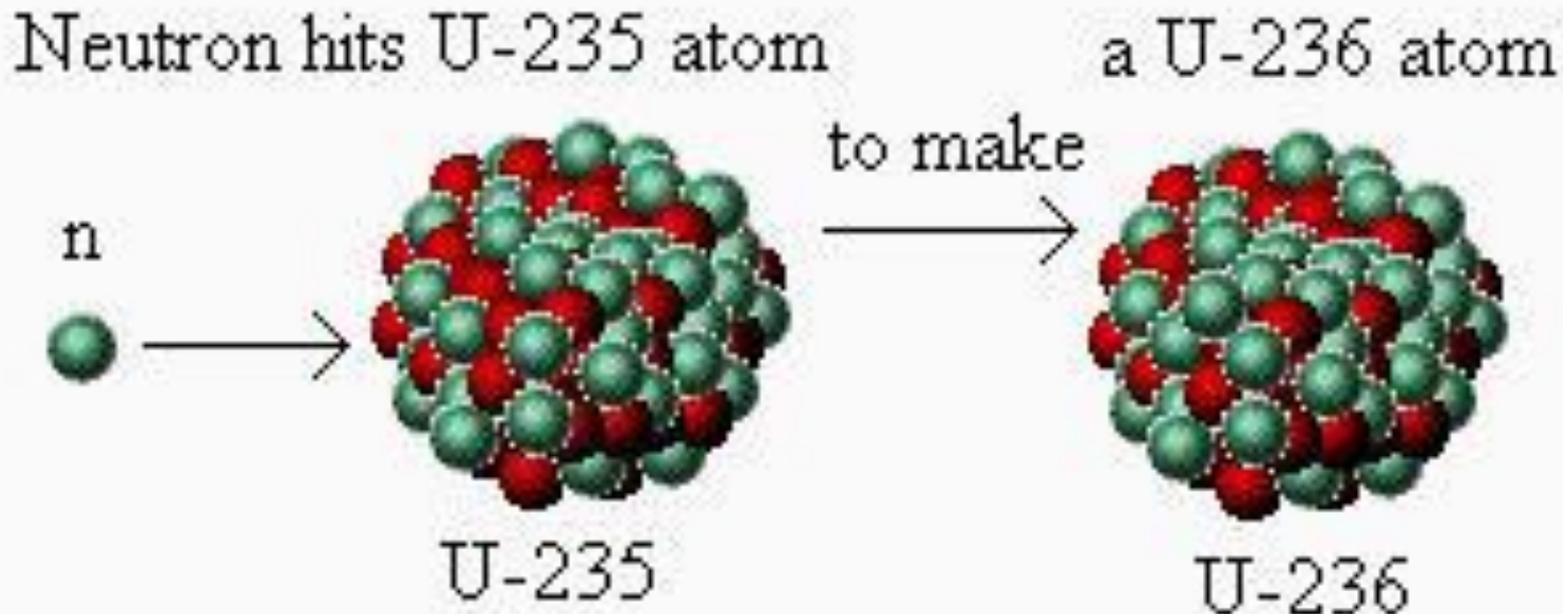
Firstly, we see a neutron striking a uranium-235 atom to make a uranium-236 atom:



Practical Application of Equivalence of mass and energy

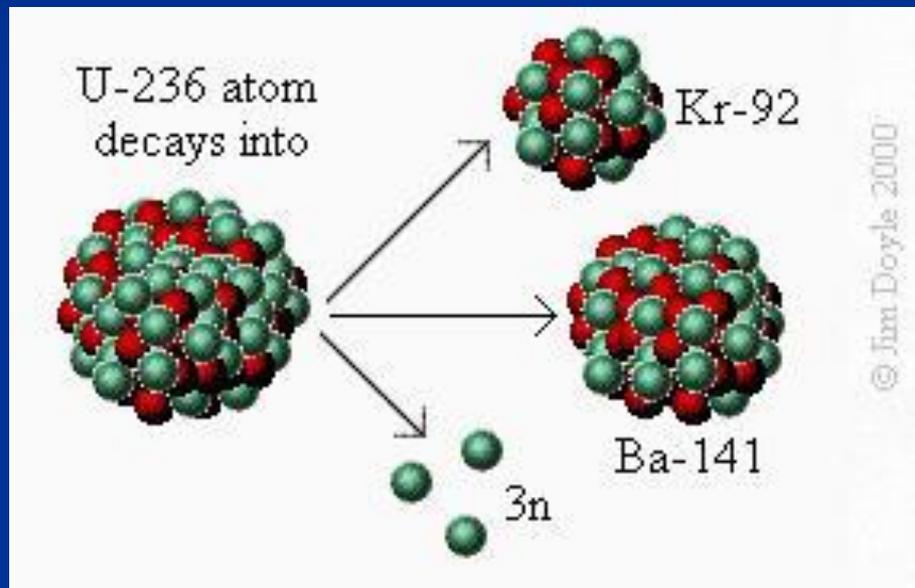
Nuclear fission takes place when a heavy atomic nucleus, such as uranium, breaks into two or more smaller pieces with the release of some energy. During this process some of the mass of the original atom is converted into energy in accordance with the equation $E = mc^2$.

First we fire a neutron (n) at the uranium-235 (U-235) atom so that it sticks to it. After a short while the uranium-235 splits into an atom of barium-141 (Ba-141), an atom of krypton-92 (Kr-92) and three neutrons. We can show this schematically. Firstly, we see a neutron striking a uranium-235 atom to make a uranium-236 atom:



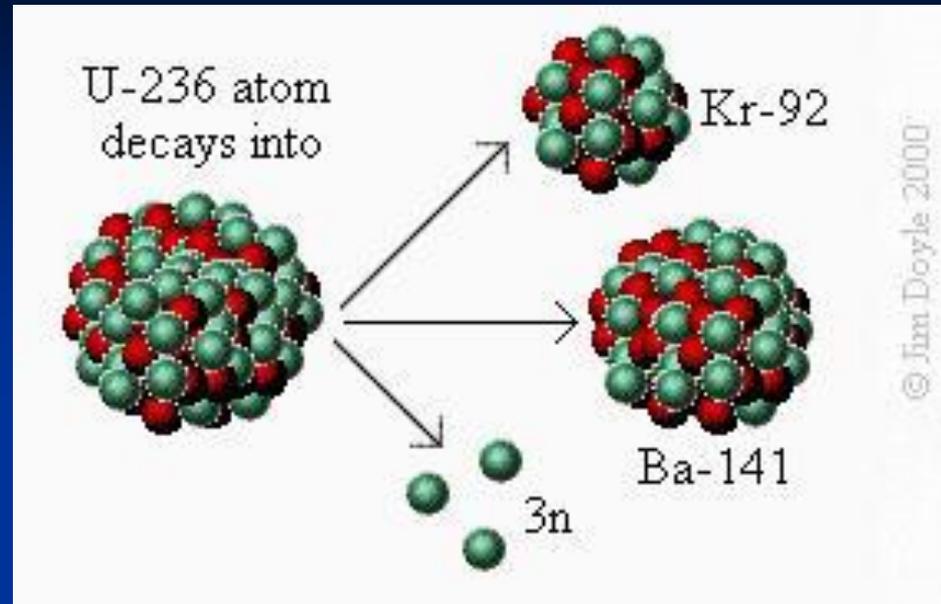
Practical Application of Equivalence of mass and energy

Secondly, the new uranium-236 atom rapidly decays into an atom of Ba-141 (barium), an atom of Kr-92 (krypton) and three neutrons:



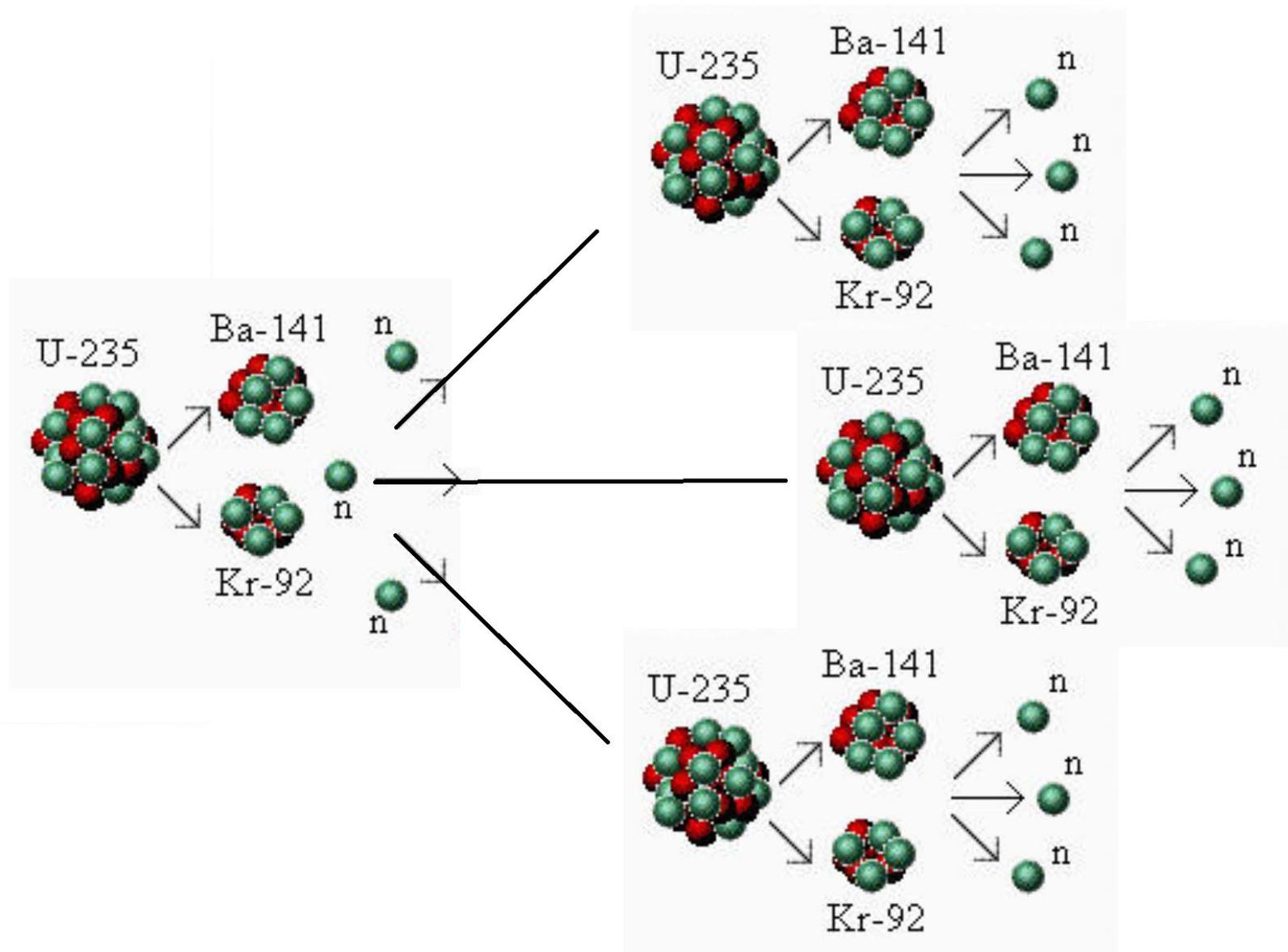
The resulting particles all have kinetic energy. This energy comes from converting a little of the mass of the original atom into energy and can be measured using $E = mc^2$. When this is done, the amount of energy typically released in the case of U-235 is around 200 MeV (0.0000000000003204 Joules).

Practical Application of Equivalence of mass and energy

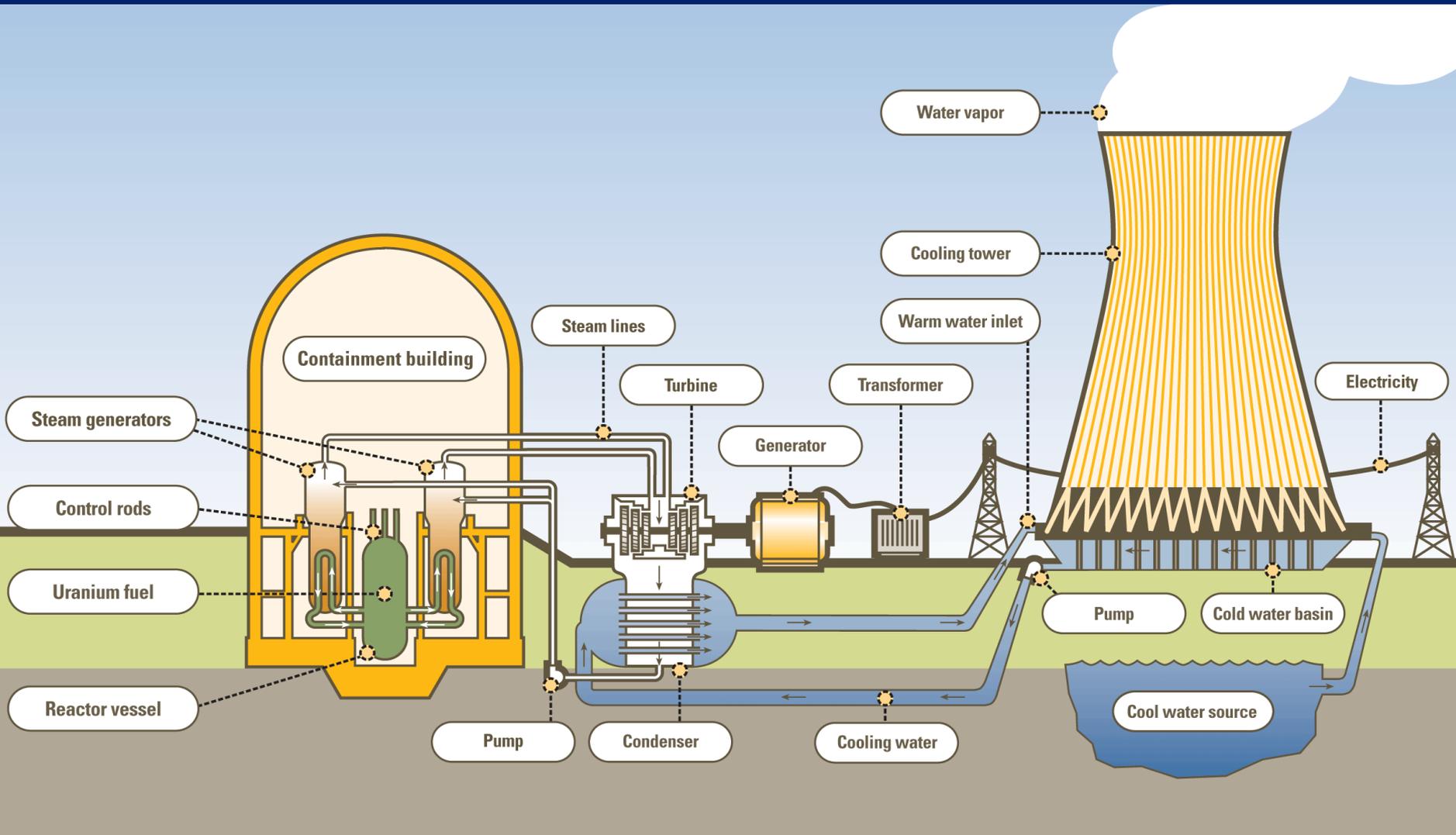


That, it seems, is a very tiny amount of energy. However, it is about a *million times* more energy than is released by the burning of one molecule of petrol (gas) in a car's engine. Put another way, if you currently use a tank of petrol each week but could use the energy provided by one tank of uranium-235 fission instead, you wouldn't need to re-fill your car for over 19,000 years.

Practical Application of Equivalence of mass and energy



Nuclear Power Stations :



Nuclear Power Stations :

Nuclear power stations use another element that can undergo nuclear fission, that of plutonium (Pu), which is also used in most modern atomic bombs. This element is slightly heavier than uranium, but doesn't occur naturally in anything like sufficient quantities to be useful and so is made, or synthesized, from uranium together with other particles. The method is surprisingly simple in that U-238 is bombarded with either neutrons until some stick and transmute (i.e. change) and so form Pu-239, or the nuclei of heavy hydrogen (1 neutron and 1 proton) are used to bombard U-238 to form Pu-238. There are intermediate stages of transmutation in each case, but what remains at the end is isotopes of plutonium, either Pu-239 or Pu-238, and with each nucleus containing 94 protons.

Nuclear Power Stations :

In this way the the vast bulk (99.3%) of mined uranium, i.e. U-238, can still be used to provide energy. The process of turning uranium into plutonium is sometimes referred to as "breeding", and "breeder reactors" are used for this purpose.

A nuclear power station works in pretty much the same way as any other power station, only the energy source is different. Generally, rods of fissionable material (plutonium) are pushed towards each other until a *controlled* amount of heat is produced. This heat is used to produce steam, which is forced at high pressure through a set of turbine wheels. The turbine wheels are connected to a generator and electricity is produced.



Chernobyl

TV Mini Series (2019)

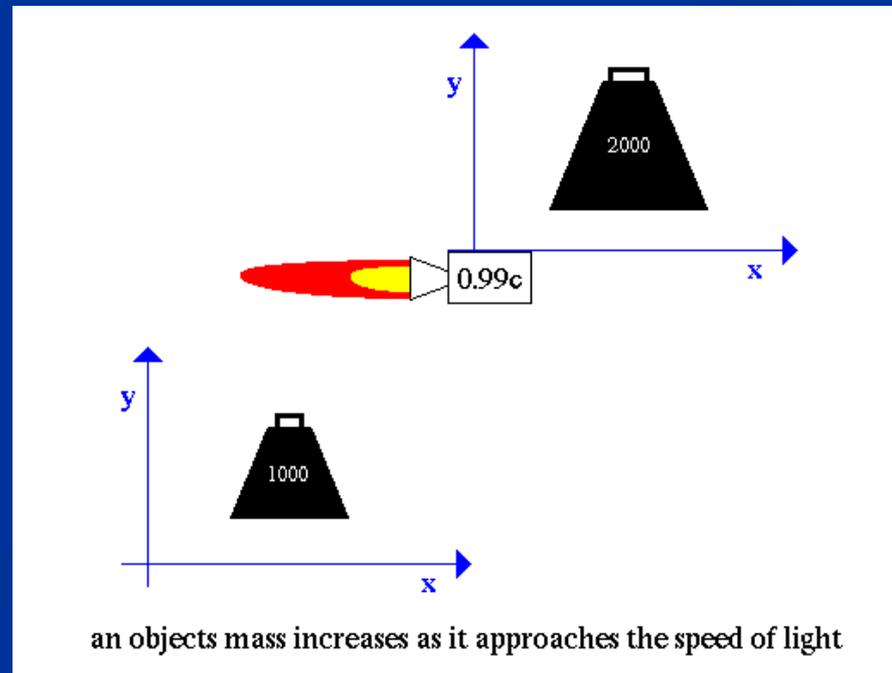
Story :

In April 1986, the city of Chernobyl in the Soviet Union suffers one of the worst nuclear disasters in the history of mankind. Consequently, many heroes put their lives on the line in the following days, weeks and months.

Relativistic Mass

If any body is moving with some velocity, that body has a mass (greater than its mass at rest) is called **Relativistic Mass** of the body!

In special theory of relativity, an object that has a mass cannot travel at the speed of light. As the object approaches the speed of light, the object's energy and momentum increase without bound.



Relativistic Mass

If we assume a body with mass m is moving with a constant velocity, v .

Using the relationship between Relativistic Energy - Momentum:

$$\rightarrow E^2 = p^2 c^2 + m_o^2 c^4$$

Where, $E = mc^2$ and, $p = mv$

$$\rightarrow (mc^2)^2 = (mv)^2 c^2 + m_o^2 c^4$$

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic Mass

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where,

m – Relativistic Mass of the body

The mass of the body which is moving with velocity v .

m_o – Rest Mass of the body

v – Relativistic Speed of the body



Relativistic Mass

Rest mass of an electron is $9.01 \times 10^{-31} \text{ kg}$. An electron is accelerated to a speed of $0.8c$ by using a particle accelerator. Calculate the mass of the moving electron.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = 9.01 \times 10^{-31} \times \frac{1}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}}$$

➔ $m = 9.01 \times 10^{-31} \times 1.66$

➔ $m = 1.5 \times 10^{-30} \text{ kg}$

The Mass is increased!

Relativistic Mass



$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = m_o \left(1 - \frac{v^2}{c^2} \right)^{-1/2}$$

Using Taylor series expansion,

$$m = m_o \left(1 - \frac{v^2}{c^2} \right)^{-1/2} = m_o \left(1 - \left(-\frac{1}{2} \right) \frac{v^2}{c^2} + \left(\frac{3}{2} \right) \left(-\frac{1}{2} \right)^2 \left(\frac{v^2}{c^2} \right)^2 - \left(\frac{5}{2} \right) \left(-\frac{1}{2} \right)^3 \left(\frac{v^2}{c^2} \right)^3 + \dots \right)$$



$$m = m_o \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \approx m_o \left(1 + \frac{1}{2} \frac{v^2}{c^2} \right)$$



$$m \approx m_o \left(1 + \frac{1}{2} \frac{v^2}{c^2} \right)$$

Because,

$$\dots \left(\frac{v^2}{c^2} \right)^2 \ll \left(\frac{v^2}{c^2} \right) \ll 1$$

Relativistic Mass

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = m_o \left(1 - \frac{v^2}{c^2} \right)^{-1/2}$$

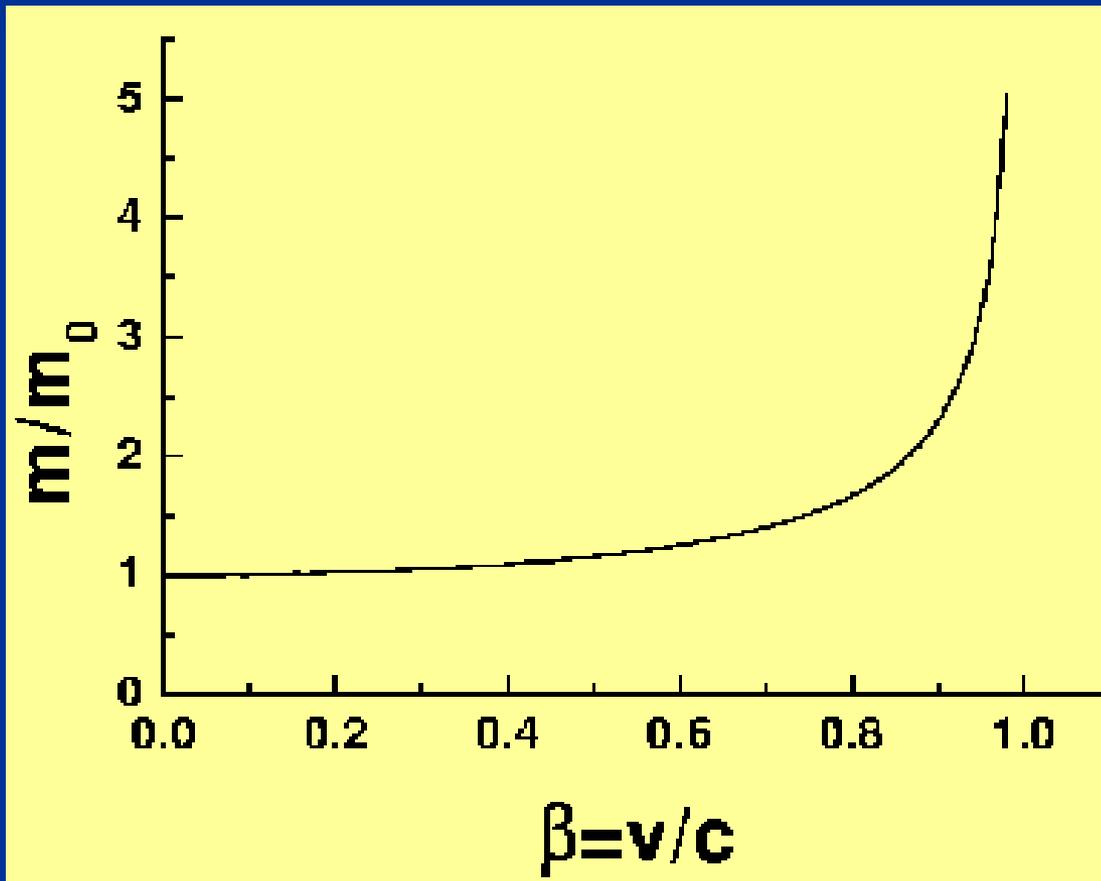


$$\frac{m}{m_o} = \left(1 - \frac{v^2}{c^2} \right)^{-1/2}$$



$$\frac{m}{m_o} = (1 - \beta^2)^{-1/2}$$

Where, $\beta = \frac{v}{c}$



Experimental Proofs :

The first verification of the increase in mass with velocity came from the experimental work of **Kaufmann** in 1902 and 1906 and particularly, that of **Bucherer** in 1909. They were working on something entirely unrelated to relativity – or so they thought. It had been known for some time that certain substances, radium for one, were constantly shooting off three different types of small particles, or rays. Such substances are called **radioactive**. They were investigating the particular type of radiation known as beta rays and were attempting to determine just what these were. In doing so, they found the velocities with which individual particles making up the radiation were ejected from radioactive substances, the amount of electric charge on each and the mass of each.

Experimental Proofs :

The velocities were found to be comparable to the velocity of light; they also found that the **higher the velocity**, the **greater the mass** of the particle. Hence, they obtained many different beta particles, each with a different mass. They found that rest mass was the same for each particle.

This result constituted the first experimental proof of ,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

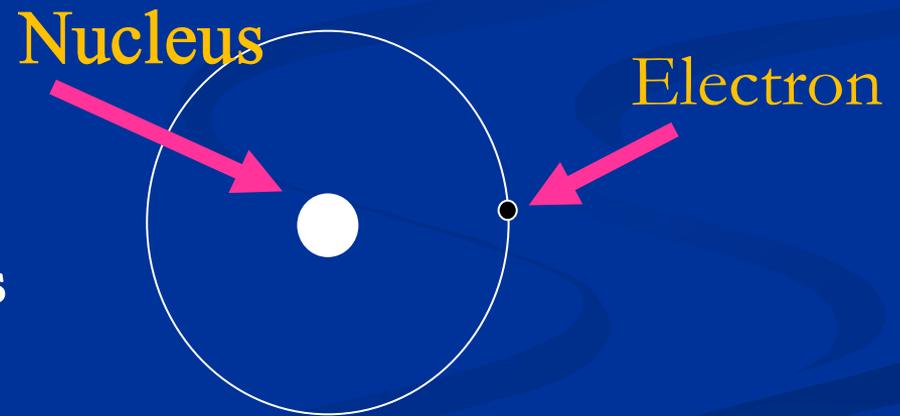
Equation and the first verification of the special theory of relativity.

Experimental Proofs :

Sommerfeld's theory of Atomic Orbits:

This verification of the mass increased predicted by the STR was proposed by Sommerfeld in 1916.

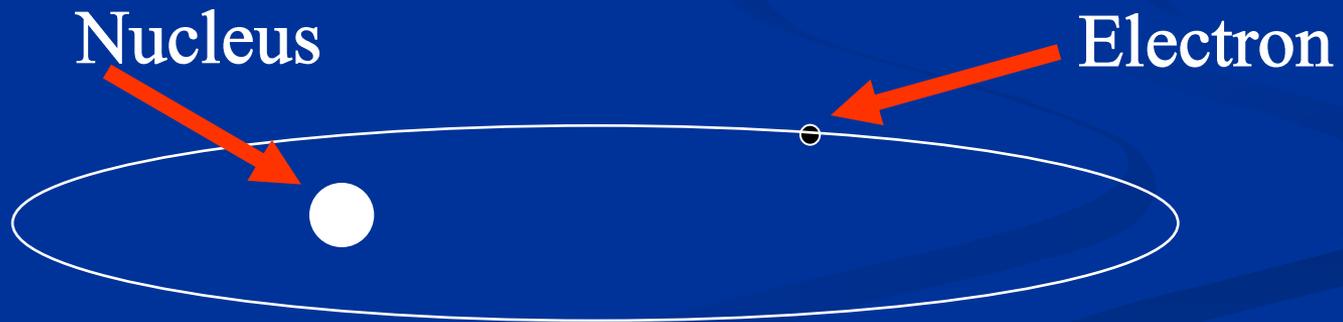
Previous to this time, the **Bohr Theory (1913)** had pictured the atom as consisting of a **nucleus at the centre** with the **electrons moving in circles about the nucleus**.



Experimental Proofs :

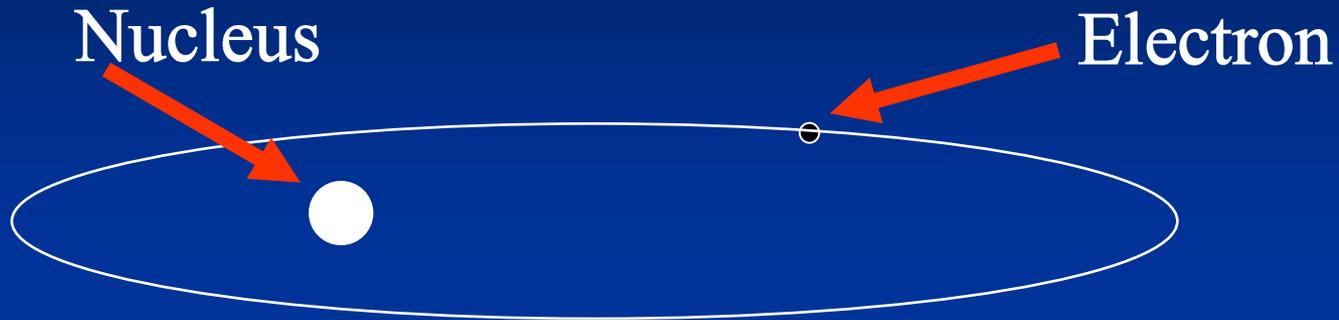
Sommerfeld's theory of Atomic Orbits...

However, Sommerfeld showed that it was more correct to assume that, in general, the electron paths were not circles but **ellipses**, and that the electron **revolved about the nucleus**, which was **situated at the one of the foci of the ellipse**, in the same way that planets revolve around the Sun as in the following figure.



Experimental Proofs :

Sommerfeld's theory of Atomic Orbits...

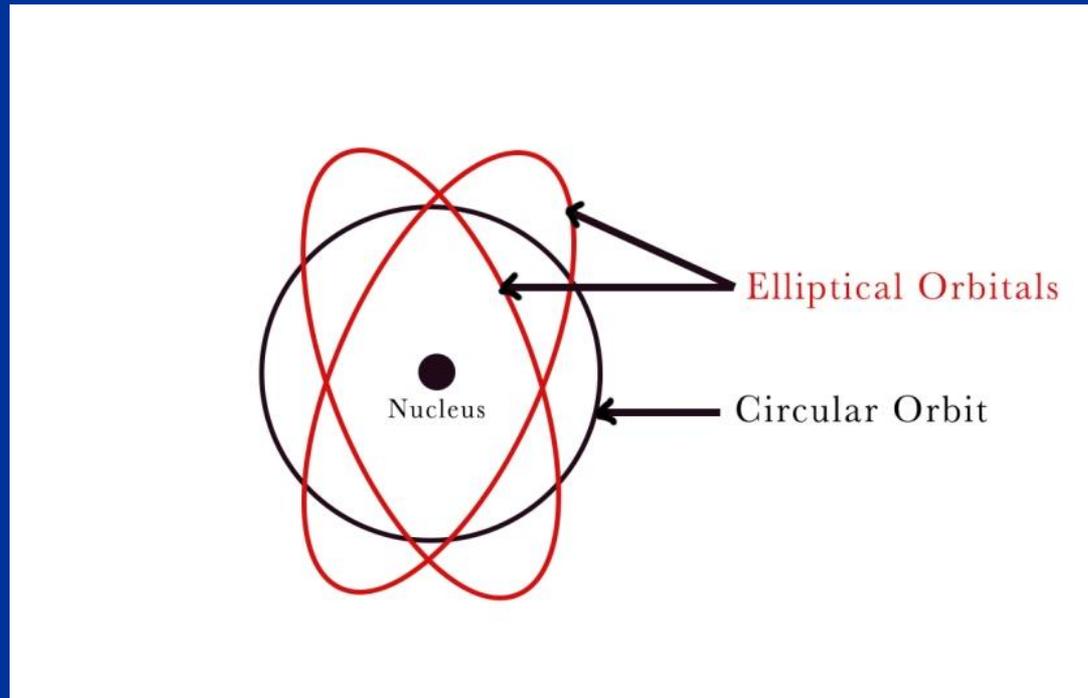


It had been shown by **Kepler** in **1609** that when a planet revolves around the Sun, the velocity of the planet changes from a minimum to maximum, that amount of the variation depending on the flatness, or ellipticity, of the orbit. Now, since the velocity changes, the mass increase formula says that the mass of the electron should change and the greater the variation in velocity, the greater will be this change in mass.

Experimental Proofs :

Sommerfeld's theory of Atomic Orbits...

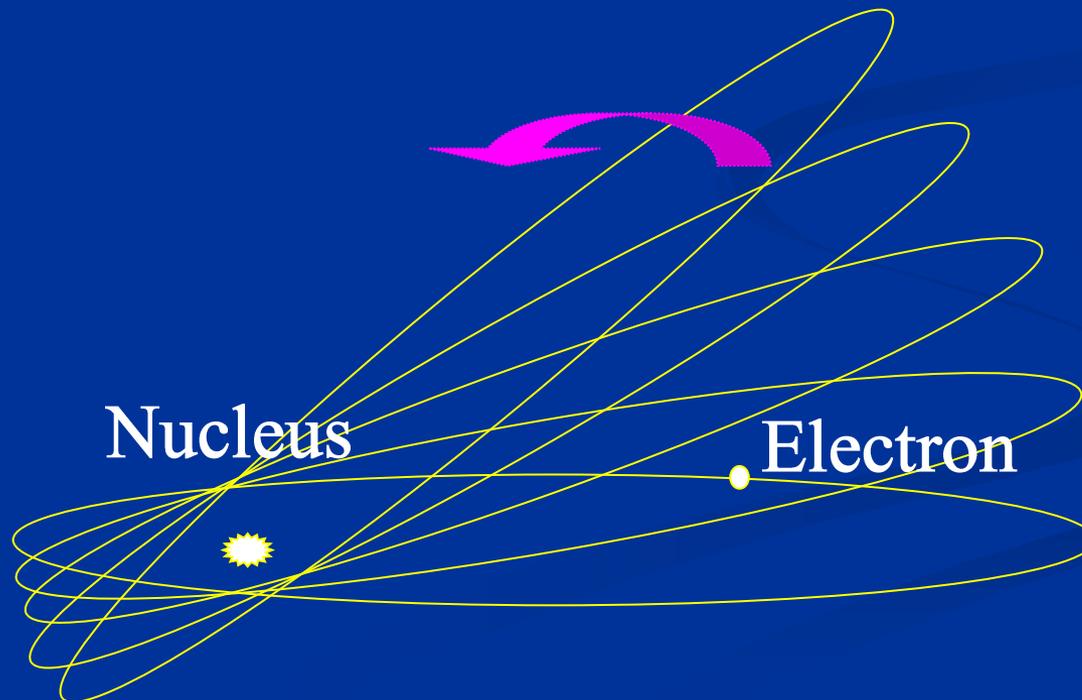
Average velocity of the electron in its orbit about the nucleus is about one one-hundredth the velocity of light, so that for a fairly flat orbit the change in velocity, and consequent change in mass, is small but detectable.



Experimental Proofs :

Sommerfeld's theory of Atomic Orbits...

Sommerfeld showed mathematically that the net effect of this change in mass is that the electron will not keep revolving around the nucleus in the same elliptical path over and over again like the Earth does around the Sun, but the ellipse will slowly rotate and the electron will describe a **Rosette Pattern** as shown.



Experimental Proofs :

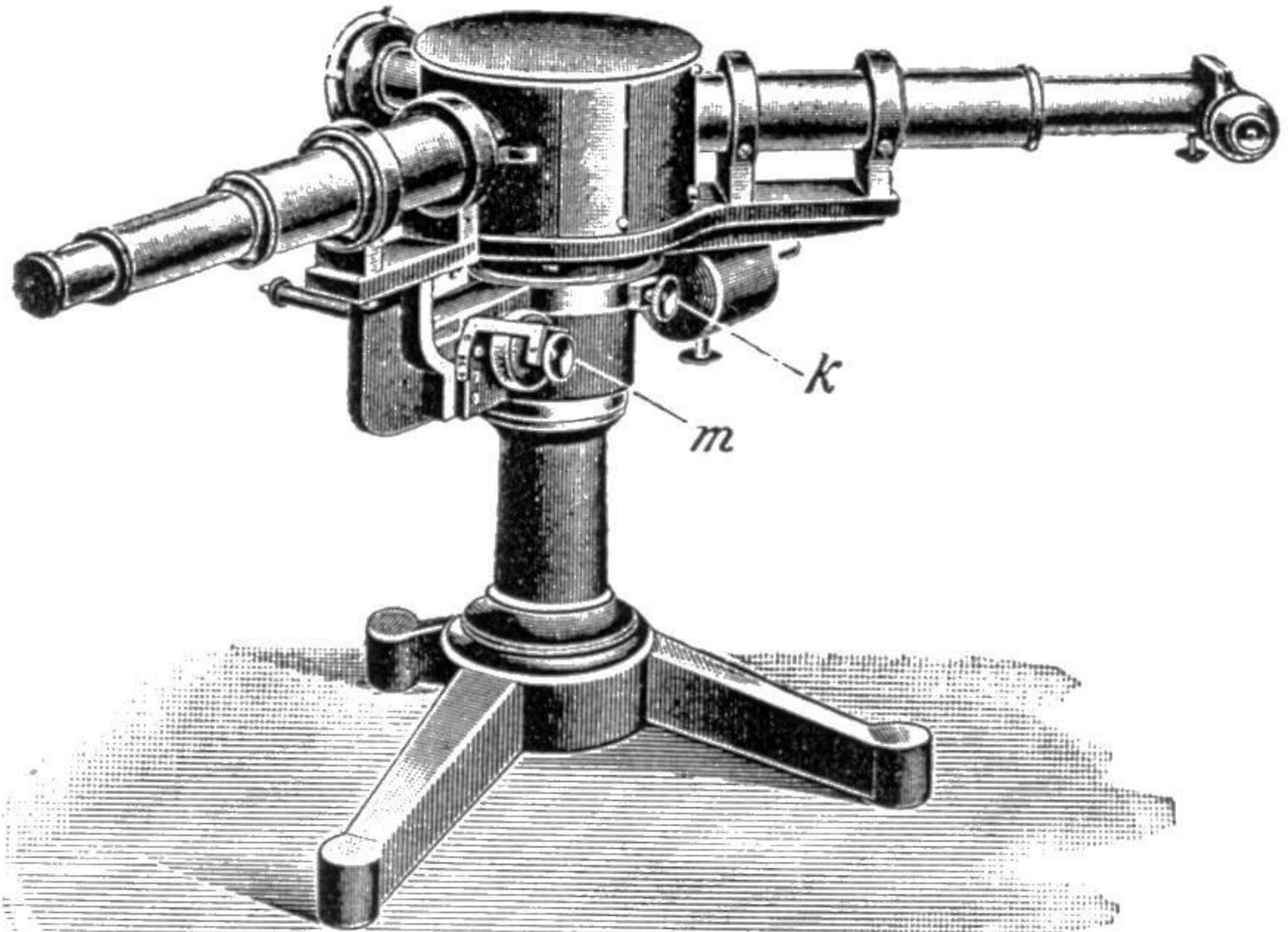
Sommerfeld's theory of Atomic Orbits...

It might seem at first glance as if it is impossible to determine the path of a single electron about a nucleus, since not only do we have no way of chopping off a single atom from a substance, but it would be impossible to see such an atom – even with the most powerful microscope we have.

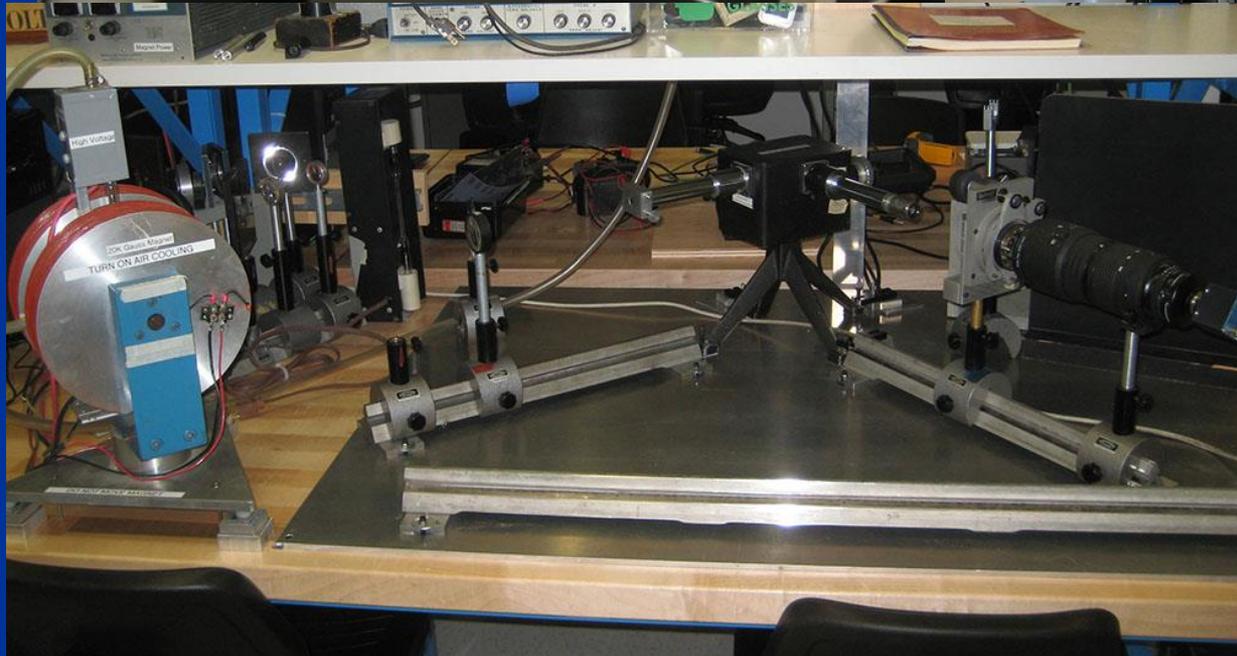
Experimental Proofs :

Sommerfeld's theory of Atomic Orbits...

In experimental work a prism by itself is insufficient for creating the best possible spectrum, since much greater precision is needed. An instrument called a **spectroscope** is used which contains a prism plus other necessary devices to help gain this high precision. The spectrum produced by a spectroscope shows a number of lines called Spectral Lines, which are scattered throughout the various colours of the spectrum.



Spectroscope
(old)

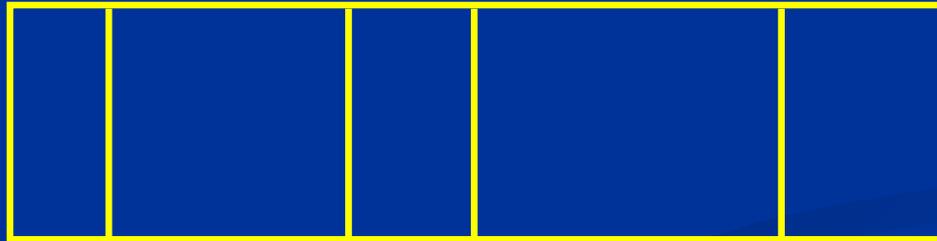


Spectroscope
(Modern)

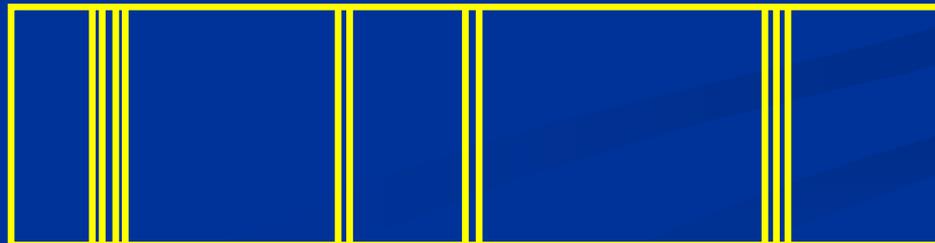
Experimental Proofs :

Sommerfeld's theory of Atomic Orbits...

Sommerfeld showed that if the path of the electron about the nucleus is an ellipse, these lines will consists of a number of single lines such shown in the following diagram.

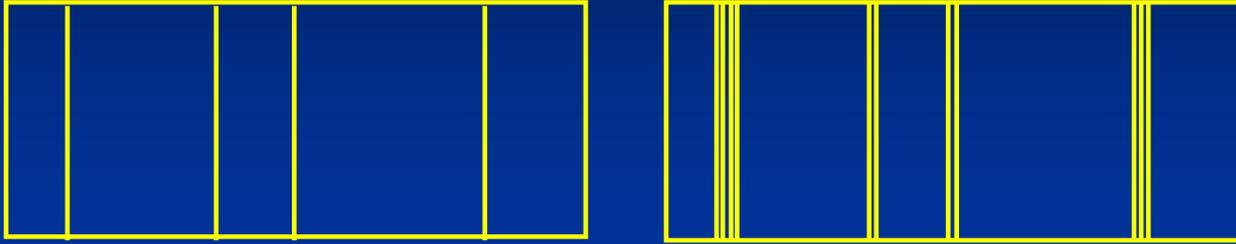


Also, for a rosette-shaped orbit these individual lines should really be split, as shown in the following diagram.



Experimental Proofs :

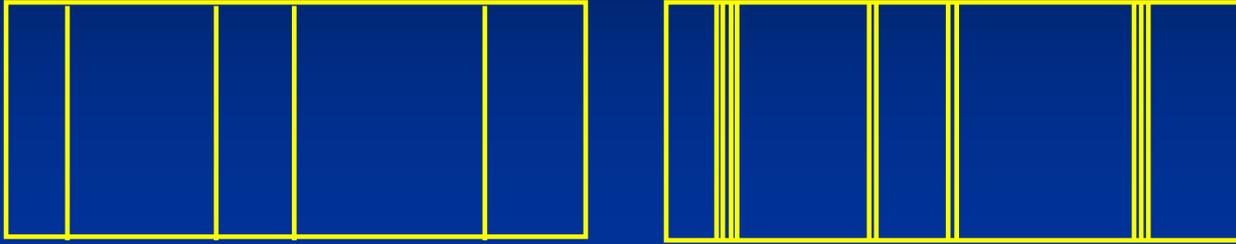
Sommerfeld's theory of Atomic Orbits...



If the spectral lines were single, then the path of the electron was an ellipse, and the mass of the electron did not change its revolution about the nucleus. But if the spectral lines were split into two or more, it would mean that the electron path was a rosette, as a result of the variation in the electron's mass, and would verify the mass-increase effect predicted by the STR.

Experimental Proofs :

Sommerfeld's theory of Atomic Orbits...



The splitting of the spectral lines was first observed and announced by **Paschen** in **1916**, when he was investigating the **spectrum of Helium**. It is interesting to note that one month after Paschen published his discovery, the sommerfeld theory was published which predicted the splitting of the spectral lines on the basis of the STR and again the mass-increase effect of the STR was verified.

Hydrogen



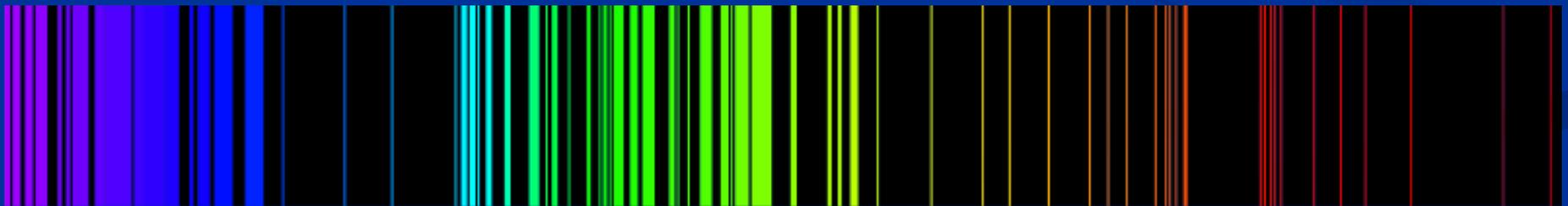
Helium



Carbon



Oxygen



Experimental Proofs :

Atomic Accelerators :

Additional examples of the mass increase with velocity have come from the spectacular results of the giant atom smashing machines which have been built to investigate the structure of the atomic nucleus. The primary purpose of these machines is to accelerate various atomic particles to high velocities; the more powerful the machine, the higher the velocities; and the higher the velocities, the greater the mass of the particles becomes, according to the mass increase formula.

These machines are called **particle accelerators**.



Experimental Proofs :

Atomic Accelerators...

Early in 1952 the **Brookhaven National Laboratory** announced its success in accelerating **protons** (nuclei of H atoms) up to **0.95 c**. As a result the mass of the proton was increased to about **three times** its original mass.

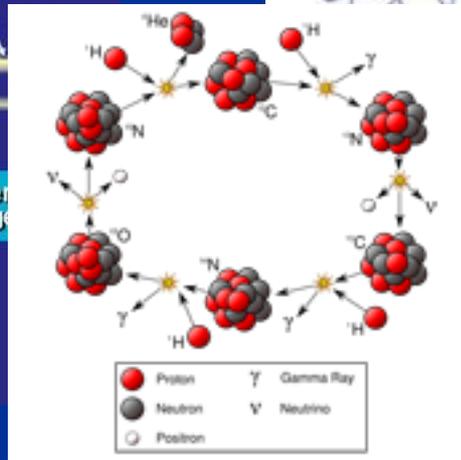
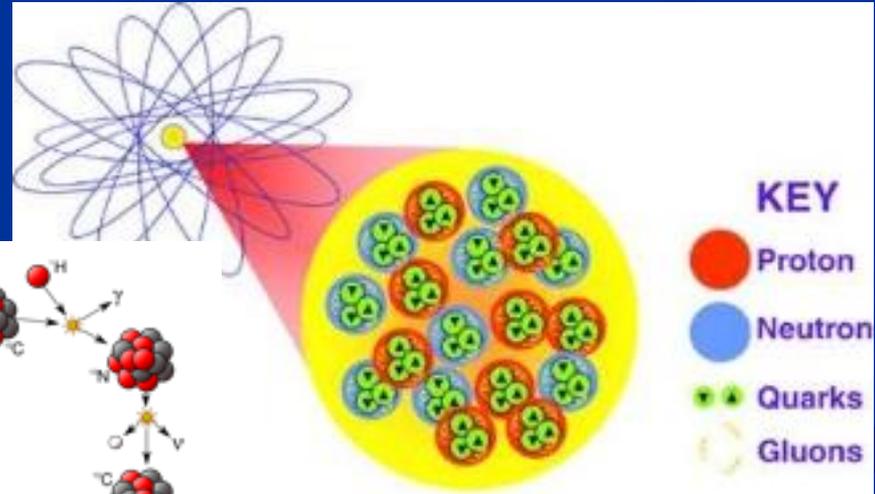
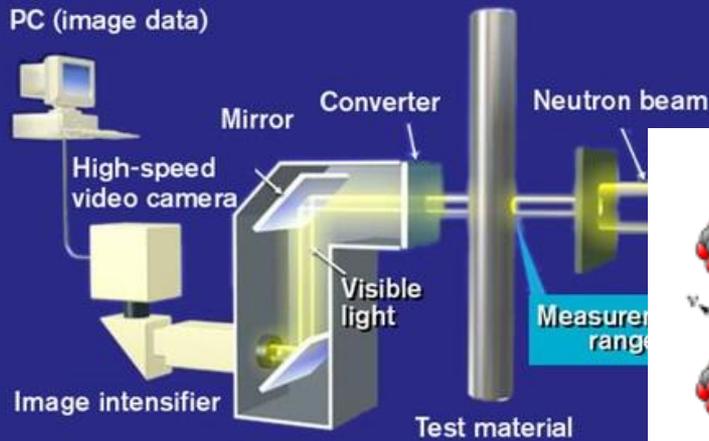


And in June 1952 the **California Institute of Technology** announced it had succeeded in accelerating **electrons** to **0.9999999 c**. The corresponding mass increase was about **900 times** its original mass.

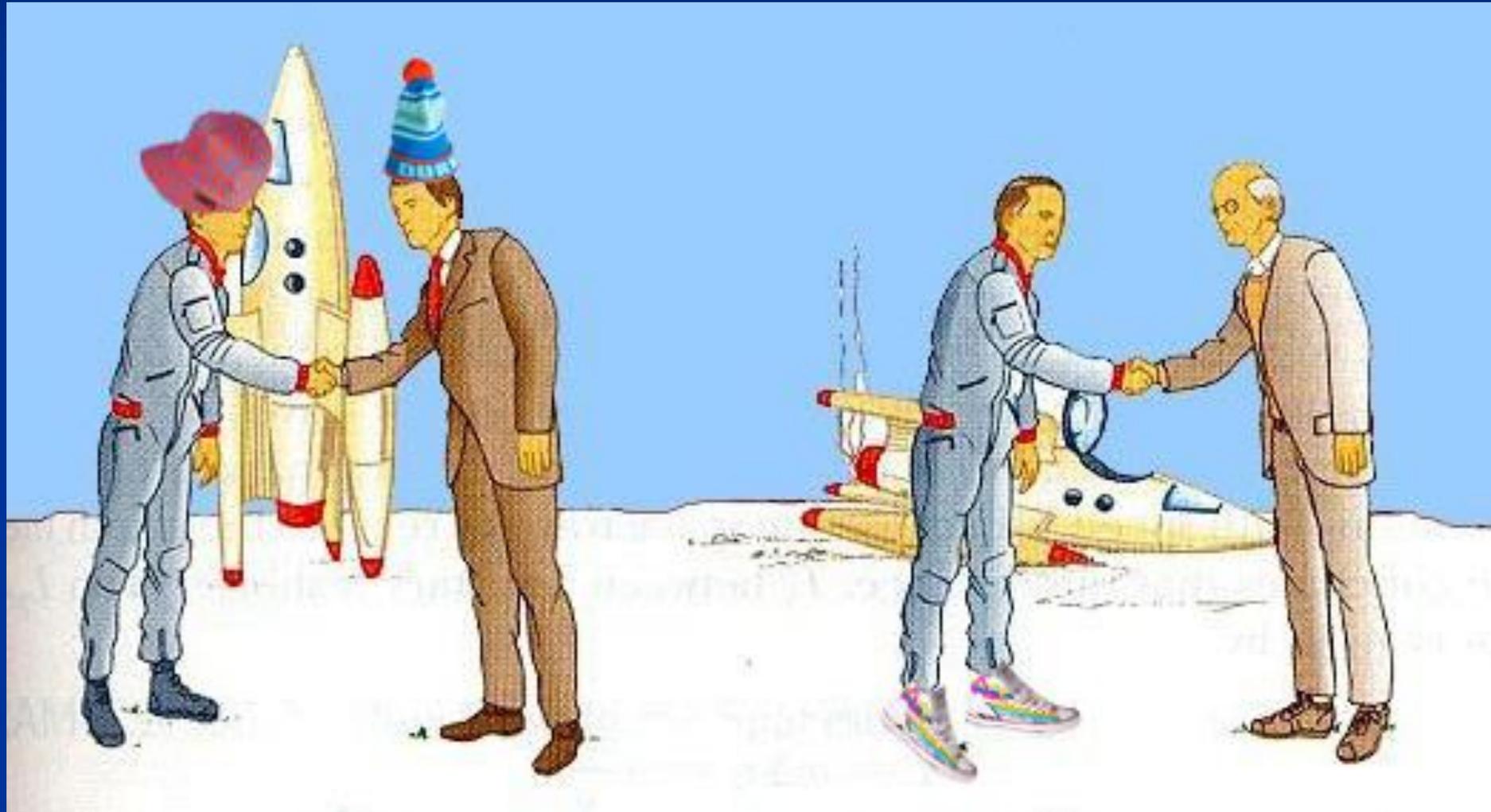
Experimental Proofs :

Atomic Accelerators...

The energy of atomic accelerators throughout the world is continually being increased, with the results that the atomic particles used for bullets in Nuclear Physics research will have large and larger effective masses as their velocities come closer and closer to the velocity of light.

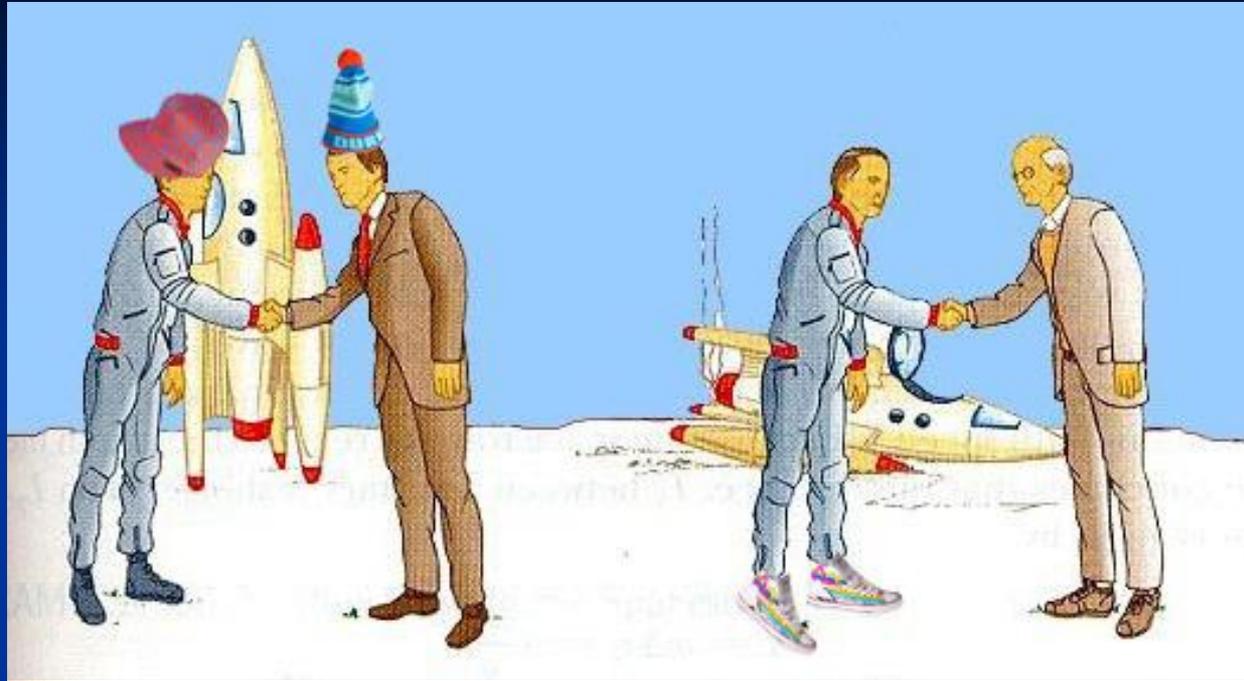


Twin Paradox



Twin Paradox

Paradox :

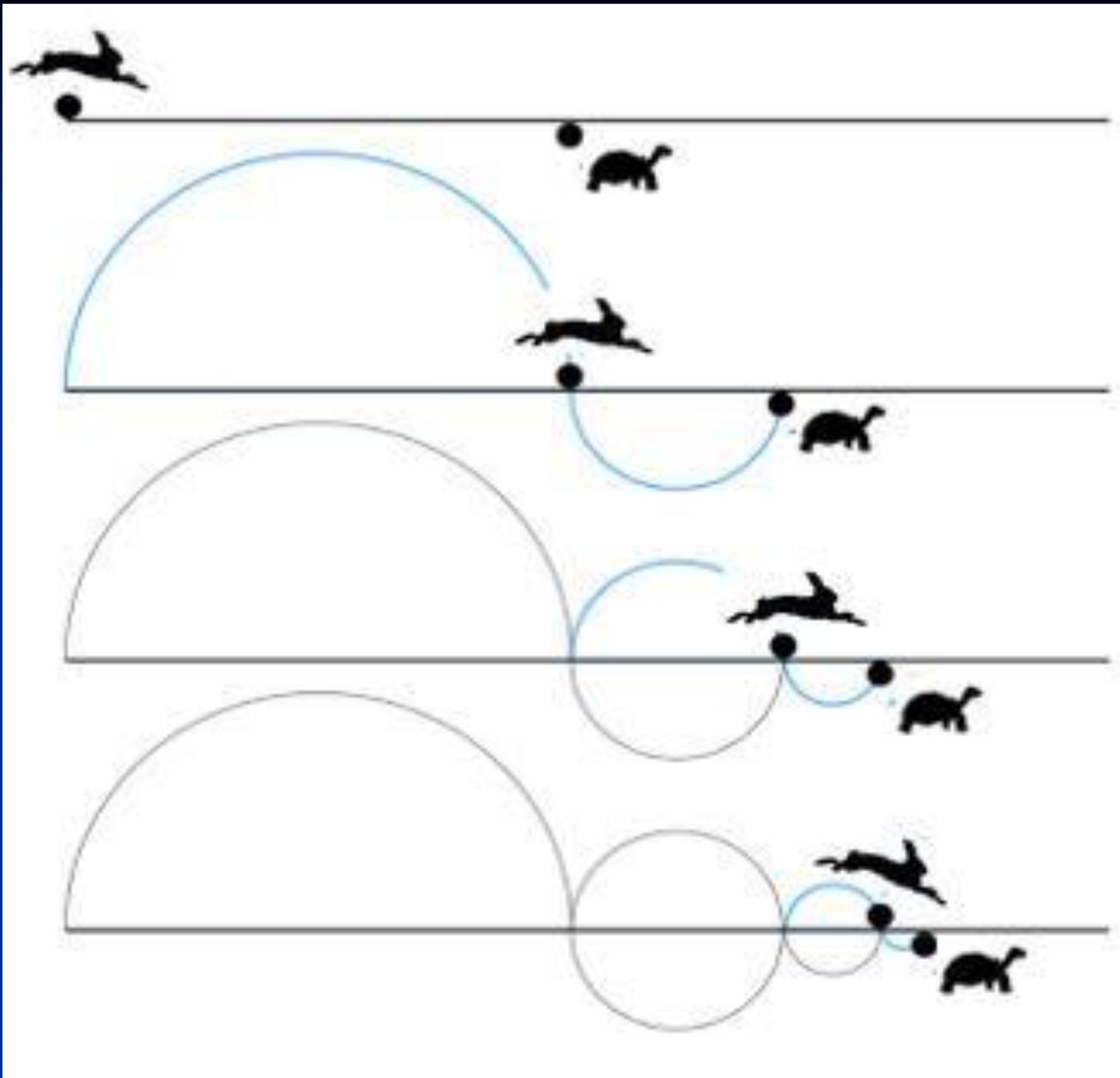
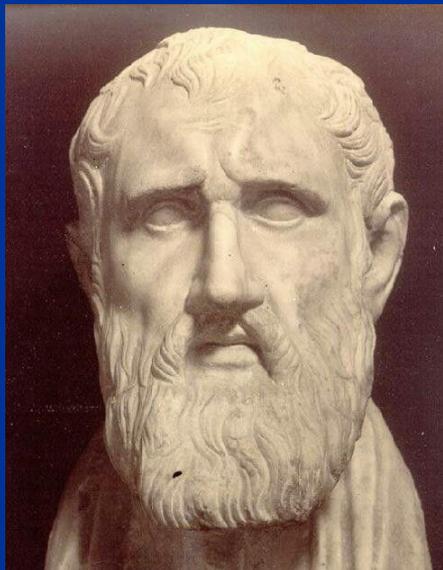


A paradox is a statement or group of statements that leads to a contradiction or a situation which defines intuition.

The term is also used for an apparent contradiction that actually expresses a non-dual truth!

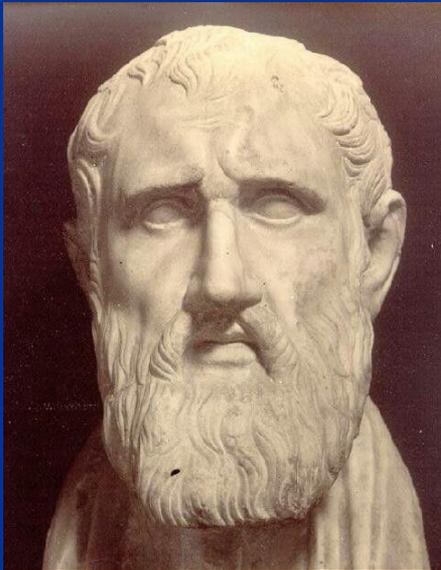
A statement or proposition seeming self-contradictory or absurd but in reality expressing a possible truth!

Zeno's Paradox



Zeno's Paradox

Zeno's Paradoxes are a set of philosophical puzzles, most famously concerning motion, proposed by ancient Greek philosopher Zeno of Elea to argue that motion and change are illusions, challenging common-sense reality with seemingly logical but absurd conclusions, like a faster runner never catching a slower one (Achilles and the Tortoise) or never reaching a destination due to infinite subdivisions (Dichotomy Paradox). These paradoxes highlight contradictions arising from applying concepts of infinite divisibility to space and time, only resolved with modern calculus and understanding of limits.

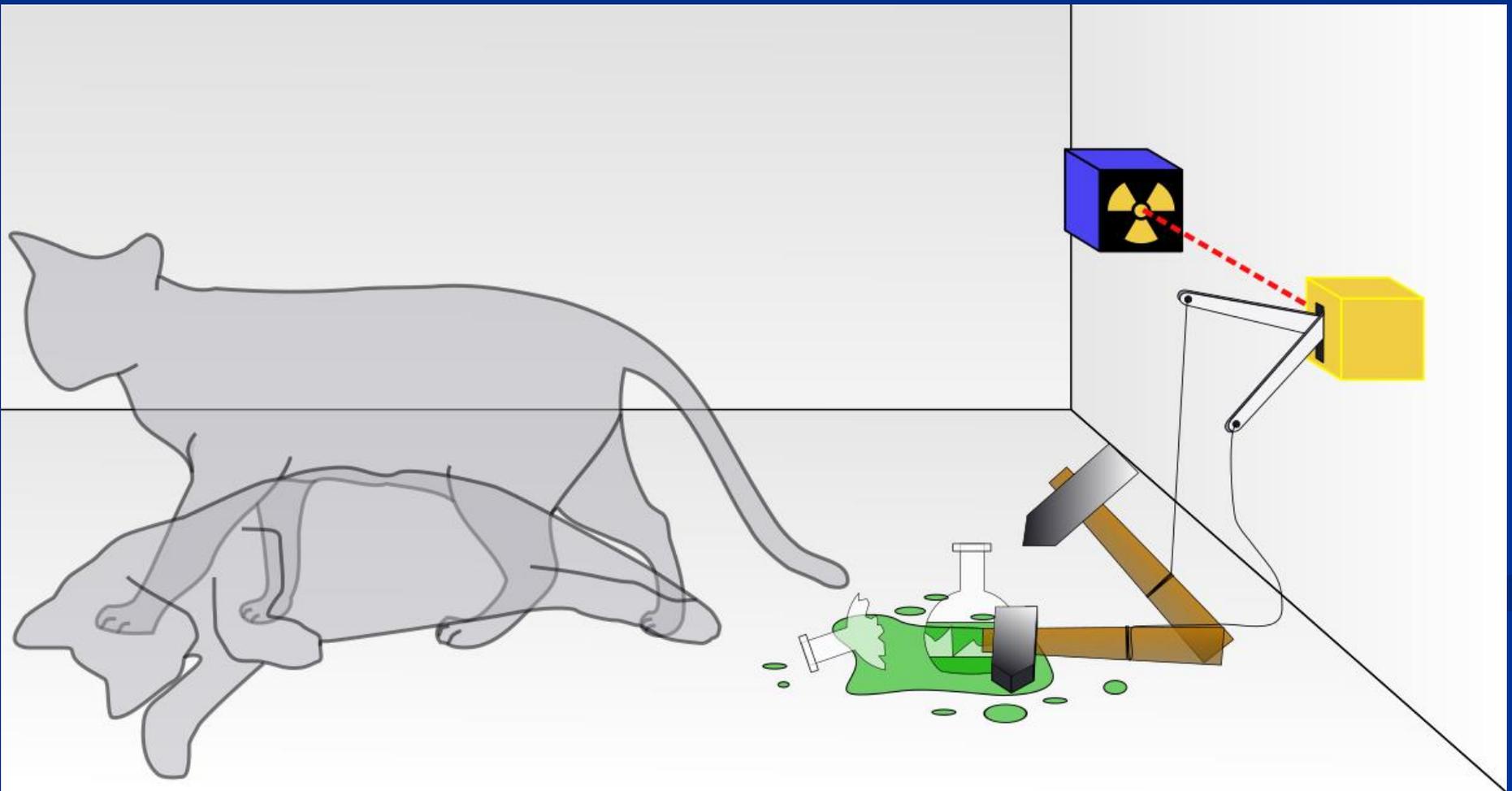
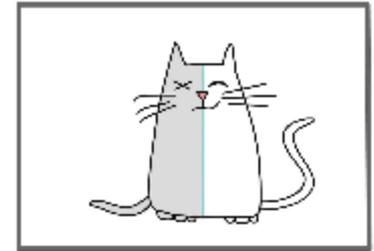


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CAT Paradox in Quantum Physics

Schrödinger's Cat



CAT Paradox in Quantum Physics

The "CAT Paradox" refers to Schrödinger's Cat, a thought experiment highlighting quantum superposition's absurdity when scaled up: a cat in a sealed box linked to a random quantum event (like an atom decaying) is considered both alive and dead simultaneously until observed, forcing a single reality. Devised by Erwin Schrödinger to criticize interpretations where macroscopic objects exist in mixed states, it illustrates the measurement problem and the transition from fuzzy quantum possibilities to definite classical outcomes, often resolved conceptually through decoherence or the many-worlds theory.

CAT Paradox in Quantum Physics

Key Concepts in the Paradox:

Quantum Superposition:

A quantum particle (like the atom) exists in all possible states (decayed/not decayed) at once.

Entanglement:

The cat's fate becomes entangled with the atom; if the atom decays, the cat dies; if not, it lives.

Measurement Problem:

Opening the box "collapses" the superposition into one definite state (alive or dead), but it's unclear why or how this happens at the macroscopic level.

Carnivorous Island in Life of Pi



MEERKAT MANOR

*That can't
be real.*

Pi

Richard Parker

Carnivorous Island in Life of Pi

The Carnivorous Island in Life of Pi is a mysterious, floating landmass of algae that appears as a paradise with fresh water and food (meerkats) but reveals a dark, predatory secret: it turns acidic at night, consuming life, symbolized by human teeth found in fruit, forcing Pi to flee this false Eden and continue his journey back to reality. It symbolizes the deceptive allure of giving up, a temptation to stay in a comforting subconscious world, which would ultimately lead to spiritual death, and also represents cannibalism and a dangerous, beautiful, yet deadly existence.

• **Carnivorous Nature:** At night, the algae becomes acidic, dissolving fish and anything else caught on its surface, revealing its true predatory nature.

• **Discovery:** Pi finds human teeth inside edible-looking flowers, confirming the island's deadly secret and its history of consuming people.

• **Symbolism:**

Carnivorous Island in *LIFE OF PI*

Key Aspects of the Island:

Appearance: A lush, vibrant island teeming with meerkats, seemingly providing everything needed for survival, a stark contrast to the open ocean.

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Symbolism:

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• **Symbolism:**

Carnivorous Island in *Life of Pi*

Symbolism:

Temptation: A seductive escape from suffering, representing a desire to surrender to despair or a blissful state of mind.

Cannibalism: A physical manifestation of Pi's own acts of cannibalism, a necessary survival mechanism he must eventually reject.

Spiritual Danger: Represents a spiritual death or stagnation; staying means losing one's soul, making escape essential.

False Eden: A corrupted paradise, like the biblical Eden, where beauty hides a deadly truth, forcing a choice between spiritual and physical death.

Twin Paradox

In Physics, the twin paradox is a thought experiment in special relativity, in which a twin makes a journey into space in a high speed rocket and returns home to find he has aged less than his identical twin who stayed on Earth.





Thank You !