Space & Atmospheric Physics

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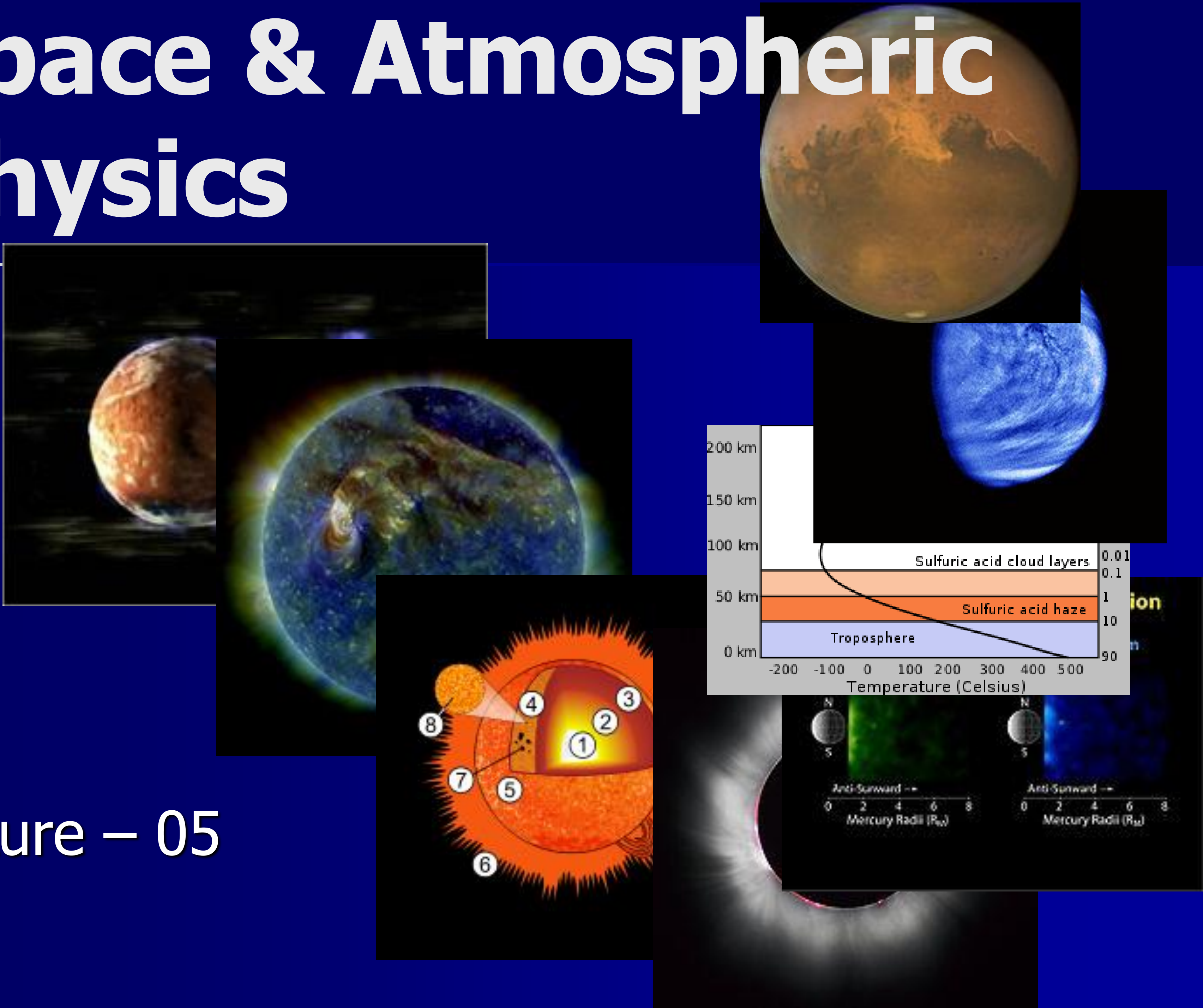
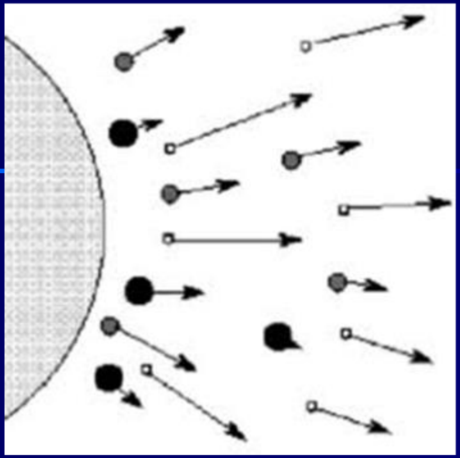


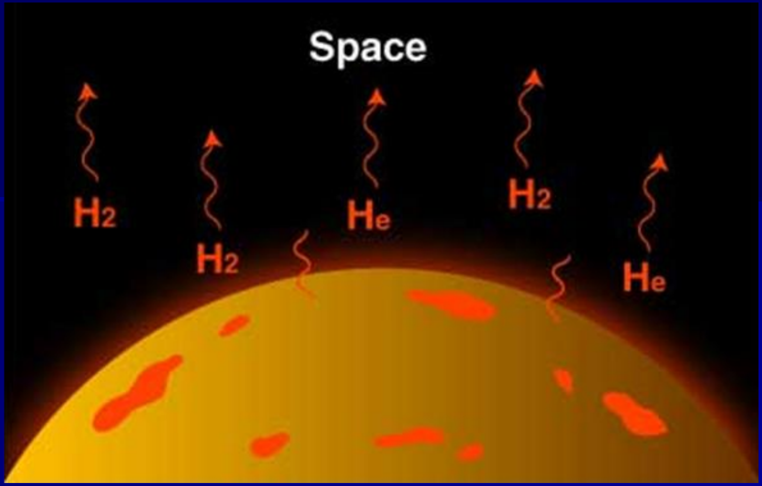
Lecture – 05

Earth Atmosphere

Retaining of Gases in the Earth
Major / Minor constituents
Barometric Equation
Scale Height
Atmospheric Regions
Temperature Profiles
Retaining of Gases
Number Density Profiles

Retaining of Gases in the Earth





The kinetic theory of gasses shows that the particle velocities of a gas in a thermal equilibrium follow a Maxwellian Distribution, which in polar coordinates is given by the expression,

$$N\,f(V)dV\,d\Omega = 4\pi\,N \cdot \frac{e^{-\left(V/V_m\right)^2}}{\left(\pi V_m^2\right)^{3/2}} V^2 dV\,Sin\theta\,d\theta\,d\phi$$

Retaining of Gases in the Earth

The most probable speed of the gas particles in the Earth (Vm)

The **most probable speed** is the speed associated with the highest point in the Maxwell distribution.

$$\frac{df(v)}{dv} = 0$$

The Maximum/Minimum value is :

$$V = \left(\frac{2kT}{M}\right)^{1/2}$$

$$\frac{d^2[f(V)]}{dV^2} = (-)ve$$

Then this V value should be the maximum value of the **Maxwellian Distribution**. This is called "The **most probable speed** "

Retaining of Gases in the Earth

When the Kinetic Energy of a particle exceeds the Potential Energy of the Gravitational Field of the Earth, this particle can in principle escape to the interplanetary space. The lowest velocity allowing the particle to escape is called the Escape Velocity Ve.

The Kinetic Energy of a particle in the Earth's atmosphere whose mass is m,

$$= \frac{1}{2} m V_e^2$$

The Potential Energy of a particle on the surface of the Earth,

$$= -\frac{GMm}{R}$$

where, M is the mass of the Earth
and R is the Radius of the Earth.

Retaining of Gases in the Earth

Kinetic Energy exceeds Potential Energy, the particle can escape ;

$$\frac{1}{2} m V_e^2 = \frac{GMm}{R}$$

$$\therefore \quad V_e = \left(\frac{2GM}{R} \right)^{1/2}$$

Where, $GM = gR^2$

Therefore, the Escape Velocity of a planet:

$$V_e = \left(\frac{2GM}{R} \right)^{1/2}$$

$$V_e = \left(\frac{2gR^2}{R} \right)^{1/2}$$

$$V_e = (2gR)^{1/2}$$

Retaining of Gases in the Earth

- For the Earth $g = 10\,ms^{-2}$ $R = 6.4 \times 10^{6}\,m$

$$v_e = (2gR)^{\frac{1}{2}} \rightarrow v_e = 11{,}200\,ms^{-1}$$

$$\frac{V_e}{V_m} = \frac{(2gR)^{1/2}}{\left(\frac{2kT}{m}\right)^{1/2}} = \left(\frac{R}{H}\right)^{1/2}$$

Where, $H = {kT}/{mg}$

The ratio of Ve : Vm

$$\frac{V_e}{V_m} = \left(\frac{R}{H}\right)^{1/2}$$

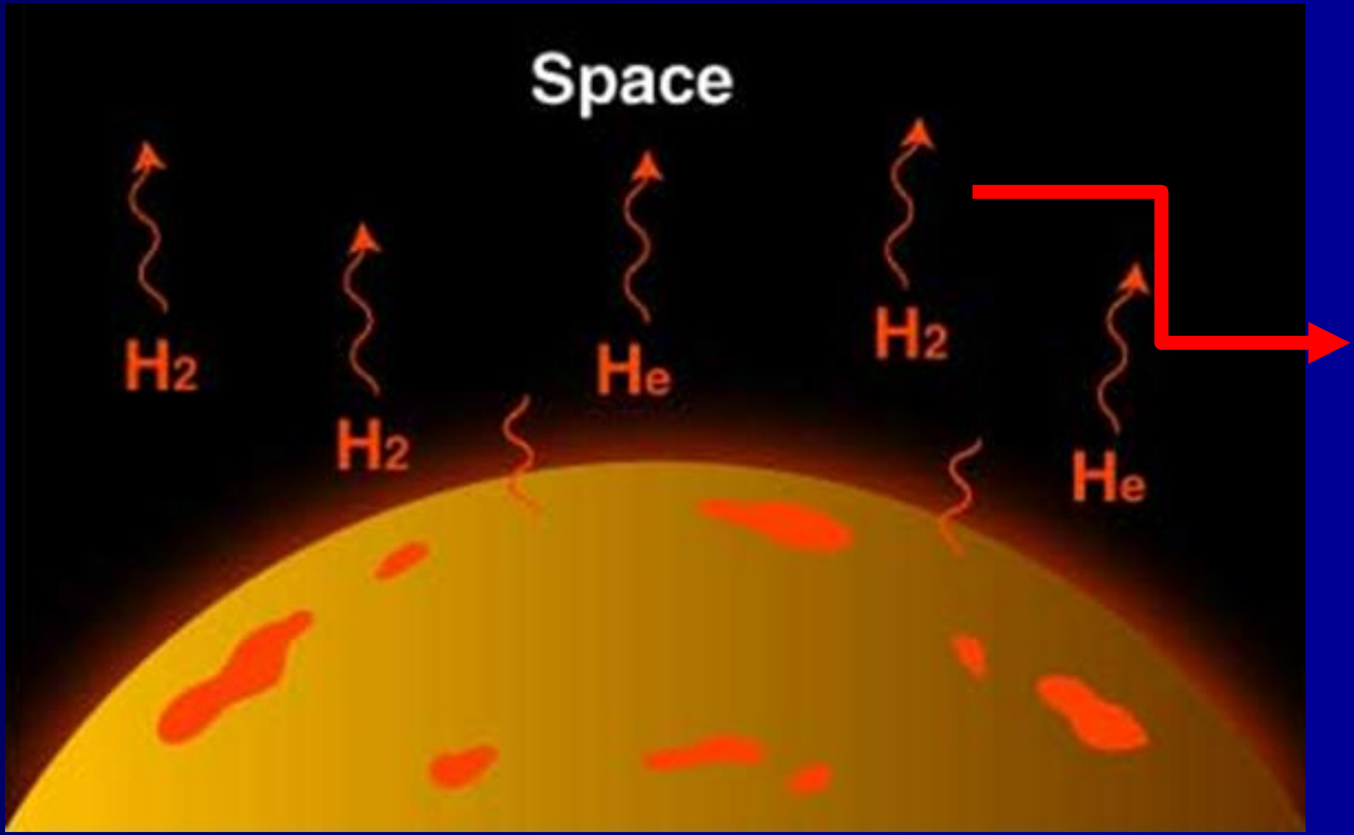
$$\frac{V_e}{V_m} = \left(\frac{6400\,km}{8.7\,km}\right)^{1/2} \rightarrow V_e \approx 28\,V_m$$

Escape Velocity

Most Probable Velocity

Retaining of Gases in the Earth

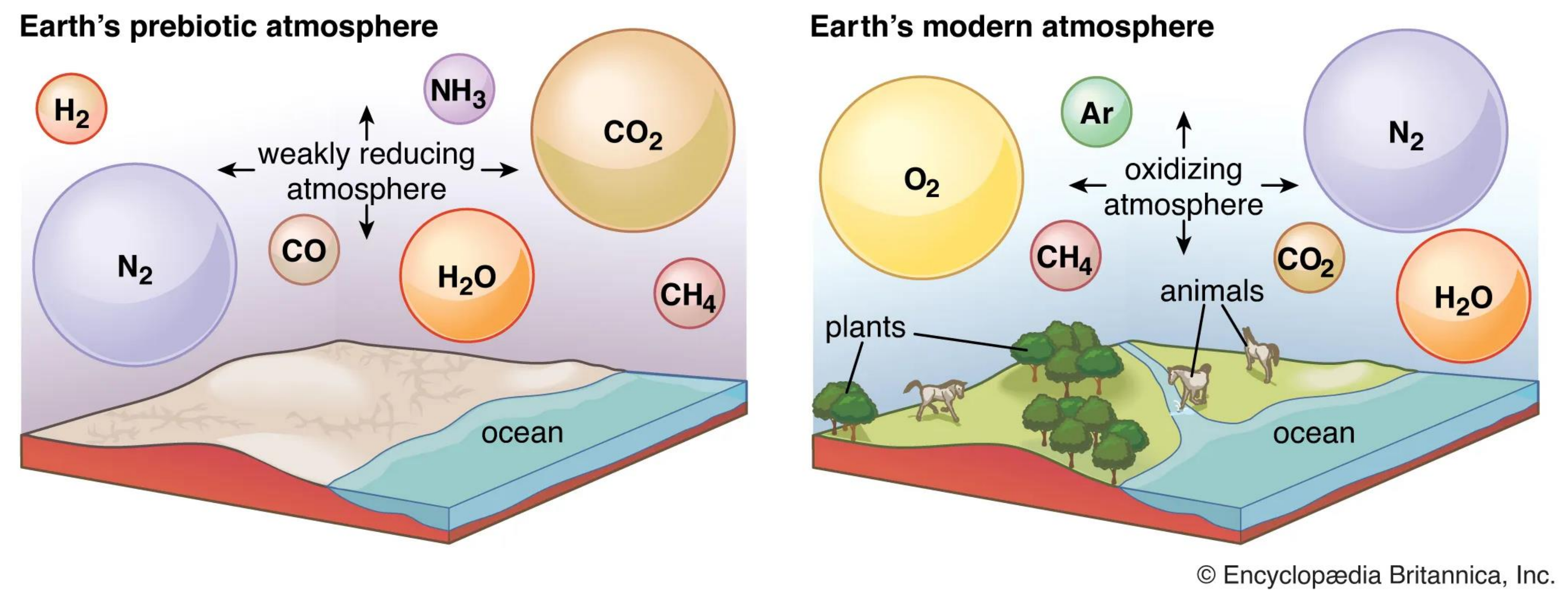
As a result, particles in the atmosphere can not escape to the interplanetary space! (But this is not the only condition necessary for the particles to escape)



Can not scape

Retaining of Gases in the Earth

The primary force responsible for the Earth's retention of atmospheric gases is **gravity**. The atmosphere is a layer of gases that surrounds the planet, and the Earth's gravitational pull is strong enough to keep these gas molecules from escaping into space.



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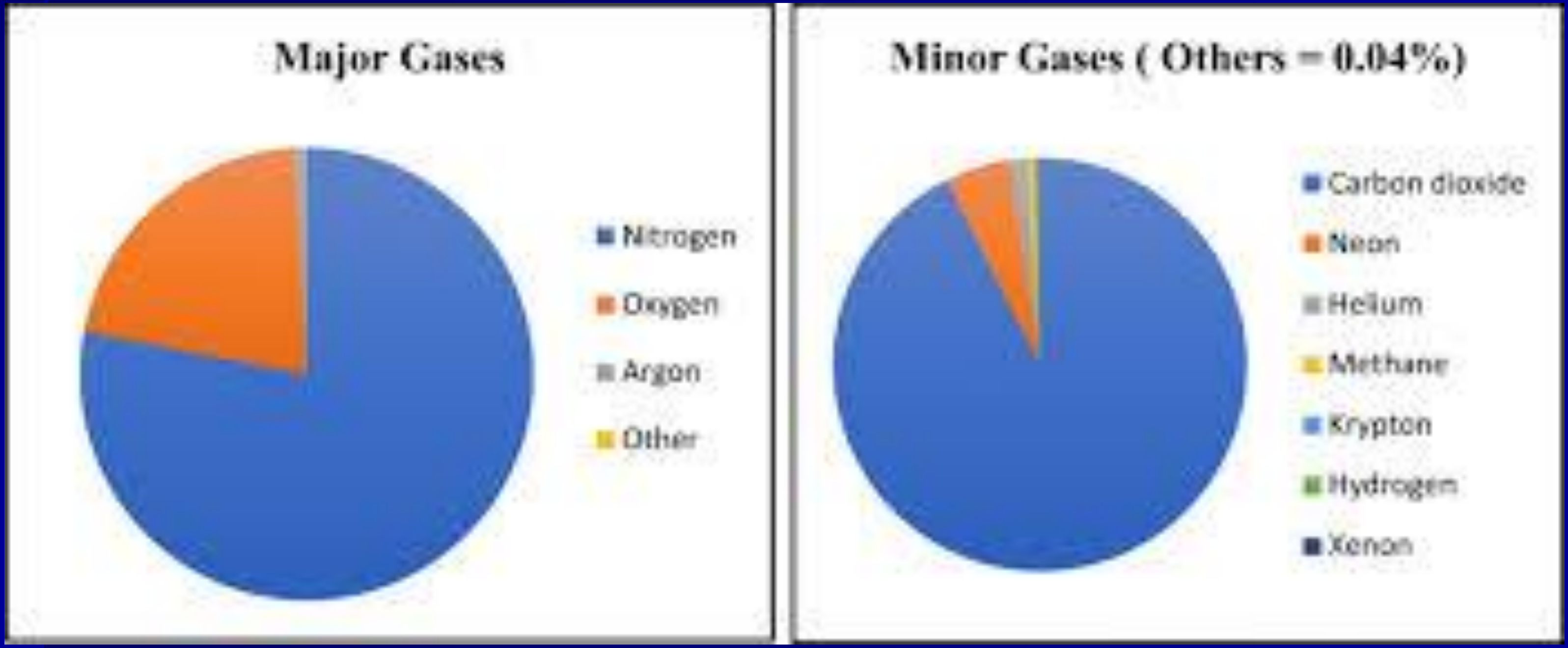
Major / Minor constituents in Earth Atmosphere

The Earth's atmosphere is primarily composed of major constituents (nitrogen and oxygen) and various minor or trace constituents (such as argon, carbon dioxide, and noble gases).



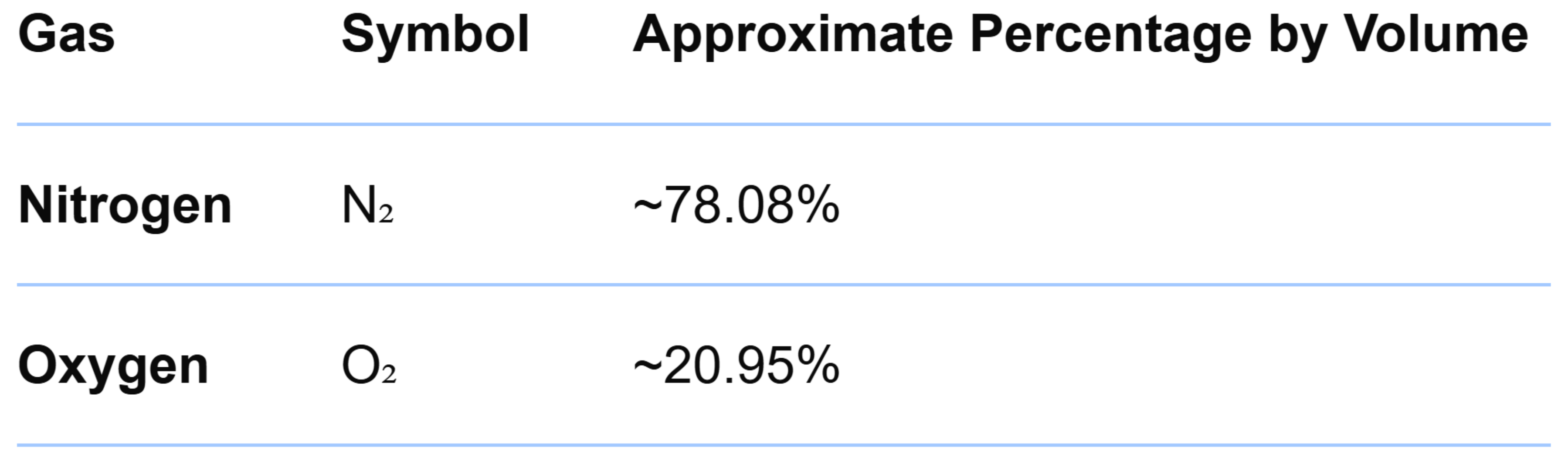
Major Gases

Minor Gases (Others = 0.04%)

Major / Minor constituents in Earth Atmosphere

Major Constituents (Constant Gases)

These two gases make up about 99% of the dry air in the atmosphere, and their concentrations are relatively stable globally.

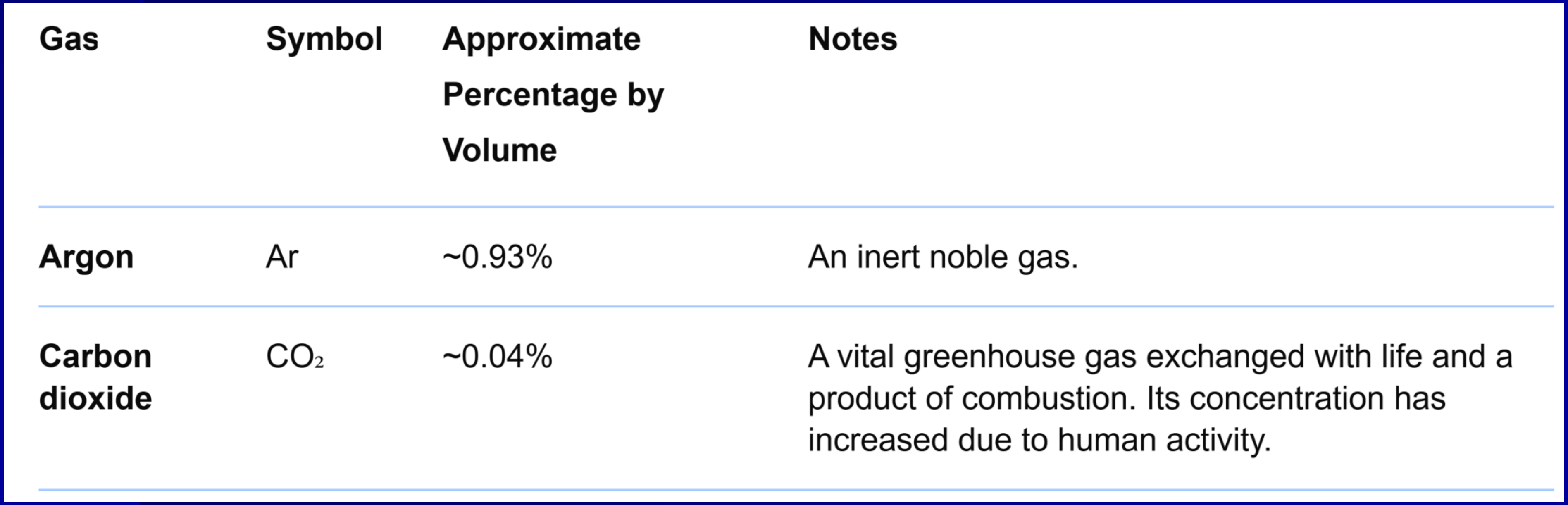


Gas	Symbol	Approximate Percentage by Volume
Nitrogen	N_2	~78.08%
Oxygen	O_2	~20.95%

Major / Minor constituents in Earth Atmosphere

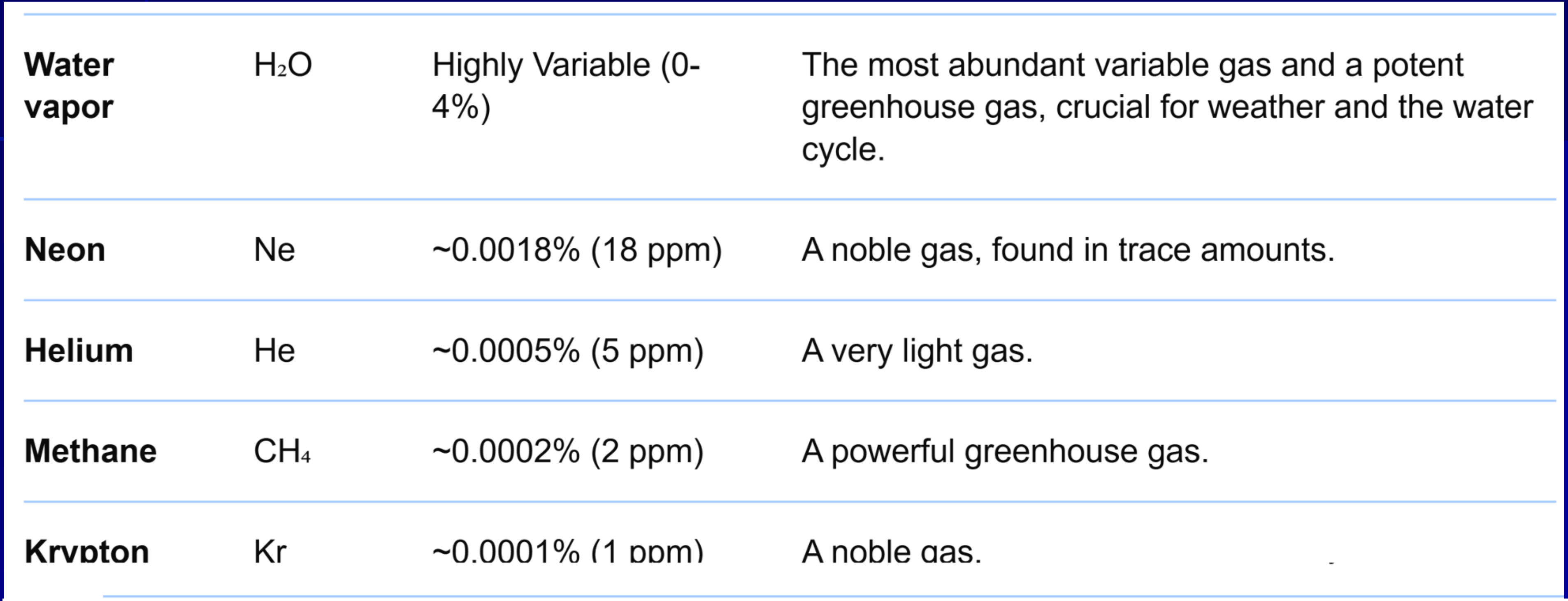
Minor Constituents (Variable and Trace Gases)

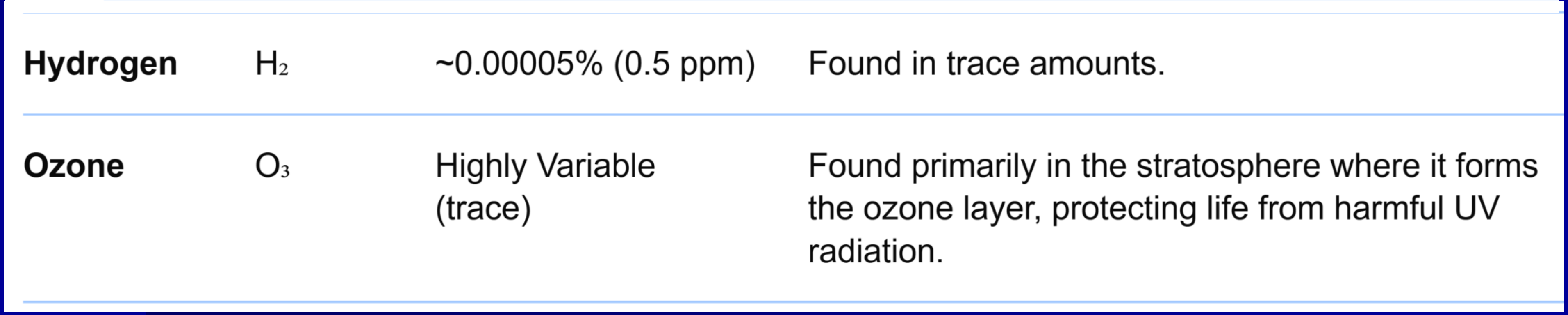
The remaining 1% consists of numerous other gases, some of which are constant in concentration while others are highly variable depending on location, time, and natural or human activities.



Gas	Symbol	Approximate Percentage by Volume	Notes
Argon	Ar	~0.93%	An inert noble gas.
Carbon dioxide	CO_2	~0.04%	A vital greenhouse gas exchanged with life and a product of combustion. Its concentration has increased due to human activity.

Major / Minor constituents in Earth Atmosphere





Water vapor	H_2O	Highly Variable (0-4%)	The most abundant variable gas and a potent greenhouse gas, crucial for weather and the water cycle.
Neon	Ne	~0.0018% (18 ppm)	A noble gas, found in trace amounts.
Helium	He	~0.0005% (5 ppm)	A very light gas.
Methane	CH_4	~0.0002% (2 ppm)	A powerful greenhouse gas.
Krypton	Kr	~0.0001% (1 ppm)	A noble gas.
Hydrogen	H_2	~0.00005% (0.5 ppm)	Found in trace amounts.
Ozone	O_3	Highly Variable (trace)	Found primarily in the stratosphere where it forms the ozone layer, protecting life from harmful UV radiation.

Major / Minor constituents in Earth Atmosphere

The atmosphere also contains tiny solid and liquid particles called aerosols (e.g., dust, pollen, salt, volcanic ash, pollutants), which play a role in cloud formation and atmospheric chemistry.

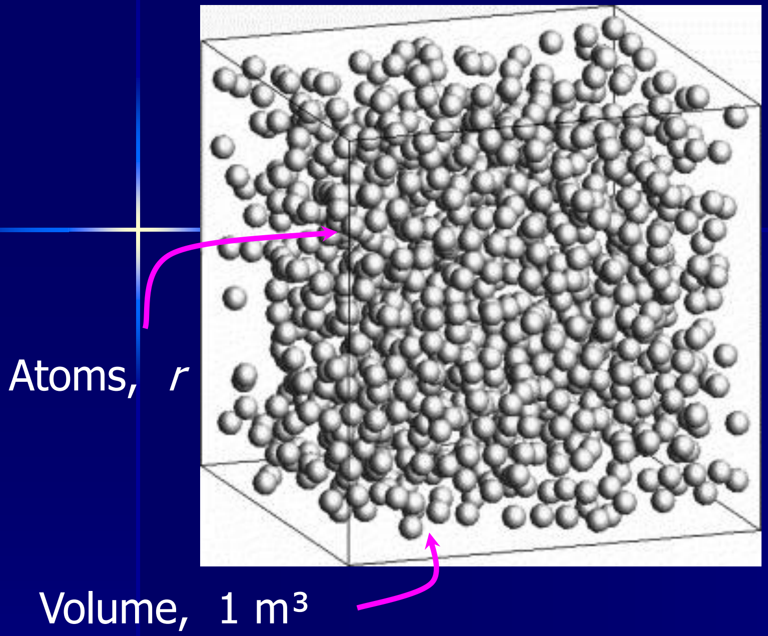
More information on atmospheric composition can be found via sources like **UCAR Center for Science Education** or the **National Oceanic and Atmospheric Administration (NOAA)**.

UCAR | CENTER FOR SCIENCE EDUCATION

NOAA
National Oceanic and
Atmospheric Administration
U.S. Department of Commerce

Barometric Equation
&
Scale Height

Density of the Atoms



Assume there are *r* atoms in this volume

Masses of the atoms are:

$$m_1, m_2, m_3, \dots, m_r$$

Number densities of those atoms are: $N_1, N_2, N_3, \dots, N_r$

Total Mass of the atoms in the above volume:

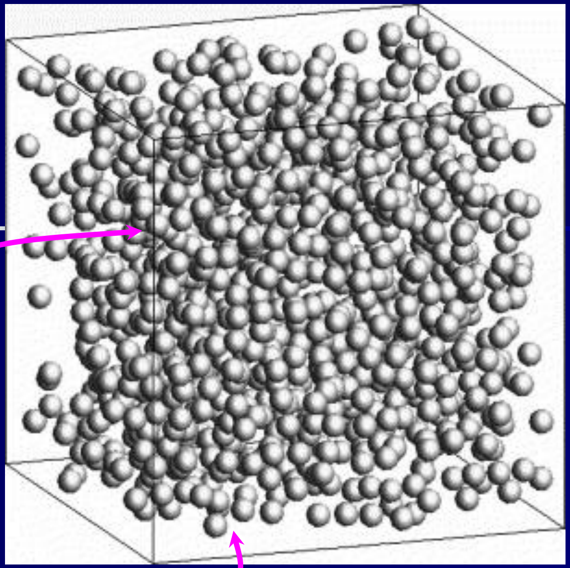
$$m_1.N_1 + m_2.N_2, + m_3.N_3, \dots, + m_r.N_r$$

(This is called the **density** because we consider the unit volume)

Total Molecular Number density:

$$N = N_1 + N_2, + N_3, \dots, + N_r$$

Density of the Atoms



Atoms, r

Volume, 1 m³

Mean Molecular mass : $\overline{m}$

$$\overline{m} = \frac{Total\ \ Mass}{Total\ \ Molecular\ \ Number\ \ Density}$$

$$\overline{m} = \frac{m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r}{N_1 + N_2 + N_3 + ... + N_r}$$

$$\overline{m} = \frac{m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r}{N}$$

Total Mass per unit volume

$$N.\overline{m} = m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r$$

Density

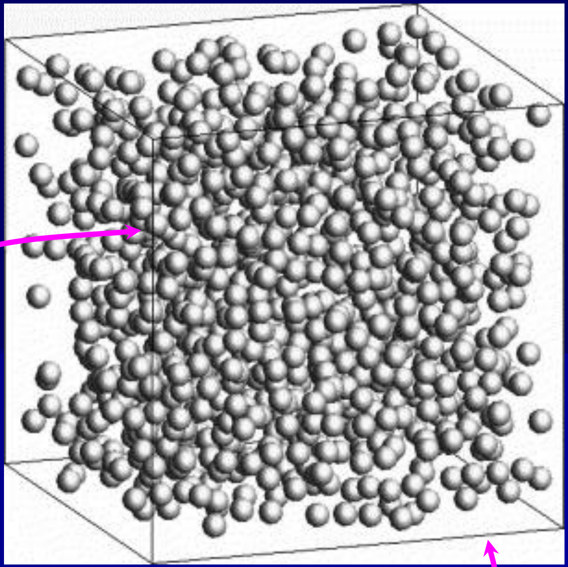
Density of the Atoms

Mean Molecular Number Density

Density

$$\rho = N \times \overline{m}$$

Total Molecular Number Density



Atoms, r

Volume, 1 m^3

For the Ideal Gas

$$PV = nRT$$

Number of molecules per volume, V

$$PV = \frac{NV}{N_o} RT$$

Avogadro Number (Number of molecules in a molecular weight)

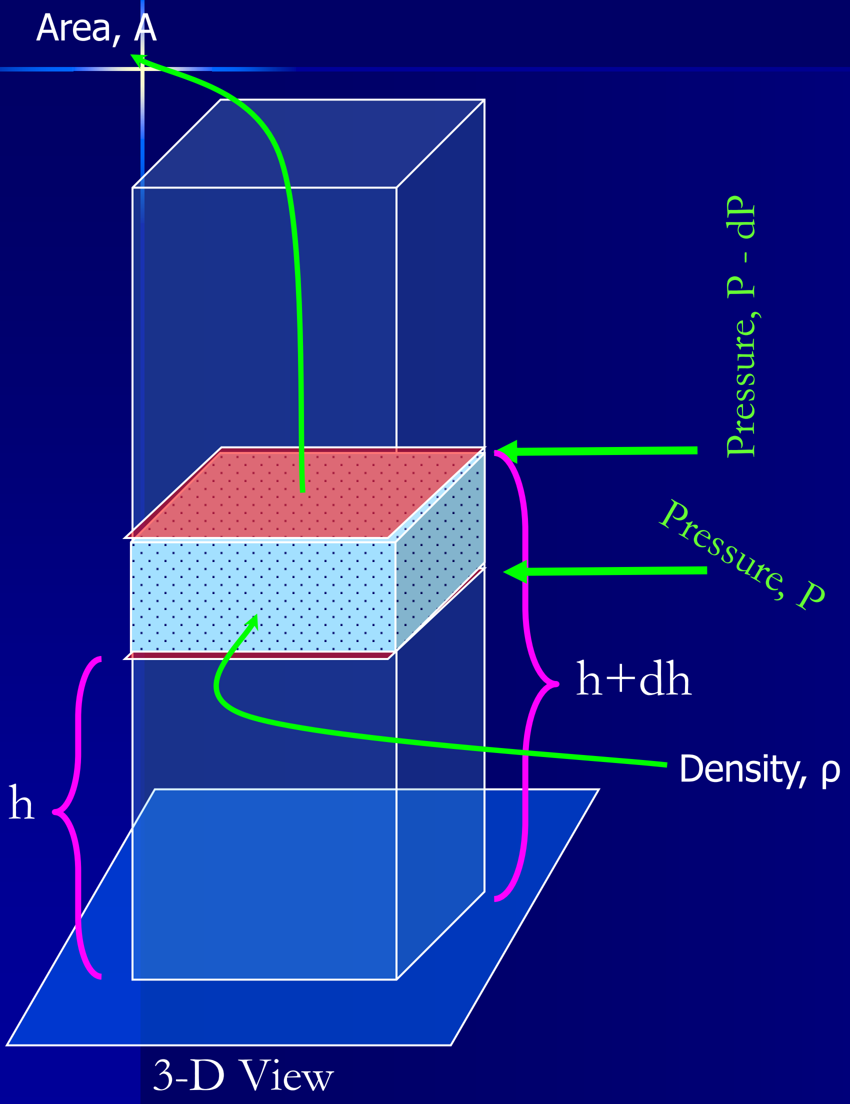
Boltzmann Constant

$$P = NkT$$

Where,

$$k = \frac{R}{N_o}$$

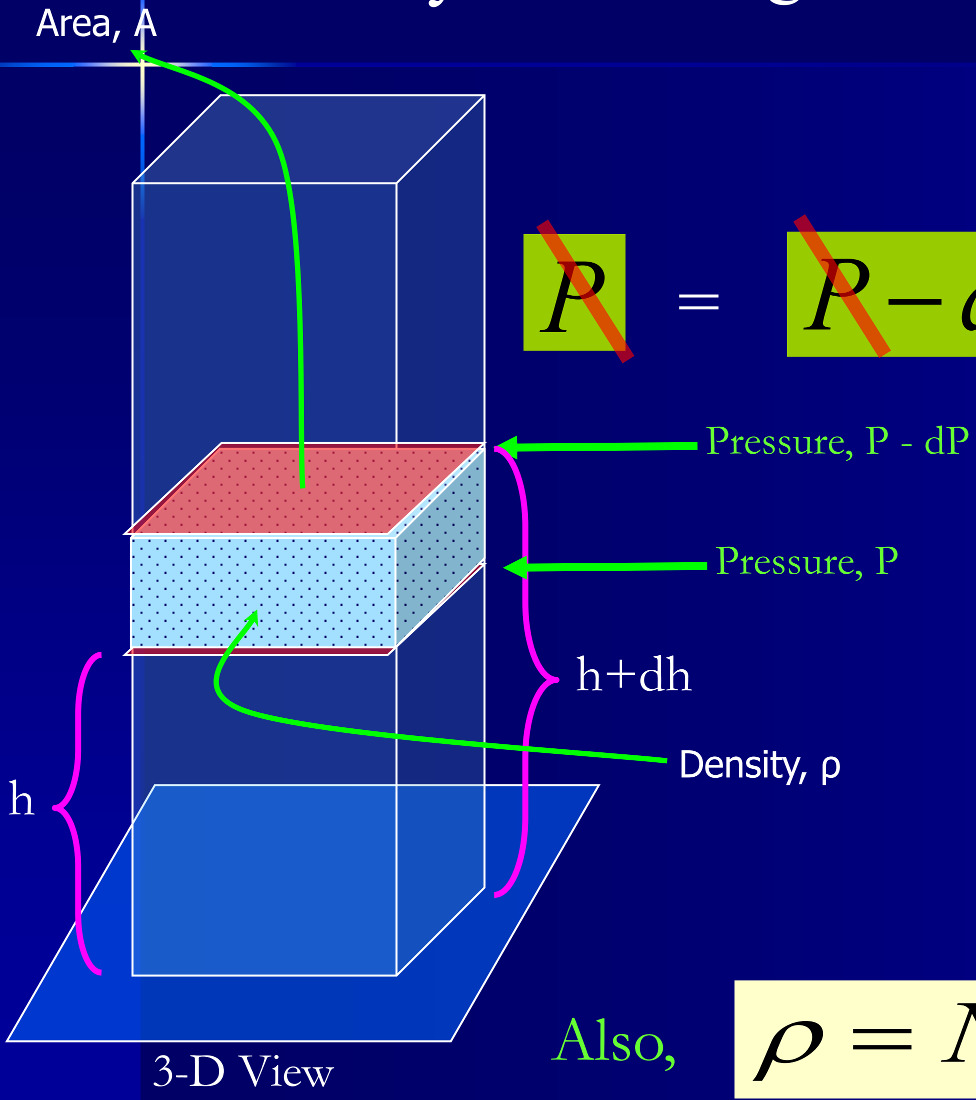
Pressure Profile



The pressure at the Earth's surface (or at higher levels) is a result of the weight of the overlying atmosphere [force per unit area]. If at a height of h the atmosphere has density ρ and pressure P then moving upwards at an infinitesimally small dh will decrease the pressure by amount dP equal to the weight of the layer of atmosphere of thickness dh.

Pressure Profile

$$\text{Pressure of the Lower Layer} = \text{Pressure of the higher Layer} + \frac{\text{Weight of the air molecules in the selected part}}{\text{Cross area of the selected part}}$$



$$\cancel{P} = \cancel{P} - dP + \frac{\cancel{A}.dh.\rho.g}{\cancel{A}}$$

$$dP = dh \times \rho \times g$$

$$dP = -\rho\, g\, dh$$

If h is increasing P is deccreasing

Also, $\rho = N \times \overline{m}$ & $P = NkT$

Pressure Profile

$$\frac{dP}{P} = -\frac{\overline{m}g}{kT}dh$$

The Pressure at height h can be written as:

$$P(h) = P_o\, e^{\frac{-\overline{m}g}{kT}h}$$

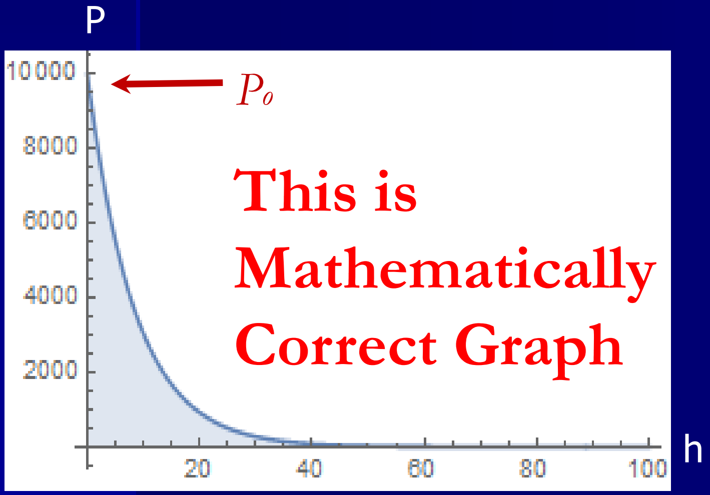
This is the general formula as the Pressure at height; This translate as the pressure decreasing exponentially with height !

If h=0 then P=Po (1); That means Po is the pressure at h=0 level or The Ground Level.

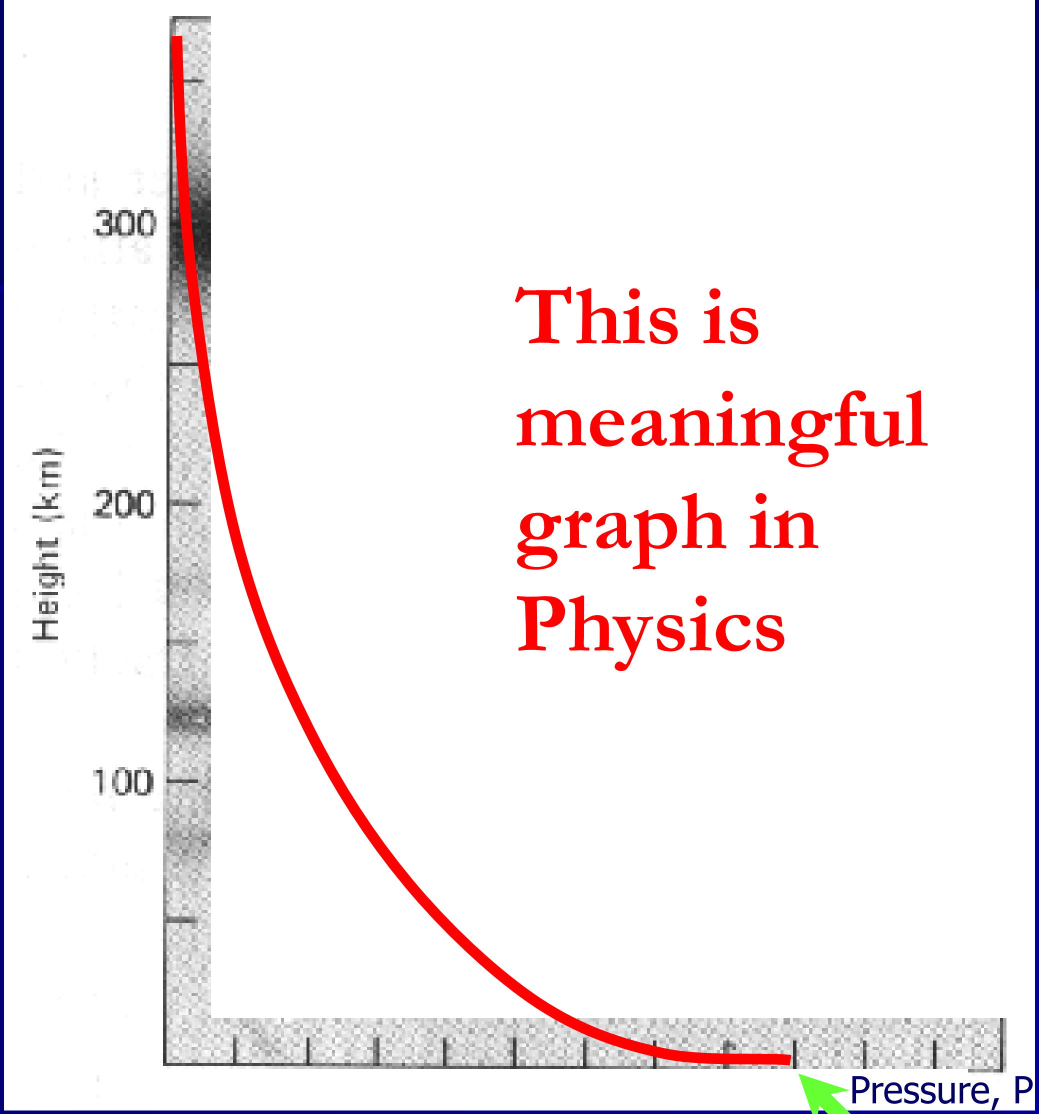
Also $\frac{-\overline{m}g}{kT}h$ is independent of the units. That means $\frac{kT}{\overline{m}g}$ is also a **some height** !

The Graph of P vs h :

$$P(h) = P_o \, e^{\frac{-\overline{m}g}{kT} h}$$



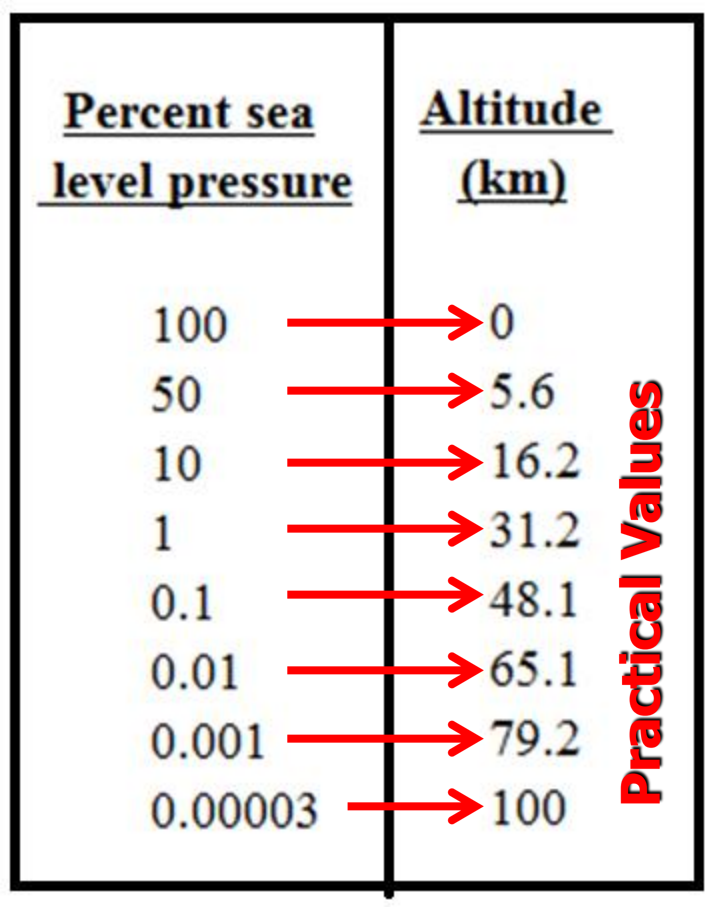
This is Mathematically Correct Graph



This is meaningful graph in Physics

The Graph of h vs P :

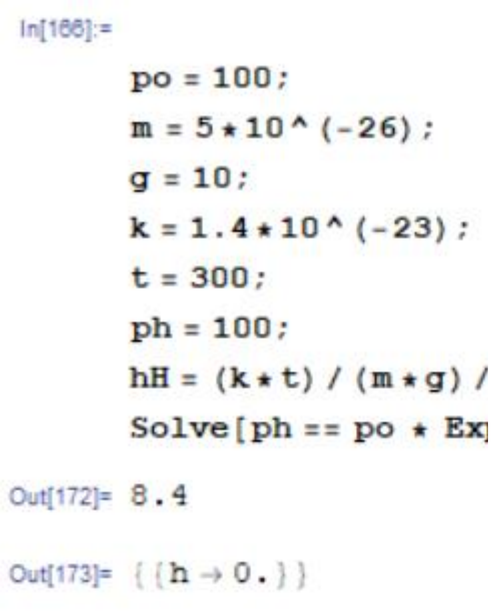
The Graph of h vs P :



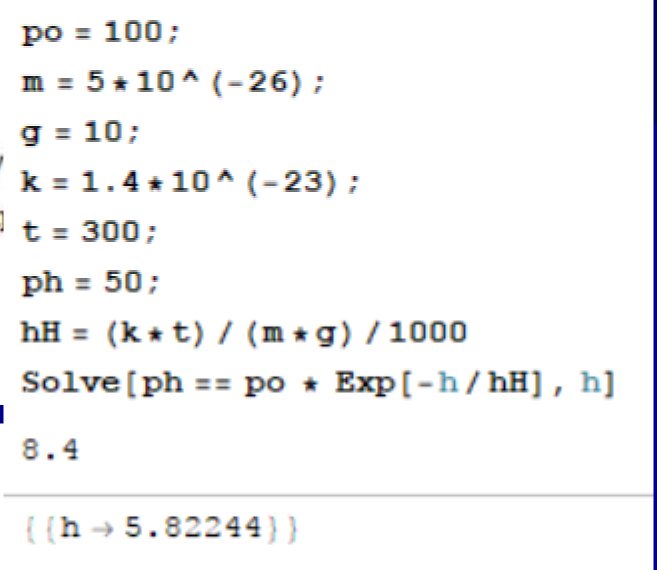
Percent sea level pressure	Altitude (km)
100	0
50	5.6
10	16.2
1	31.2
0.1	48.1
0.01	65.1
0.001	79.2
0.00003	100

Practical Values

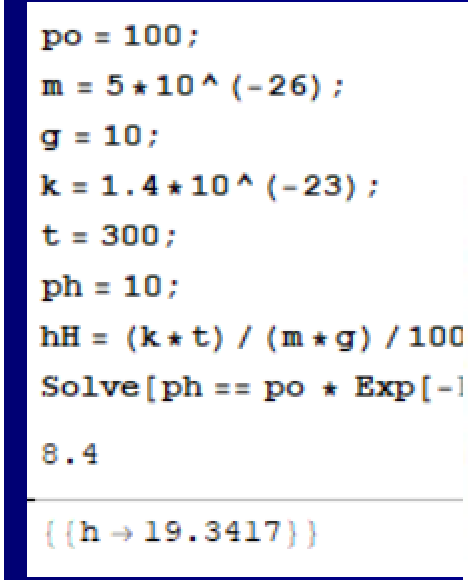
Theoretical Values...



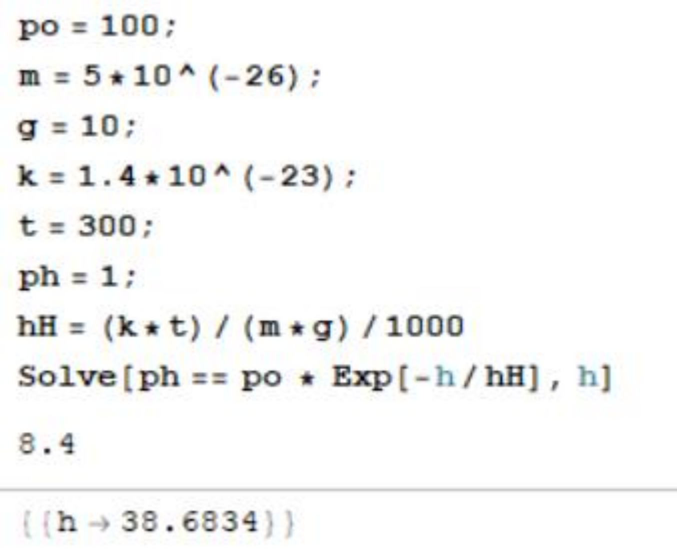
```
In[166]:=
        po = 100;
        m = 5*10^(-26);
        g = 10;
        k = 1.4*10^(-23);
        t = 300;
        ph = 100;
        hH = (k*t) / (m*g) /
        Solve[ph == po * Ex
Out[172]= 8.4

Out[173]= {{h → 0.}}
```



```
po = 100;
m = 5*10^(-26);
g = 10;
k = 1.4*10^(-23);
t = 300;
ph = 50;
hH = (k*t) / (m*g) / 1000
Solve[ph == po * Exp[-h / hH], h]

8.4

{{h → 5.82244}}
```



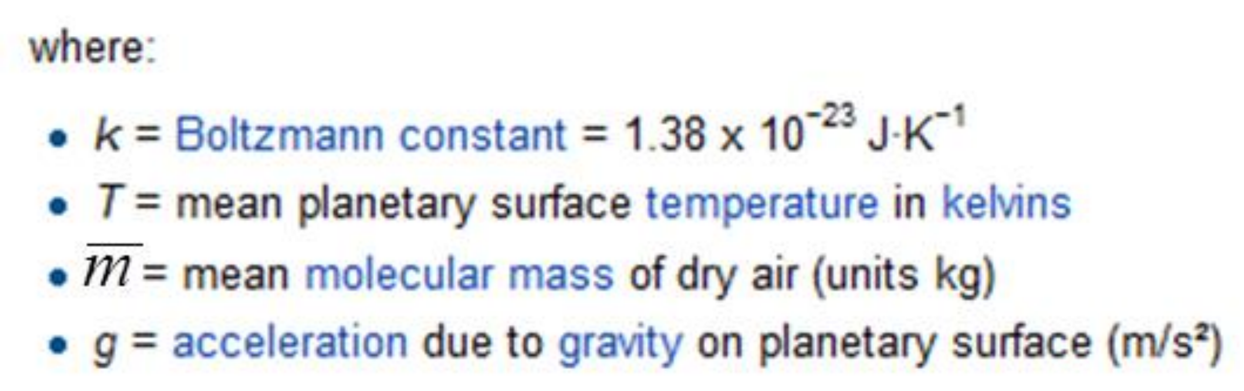
```
po = 100;
m = 5*10^(-26);
g = 10;
k = 1.4*10^(-23);
t = 300;
ph = 10;
hH = (k*t) / (m*g) / 100
Solve[ph == po * Exp[-

8.4

{{h → 19.3417}}
```



```
po = 100;
m = 5*10^(-26);
g = 10;
k = 1.4*10^(-23);
t = 300;
ph = 1;
hH = (k*t) / (m*g) / 1000
Solve[ph == po * Exp[-h / hH], h]

8.4

{{h → 38.6834}}
```

Scale Height (H)

$$H = \frac{kT}{\overline{m}g}$$



where:

- k = Boltzmann constant = 1.38 x 10^{-23} J·K^{-1}
- T = mean planetary surface temperature in kelvins
- $\overline{m}$ = mean molecular mass of dry air (units kg)
- g = acceleration due to gravity on planetary surface (m/s²)

$$H = \frac{\left(1.4\times10^{-23}\right)\times(300)}{\left(5.0\times10^{-26}\right)\times(10)}$$

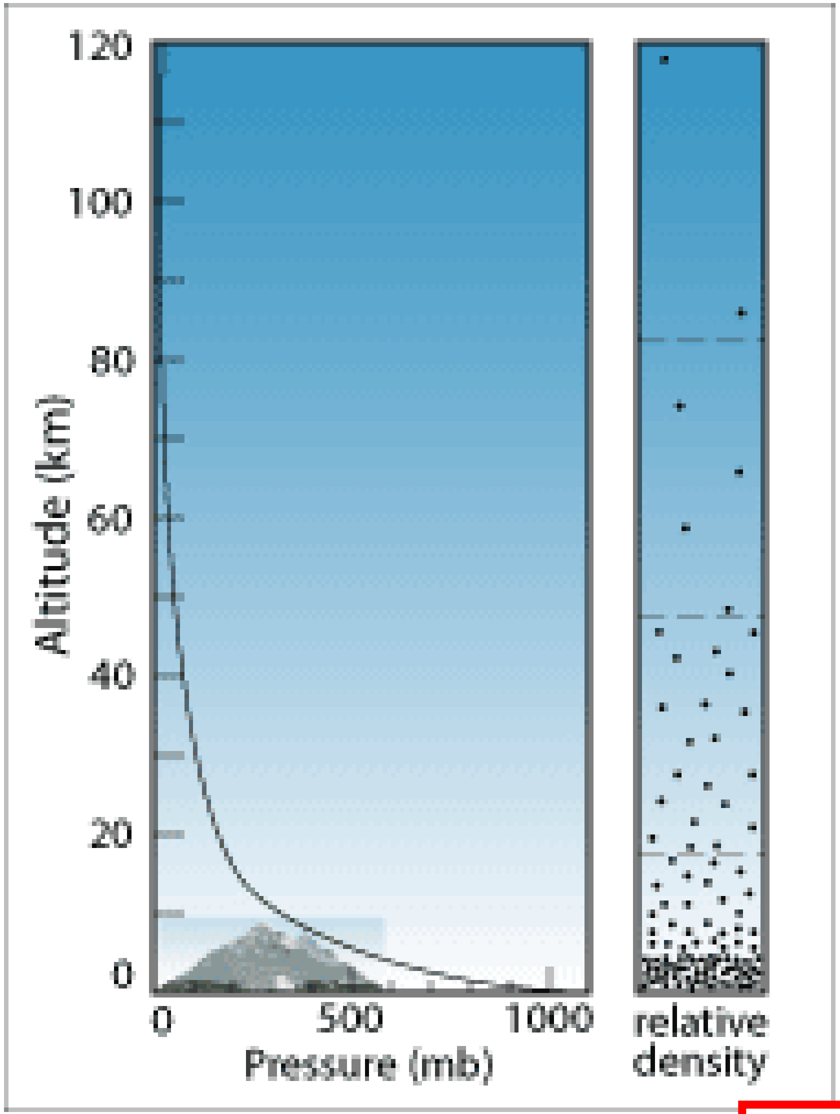
$$H = 8.4km$$

Theoretically this *H* is a constant. But practically this *H* is not a constant. Because, the values of "**mean molecular mass**", "**acceleration due to gravity**" and "**mean planetary surface temperature**" are changing with respect to height from the Earth surface.

The Graph of Scale Heights vs P :

Height	Pressure	
H	Po / e	0.36 Po
2 H	Po / e^2	0.13 Po
3 H	Po / e^3	0.04 Po
4 H	Po / e^4	0.01 Po
5 H	Po / e^5	0.006 Po
………	…….	
n H	Po / e^n	

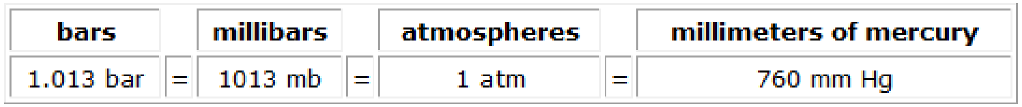
Scale Height of the Earth, H



Pressure and density decrease rapidly with altitude.

Temp, T vs Sca Hght, H

T (K)	H (m)
290	8500
273	8000
260	7610
210	6000



bars		millibars		atmospheres		millimeters of mercury
1.013 bar	=	1013 mb	=	1 atm	=	760 mm Hg

Correspondence of atmospheric measurement units.

	Height (km)	Pressure	
6 x 1	6	Po / 2	Po / 2^1
6 x 2	12	Po / 4	Po / 2^2
6 x 3	18	Po / 8	Po / 2^3
6 x 4	24	Po / 16	Po / 2^4
6 x 5	30	Po / 32	Po / 2^5
	………	…….	
	6 n	Po / 2^n	

Molecular Number Density

Using the Pressure Equation : $P(h) = P_o\, e^{\frac{-h}{H}}$

Where, $H = 8.4km$

For the Ideal Gas

$$PV = nRT$$

$$P = NkT$$

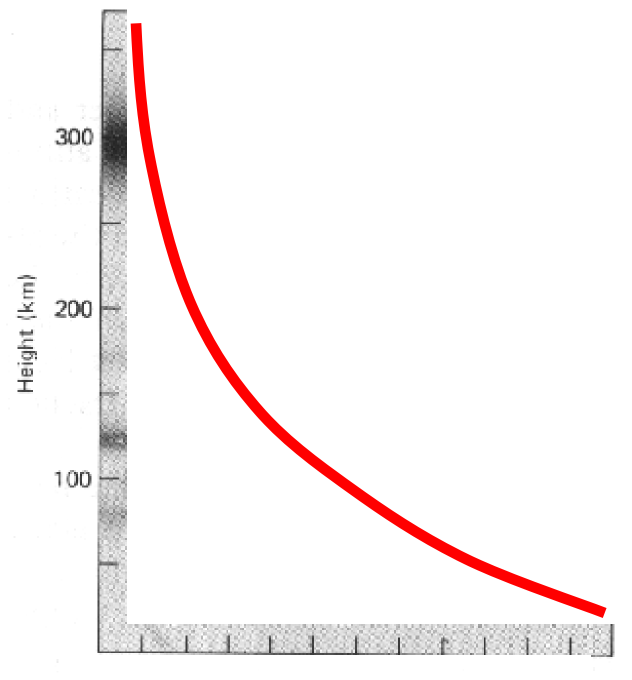
$$N = \frac{P}{kT}$$

$$N(h) = \frac{P(h)}{kT}$$

&

$$No = \frac{Po}{kT}$$

$$N(h) = N_o\, e^{-\frac{h}{H}}$$



Molecular Number Density

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$

If h = H,

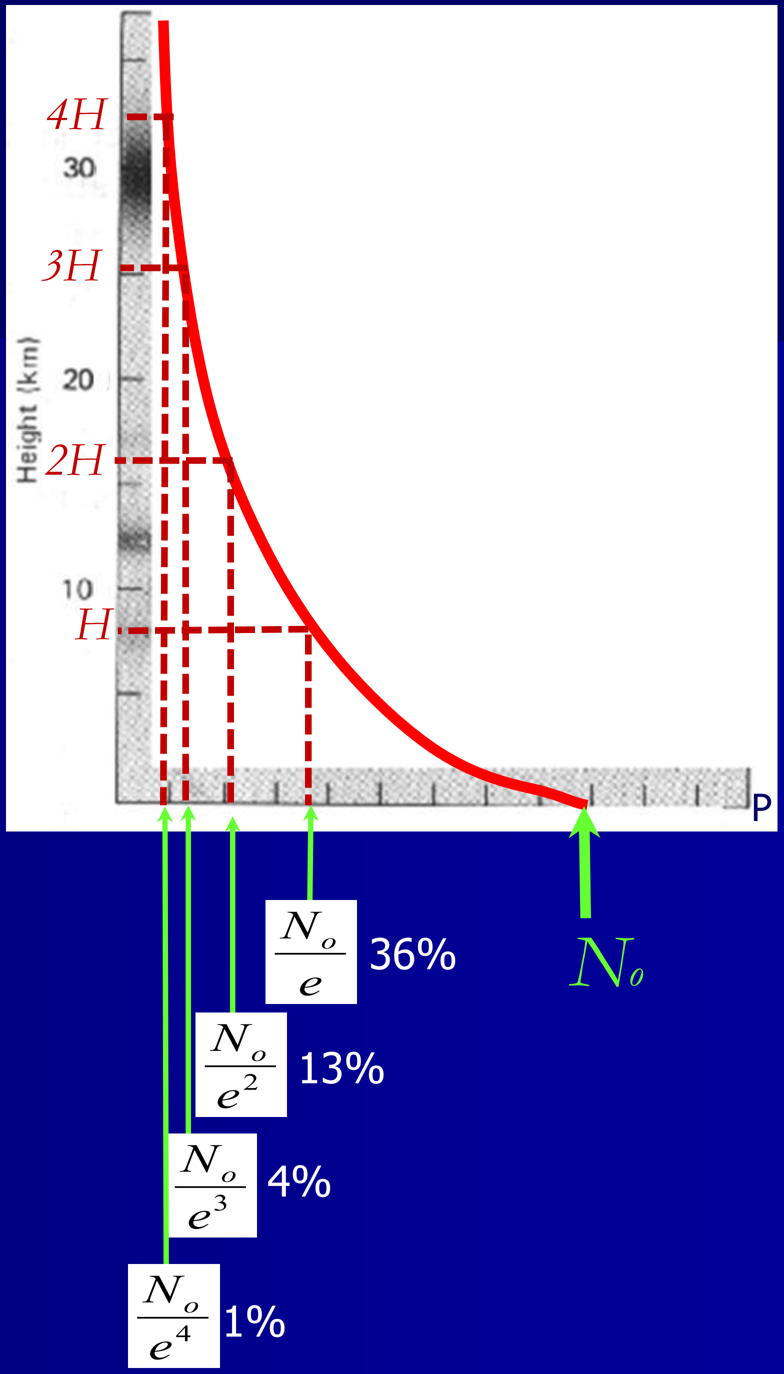
$$N(H) = N_o e^{\frac{-H}{H}}$$

$$\Rightarrow N(H) = \frac{N_o}{e}$$

$$\Rightarrow 0.36 N_o$$

Height	Mol Num Den	
H	No / e	0.36 No
2 H	No / e^2	0.13 No
3 H	No / e^3	0.04 No
4 H	No / e^4	0.01 No
5 H	No / e^5	0.006 No
………	…….	
n H	No / e^n	

The Graph of H vs N :



Always Molecular Number Density is decreasing by a factor of e when height is increasing by a multiplies of H

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$

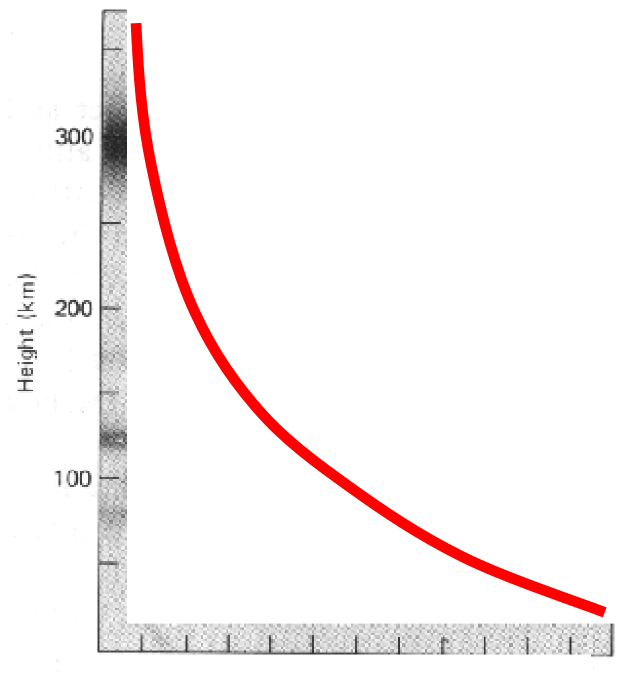
Eg:

At which height from the surface of the Earth, which you can expect the Molecular Number Density which is half of that of the initial value of the Molecular Number Density ?

If N(h) = No/2 when h=h,

$$\frac{N_o}{2} = N_o e^{\frac{-h}{H}}$$

$$h = \sim 6km$$



Molecular Number Density

	Height (km)	Mol Num Density	
6 x 1	6	No / 2	No / 2^1
6 x 2	12	No / 4	No / 2^2
6 x 3	18	No / 8	No / 2^3
6 x 4	24	No / 16	No / 2^4
6 x 5	30	No / 32	No / 2^5
	………	…….	
	6 n	No / 2^n	

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$

If h=6 km Then N(h) = ?, → $N = \frac{N_o}{2}$

If h=60 km Then N(h) = ?, → $N = \frac{N_o}{2^{10}} =\sim \frac{N_o}{1000}$

If h=600 km Then N(h) = ?, → $N = \frac{N_o}{2^{100}} =\sim \frac{N_o}{10^{30}}$

That means at 600 km height, the Molecular Number Density is (1/(10^30)) from its initial value.

Consider Linear Distance ;
At 600 km height, the Molecular Linear Distance is (1/(10^30))^(1/3) = (1/(10^10)) from its initial value.

$$= \left(\frac{1}{10^{30}} \right)^{\frac{1}{3}}$$

Molecular Number Density

$$=\left(\frac{1}{10^{30}}\right)^{\frac{1}{3}}$$

Linear Distance of the molecules = **Mean Free Path** ;
This is "Separation between two atoms"

Mean Free Path on the ground level m $= 6.0 \times 10^{-8}$

Mean Free Path at altitude 600 km height from the ground level :

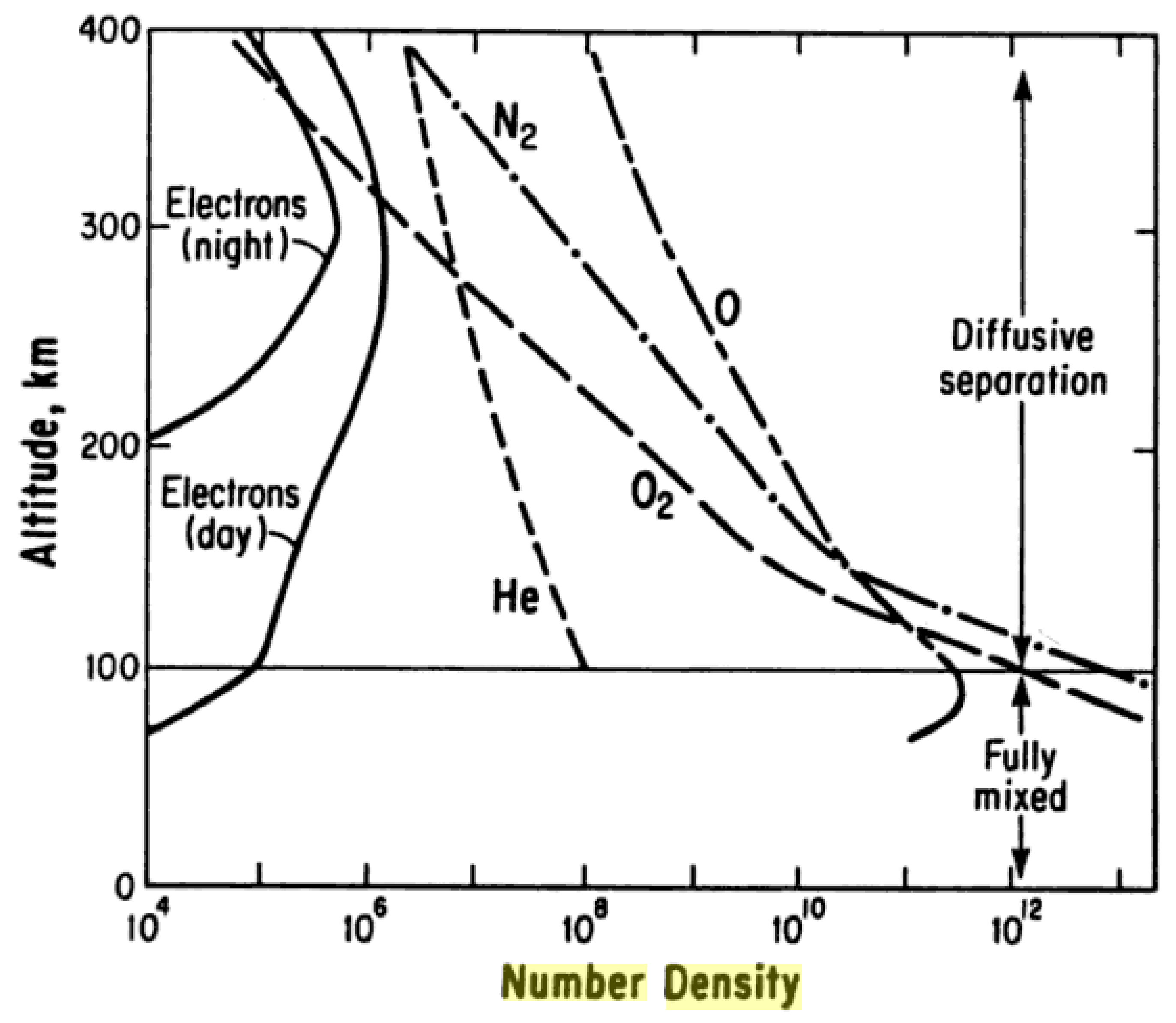
$$= 6 \times 10^{-8} \times \left(10^{30}\right)^{\frac{1}{3}}$$

$$= 6 \times 10^{-8} \times 10^{10}$$

$$= 600\,m$$

That means the **gap between two atoms** on that 600 km height (altitude) from the ground level is very high ! At that level there is no mean "**The gas**", because the **mean free path is very high** (600 m)

Molecular Number Density



Density

Using the Molecular Number Density Equation :

$$N(h) = N_o e^{-\frac{h}{H}}$$

Where, $H = 8.4km$

Mean Molecular Number Density

Density

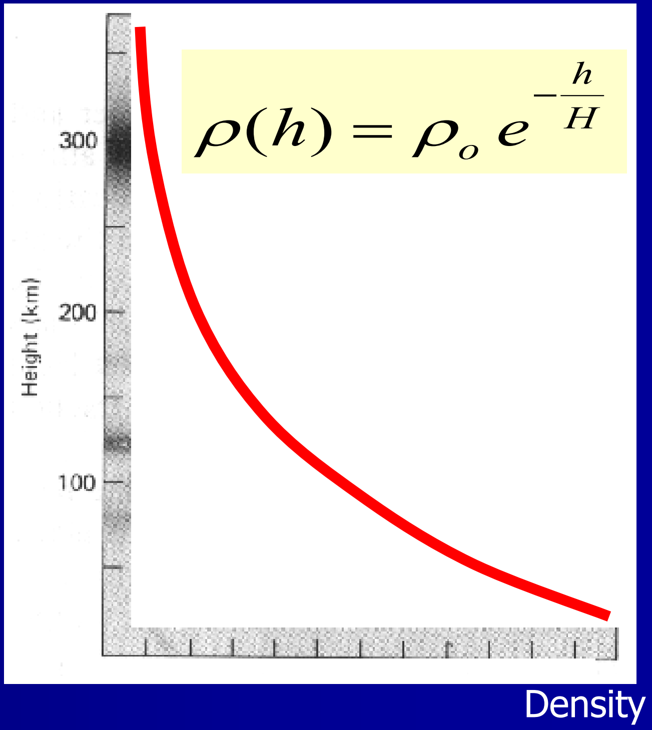
$$\rho = N \times \overline{m}$$

Total Molecular Number Density

$$\rho(h) = N(h) \times \overline{m}$$

&

$$\rho_o = N_o \times \overline{m}$$

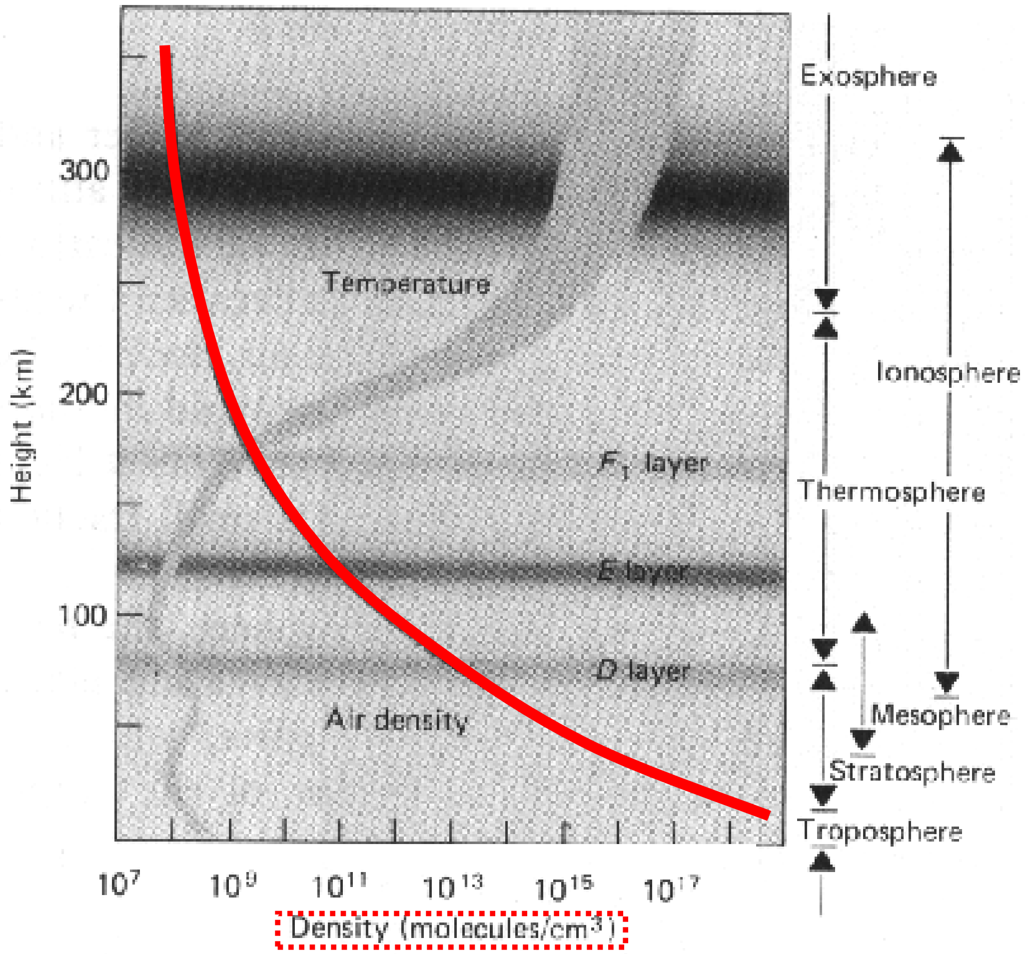


$$\rho(h) = \rho_o e^{-\frac{h}{H}}$$

Density

$$\rho(h) = \rho_o \, e^{-\frac{h}{H}}$$

Where, $H = 8.4km$



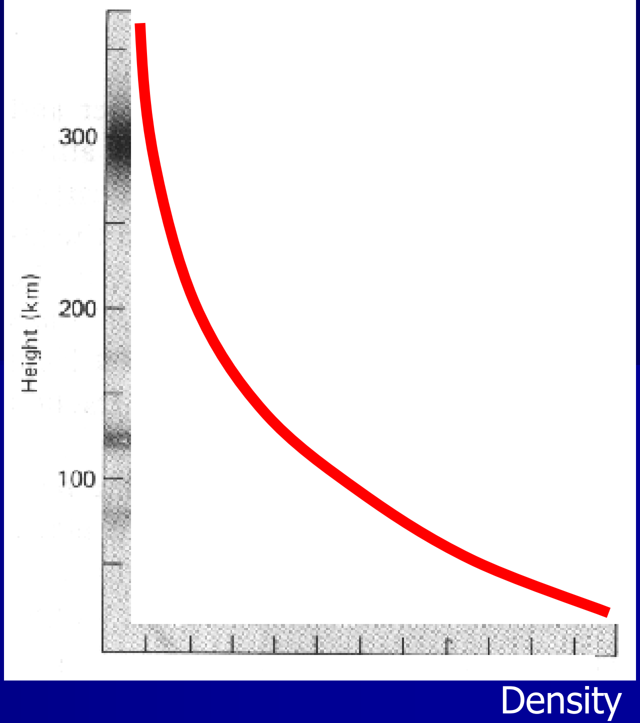
Variations in the density of
Earth's neutral atmosphere with altitude

If h = H,

$$\rho(H) = \rho_o \, e^{\frac{-H}{H}} \Rightarrow \rho(H) = \frac{\rho_o}{e} \Rightarrow 0.36\rho_o$$

Density

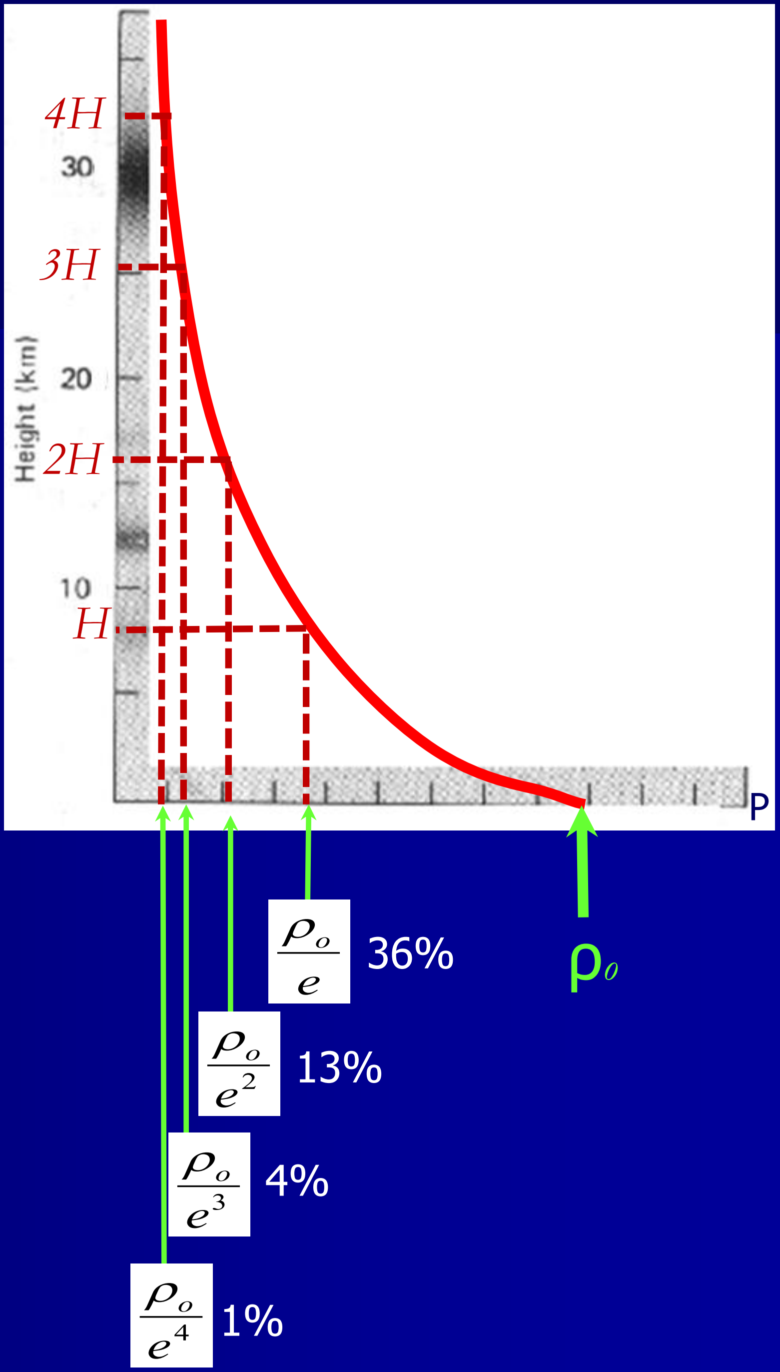
$$\rho(h) = \rho_o \, e^{-\frac{h}{H}}$$



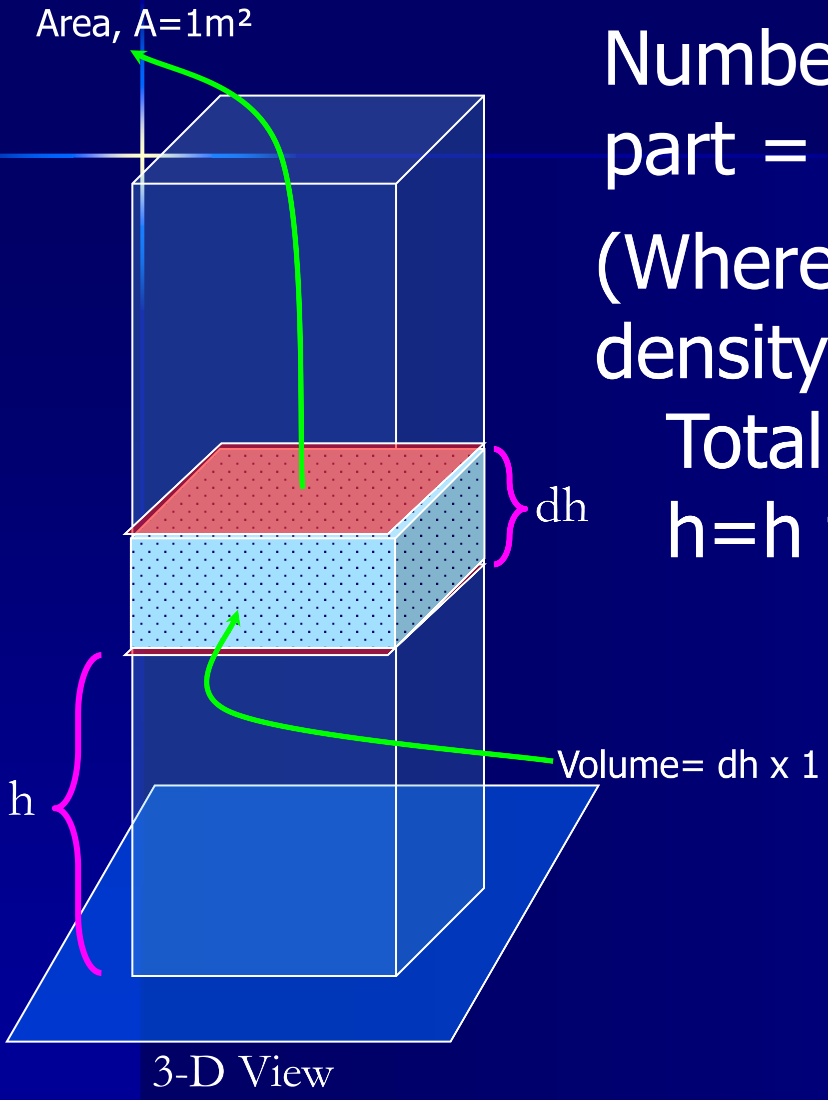
Height	Density	
H	ρo / e	0.36 ρo
2 H	ρo / e^2	0.13 ρo
3 H	ρo / e^3	0.04 ρo
4 H	ρo / e^4	0.01 ρo
5 H	ρo / e^5	0.006 ρo
………	…….	
n H	ρo / e^n	

The Graph of H vs ρ :

Always Density is decreasing by a factor of e when height is increasing by a multiplies of H



Total Number of Molecules from Earth Surface to altitude h :



Number of molecules in a selected part = $N \times dh \times 1$

(Where N is the molecular number density)

Total Number of molecules from h=h to h=∞

$$\int_{h=h}^{\infty} N.dh$$

Where,

$$N(H) = N_o e^{\frac{-h}{H}}$$

$$\int_{h=h}^{\infty} N_o e^{\frac{-h}{H}} .dh$$

Total Number of Molecules from Earth Surface to altitude h :

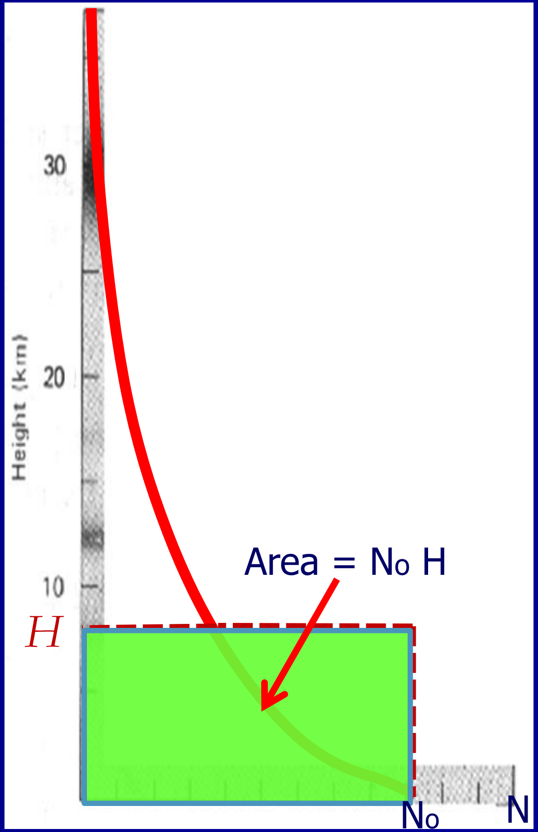
$$N_{\underset{h\rightarrow\infty}{Total}} = N_o H \ e^{-\frac{h}{H}}$$

Case I :

$$N_{\underset{0\rightarrow\infty}{Total}} = N_o H$$

That means, if the molecular number density of the atmosphere of the Earth varies **linearly** without varying **exponentially**, the atmosphere of the Earth will diminish after ~8.4 km (a scale height).

This gives to us another definition for the Scale Height !



Total Number of Molecules from Earth Surface to altitude h :

$$N_{\underset{h\to\infty}{Total}} = N_o H\ e^{-\frac{h}{H}}$$

Case II :

$$\frac{N_{\underset{h\to\infty}{Total}}}{N_{\underset{0\to\infty}{Total}}} = \frac{N_o H\ e^{-h/H}}{N_o H} = e^{-h/H}$$

Fraction of the Number of Molecules from the specific height h.

If h=H km Then RATIO = ?, ➡ $\left(e^{-h/H}\right)_{h\to H} = e^{-\ H/H} = e^{-1}$

~ 40 %

60 % of the total molecules exist bellow H (8.4 km) !

Total Number of Molecules from Earth Surface to altitude h :

If h=2H km Then
RATIO = ?,

$$\left(e^{-h/H}\right)_{h\to 2H} = e^{-\ 2H/H} = e^{-2}$$

~ 15 %

85 % of the total molecules exist bellow 2H (16.8 km) !

If h=3H km Then
RATIO = ?,

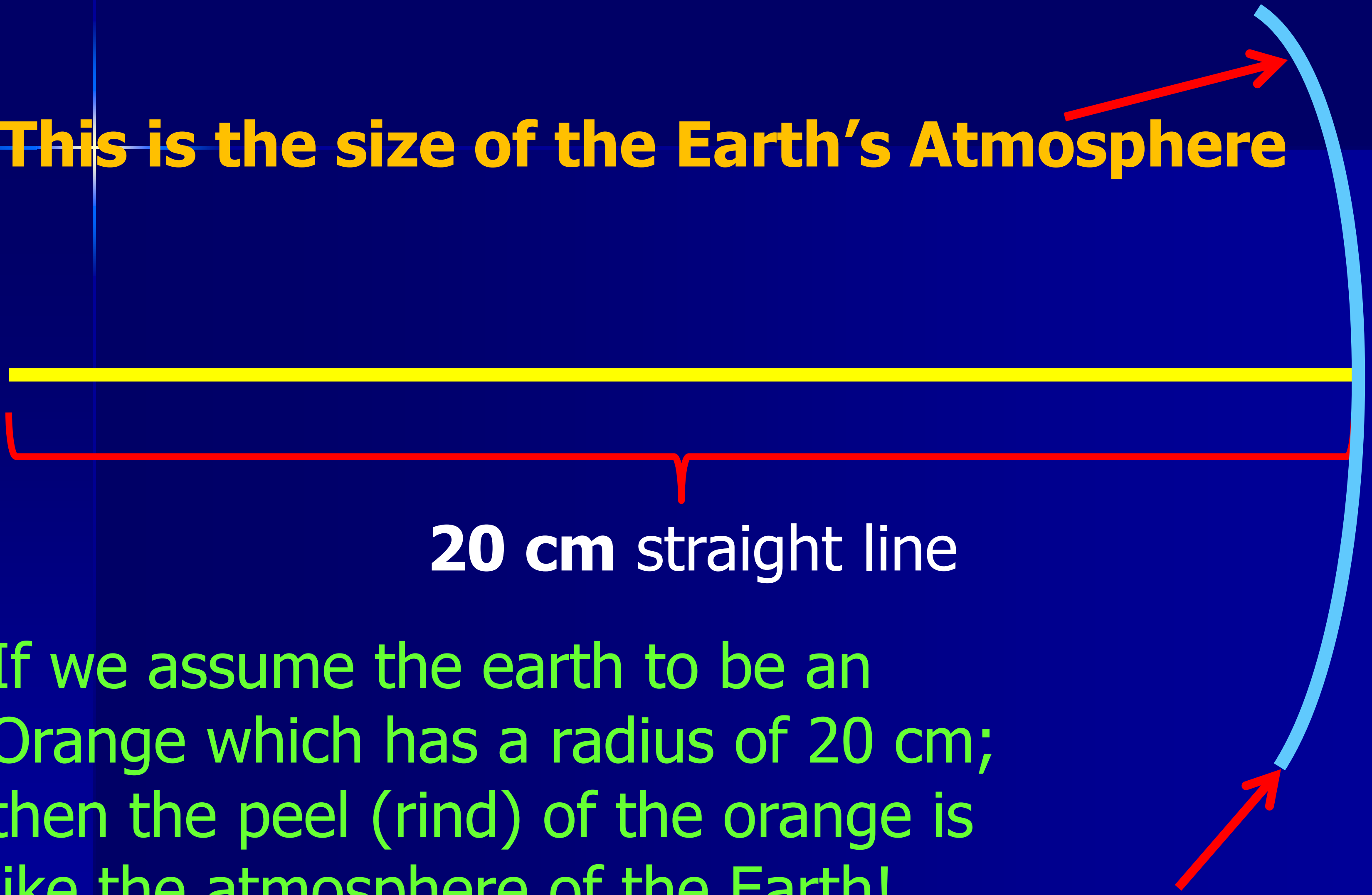
$$\left(e^{-h/H}\right)_{h\to 3H} = e^{-\ 3H/H} = e^{-3}$$

~ 5 %

95 % of the total molecules exist bellow 3H (16.8 km) !

h (km)		$N(h \to \infty) / N(0 \to \infty)$	% below h
H	08.4	36.78	63.21
2 H	16.8	13.53	86.46
3 H	25.2	4.97	95.02
4 H	36.6	1.83	98.16
5 H	42.0	0.67	99.32
6 H	50.4	0.24	99.75
7 H	58.8	0.09	99.90
8 H	67.2	0.03	99.96
9 H	75.6	0.01	99.98
10 H	84.0	0.004	99.995

Sketch the size of the Earth’s Atmosphere



This is the size of the Earth’s Atmosphere

20 cm straight line

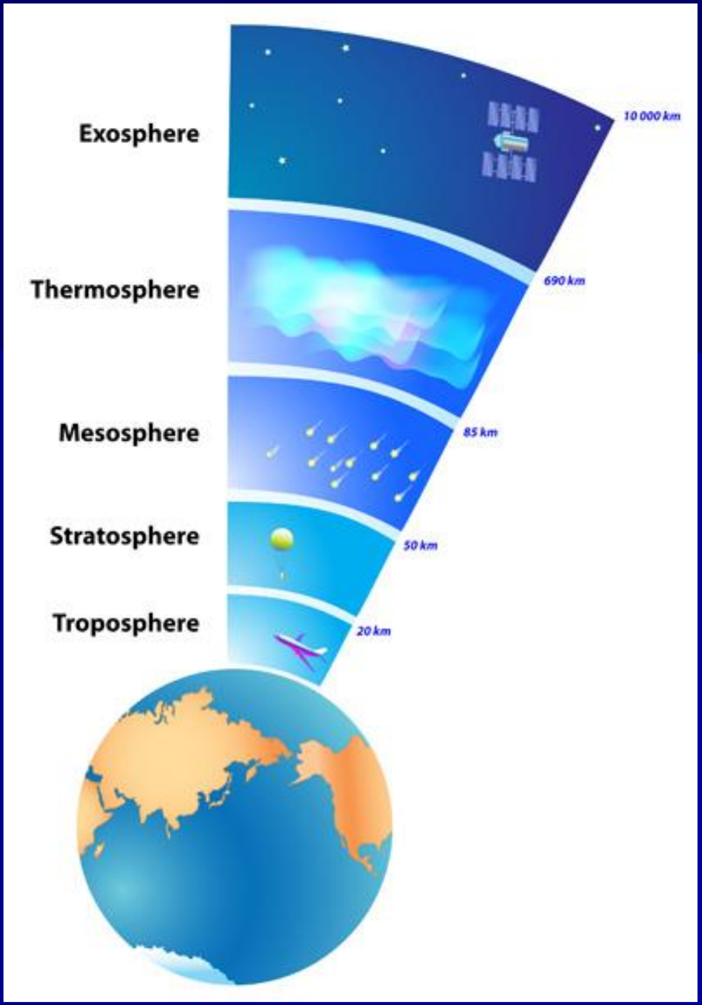
If we assume the earth to be an Orange which has a radius of 20 cm; then the peel (rind) of the orange is like the atmosphere of the Earth!

1 mm thick line

Earth Atmosphere

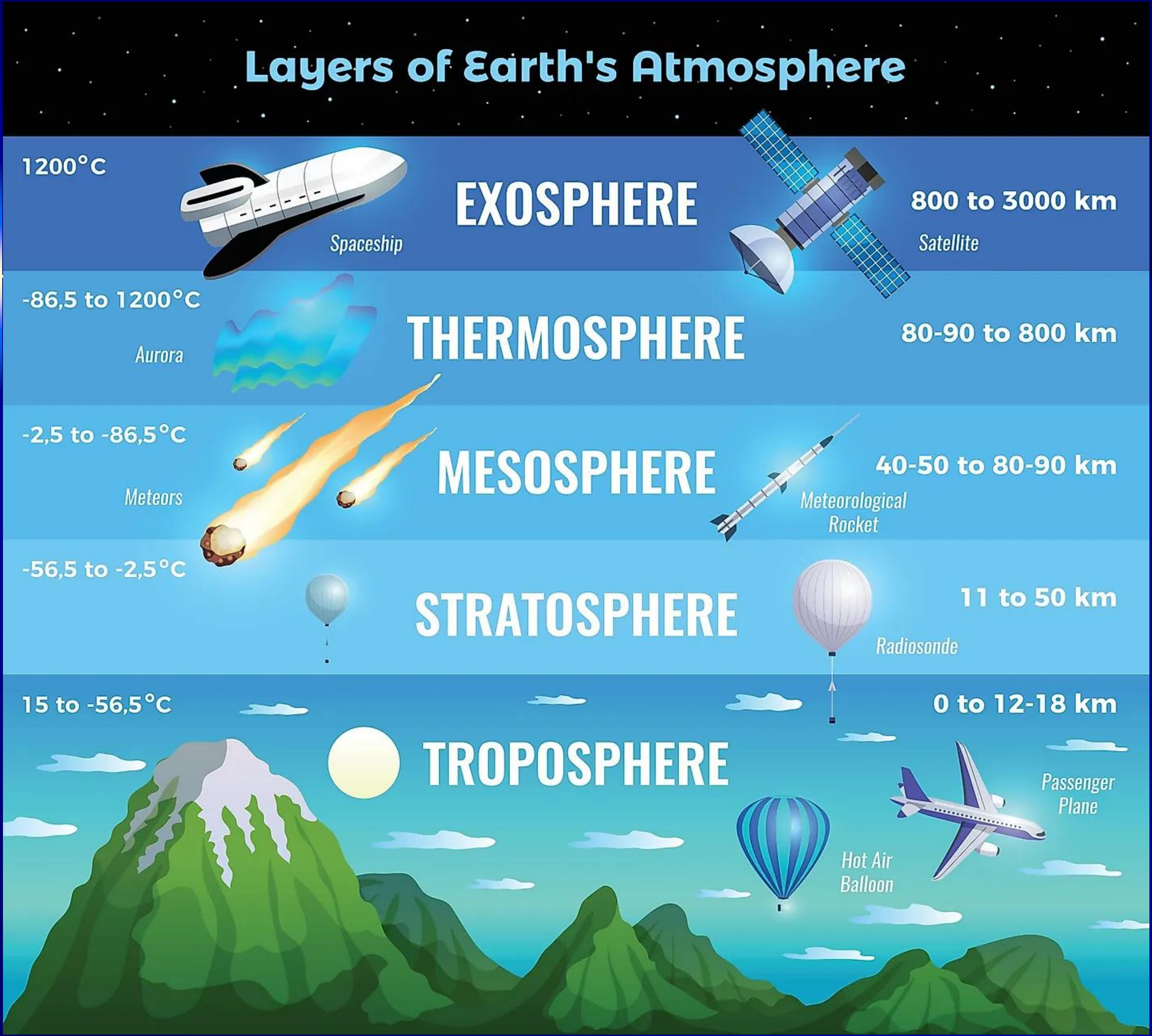
- Retaining of Gases in the Earth
- Major / Minor constituents
- Barometric Equation
- Scale Height
- Atmospheric Regions
- Temperature Profiles
- Retaining of Gases
- Number Density Profiles

Atmospheric Regions



The properties of the Earth's atmosphere vary with altitude. Based on these properties, the atmosphere may be regarded as having different layers or zones. According to one system of nomenclature, there are five layers:

the troposphere, stratosphere, mesosphere, thermosphere, and exosphere. The boundaries between these regions are called the tropopause, stratopause, mesopause, and exobase.


Layers of Earth's Atmosphere
1200°C
Spaceship
EXOSPHERE
800 to 3000 km
Satellite
-86,5 to 1200°C
Aurora
THERMOSPHERE
80-90 to 800 km
-2,5 to -86,5°C
Meteors
MESOSPHERE
40-50 to 80-90 km
Meteorological Rocket
-56,5 to -2,5°C
STRATOSPHERE
11 to 50 km
Radiosonde
15 to -56,5°C
TROPOSPHERE
0 to 12-18 km
Passenger Plane
Hot Air Balloon

Earth Atmosphere

Retaining of Gases in the Earth
Major / Minor constituents
Barometric Equation
Scale Height
Atmospheric Regions
Temperature Profiles
Retaining of Gases
Number Density Profiles

Temperature Profile of the Earth

The temperature of the atmosphere of the Earth varies with the **distance from the equator** (latitude) and **height above the surface** (altitude). It also changes in **time**, **varying from season to season**, **from day to night** and **irregularly due to passing weather systems**. If these variations are averaged out on a global basis, a pattern of average temperatures emerges for the atmosphere.

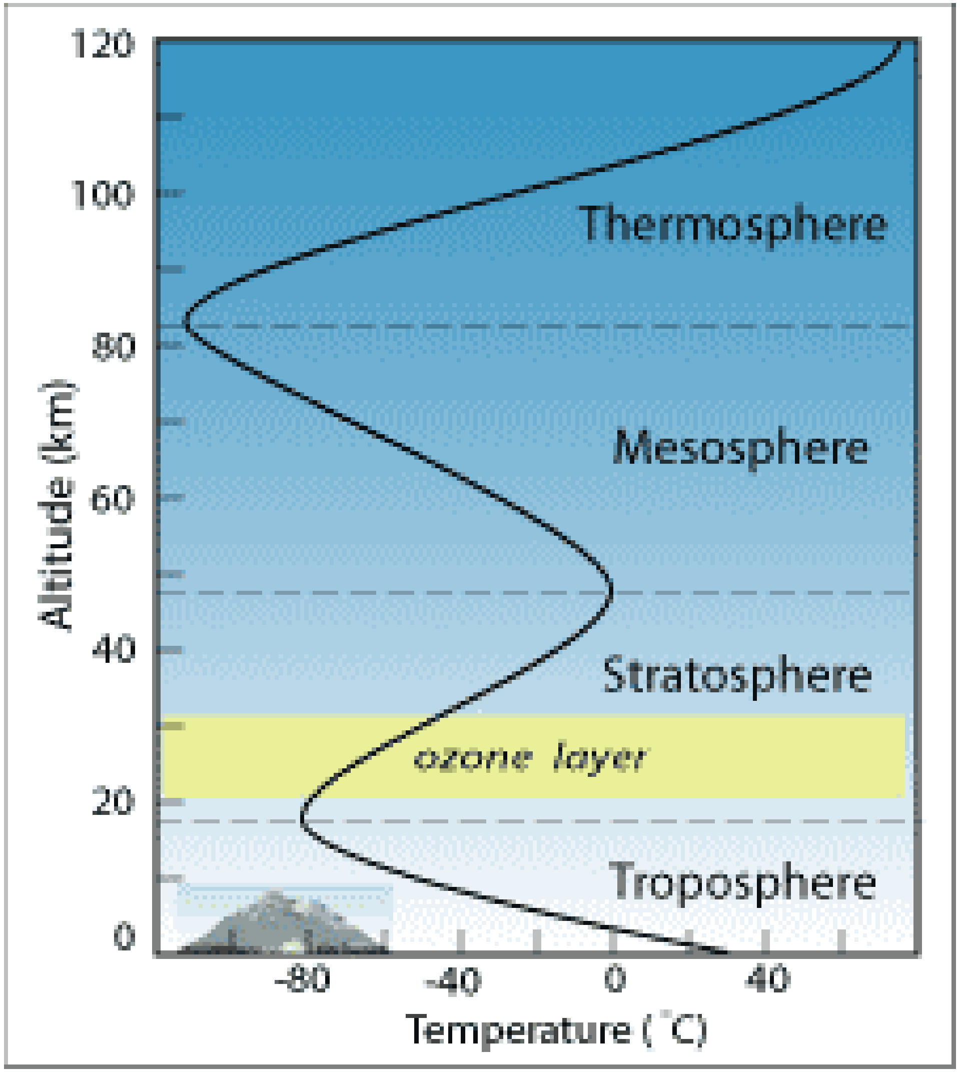
Temperature Profile of the Earth

The vertical temperature profile (the way temperature changes with height) divides the atmosphere into four layers:

The troposphere,
The stratosphere,
The mesosphere,
The thermosphere.

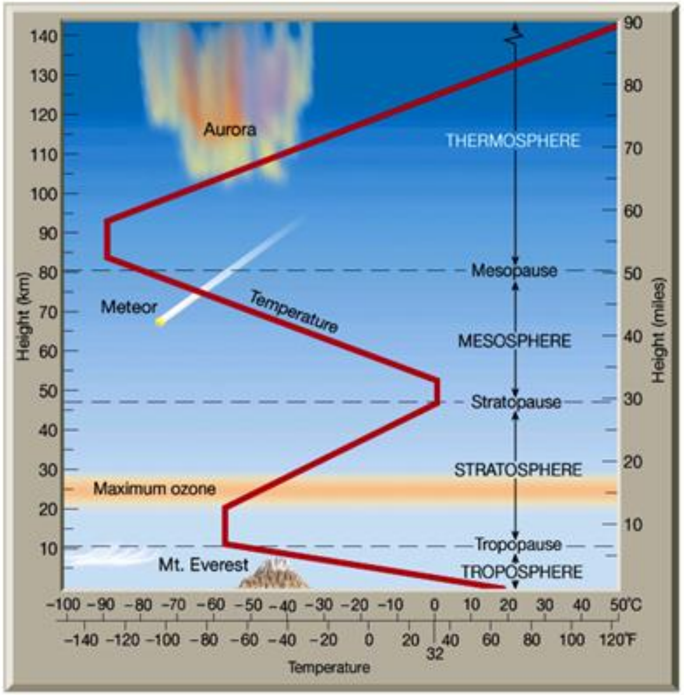
The boundaries between these regions / layers are called tropopause, stratopause and mesopause.

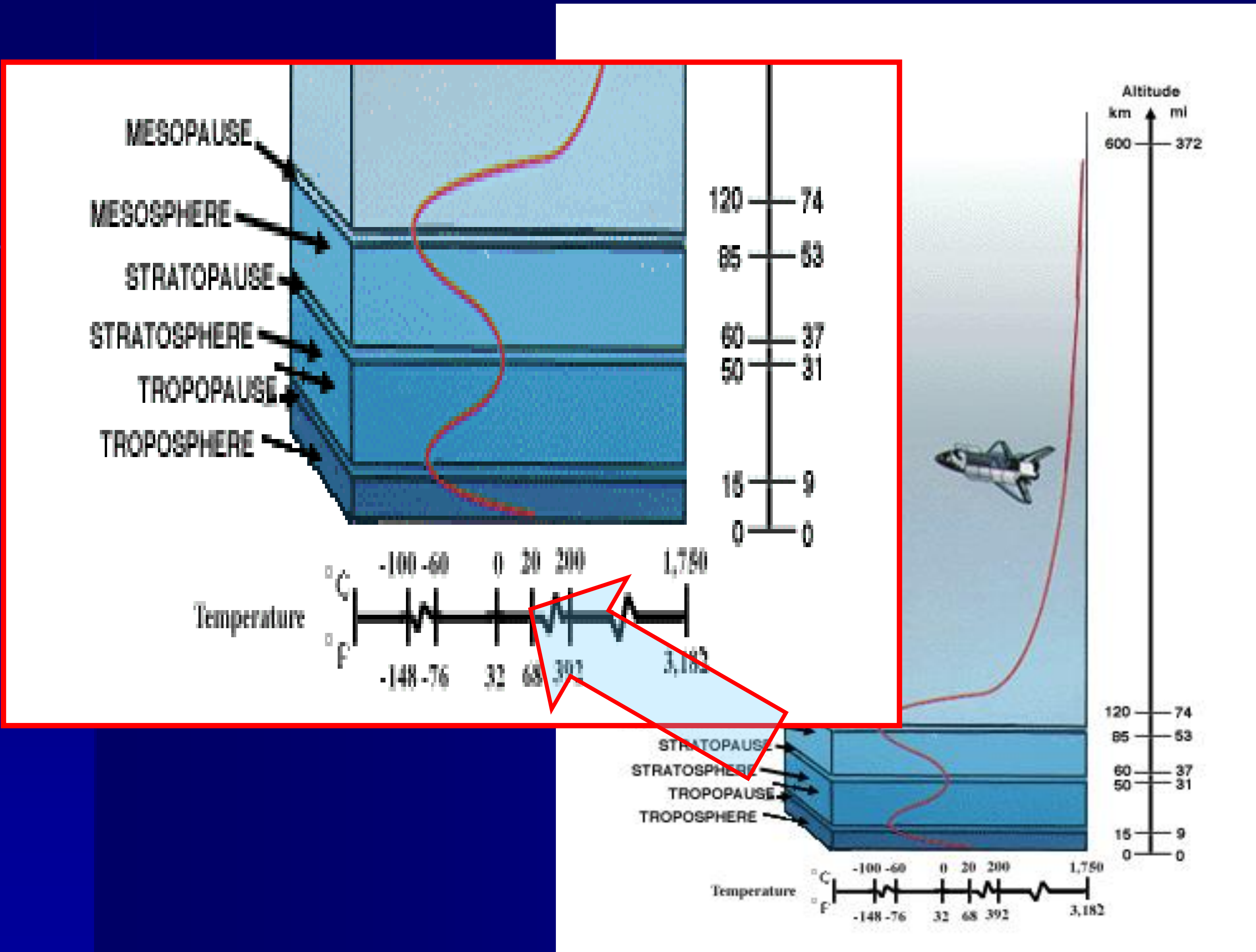
Temperature Profile



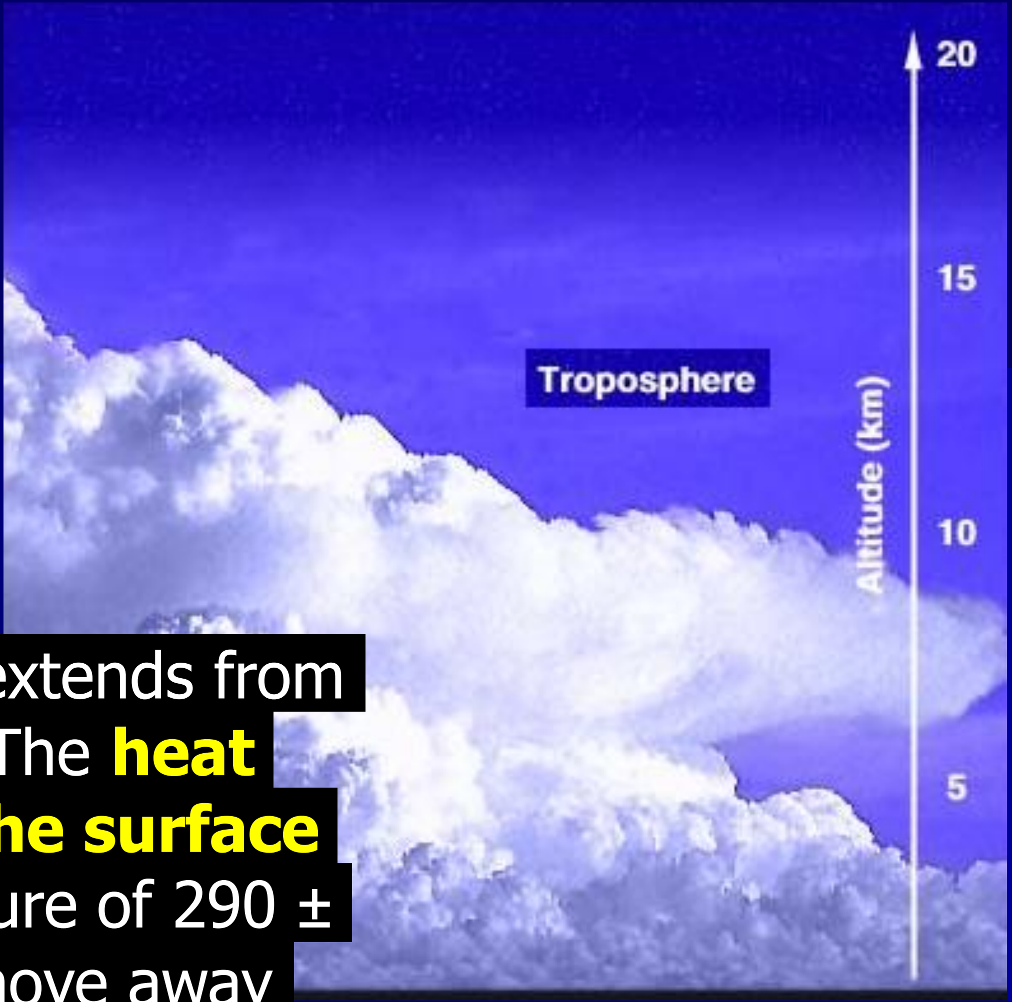
This graph shows how temperature varies with altitude in earth's atmosphere.

Vertical structure of the atmosphere




MESOPAUSE
MESOSPHERE
STRATOPAUSE
STRATOSPHERE
TROPOPAUSE
TROPOSPHERE
120 — 74
85 — 53
60 — 37
50 — 31
15 — 9
0 — 0
Temperature
°C -100 -60 0 20 200 1,750
°F -148 -76 32 68 392 3,182
Altitude
km mi
600 — 372
120 — 74
85 — 53
60 — 37
50 — 31
15 — 9
0 — 0
STRATOPAUSE
STRATOSPHERE
TROPOPAUSE
TROPOSPHERE
Temperature
°C -100 -60 0 20 200 1,750
°F -148 -76 32 68 392 3,182

Troposphere

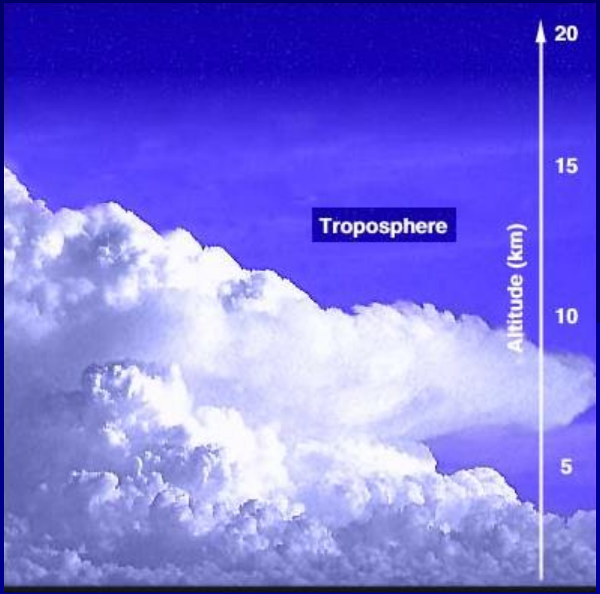


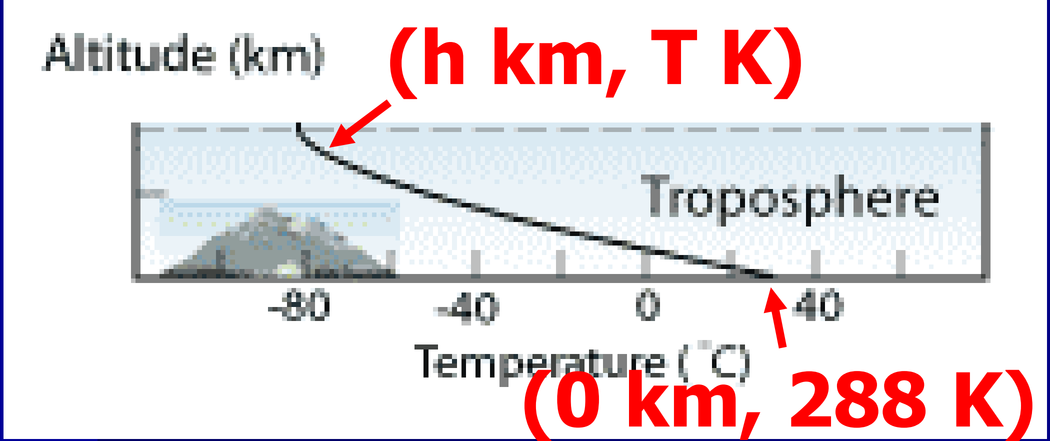
This is the lowest layer and extends from the ground to about 13 km. The **heat source for this region is the surface of the Earth**, at a temperature of 290 ± 20 K and, therefore, as we move away from the ground, the temperature decreases at a rate of reaching a minimum of 210 ± 20 K at the tropopause.

Troposphere

This level, is just above the cruising altitude of large commercial jet aircraft.

The drop of temperature with height is called the lapse rate, is nearly steady throughout the troposphere at 6.5°C/ km.



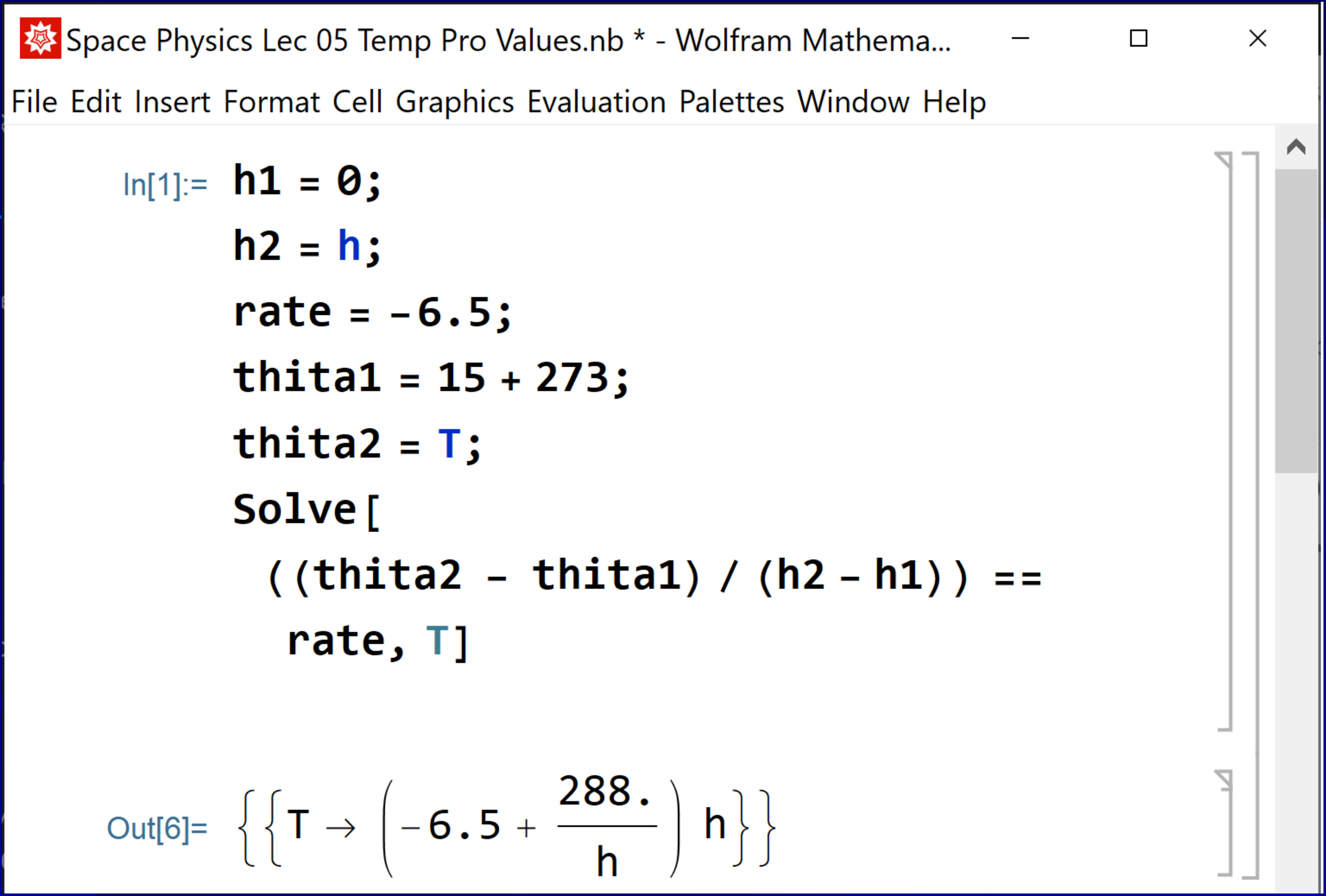


(h km, T K)

(0 km, 288 K)

The drop of Temperature with height :

Lapse Rate : 6.5 °C / km

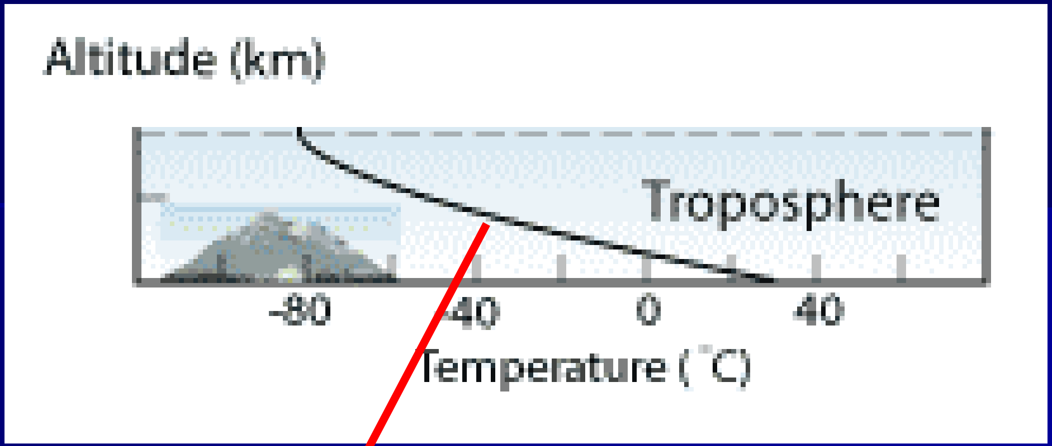


```
In[1]:= h1 = 0;
        h2 = h;
        rate = -6.5;
        thita1 = 15 + 273;
        thita2 = T;
        Solve[
         ((thita2 - thita1) / (h2 - h1)) ==
          rate, T]
```

Out[6]= $\left\{\left\{T \to \left(-6.5 + \frac{288.}{h}\right) h\right\}\right\}$

Troposphere

Altitude (km)	Temperature (K)
0	288
2	275
4	262
6	249
8	236
10	223
12	210



Lapse Rate : 6.5 °C / km

$$\underset{(\mathrm{K})}{T(h)} = - 6.5 \overset{(\mathrm{km})}{h} + \underset{(\mathrm{K})}{288}$$

1 °C = 1 K

Tropopause:

The upper boundary of the troposphere occurring at an altitude of 13 ± 5 km.

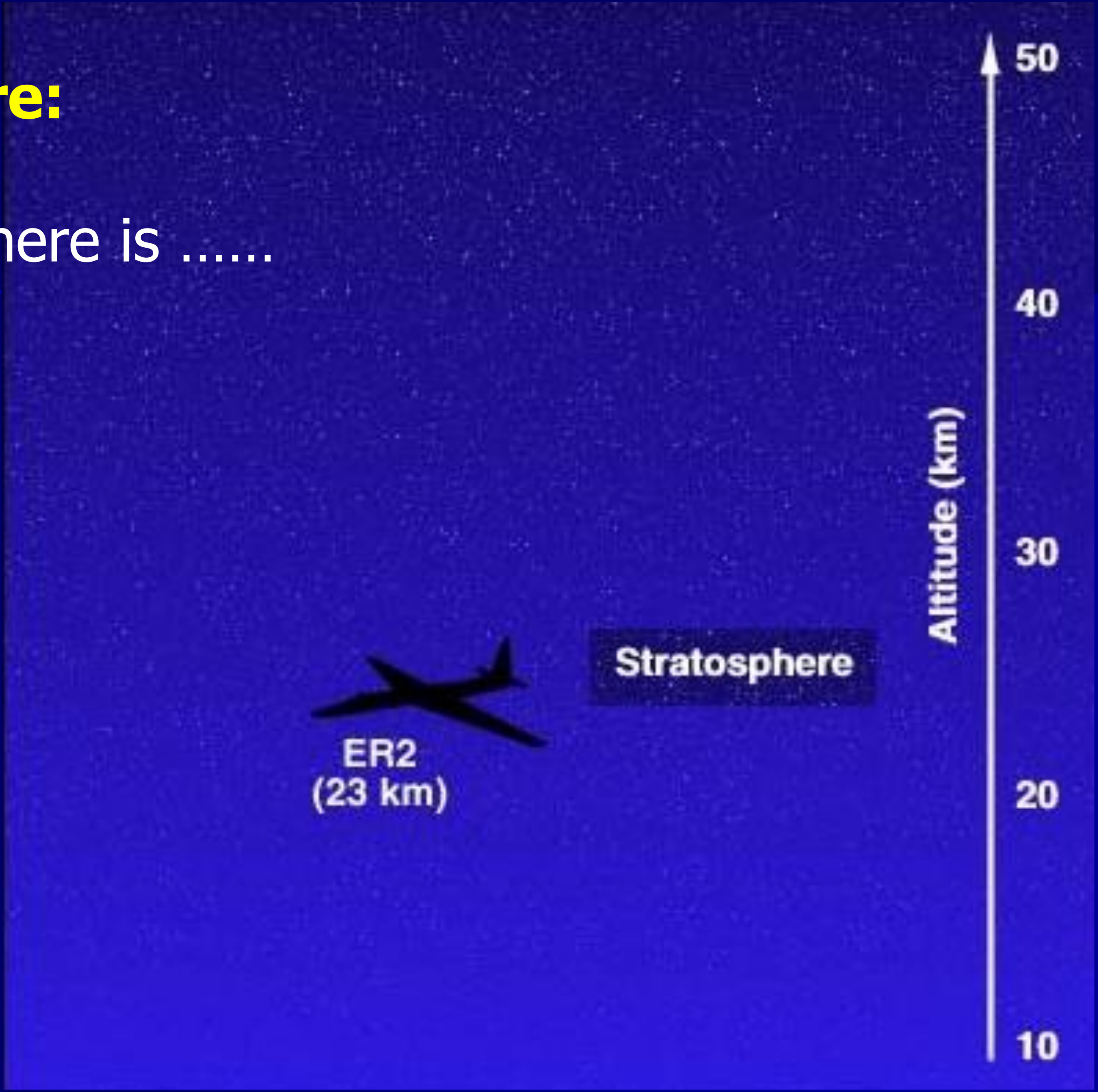
Pidurutalagala, or Mount Pedro in English, is an ultra prominent peak, and the tallest mountain in Sri Lanka, at **2,524 m. Find the temperature at the top of the mountain.** **Answer : 10.5 °C**

Mount Everest, is Earth's highest mountain. Its peak is **8,848 meters** above sea level. **Find the temperature at the top of the mountain.**

Answer : - 30.5 °C

Stratosphere:

The stratosphere is ……



Thank You !

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