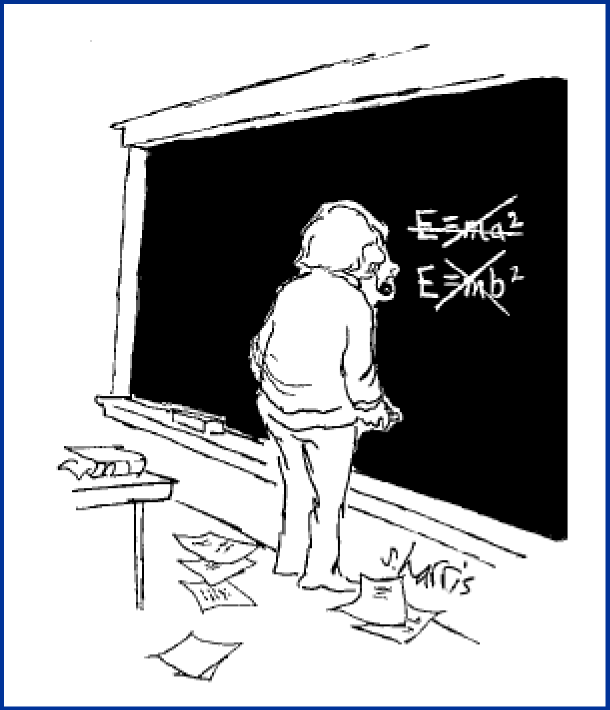
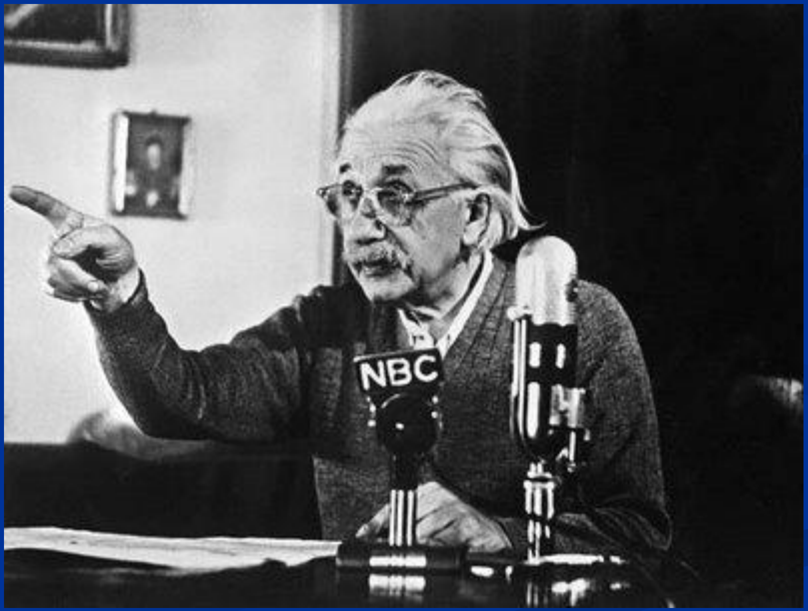
Special Theory of Relativity





4th & 5th Lectures

Special Theory of Relativity

Einstein's Two Postulates in STR

Postulate 01 : The Principle of Relativity:

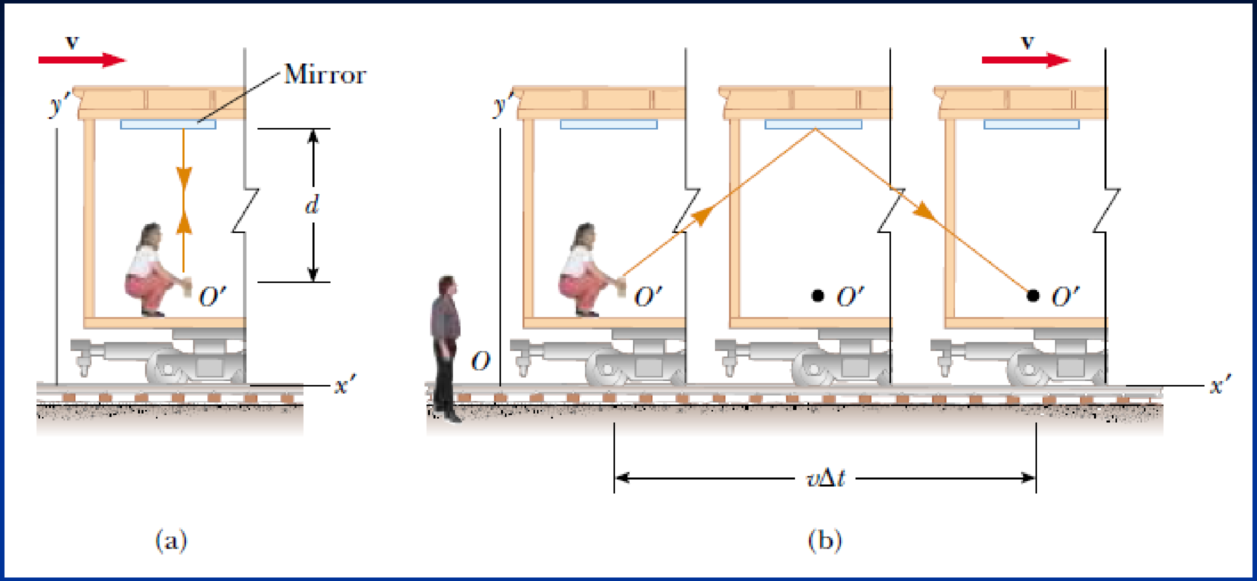
The laws of physics must be the same in all inertial reference frames.

The laws of Physics are the same for all observers in uniform motion relative to one another.

Postulate 02 : The constancy of the speed of light :

The speed of light in vacuum has the same value, $c = 3 \times 10^8$ m/s in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Measurement of Time in STR



$$\Delta t_O = \Delta t_P \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where, v is the Relative Speed of the Two Frames

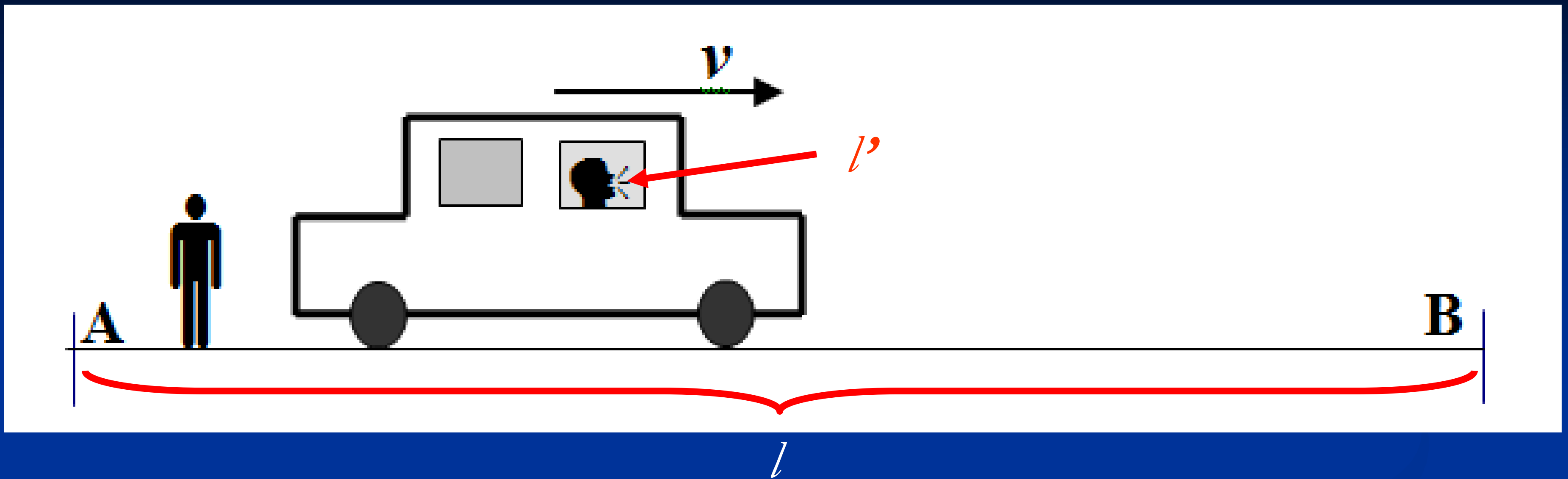
$$\Delta t_O \rangle \Delta t_P$$

Time interval w. r. t the stationary frame

Time interval w. r. t the moving frame

This is called Time Dilation !

Measurement of Length in STR



$$t_{im} = \frac{l}{v}$$

$$t_{pro} = \frac{l^1}{v}$$

$$t_{im} = t_{pro} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \quad l^1 = l \sqrt{1 - \frac{v^2}{c^2}}$$

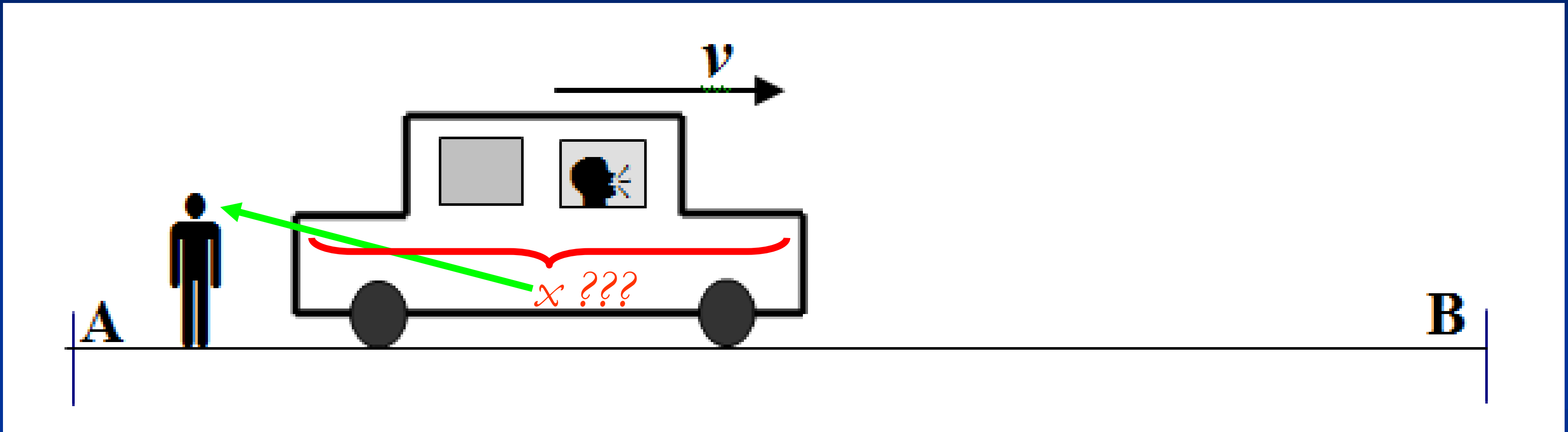
If $v > 0$ $\Rightarrow$ $l^1 < l$

l^1: Length measured by an observer in the car

l: Length measured by observer in the Earth

This is called
Length Contraction !

What is the length of the car as seen by an observer on the Earth ???



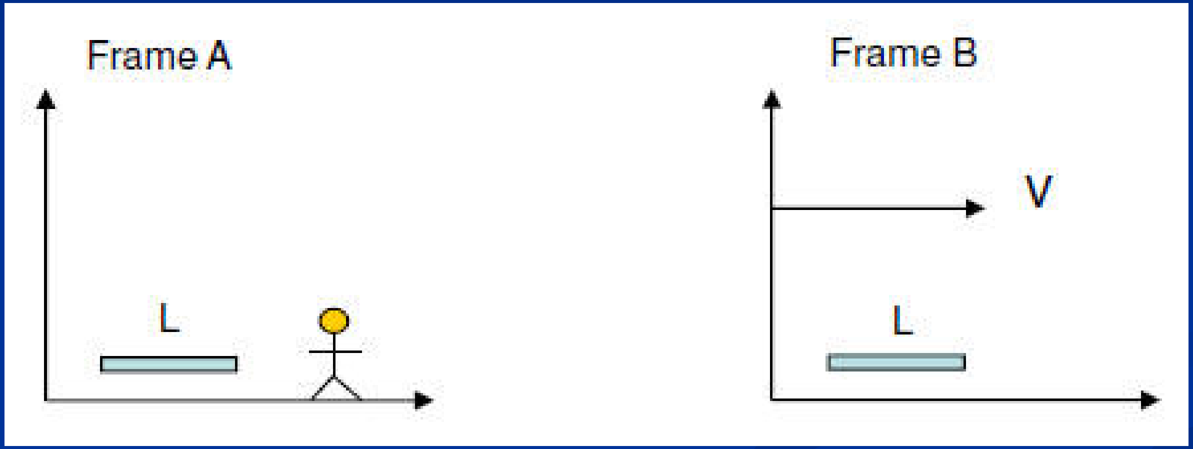
Is it less than the original length ???

Is it greater than the original length ???

Is it equal to the original length ???

Length Contraction

Length contraction is the observation that a moving object appears shorter than a stationary object.



Length contraction – an observer in frame A sees the stick in frame B as shorter than the stick in frame A.

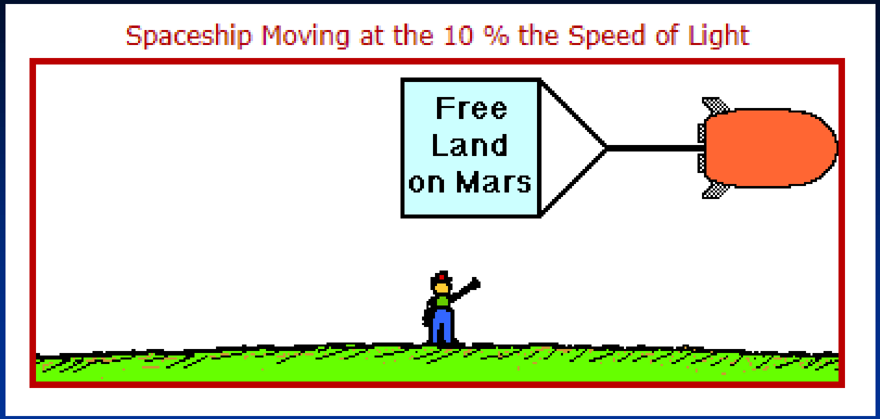
$$l^1 < l$$

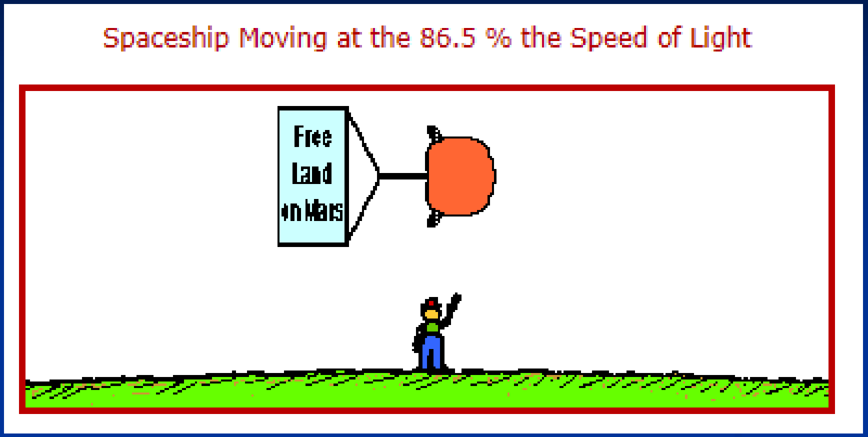
Length measured by an observer in the Frame A

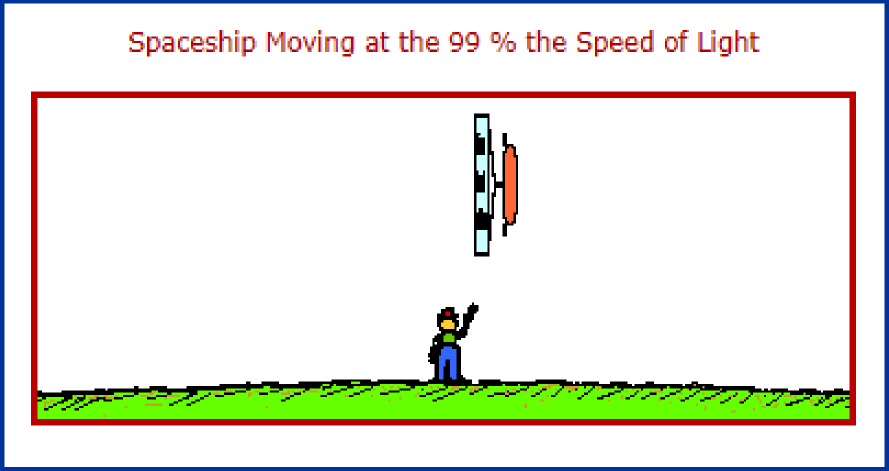
Length measured by an observer in the Frame B

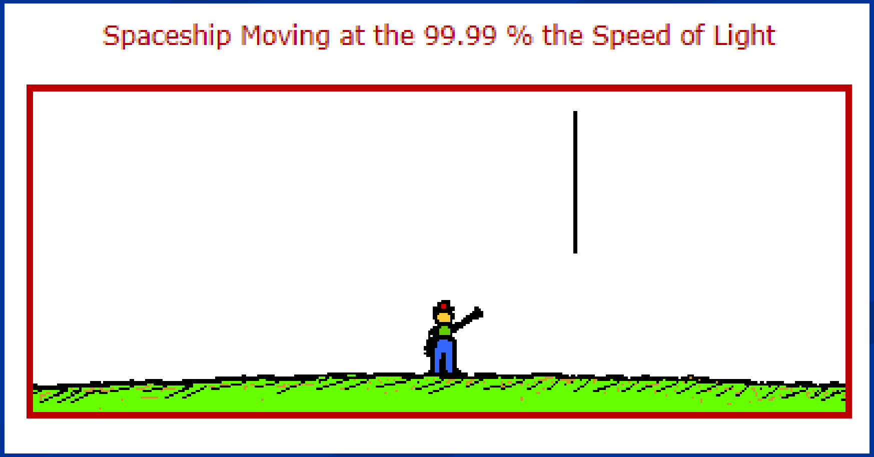
One of the peculiar aspects of Einstein's theory of special relativity is that the length of objects moving at relativistic speeds undergoes a contraction along the dimension of motion. An observer at rest (relative to the moving object) would observe the moving object to be shorter in length. That is to say, that an object at rest might be measured to be 200 feet long; yet the same object when moving at relativistic speeds relative to the observer/measurer would have a measured length which is less than 200 ft. This phenomenon is not due to actual errors in measurement or faulty observations. The object is actually contracted in length as seen from the *stationary reference frame*. The amount of contraction of the object is dependent upon the object's speed relative to the observer.

http://www.physicsclassroom.com/mmedia/specrel/lc.cfm









http://www.physicsclassroom.com/mmedia/specrel/lc.cfm

Example :

A *200 m* long train passes through a tunnel *100 m* long with the constant speed *c/4* .

(*a*) What is the length of the tunnel as seen by an observer in the train ???

(*b*) What is the length of the train as seen by an observer on the Earth ???

(*a*)

Length (Tunnel, E) $= 100\ m$

Length (Tunnel, Train) $= l\ ?$

Using relativistic length equation : $l^1 = l\sqrt{1-\frac{v^2}{c^2}}$

$$l = 100\sqrt{1-\frac{\left(\frac{c}{4}\right)^2}{c^2}} \Rightarrow l = 100 \times 0.9682 \Rightarrow l = 96.82\,m$$

Example :

A *200 m* long train passes through a tunnel *100 m* long with the constant speed $c/4$.

(a) What is the length of the tunnel as seen by an observer in the train ???

(b) What is the length of the train as seen by an observer on the Earth ???

(b)

Length (Train, E) $= 200\ m$ *at rest*

Length (Train, E) $= l$? *is moving*

Using relativistic length equation :

$$l^1 = l\sqrt{1-\frac{v^2}{c^2}}$$

$$l = 200\sqrt{1-\frac{\left(\frac{c}{4}\right)^2}{c^2}} \Rightarrow l = 200 \times 0.9682 \Rightarrow l = 193.64\,m$$

Doppler's Effect in STR

Doppler's Effect for Sound Waves :

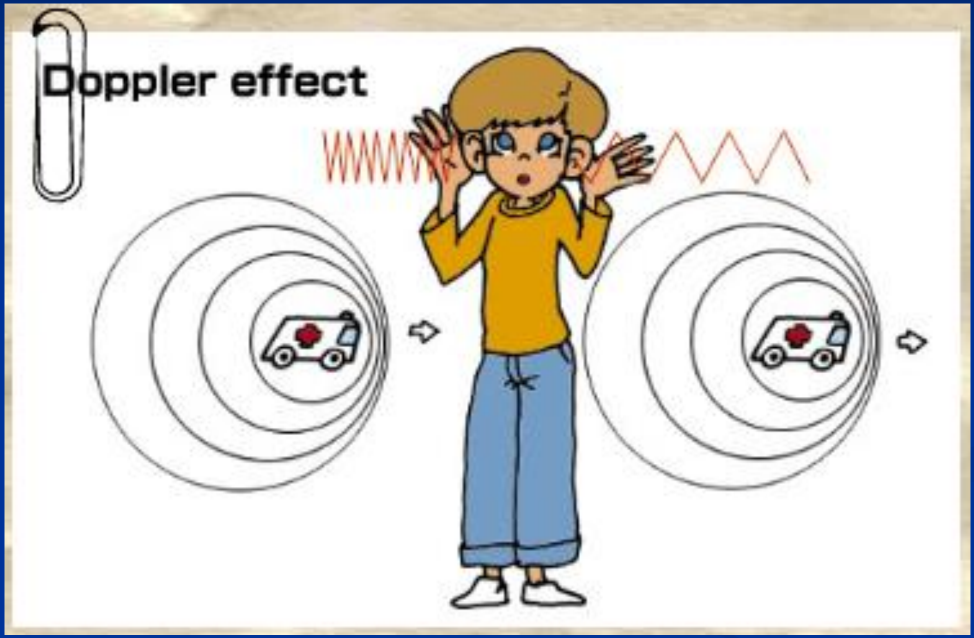
The Doppler's effect (or Doppler's shift), named after Austrian physicist Christian Doppler who proposed it in 1842, is the change in frequency of a wave for an observer moving relative to the source of the wave.

It is commonly heard when a vehicle sounding a siren or horn approaches, passes and recedes from the observer.

P.T.O

Doppler's Effect in STR

Doppler's Effect for Sound Waves :

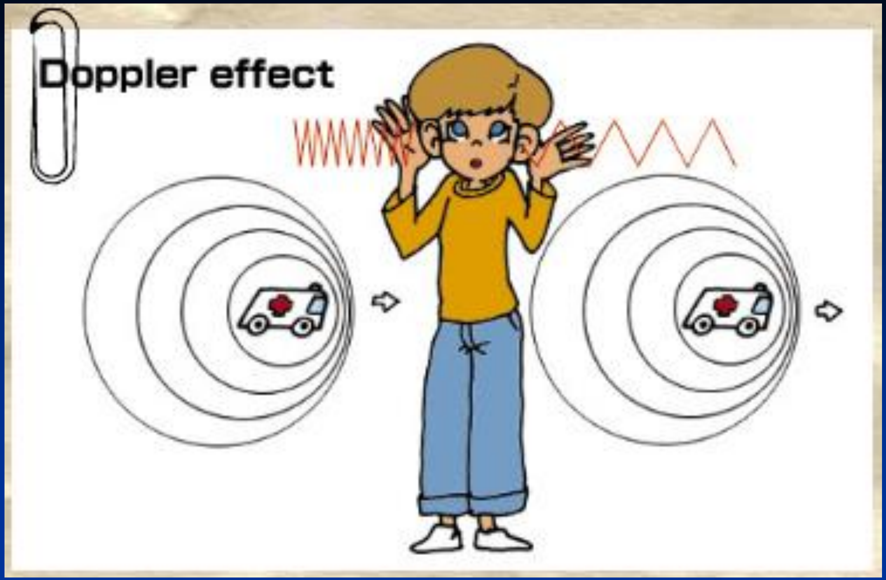


In classical physics (wave in a medium), where the source and the receiver velocities are not supersonic, the relationship between observed frequency f_0 and emitted frequency (or source frequency) f_s is given by,

P.T.O

Doppler's Effect for Sound Waves :

$$f_o = f_s\left(\frac{v \pm v_1}{v \mp v_2}\right)$$



Where,

v is the velocity of waves in the medium.

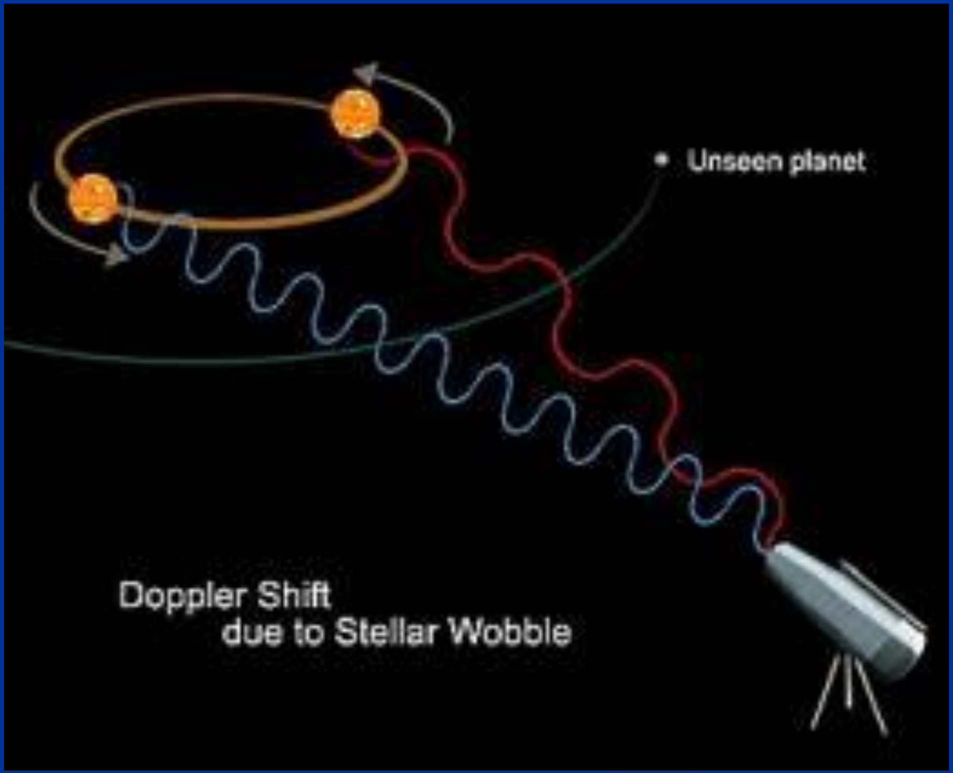
v1 is the velocity of the receiver relative to the medium; positive if the receiver is moving towards the source.

v2 is the velocity of the source relative to the medium; positive if the source is moving away from the receiver.

The frequency is decreased if either is moving away from the other!

Doppler's Effect in STR

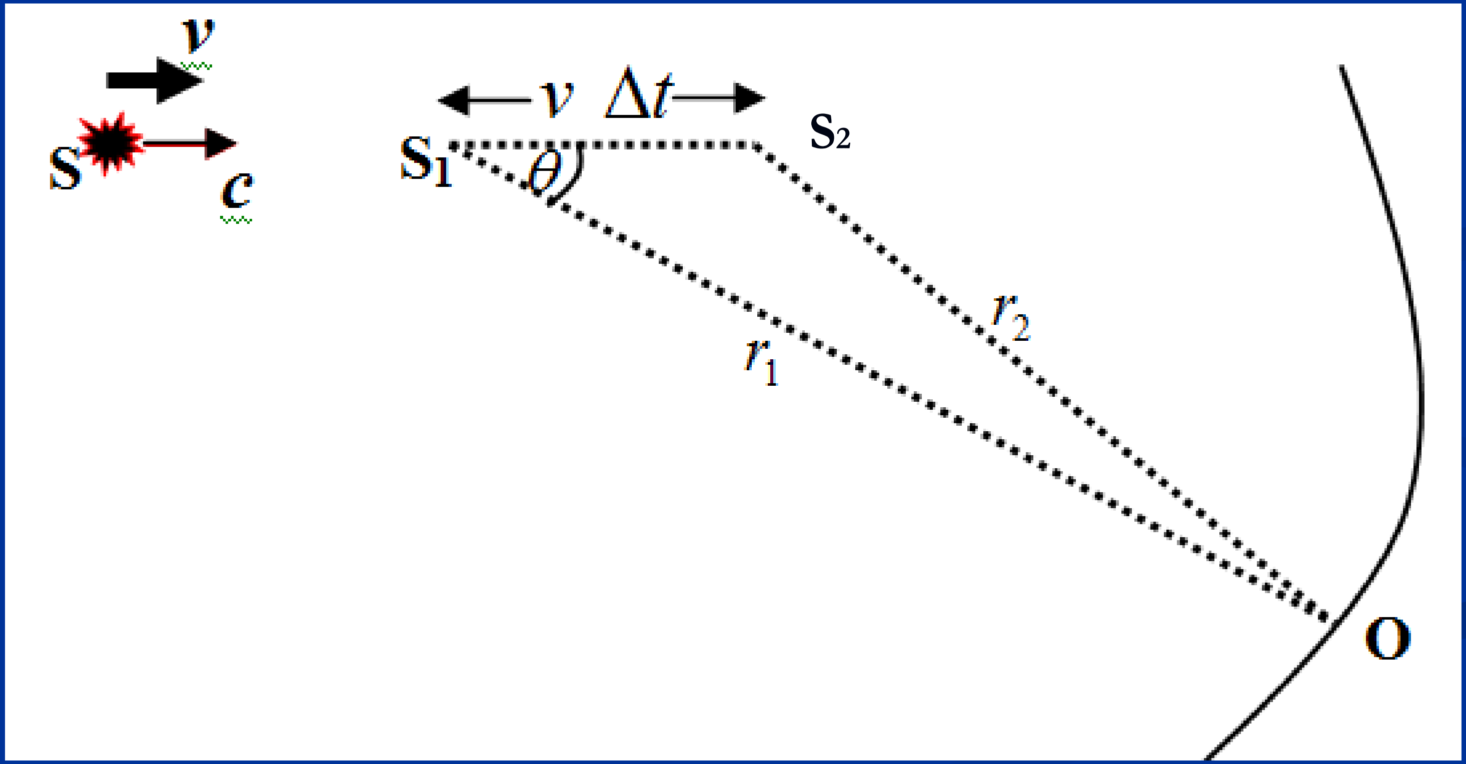
The relativistic Doppler's effect is the change in frequency (and the wavelength) of light, caused by the relative motion of the source and the observer (as in the classical Doppler's effect), when taking into account effects of the Special Theory of Relativity!



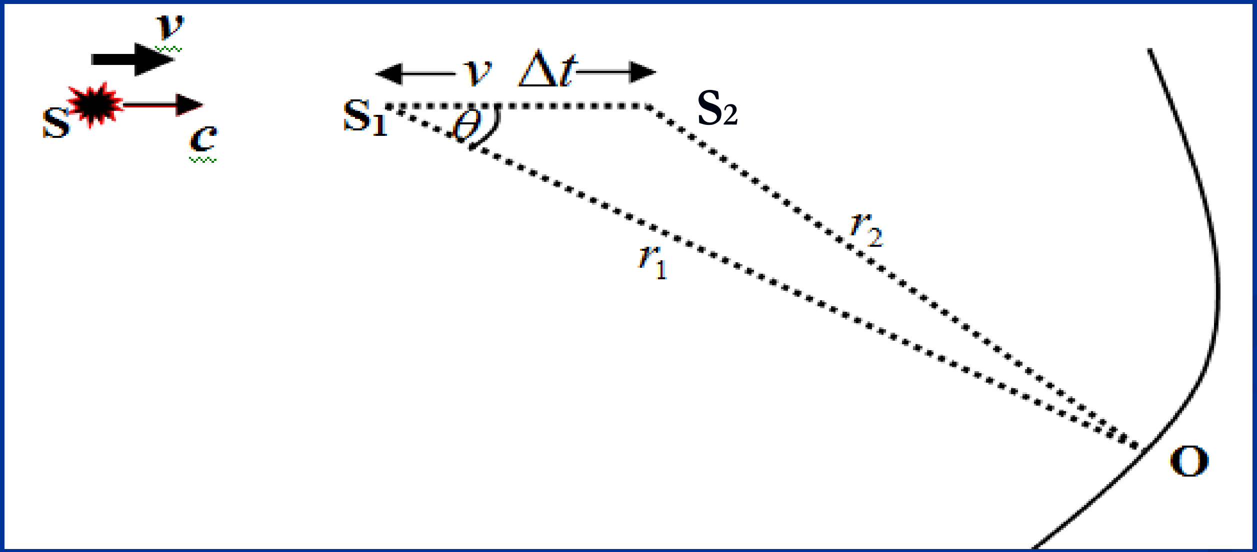
If a light source in uniform motion approaches or recedes a stationary observer then the frequency of light is observed to change. This is known as **Doppler effect** in STR.

P.T.O

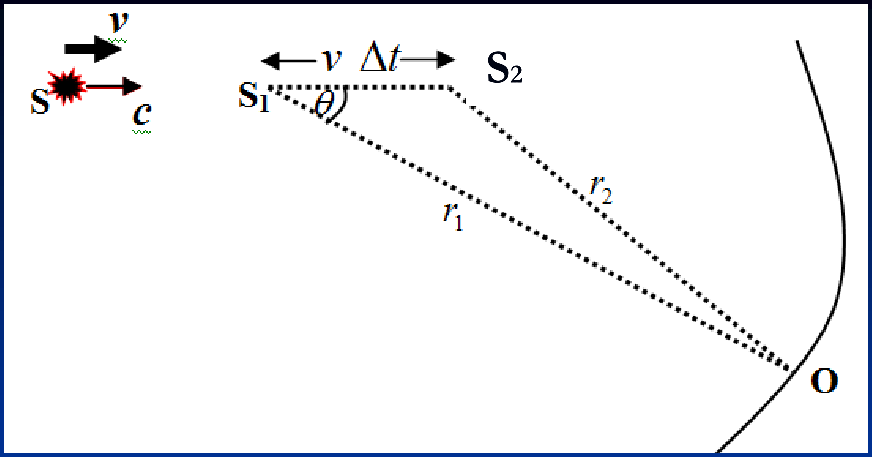
We assume a frame in which the source is moving and observer is stationary. Consider a source *S* of frequency *fs* moving with velocity *v*. The source emits periodic waves. Suppose that when the source is at *S1* it emits a signal. This reaches the observer *O* after a time *r1/c*.



We assume a frame in which the source is moving and observer is stationary. Consider a source S of frequency fs moving with velocity v. The source emits periodic waves. Suppose that when the source is at $S1$ it emits a signal. This reaches the observer O after a time $r1/c$.



When the signal reaches O the source has moved $S2$. Suppose the time taken for the source to move from $S1$ to $S2$ is Δt. Then $S1S2 = v\,\Delta t$.



$$S_1S_2 = v\,\Delta t$$

This is the distance covered by the source before giving out the next signal. When the source is at *S2* it gives out a signal which is received by the observer at *O*.

Thus, the time for the second signal to be received at O is,

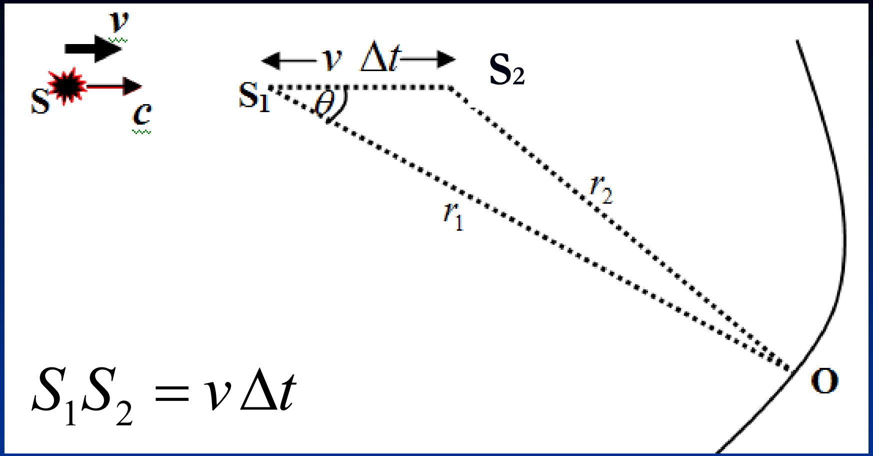
$$\Delta t + \frac{r_2}{c}$$

The difference in time for the two signals to reach O is then,

$$\Delta t_o = \Delta t + \frac{r_2}{c} - \frac{r_1}{c}$$

$$\Delta t_o = \Delta t + \frac{(r_2 - r_1)}{c}$$

→ 01



From the figure we note that, (Using Cosine Theorem)

$$r_2^2 = r_1^2 + (v\Delta t)^2 - 2r_1(v\Delta t)\cos\theta$$

As $r_1 >> v\Delta t$, we get , $v\Delta t \approx (v\Delta t)\cos\theta$

$$\Rightarrow r_2^2 = r_1^2 - 2r_1(v\Delta t)\cos\theta + ((v\Delta t)\cos\theta)^2$$

$$\Rightarrow r_2^2 = (r_1 - (v\Delta t)\cos\theta)^2 \quad \Rightarrow r_2 = r_1 - (v\Delta t)\cos\theta$$

$$\Rightarrow r_2 - r_1 = -(v\Delta t)\cos\theta$$

→ 02

Connect equation 01 and 02 :

$$\Delta t_o = \Delta t + \frac{(r_2 - r_1)}{c}$$

$$\Rightarrow \Delta t_o = \Delta t + \frac{(-(v\Delta t)\cos\theta)}{c}$$

$$\Rightarrow \Delta t_o = \Delta t\left(1 - \frac{v\cos\theta}{c}\right) \longrightarrow 03$$

This is the time difference between two successive signals received at O, i.e.: $1 / \Delta to$ is the observed frequency of emission.

$$f_o = \frac{1}{\Delta t_o}$$

P.T.O

This is the time difference between two successive signals received at O, i.e.: $1 / \Delta t_o$ is the observed frequency of emission.

$$f_o = \frac{1}{\Delta t_o}$$

We now need to connect t_s with the actual frequency of emission by the source, f_s. We realize that $1/f_s$ is the time difference between two successive signals emitted by the source and is therefore a proper time in the frame of the source. On the other hand, Δt is the time interval between the emission of two consecutive signals as seen from the stationary frame.

P.T.O

Using the Relativistic Time equation :

$$\Delta t = \Delta t_s \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \Delta t_s = \Delta t\sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow \frac{1}{f_s} = \Delta t\sqrt{1-\frac{v^2}{c^2}}$$

Using equation 03 ;

$$\Delta t_o = \Delta t\left(1-\frac{v\cos\theta}{c}\right)$$

We get,

$$\Delta t = \Delta t_o \frac{1}{\left(1-\frac{v}{c}\cos\theta\right)}$$

Then we have,

$$\frac{1}{f_s} = \Delta t\sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{1}{f_s} = \Delta t_o \frac{1}{\left(1-\frac{v}{c}\cos\theta\right)}\sqrt{1-\frac{v^2}{c^2}} \Rightarrow$$

P.T.O

$$\frac{1}{f_s} = \Delta t_o \frac{1}{\left(1 - \frac{v}{c}\cos\theta\right)} \sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{1}{f_s} = \Delta t_o \frac{1}{\gamma(1 - \beta\cos\theta)}$$

where, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and, $\beta = \frac{v}{c}$

As the frequency f_o observed by the observer is $1/\Delta t_o$, we obtain,

$$f_o = \frac{1}{\Delta t_o}$$

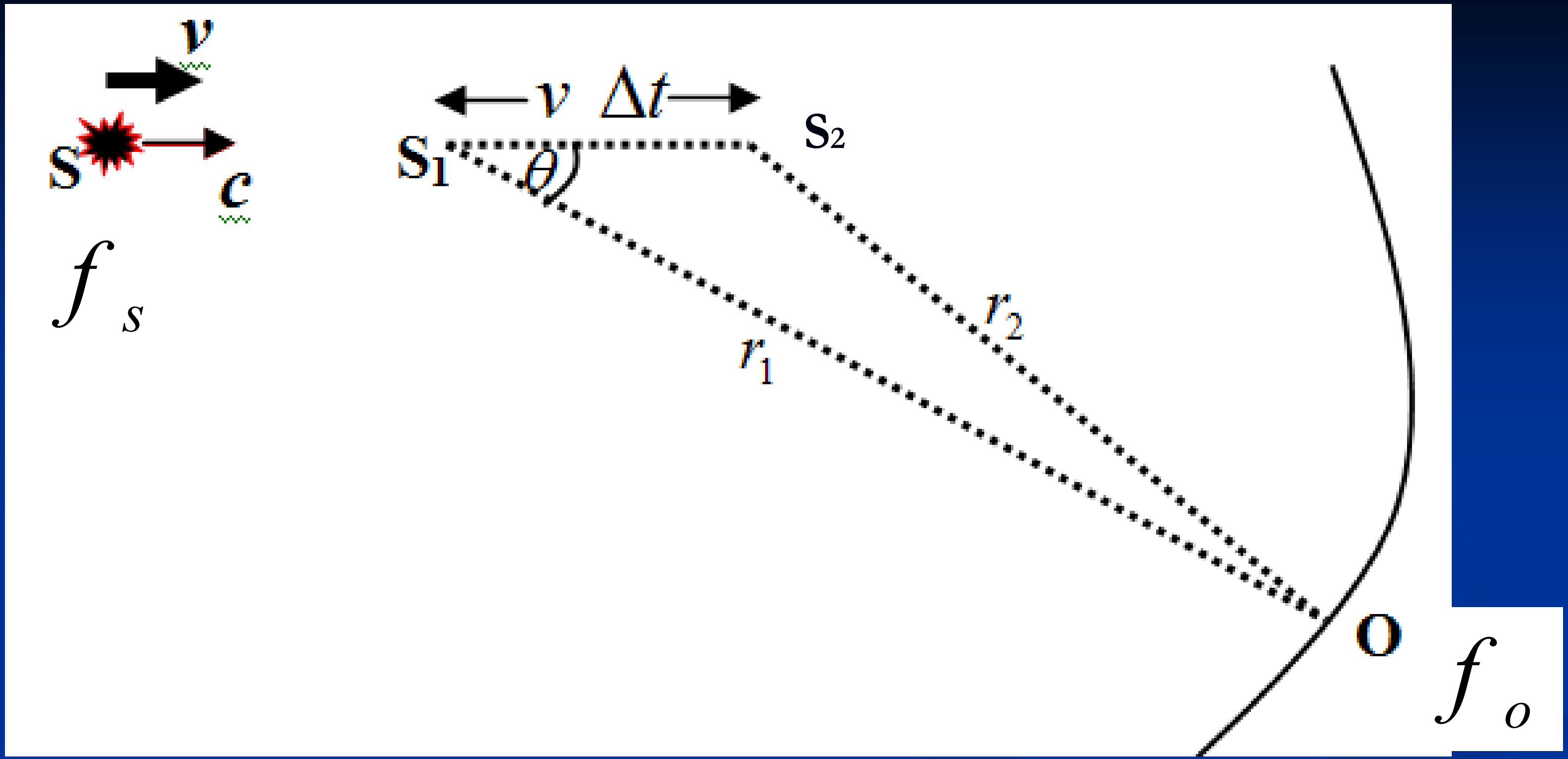
$$\frac{1}{f_s} = \Delta t_o \frac{1}{\gamma(1 - \beta\cos\theta)}$$

$$\frac{1}{f_s} = \frac{1}{f_o} \frac{1}{\gamma(1 - \beta\cos\theta)}$$

$$f_o = f_s \frac{1}{\gamma(1 - \beta\cos\theta)}$$

where, $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$ $\beta = \frac{v}{c}$

This is the general form of the Doppler's Effect



$$f_o = f_s \frac{1}{\gamma\left(1-\beta\cos\theta\right)}$$

where, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

and $\beta = \dfrac{v}{c}$

This is the general form of the Doppler's Effect in STR!

Special Cases:

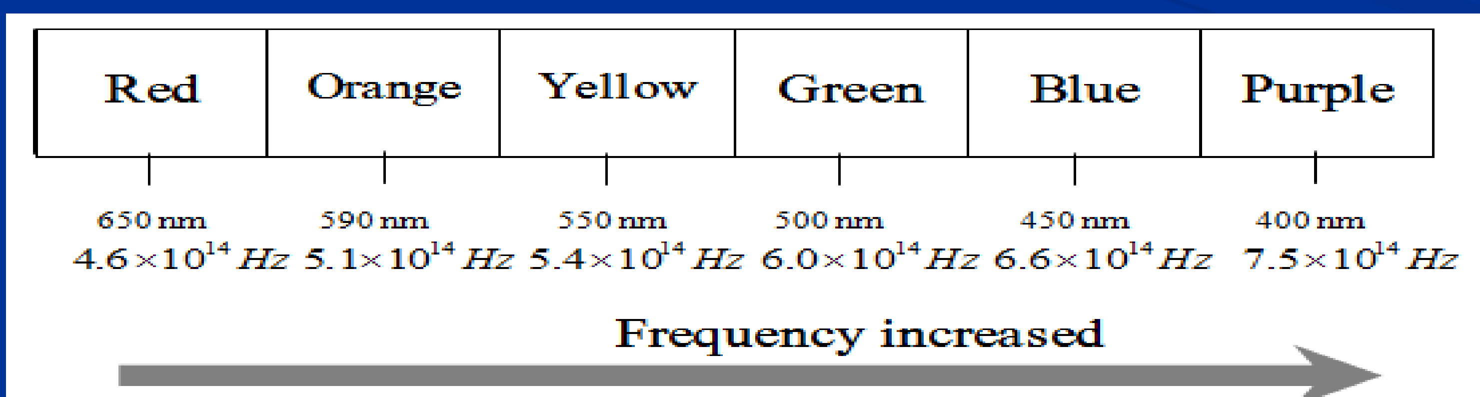
(a) If the source is directly approaching the observer:

Then, $\theta = 0$ and $\cos\theta = 1$ ➡ $f_o = f_s \dfrac{1}{\gamma(1-\beta)}$ where, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

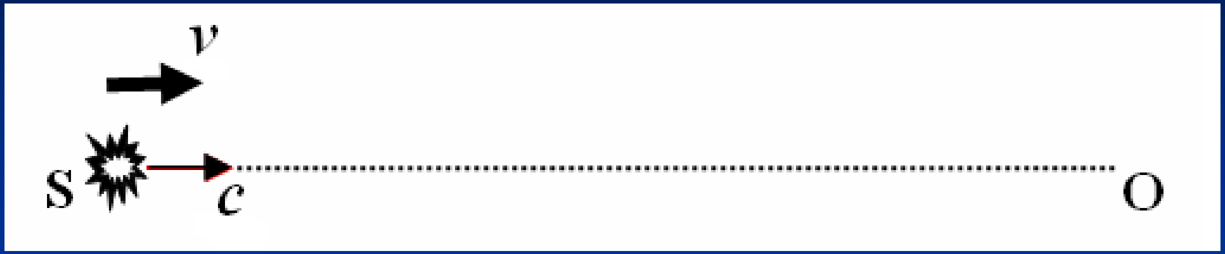
$\beta = \dfrac{v}{c}$

➡ $$f_o = f_s \sqrt{\frac{1+\beta}{1-\beta}}$$ ➡ $f_o > f_s$

➡ **Thus, the frequency appears to increase !**

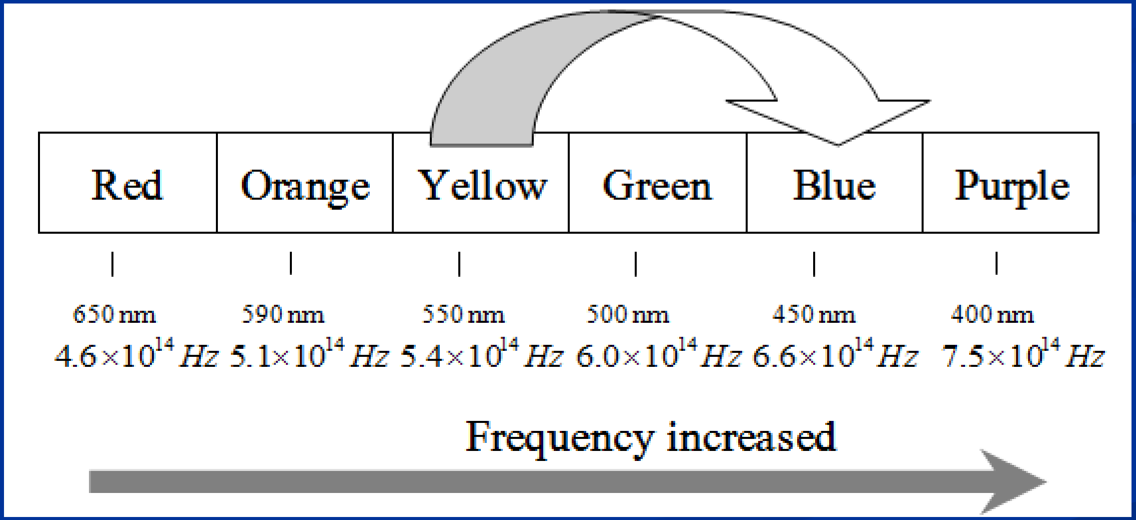


(a) If the source is directly approaching the observer:



$f_o > f_s$

Thus, the frequency appears to increase !



This is called frequency shift !

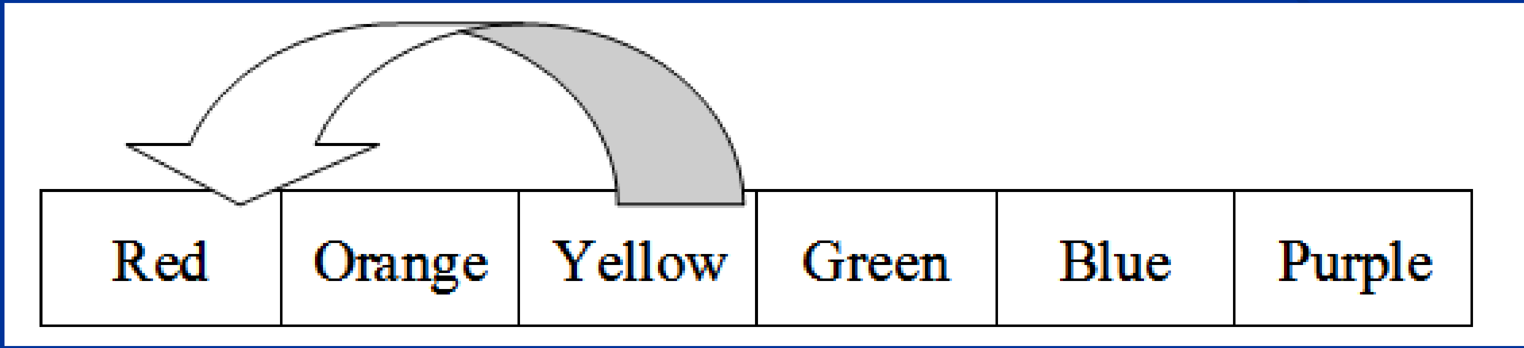
Special Cases:

(b) If the source is receding directly from the observer:

Then, $\theta = \pi$ and $\cos\theta = -1$ ➡ $f_o = f_s \dfrac{1}{\gamma(1+\beta)}$ where, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$, $\beta = \dfrac{v}{c}$

➡ $$f_o = f_s \sqrt{\frac{1-\beta}{1+\beta}}$$

➡ **Thus, the frequency appears to decrease !**



Example :

A yellow coloured vehicle appears as a green coloured vehicle to a stationary observer due to its speed. Find the velocity of the vehicle. (Wavelengths of yellow and green light are 550nm and 500nm respectively)

Is the above incident practically possible??? Briefly explain your answer.

Example :

A yellow coloured vehicle appears as a green coloured vehicle to a stationary observer due to its speed. Find the velocity of the vehicle. (Wavelengths of yellow and green light are 550nm and 500nm respectively)

Is the above incident practically possible??? Briefly explain your answer.

Source frequency = true colour of the vehicle

fs = frequency of the yellow colour

For E-M Waves : $v = f\lambda$ → $f = \dfrac{c}{\lambda}$

→ $f_s = \dfrac{3\times10^{8}\,ms^{-1}}{550\times10^{-9}\,m}$

→ $f_s = 5.45\times10^{14}\,Hz$

Example :

Observed frequency = appeared colour of the vehicle

fo = frequency of the green colour

For E-M Waves : $v = f\lambda$ ➡ $f = \frac{c}{\lambda}$

➡ $f_o = \frac{3\times10^8\ ms^{-1}}{500\times10^{-9}\ m}$

➡ $f_o = 6.00\times10^{14}\ Hz$

$f_o = 6.00\times10^{14}\ Hz$ > $f_s = 5.45\times10^{14}\ Hz$

That means, $f_o > f_s$

Then, the frequency appears to increase! That means, the car is directly approaching to the observer! ∴ Using the Doppler's equation, ➡ $f_o = f_s\sqrt{\frac{1+\beta}{1-\beta}}$

Example :

$$f_o = f_s \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\Rightarrow 6.00\times10^{14} = 5.45\times10^{14} \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\Rightarrow 1.1 = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\Rightarrow 1.21 = \frac{1+\beta}{1-\beta}$$

$$\Rightarrow \beta = \frac{0.21}{2.21}$$

$$\Rightarrow \beta = \frac{0.21}{2.21}$$

$$\Rightarrow \frac{v}{c} = \frac{0.21}{2.21}$$

$$\Rightarrow \frac{v}{c} = 0.095$$

$$\Rightarrow v = 0.095\,c$$

$$\Rightarrow v = 2.8\times10^{7}\, ms^{-1}$$

This is a practically impossible velocity ! Why ?

World fastest vehicle on air in 2025

Category	Speed (km/h)	Vehicle
Uncrewed aerial vehicle	21,245	**HTV-2**
Crewed, rocket-powered	7,270	North American X-15A-2
Crewed, air-breathing	3,529.56	Lockheed SR-71A Blackbird #61-7958
Commercial	2,428.5	Tupolev Tu-144

21245 km/h → 21245 x (1000/3600) m/s

Untitled-1 * - Wolfram Mathematica 12.1

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```
In[39]:= vv = 21425;
         fact = 1000 / 3600;
         vvnew = vv * fact // N;
         Print["The velocity ", vv, " km/h is to ", vvnew, " m/s"]
         Print["It is in velocity of sound times ", N[vvnew / 330]]

         The velocity 21425 km/h is to 5951.39 m/s

         It is in velocity of sound times 18.0345
```

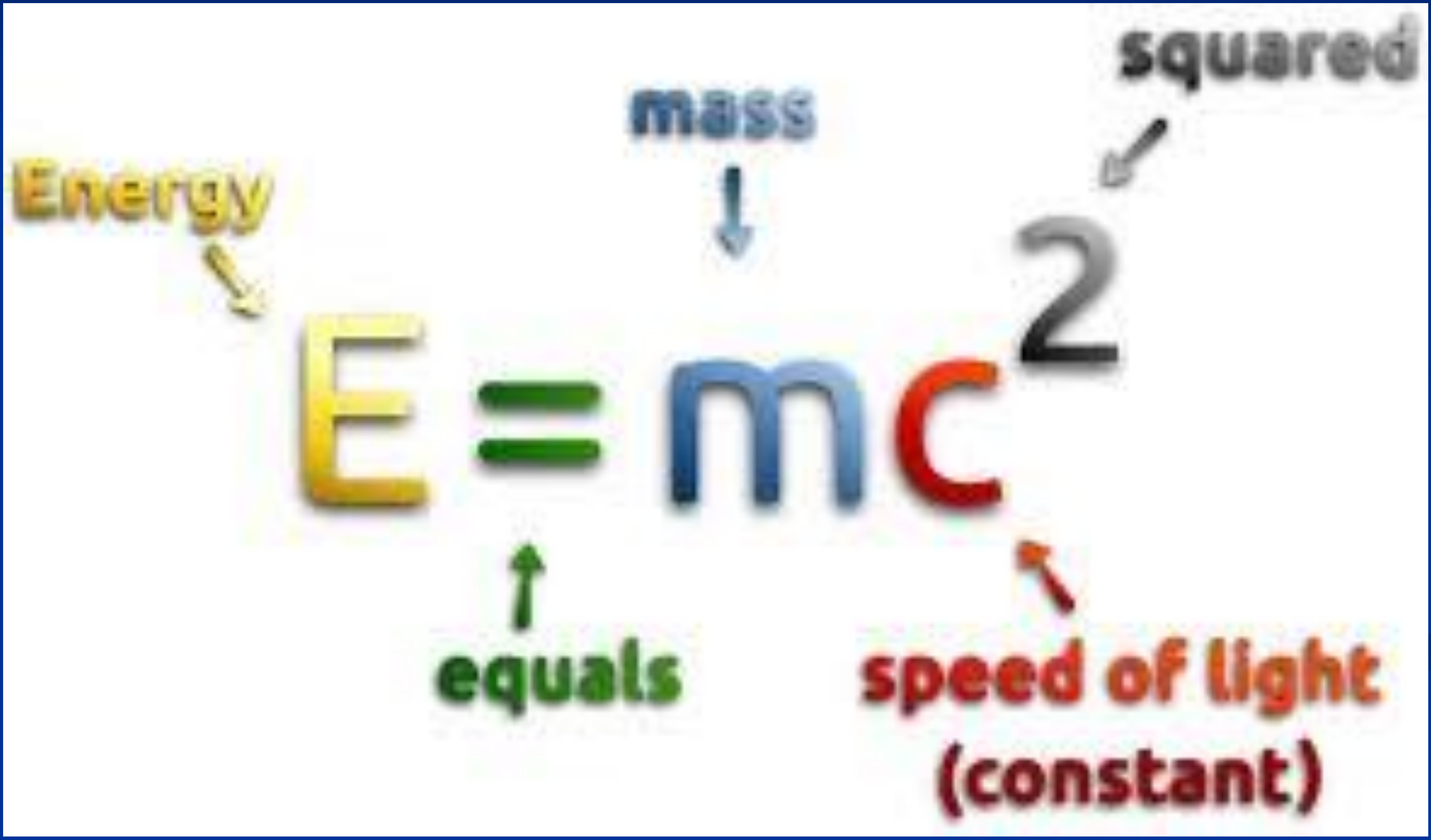
21245 km/h → 21245 x (1000/3600) m/s

→ **18** x (330 m/s)

This is the fastest vehicle exist in the world in 2025….

Every event is depend on the frame !!!!

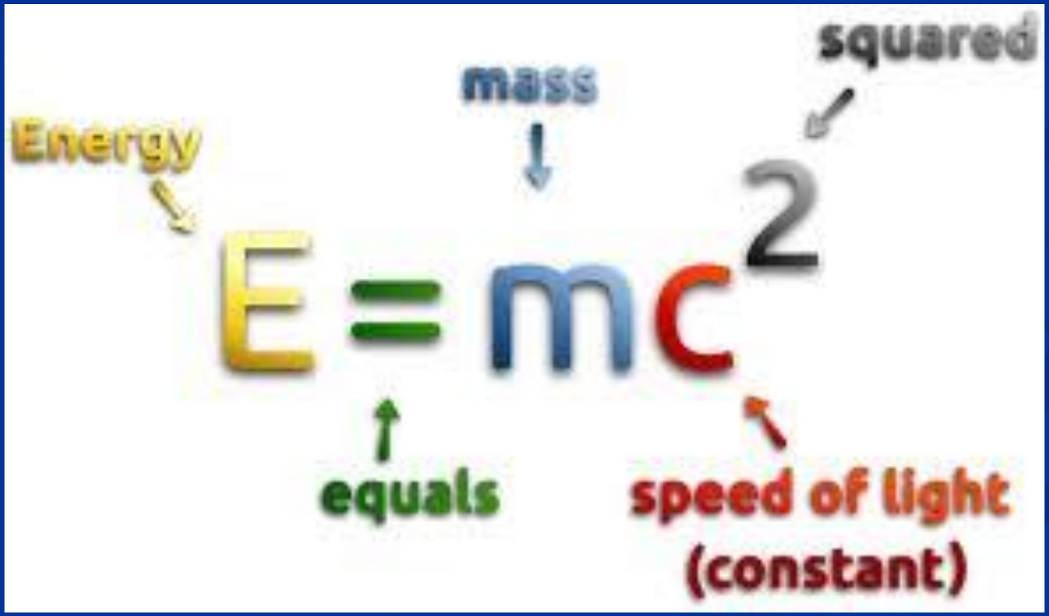
The mass – energy equivalence



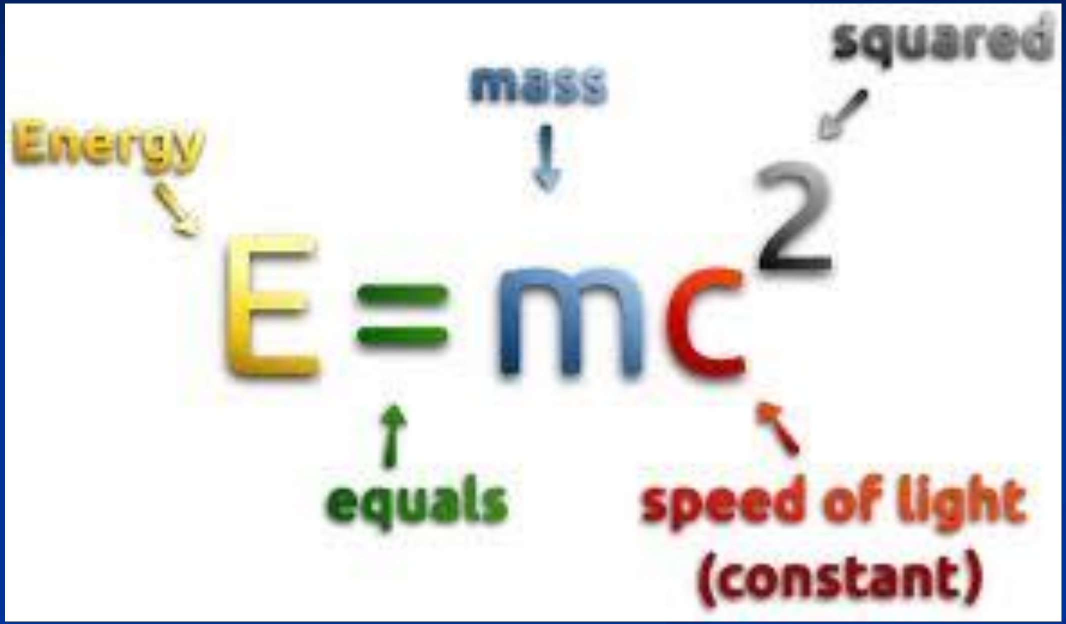
The mass – energy equivalence

In Physics, mass – energy equivalence is the concept that the mass of a body is a measure of its energy content.

Albert Einstein proposed mass – energy equivalence in 1905. The equivalence is described by the famous equation,



The mass – energy equivalence

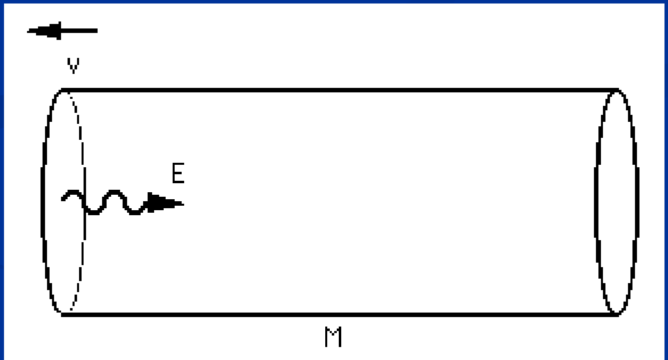


The equation **$E = m c^2$** indicates that energy always exhibits mass in whatever from the energy takes. It does not imply that mass may be "**converted**" to energy, for modern theory holds that neither mass or energy may be destroyed, but only moved from one location to another.

Proof of $E = m c^2$ [Einstein's Box]

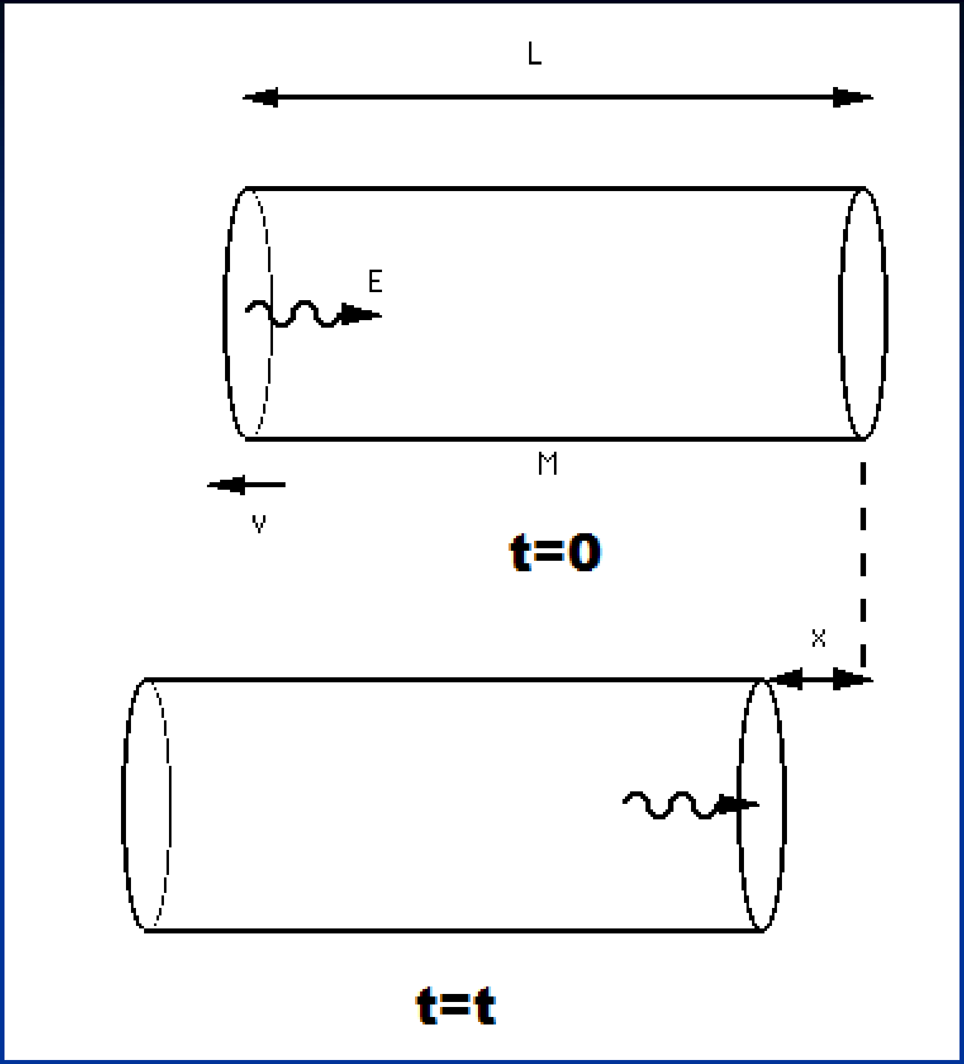
This is a Hypothetical Experiment.

Before Einstein, it was known that a beam of light pushes against matter; that is known as Radiation Pressure. This means the light has momentum, E/c. Einstein used this fact to show that radiation (light) energy has an equivalent mass.



Consider a cylinder of mass M. A pulse of light with energy E is emitted from the left side. The cylinder recoils to the left with velocity V. If the mass of cylinder is large, it doesn't move far before the light reaches the other side. So, the light must travel a distance L, requiring time $t = L/c$. In this time, the cylinder travels a distance x.

P.T.O



Momentum of the photon give the momentum to the cylinder.

Momentum of the photon = Momentum of the Box

$$\frac{E}{c} = M V$$ → 01

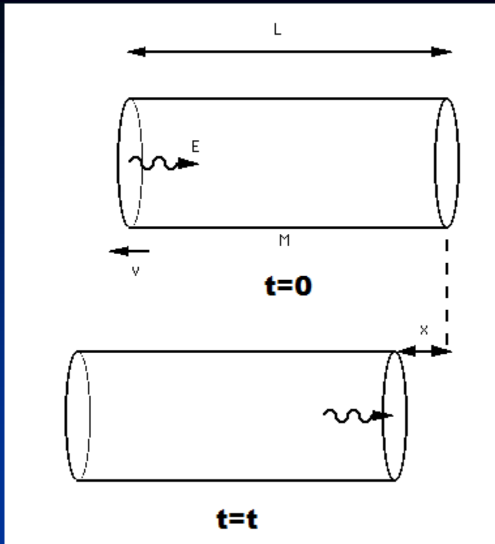
Time for light beam to cross the cylinder =

$$t = \frac{L}{c}$$ → 02

Distance traveled by the cylinder =

$$x = V t$$ → 03

P.T.O



Lets assume the photon (light pulse) has a mass *m*.

Using the law of conservation of momentum;

$$mc + M\,(-V) = 0$$

Where, $V = \frac{x}{t}$ *and* $c = \frac{L}{t}$

$$\Rightarrow M x = mL$$

Using the equation 01 & 03, $M = \frac{E}{Vc}$ *and* $x = Vt$

$$\Rightarrow E = mc^2$$

Einstein was not the first to propose a mass-energy relationship. However, Einstein was the first scientist to propose the **$E = m c^2$** formula and the first to interpret mass-energy equivalence as a fundamental principle that follows from the relativistic symmetries of Space & Time!

The mass – energy equivalence

4-meter-tall sculpture of Einstein's 1905 $E = mc^2$ formula at the 2006 Walk of Ideas, Berlin, Germany.

Find the mass-equivalence energy of a 1kg.

Using, $E = mc^2$

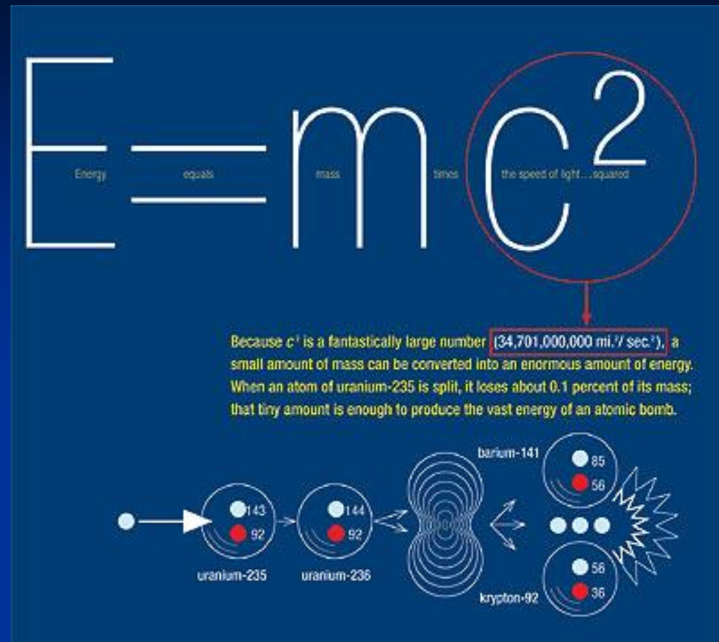
$$E = (1)\,(3\times 10^8)^2$$

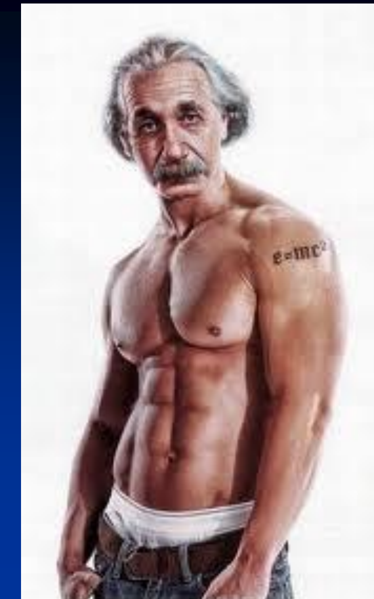
$$E = 9\times 10^{16}\ J$$

This is a very large energy. Using this energy, we can vaporize **~10^10 kg** of water at the room temperature (**30°C**)!

$\because E = ms\theta + mL$

Equivalence of mass and energy





Einstein put forward new ideas regarding the relationship between space, time, mass and energy which have come to be known as the theory of relativity. It had long been accepted that matter could not be destroyed. This assumption was expressed in the *law of conservation of matter*, which states that the total quantity of matter in the universe is fixed and cannot be increased or decreased by human agency.

Thank You !