# PHY 359 2.0 / ASP 487 2.0
# Telecommunication

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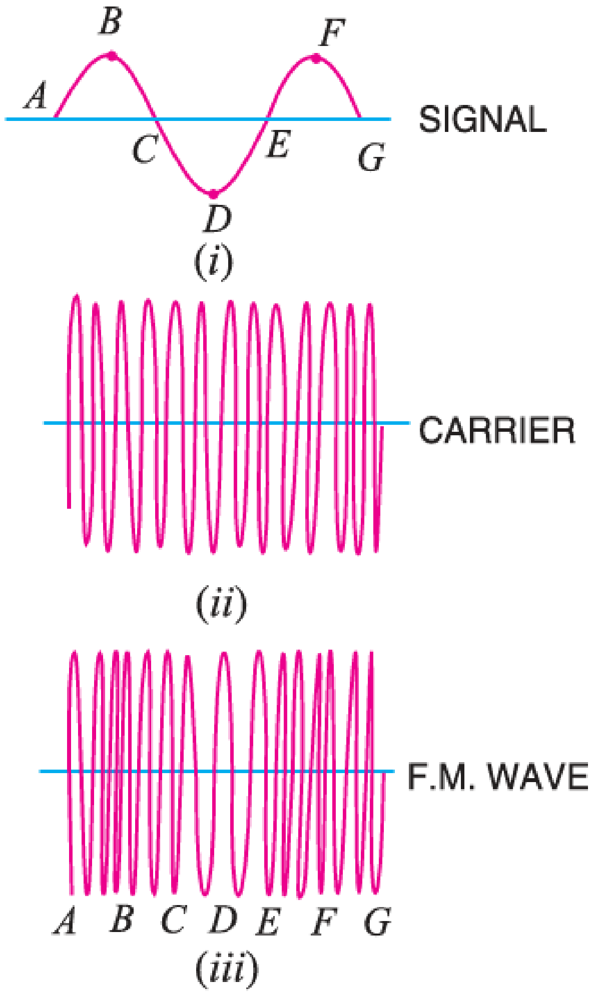
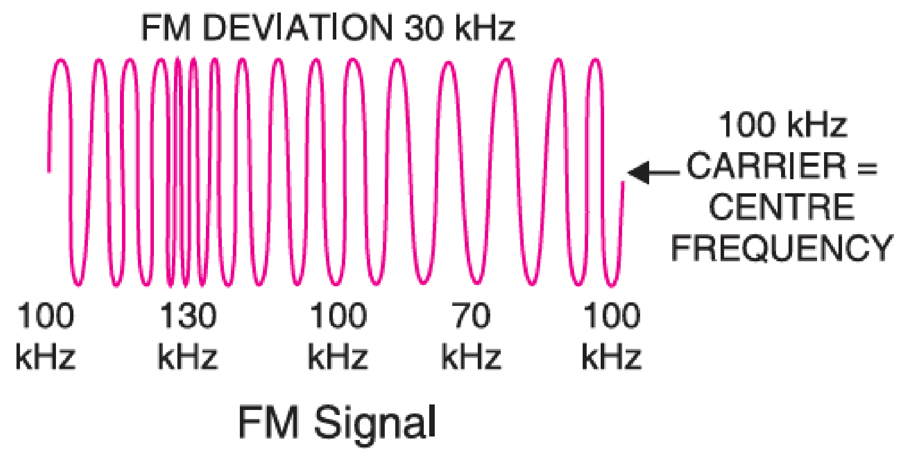
# Frequency Modulation
# and
# Demodulation

- When the frequency of the carrier wave is changed in accordance with the intensity of the signal, it is called frequency modulation (FM).
- However, the amplitude of the modulated wave remains the same i.e. carrier wave amplitude.
- The frequency variations of carrier wave depend upon the instantaneous amplitude of the signal as shown in the following figure.
- When the signal voltage is zero as at A, C, E and G, the carrier frequency is unchanged. When the signal approaches its positive peaks as at B and F, the carrier frequency is increased to maximum as shown by the closely spaced cycles.
- However, during the negative peaks of the signal as at D, the carrier frequency is reduced to the minimum as shown by the widely spaced cycles.

# Frequency Modulation ???



## Key points about frequency modulation (FM)

- The frequency deviation of the FM signal depends on the amplitude of the modulating signal.
- The center frequency is the frequency without modulation or when the modulating voltage is zero.
- The audio frequency (that is frequency of modulating signal) does not determine frequency deviation.

## The advantages of FM over AM.

- It gives noiseless reception. As discussed before, noise is a form of amplitude variations, and a FM receiver will reject such signals.
- The operating range is quite large.
- It gives high-fidelity reception.
- The efficiency of transmission is very high.

**Example:**

The maximum frequency deviation is $(f_{c\ (max)} – f_c)$ and occurs at the peak voltage of the modulating signal. Suppose we modulate a 100 MHz carrier by $2V_{pp}$, 1 kHz modulating signal, and the maximum frequency deviation is 30 kHz.

1. What is the minimum and maximum frequency of the carrier wave?
2. If the amplitude of the modulating signal is increased to $4V_{pp}$, what is the maximum frequency deviation and minimum and maximum frequency of the carrier wave?

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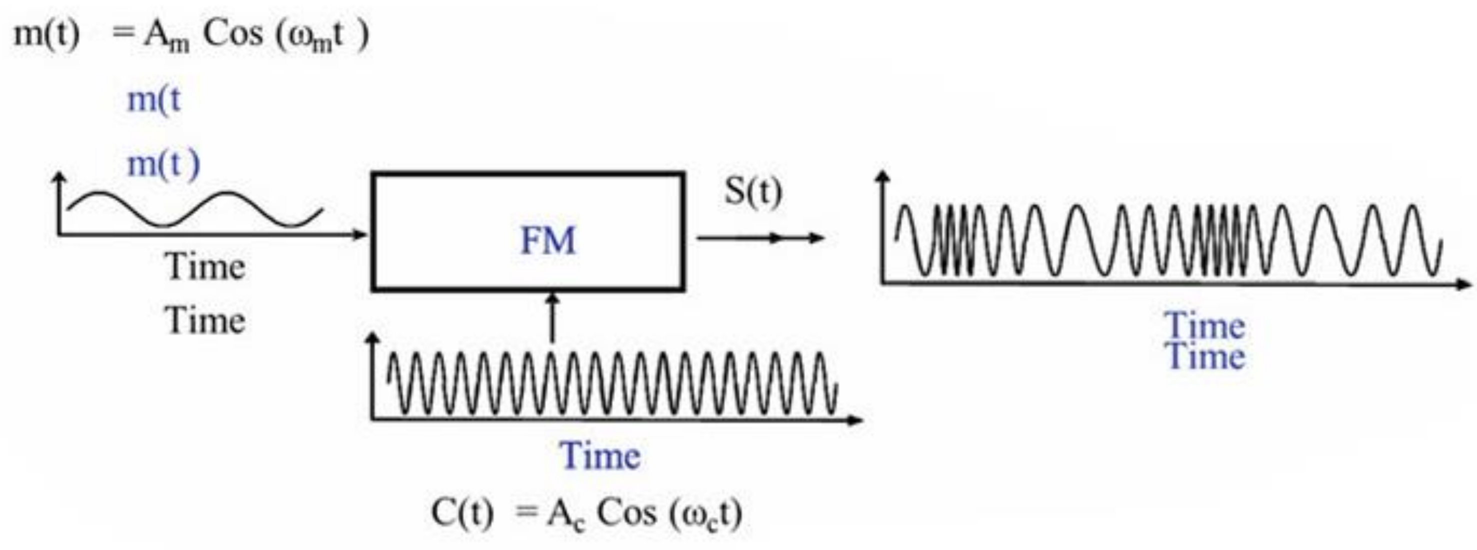
# The Basic Theory of FM

- The modulated carrier frequency $f_c$ varies back and forth and depends on amplitude $A_m$ and frequency $f_m$ of the input signal.
- The bellow figure shows the functional diagram of a typical FM, using a single-tone modulating signal.
- Here, m(t) is the input analog signal we want to transmit, C(t)is the carrier frequency without modulation, and S(t) is the output FM-modulated carrier frequency. These parameters are described below.



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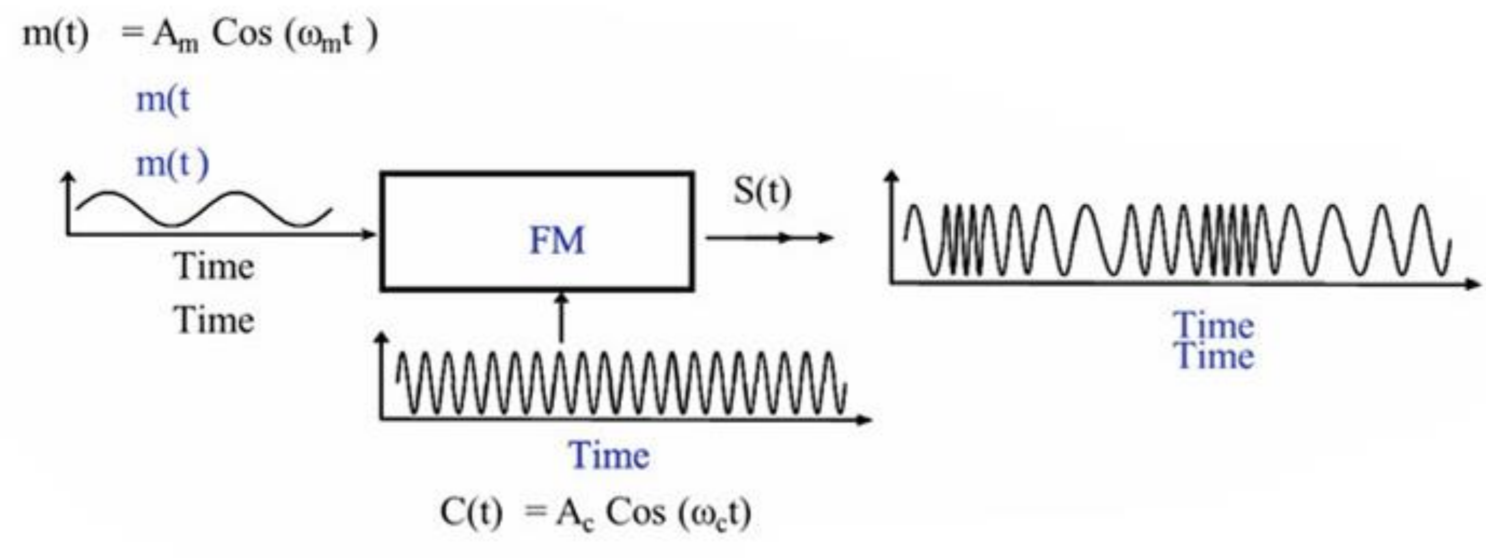
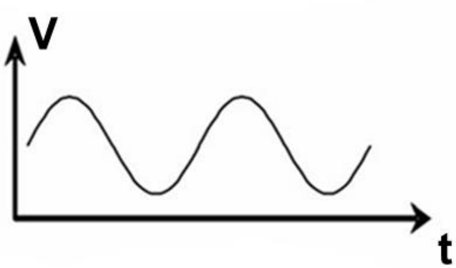
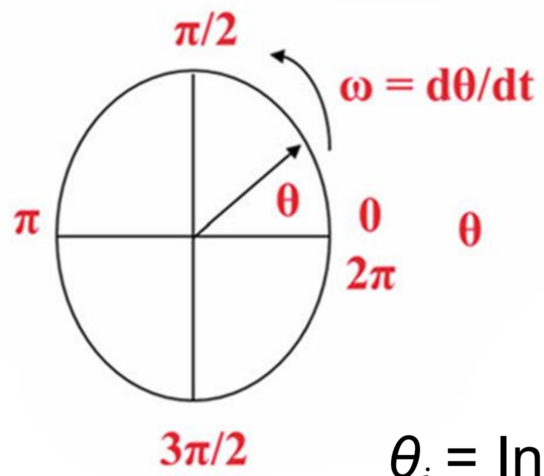


# The Basic Theory of FM

- A single frequency input modulating signal and its angular frequency as shown in bellow Figure. Since the frequency of the carrier varies in accordance with the input signal, the instantaneous frequency of the carrier is given by

$$f_i(t) = f_c(t) + D_f m(t)$$




$$\theta_i(t) = 2\pi \int_0^t f_i dt$$

$\theta_i$ = Instantaneous phase angle

$f_i$ = Instantaneous frequency

# The Basic Theory of FM

- A single frequency input modulating signal and its angular frequency as shown in bellow Figure. Since the frequency of the carrier varies in accordance with the input signal, the instantaneous frequency of the carrier is given by

**$f_i$ = Instantaneous frequency**
**$f_c$ = Carrier frequency**
**$D_f$ = Constant**
**$m(t)=A_m \cos(\omega_m t)$**

$$f_i(t) = f_c(t) + D_f m(t)$$

**The FM-modulated signal is given by**

**$A_c$ - The amplitude of the carrier frequency**
**$\theta_i$ - Instantaneous phase angle**

$$S(t) = A_c \cos(\theta_i)$$

**Substituting,**

$$S(t) = A_c \cos[2\pi \int_0^t f_i dt]$$

$$S(t) = A_c \cos[2\pi \int_0^t \{fc(t) + Df\, m(t)\} dt]$$

# The Basic Theory of FM

- where m(t) is the input modulating signal. Integrating the above equation, obtain the desired FM signal as follows:

$$S(t) = A_c \cos\left[2\pi f_c t + \left(\frac{\Delta f}{f_m}\right)\sin(2\pi f_m t)\right]$$
$$= A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

$$\beta = \frac{\Delta f}{f_m}$$

$$\Delta f = D_f A_m$$

- $S(t)$ = FM-modulated carrier signal,
- $f_c$ = Frequency of the carrier,
- $A_c$ = Amplitude of the carrier frequency,
- $\Delta f = D_f A_m$ = Frequency deviation,
- $f_m$ = Input modulating frequency,
- $Am$ = Amplitude of the input modulating signal,
- $D_f$ = A constant parameter, and
- $\beta = \Delta f / f_m$ = Modulation index.

- The modulation index β is an important design parameter in FM. It is directly related to FM bandwidth. It may also be noted that FM bandwidth depends on both frequency and amplitude of the input modulating signal.

# Comparison of FM and AM

| FM | AM |
|---|---|
| The amplitude of carrier remains constant with modulation. | The amplitude of carrier changes with modulation. |
| The carrier frequency changes with modulation. | The carrier frequency remains constant with modulation. |
| The carrier frequency changes according to the strength of the modulating signal. | The carrier amplitude changes according to the strength of the modulating signal. |
| The value of modulation index ($m_f$) can be more than 1. | The value of modulation factor ($m$) cannot be more than 1 for distortionless AM signal. |

# Example:

1. The equation gives a frequency-modulated voltage wave: $s(t) = 10 \cos(7 \times 10^8 t + 6 \sin(1000t))$ Find,
   (i) carrier frequency. (ii) signal frequency. (iii) modulation index.
   (iv) maximum frequency deviation.
   (v) power dissipated by the FM wave in a 10 Ω resistor.

2. A 500 Hz audio sine wave modulates with a 30 MHz carrier wave. If the carrier voltage is 5V and the maximum frequency deviation is 15 kHz, write down the voltage equation of the FM wave.

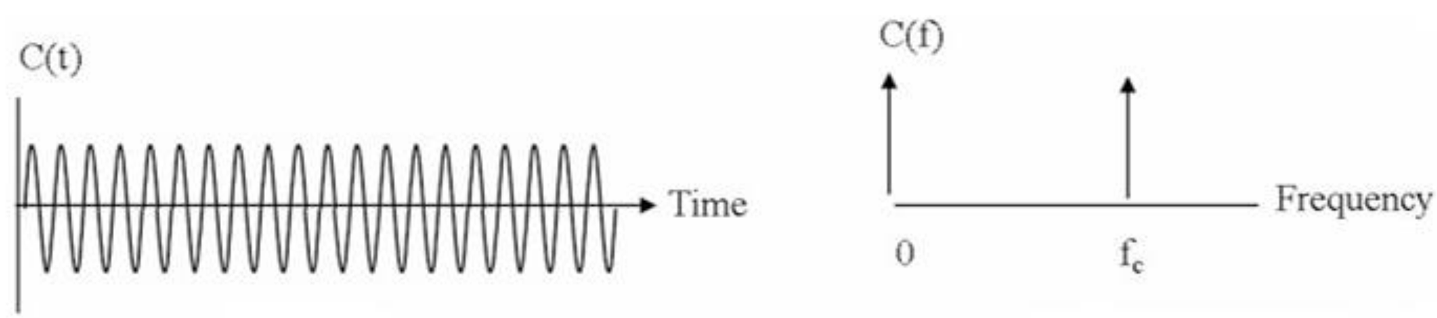
3. The carrier frequency in an FM modulator is 2 MHz. If the modulating frequency is 12 kHz, what are the first four upper sidebands and lower sideband frequencies?

4. In an FM system, when the audio frequency (AF) is 1000 Hz and the AF voltage is 2.5V, the frequency deviation is 4.8 kHz. If the AF voltage is now increased to 7.5V, what is the new frequency deviation? If the AF voltage is raised to 10V while the AF is dropped to 200 Hz, what is the deviation? Find the modulation index in each case.

# FM Spectrum and Bandwidth

- **Needs advanced mathematics to derive the spectrum of FM wave.**
- **After derivation: If $f_c$ and $f_m$ are the carrier and signal frequencies respectively, then FM spectrum will have the following frequencies : fc ; $f_c \pm f_m$ ; $f_c \pm 2f_m$ ; $f_c \pm 3f_m$ and so on.**
- **$f_c + f_m$, $f_c + 2f_m$, $f_c + 2f_m$...... are the upper sideband frequencies**
- **$f_c - f_m$, $f_c - 2f_m$, $f_c - 2f_m$...... are the lower sideband frequencies**

**A high-frequency carrier wave and its frequency response before modulation**



# FM Spectrum and Bandwidth

- Input Signal : $m(t) = A_m \cos(\omega_m t)$
- Carrier Frequency : $C(t) = A_c \cos(\omega_c t)$
- Modulated Carrier : $S(t) = A_c \cos[(\omega_c t) + \beta \sin(\omega_m t)]$

$S(t)$ = The modulated carrier,
$Ac$ = Frequency of the carrier,
$\omega_c$ = Nominal frequency of the carrier frequency,
$\beta = \Delta f / fm$ = Modulation index,
$\Delta f = D_f A_m$ = Frequency deviation,
$D_f$ = Constant, and
$A_m$ = Amplitude of the input modulating signal.

**The modulated carrier frequency S(t) depends on both the frequency and amplitude of the modulating signal.**

- As time passes, the carrier moves back and forth in frequency in exact step with the input signal.
- Frequency deviation is proportional to the input signal voltage.

# FM Spectrum and Bandwidth

- A group of many sidebands is created, spaced from carrier by amounts $N \times f_i$.
- Relative strength of each sideband depends on Bessel function.
- Strength of individual sidebands far away from the carrier is proportional to (frequency deviation × input frequency).
- Higher order spectral components are negligible.
- Carson's rule can be used to determine the approximate bandwidth:

**Bandwidth required = 2 (highest input frequency + frequency deviation)**

- **Carson's rule states that more than 98 % of the power of an FM signal lies within a bandwidth given by the following approximation:**

**FM Bandwidth (BW) = $2f_m(1+\beta)$**

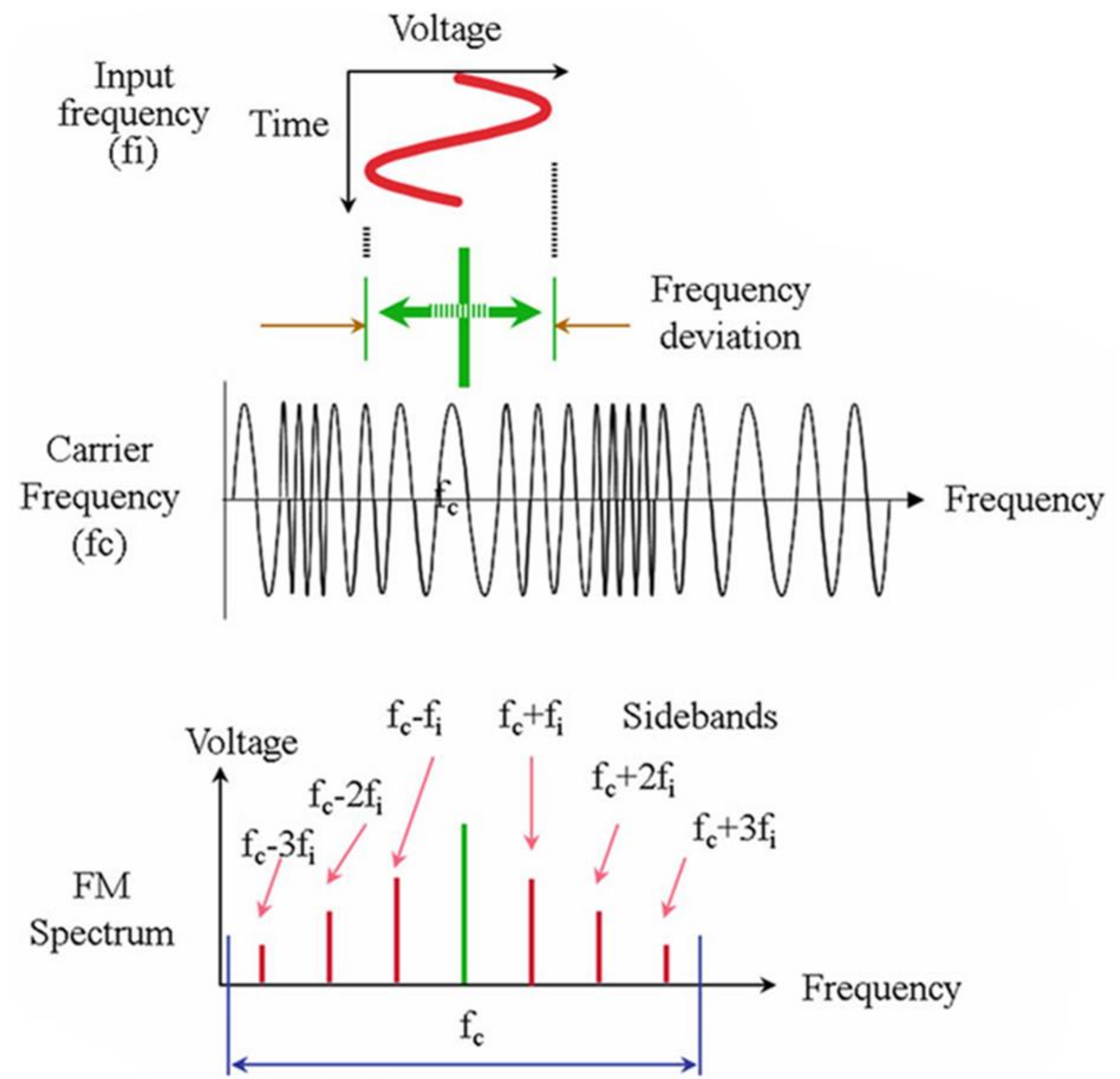
**$\beta = \Delta f/f_m$ = Modulation index**

**$\Delta f$ = Peak deviation of the instantaneous frequency from the center of the carrier frequency**

**$f_m$ = Highest frequency of the modulating signal**

# FM Spectrum and Bandwidth

- As time passes, the carrier moves back and forth in frequency in exact step with the input signal and generates an infinite number of sidebands.



# FM Spectrum and Bandwidth

## FM Bandwidth and Bessel Function

- FM bandwidth can be estimated using the Bessel function. For a single-tone modulation, It can be obtained as a function of the sideband number and the modulation index.
  For a given β, the solution for S(t) is given as follows:

**"J" = The Bessel functions representing the sidebands**

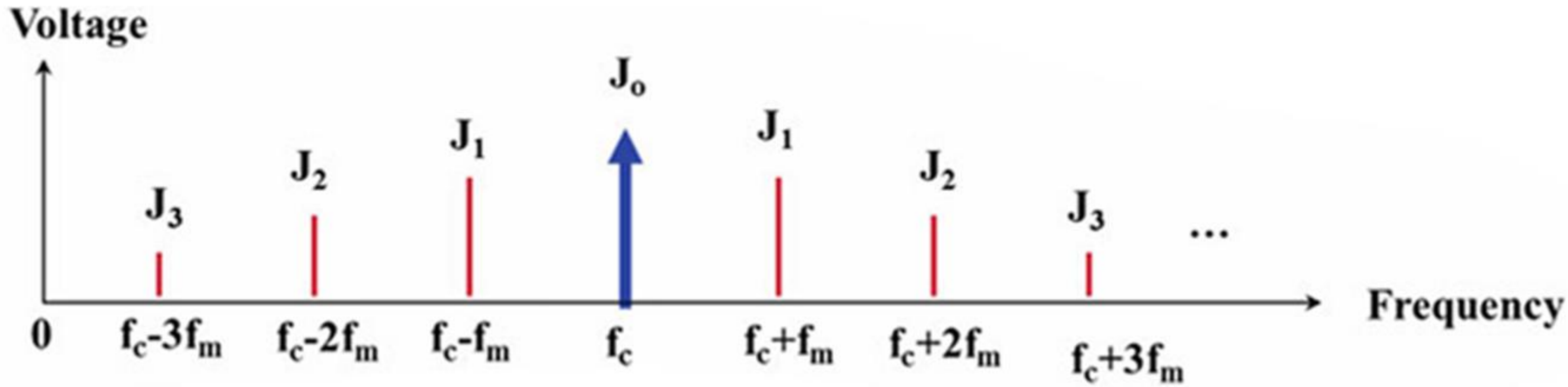
$$
\begin{aligned}
S(t) &= A_c \cos[(\omega_c t) + \beta \sin(\omega_m t)] \\
&= J_0(\beta)\cos(\omega_c t) \\
&\quad + J_1(\beta)\cos(\omega_c t + \omega_m t) + J_2(\beta)\cos(\omega_c t + 2\omega_m t) + J_3(\beta)\cos(\omega_c t + 3\omega_m t) + \ldots \\
&\quad - J_1(\beta)\cos(\omega_c t - \omega_m t) - J_2(\beta)\cos(\omega_c t - 2\omega_m t) - J_3(\beta)\cos(\omega_c t - 3\omega_m t) + \ldots
\end{aligned}
$$

- $J_0(\beta)$ - The amplitude of the fundamental spectral component
- $J_n(\beta)$ (n = 1,2,3,…) - The remaining spectral components for the sidebands.
- Each sidebands are separated by the input modulating frequency
- These values are also available as a standard Bessel function table

# FM Spectrum and Bandwidth

## FM Bandwidth and Bessel Function

- Power is taken from the carrier $J_0$ and distributed among the sidebands $J_1$, $J_2$, $J_3$, $J_4$,…, etc. Each sideband is separated by the modulating frequency $f_m$

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&\quad - J_1(\beta)\cos(\omega_c t - \omega_m t) - J_2(\beta)\cos(\omega_c t - 2\omega_m t) - J_3(\beta)\cos(\omega_c t - 3\omega_m t) + \dots
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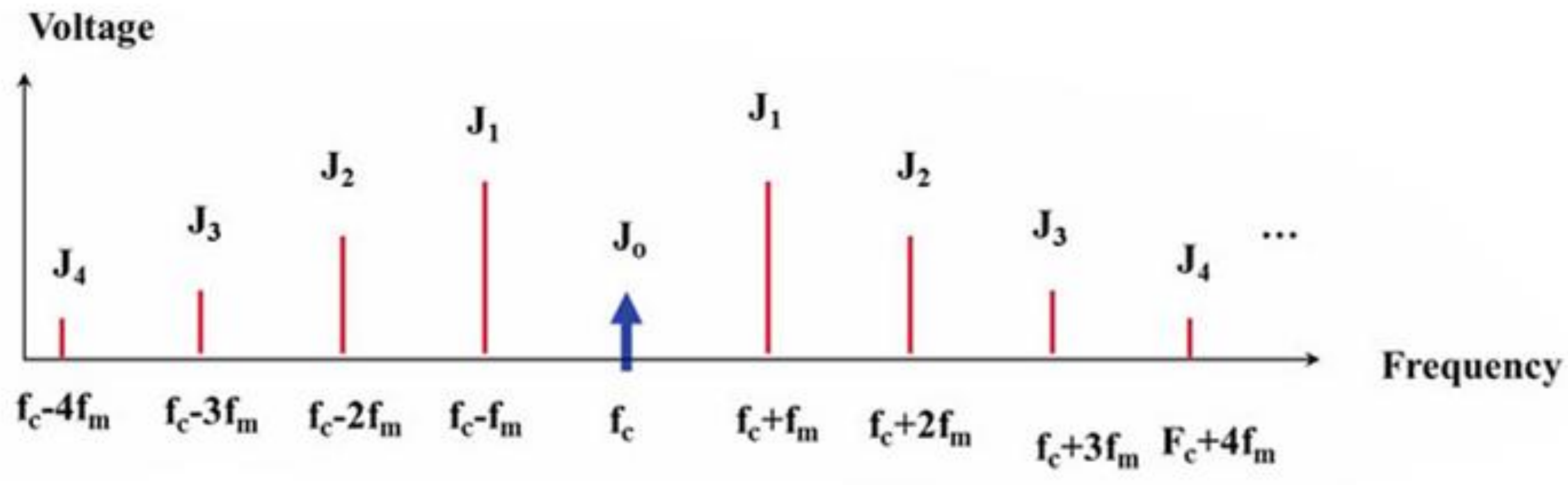
# FM Spectrum and Bandwidth

## FM Bandwidth and Bessel Function




- FM sidebands for large β. More power is taken from the carrier $J_0$ and distributed among the sidebands $J_1$, $J_2$, $J_3$, $J_4$,… Each sideband is separated by the modulating frequency $f_m$

# FM Spectrum and Bandwidth

## FM Bandwidth and Bessel Function

**Bessel Function Table**

| β | J0 | J1 | J2 | J3 | J4 | J5 |
|---|---|---|---|---|---|---|
| 0 | 1 | | | | | |
| 0.25 | 0.98 | 0.12 | | | | |
| 0.5 | 0.94 | 0.24 | 0.03 | | | |
| 1 | 0.77 | 0.44 | 0.11 | 0.02 | | |
| 1.5 | 0.51 | 0.56 | 0.23 | 0.06 | 0.01 | |
| 2 | 0.22 | 0.58 | 0.35 | 0.13 | 0.03 | |

## Next…

# AM and FM Radio Receivers