MATHEMATICAL MODELING

Introduction

- **A mathematical modelling the process of developing a description of a system using mathematical concepts and language.**

- For simplicity we consider only time dependent systems, where other influential factors are neglected and assumed that the parameters are only depends on time.

Importance of Mathematical Modeling

- **To predict the future of a system.**
- **To do necessary corrections to a system to achieve a specific target.**
- **Simulating a system with variable parameters to identify the optimal conditions of a system.**

Main Steps in Modeling

1. **Collecting a past data set.**
   a. **Checking the authenticity and accuracy of the data set.**
   b. **Analyzing the data points and correcting the fault data points.**
2. **Determining the suitable function to explain the data set.**
3. **Checking the model with near future and far future values.**

Mathematical Modeling with Mathamatica.

- **Inserting data into a 2-D table.**
  - **In order to create a 2 x n table with 2 Lists following method could be used**
    - Data set = Transpose[{List1,List2}]
- **Plotting the dataset using ListPlot command.**
- **Fit a function to explain the data points using "Fit[ ]" command.**
- **Plot both dataset and the function on same graph to check whether the function is explaining the dataset.**
- **Check the function for near future and far future values using "/.vairable-> year(time)"**

Example

Variation of Average Petrol Prices per year in Sri Lanka with Time is represented in the following table.

Time (Year)	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Petrol Price (Rs:)	48	50	55	55	59	69	82	91	102	110	115	120
Time (Year)	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Petrol Price (Rs:)	125	133	145	159	150	117	117	117	142	133	137 (?)	

The First row in the above table represent Time (in years) and the second row represent Average Prices of Petrol (in Rupees).

(i) Draw the graph of Petrol Price vs. time (in years).

(ii) Determine a **suitable model** which fits the above data set.

(iii) Find the average Petrol Price in 2020, 2021, 2022 and 2020 using your model.

(iv) By using the above model, estimate the average Petrol Price in year 2050.
Is it reasonable to estimate the average Petrol Price in 2050 ? Explain your answer.

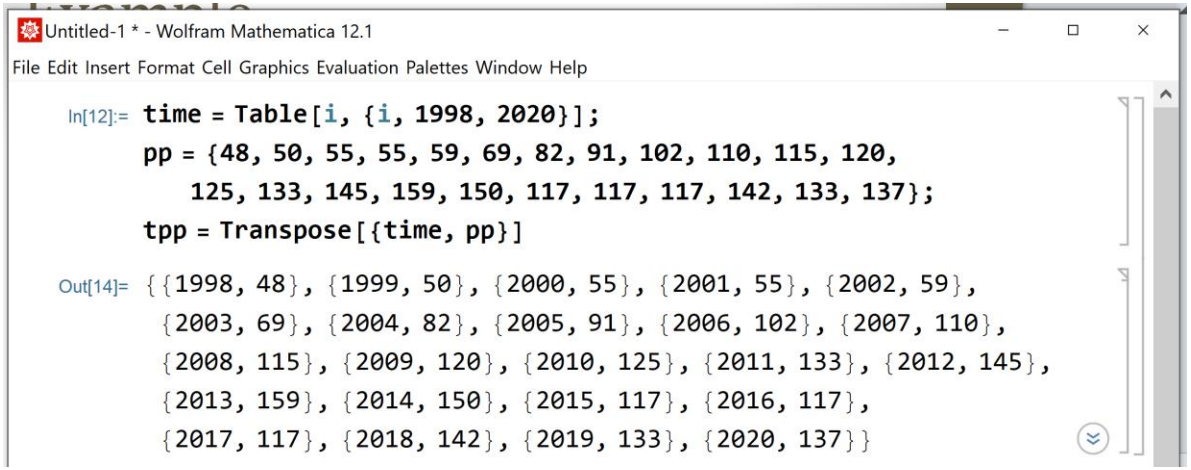
Example

Variation of Petrol Prices with Time is represented in the following table.

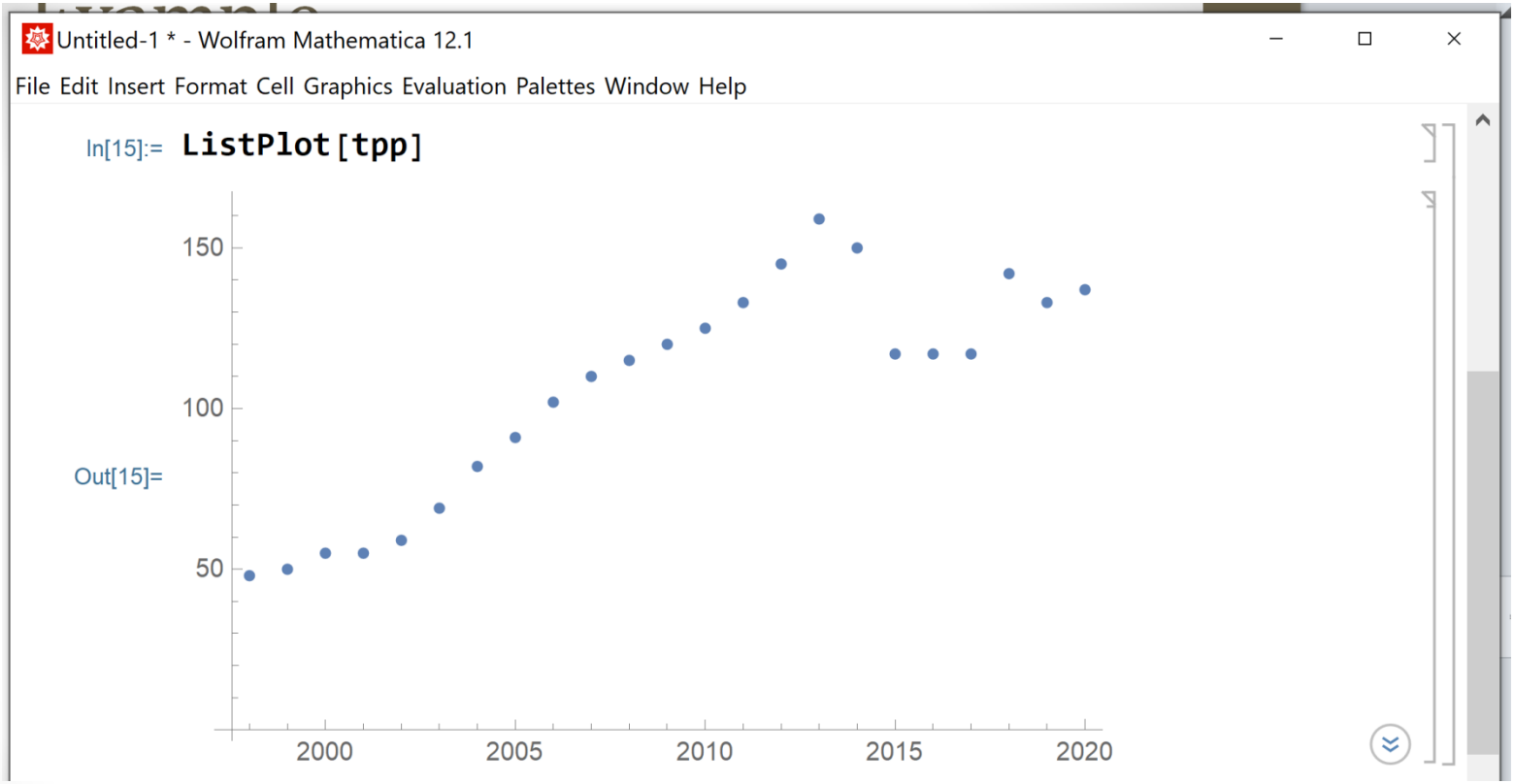
Time (Year)	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Petrol Price (Rs:)	48	50	55	55	59	69	82	91	102	110	115	120
Time (Year)	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Petrol Price (Rs:)	125	133	145	159	150	117	117	117	142	133	137 (?)	

The First row in the above table represent Time (in years) and the second row represent Average Prices of Petrol (in Rupees).

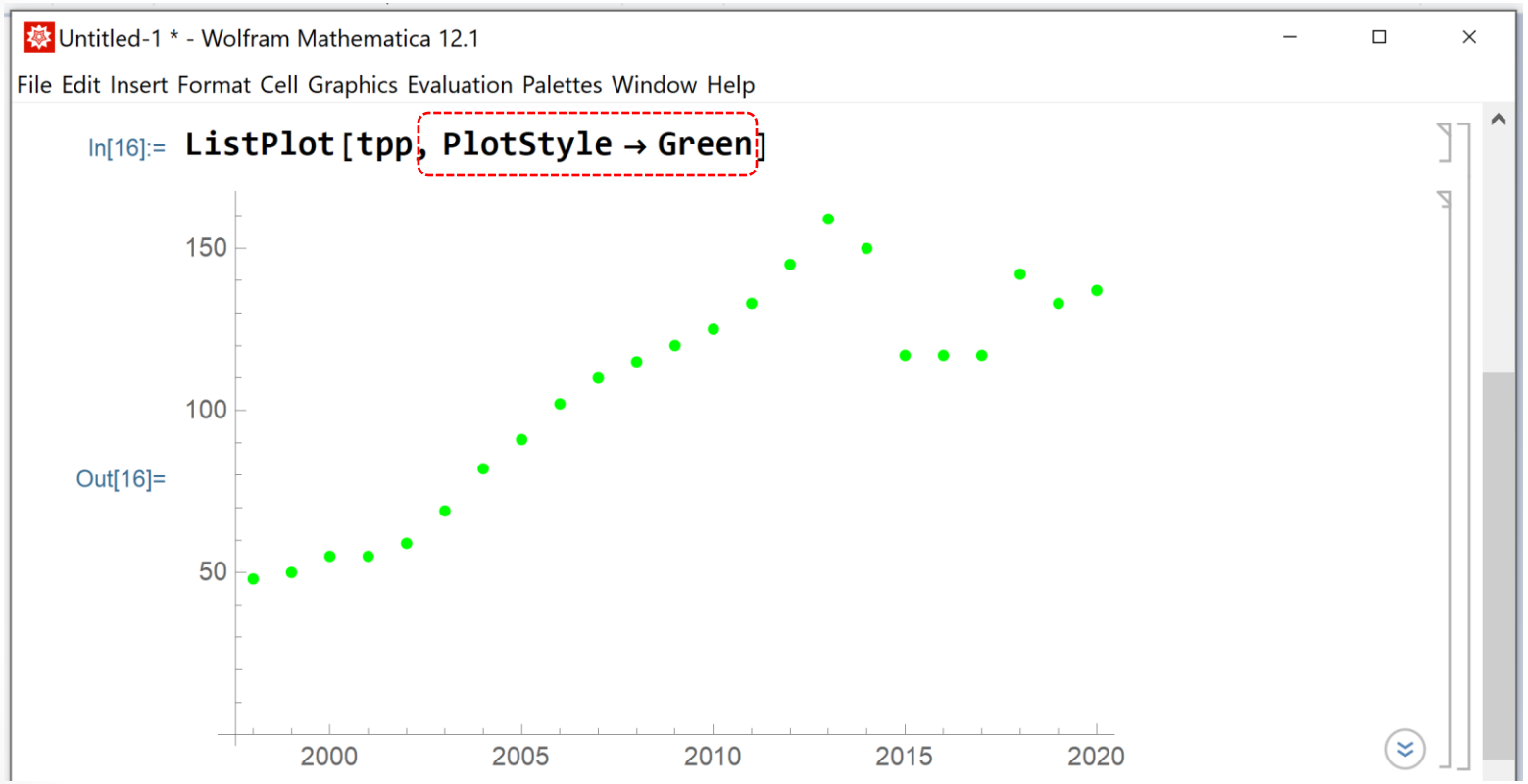
(a) Enter the above data into a table form as Petrol Price vs Time.



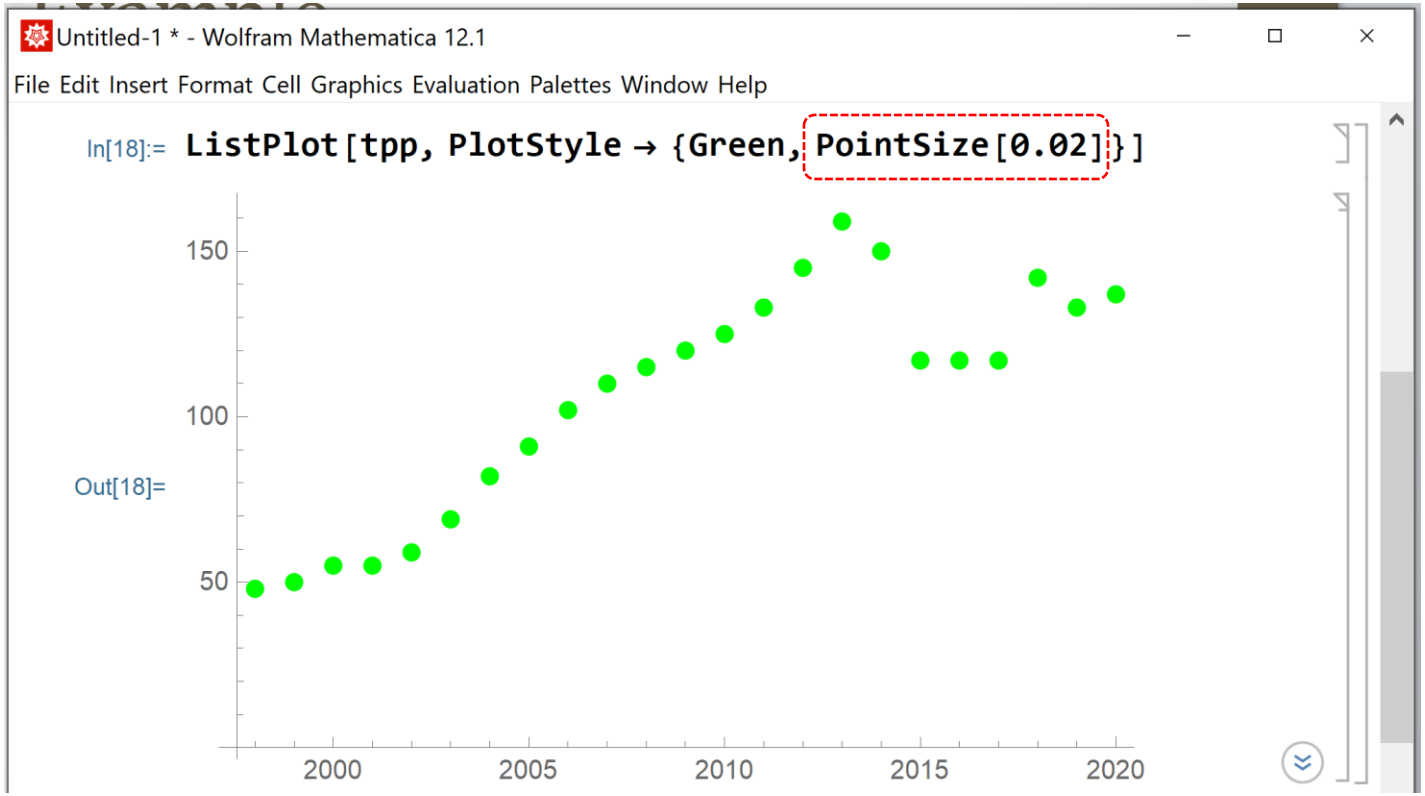
(b) Plot the graph of Petrol Prices vs Time.



(b) Plot the graph of Petrol Prices vs Time.

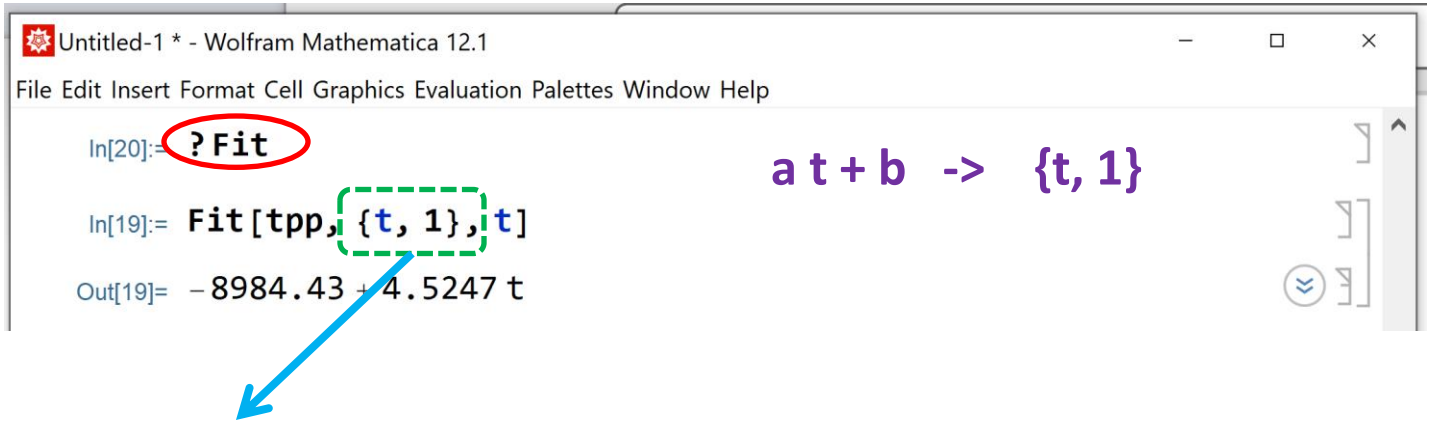


(b) Plot the graph of Petrol Prices vs Time.



(c) Find the suitable equation passing through the points of the above graph.

Fit[data-set, {Model}, model-parameter]



fyp = Fit[tpp, {t, 1}, t] – *This is for a Linear (Simple) Function*

fyp = Fit[tpp, {t^2, t, 1}, t] – *This is for a Quadratic Function*

fyp = Fit[tpp, {t^3, t^2, t, 1}, t] – *This is for a Cubic Function*

.....

fyp = Fit[tpp, {t^100, t^99, t^98, … t^2, t, 1}, t]

– *This is for a 100th order Polynomial Function.*

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In[5]:=
```
Fit[tpp, {t^2, t, 1}, t]
```

Out[5]=
$$-1.23495 \times 10^{6} + 1225.01\, t - 0.303755\, t^{2}$$

In[6]:=
```
Fit[tpp, {t^3, t^2, t, 1}, t]
```

Out[6]=
$$1.40149 \times 10^{8} - 209\,904.\, t + 104.789\, t^{2} - 0.0174369\, t^{3}$$

In[7]:=
```
Fit[tpp, {t^4, t^3, t^2, t, 1}, t]
```

Out[7]=
$$8.1171 \times 10^{10} - 1.61548 \times 10^{8}\, t + 120\,567.\, t^{2} - 39.9919\, t^{3} + 0.00497442\, t^{4}$$

In[10]:=
```
mod = Table[t^i, {i, 0, 20}]
Fit[tpp, mod, t]
```

Out[10]=
$$\{1, t, t^{2}, t^{3}, t^{4}, t^{5}, t^{6}, t^{7}, t^{8}, t^{9}, t^{10}, t^{11}, t^{12}, t^{13}, t^{14}, t^{15}, t^{16}, t^{17}, t^{18}, t^{19}, t^{20}\}$$

Out[11]=
$$\begin{aligned} &-2.54098 \times 10^{10} + 6.38092 \times 10^{6}\, t + 5652.67\, t^{2} + 2.11476\, t^{3} + 0.000310514\, t^{4} - \\ &2.17855 \times 10^{-7}\, t^{5} - 2.37158 \times 10^{-10}\, t^{6} - 1.40737 \times 10^{-13}\, t^{7} - 6.0465 \times 10^{-17}\, t^{8} - \\ &1.71257 \times 10^{-20}\, t^{9} + 1.46994 \times 10^{-25}\, t^{10} + 4.37262 \times 10^{-27}\, t^{11} + 3.75281 \times 10^{-30}\, t^{12} + \\ &2.14442 \times 10^{-33}\, t^{13} + 8.8681 \times 10^{-37}\, t^{14} + 1.96442 \times 10^{-40}\, t^{15} - 7.60934 \times 10^{-44}\, t^{16} - \\ &1.23051 \times 10^{-46}\, t^{17} - 8.0692 \times 10^{-50}\, t^{18} - 2.22849 \times 10^{-53}\, t^{19} + 2.24147 \times 10^{-56}\, t^{20} \end{aligned}$$

Plot the above **graph (data set)** and the **equation obtained in the previous part** in the **same graph** by using **different two colours**.

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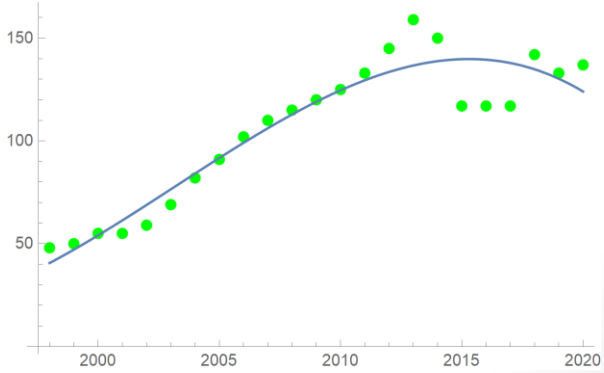
```
In[ ]:= time = Table[i, {i, 1998, 2020}];
        pp = {48, 50, 55, 55, 59, 69, 82, 91, 102, 110, 115, 120,
            125, 133, 145, 159, 150, 117, 117, 117, 142, 133, 137};
        tpp = Transpose[{time, pp}];

        g1 = ListPlot[tpp, PlotStyle → {Green, PointSize[0.02]}];
        f = Fit[tpp, {t^3, t^2, t, 1}, t]
        g2 = Plot[f, {t, time[[1]], Last[time]}];
        Show[{g1, g2}]
```

Initial Point

Final Point



If you want to plot the corresponding function on the data set then, this is the way to it....

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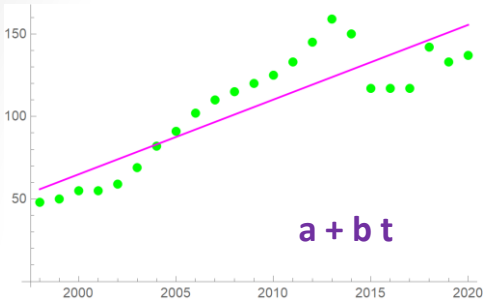
In[8]:=
```
time = Table[i, {i, 1998, 2020}];
pp = {48, 50, 55, 55, 59, 69, 82, 91, 102, 110, 115, 120, 125, 133, 145, 159,
   150, 117, 117, 117, 142, 133, 137};
tpp = Transpose[{time, pp}];

g1 = ListPlot[tpp, PlotStyle → {Green, PointSize[0.02]}];
f = Fit[tpp, {t^3, t^2, t, 1}, t]
g2 = Plot[f, {t, time[[1]], Last[time]}, PlotStyle → Magenta];
Show[{g1, g2}]
```

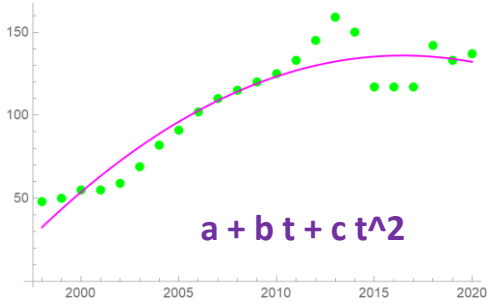
Out[12]= $1.40149 \times 10^8 - 209\,904.\,t + 104.789\,t^2 - 0.0174369\,t^3$

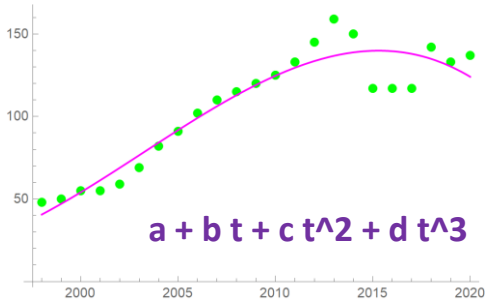
Assign colours to the second graph

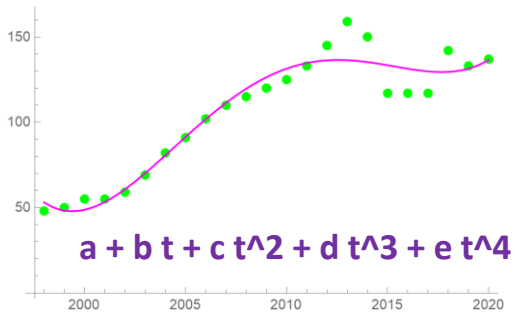
Out[14]=

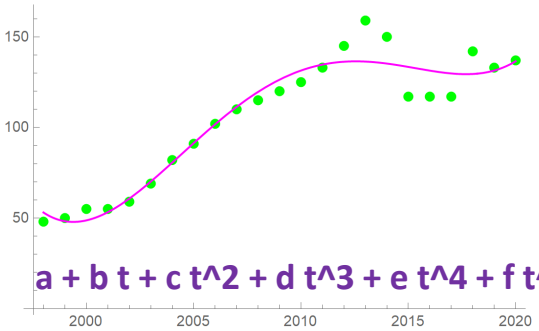
*Using **Trial and Error Method** to **find the suitable function** for the data set !!!*





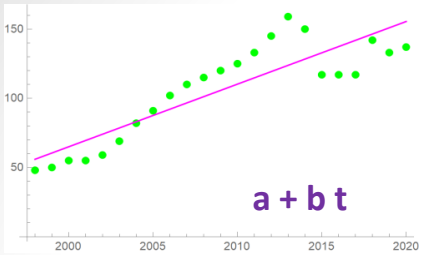


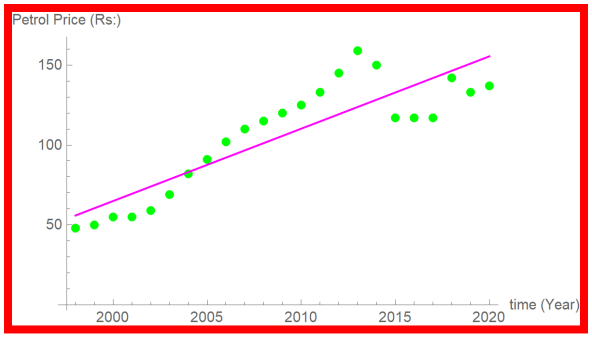


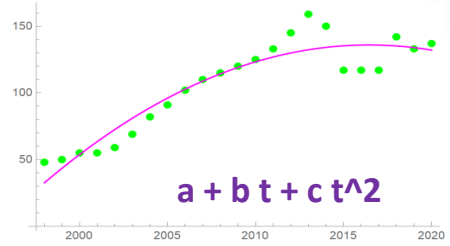


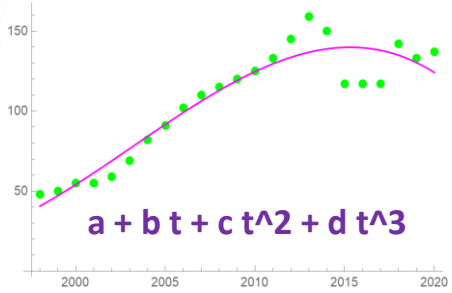

```
Fit[tpp, {t^4, t^3, t^2, t, 1}, t]
```

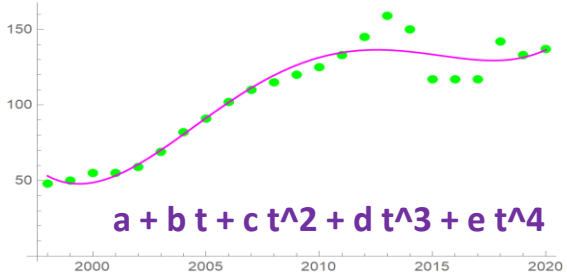
$$8.1171 \times 10^{10} - 1.61548 \times 10^{8}\, t + 120\,567.\, t^2 - 39.9919\, t^3 + 0.00497442\, t^4$$

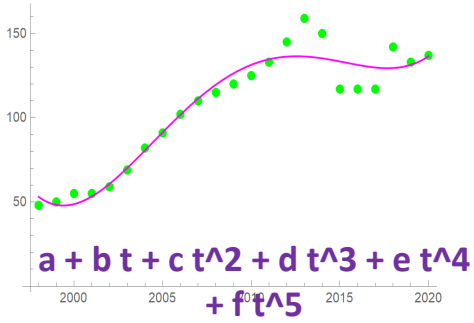












All these models which we developed using Mathematica might not be the accurate/correct model. The output models which are provided by Mathematica depend on the input structure that we gave. Therefore, the objective of this lesson is not to obtain the accurate/correct model, but to obtain the optimum model that best fits to the input structure given by the student. Moreover, the final output model will completely depend on the general knowledge; the student has in mathematical modeling. **Due to this reason, the student should have a good common sense to test the accuracy of the output and to decide whether the selected model is accepted or not.**

(d) Find the Average Price of Petrol in 2020, 2021, 2022 and 2050 using the equation that you obtained by Mathematical Modeling.

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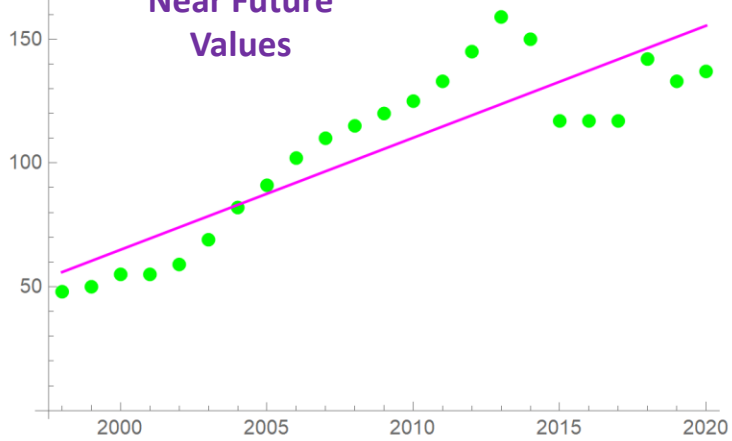
```
In[57]:= time = Table[i, {i, 1998, 2020}];
         pp = {48, 50, 55, 55, 59, 69, 82, 91, 102, 110, 115, 120, 125, 133, 145, 159,
            150, 117, 117, 117, 142, 133, 137};
         tpp = Transpose[{time, pp}];

         g1 = ListPlot[tpp, PlotStyle → {Green, PointSize[0.02]}];
         f = Fit[tpp, {t, 1}, t]
         g2 = Plot[f, {t, time[[1]], Last[time]}, PlotStyle → Magenta];
         Show[{g1, g2}]

         f /. t → {2020, 2021, 2022, 2050}

Out[61]= -8984.43 + 4.5247 t
```

Far Future Values

Near Future Values



Out[63]=

Out[64]= {155.467, 159.992, 164.517, 291.208}

(e) Find the corresponding year when Average Price of Petrol is Rs: 500.00 using the suitable equation that you found in the previous part.

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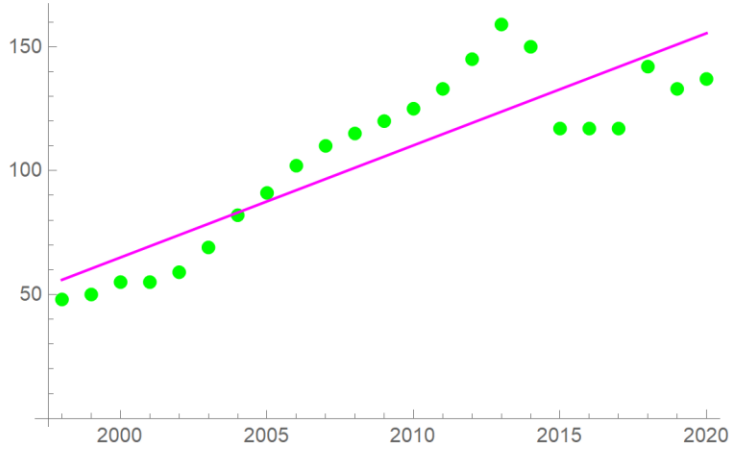
```
In[65]:= time = Table[i, {i, 1998, 2020}];
         pp = {48, 50, 55, 55, 59, 69, 82, 91, 102, 110, 115, 120, 125, 133, 145, 159,
            150, 117, 117, 117, 142, 133, 137};
         tpp = Transpose[{time, pp}];
         g1 = ListPlot[tpp, PlotStyle → {Green, PointSize[0.02]}];
         f = Fit[tpp, {t, 1}, t]
         g2 = Plot[f, {t, time[[1]], Last[time]}, PlotStyle → Magenta];
         Show[{g1, g2}]
         f /. t → {2020, 2021, 2022, 2050}

         Solve[f == 500.00, t]
Out[69]= -8984.43 + 4.5247 t
```

Out[71]=




```
Out[72]= {155.467, 159.992, 164.517, 291.208}

Out[73]= {{t → 2096.14}}
```

This is a REAL Value; then we can accept the answer !!!

Complete Source Code :

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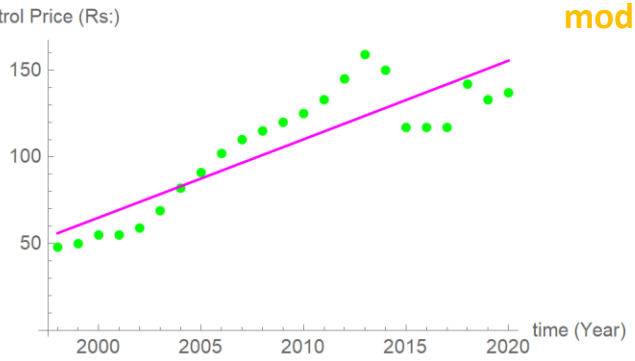
In[359]:=
```
(* Problem of Petrol Price in Sri Lanka 1998 to 2020 *)
time = Table[i, {i, 1998, 2020}];
pp = {48, 50, 55, 55, 59, 69, 82, 91, 102, 110, 115, 120, 125, 133, 145, 159, 150, 117, 117, 117, 142, 133, 137};
tpp = Transpose[{time, pp}];
g1 = ListPlot[tpp, PlotStyle → {Green, PointSize[0.02]}, AxesLabel → {"time (Year)", "Petrol Price (Rs:)"}];
f = Fit[tpp, {t^2, 1}, t];
Print["Suitable Model is : ", f]
g2 = Plot[f, {t, time[[1]], Last[time]}, PlotStyle → Magenta];
Show[{g1, g2}]
out = f /. t → {2020, 2021, 2022, 2050};
Print["Petrol Price in 2020, 2021, 2022 and 2050 respectrively are : Rs ", out]
sol = Solve[f == 500.00, t]
new = t /. sol;
Print["Petrol Price will be Rs: 500 on year ", new[[2]]]
```

Label the x & y axis of graph

Suitable Model is : $-4436.75 + 0.00112545\, t^2$

This is not the suitable model ! You should find the suitable model according to your knowledge !!!

Out[366]=



Petrol Price in 2020, 2021, 2022 and 2050 respectrively are : Rs {155.525, 160.073, 164.623, 292.942}

Out[369]= {{t → −2094.39}, {t → 2094.39}}

Petrol Price will be Rs: 500 on year 2094.39

Key features of a Good Model

- **The model should be simple,**
  - If the order of the fitted function is high it is not a good model .
  - Function does not need to satisfy all data points.
  - If a any complex term to be introduced it needed to be justified.

- **Far future and near future values given from the function should be acceptable.**
  - With the knowledge available about the system it needed to be confirm that the guessed model is accepted for known future.

Example

The variation of the monthly average temperature at the University premises for the last 13 months is given in the following table.

Month	Temperature (°C)
1 – May 2019	33
2 – June 2019	30
3 – July 2019	31
4 – Aug 2019	32
5 – Sept 2019	33
6 – Oct 2019	33
7 – Nov 2019	32
8 – Dec 2019	32
9 – Jan 2020	32
10 – Feb 2020	31
11 – Mar 2020	32
12 – Apr 2020	33
13 – May 2020	33

1) Draw the graph of temperature vs. time (in months).
2) Determine a suitable model which fits the above data set.
3) Find the average temperature in the months, June 2020 and July 2020.
4) By using the above model, is it reasonable to estimate the average temperature for the month January 2025? Explain your answer.

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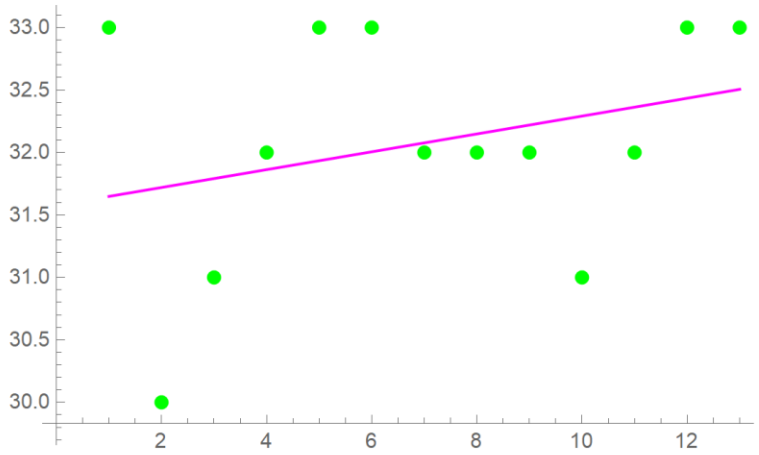
In[174]:=
```
tt = Table[i, {i, 1, 13}];
temp = {33, 30, 31, 32, 33, 33, 32, 32, 32, 31, 32, 33, 33};
Print["Maximum Temperature : ", Max[temp], "°C   and   Minimum Temperature : ", Min[temp], "°C"]
tttemp = Transpose[{tt, temp}]
g1 = ListPlot[tttemp, PlotStyle → {Green, PointSize[0.02]}];
f = Fit[tttemp, {t, 1}, t]
(*  {t,1}   {t^2,t,1}   {t^3,t^2,t,1}   {Sin[t],1}  {Sin[t],t^3,t^2,t,1}   *)
g2 = Plot[f, {t, tt[[1]], Last[tt]}, PlotStyle → Magenta];
Show[{g1, g2}]
f /. t → {14, 15, 44}
```

```
Maximum Temperature : 33°C   and   Minimum Temperature : 30°C
```

Out[177]=
```
{{1, 33}, {2, 30}, {3, 31}, {4, 32}, {5, 33}, {6, 33},
 {7, 32}, {8, 32}, {9, 32}, {10, 31}, {11, 32}, {12, 33}, {13, 33}}
```

Out[179]=
$$31.5769 + 0.0714286\,t$$

Out[181]=



Out[182]=
```
{32.5769, 32.6484, 34.7198}
```

Complete Source Code :

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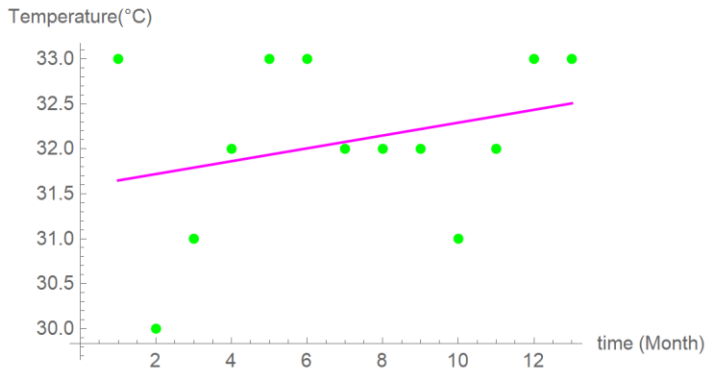
In[449]:=
```
(* Problem of Average Temperature varies with time at USJ from May-2019 tp May-2020 *)
tt = Table[i, {i, 1, 13}];
temp = {33, 30, 31, 32, 33, 33, 32, 32, 32, 31, 32, 33, 33};
Print["Maximum Temperature : ", Max[temp], "°C   and   Minimum Temperature : ", Min[temp], "°C"]
tttemp = Transpose[{tt, temp}];
g1 = ListPlot[tttemp, PlotStyle → {Green, PointSize[0.02]}, AxesLabel → {"time (Month)", "Temperature(°C)"}];
f = Fit[tttemp, {t, 1}, t];
Print["Suitable Model is : ", f]
(*  {t,1}   {t^2,t,1}   {t^3,t^2,t,1}   {Sin[t],1}  {Sin[t],t^3,t^2,t,1}   *)
g2 = Plot[f, {t, tt[[1]], Last[tt]}, PlotStyle → Magenta];
Show[{g1, g2}]
sol2 = f /. t → {14, 15, 44};
Print["Average Temperature at USJ on June 2020, July 2020 and Jan 2025 are ", sol2, " in °C"]
Print["The temp value on Jan 2025 is not suitable because it is very far from the domain of the data set !!!"]
```

```
Maximum Temperature : 33°C   and   Minimum Temperature : 30°C

Suitable Model is : 31.5769 + 0.0714286 t
```

Out[457]=




```
Average Temperature at USJ on June 2020, July 2020 and Jan 2025 are {32.5769, 32.6484, 34.7198} in °C

The temp value on Jan 2025 is not suitable because it is very far from the domain of the data set !!!
```

Thank you!!!