PHY 359 2.0 / ASP 487 2.0 Telecommunication

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Electronic Filters

Introduction

- Filters are circuits that are capable of *passing signals within a band* of frequencies while *rejecting or blocking* signals of frequencies *outside this band*. This property of filters is also called “frequency selectivity”.

- Filter can be divide in to two parts

Passive filters: The circuits built using RC, RL, or RLC circuits

Active filters: The circuits that employ one or more op-amps in the design an addition to resistors and capacitors

Passive devices:

- If a device does not have a built in power source it is termed passive device.

  Eg: Resistors, capacitors, inductors, diodes, transformers, switches, relays etc.

Active devices:

- It is a device that can amplify, producing an output signal with more power in it than the input signal. The additional power comes from an external source (Power supply). The devices with pow gain are distinguishable by their ability to make oscillators, by feeding some output power back into the input.

  Eg: Transistors, and circuits containing transistors, ICs.

Advantages of Active Filters over Passive Filters

- Active filters can be designed to provide required gain, and hence no attenuation as in the case of passive filters
- No loading problem, because of high input Impedance and low output Impedance of op-amp.
- Active Filters are cost effective as a wide variety of economical op-amps are available.

Applications

- Active filters are mainly used in communication, Analog Instruments and signal processing circuits.
- They are also employed in a wide range of applications.

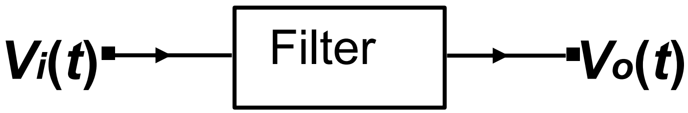
  Eg: entertainment, medical electronics, modern physics applications, Telecommunication Devices, etc.

- **There are 4 basic categories of active filters**

    1. Low-pass filters
    2. High-pass filters
    3. Band-pass filters
    4. Band-reject filters

- **Each of these filters can be constructed by using op-amp as the active element combined with RC, RL or RLC circuit as the passive elements.**

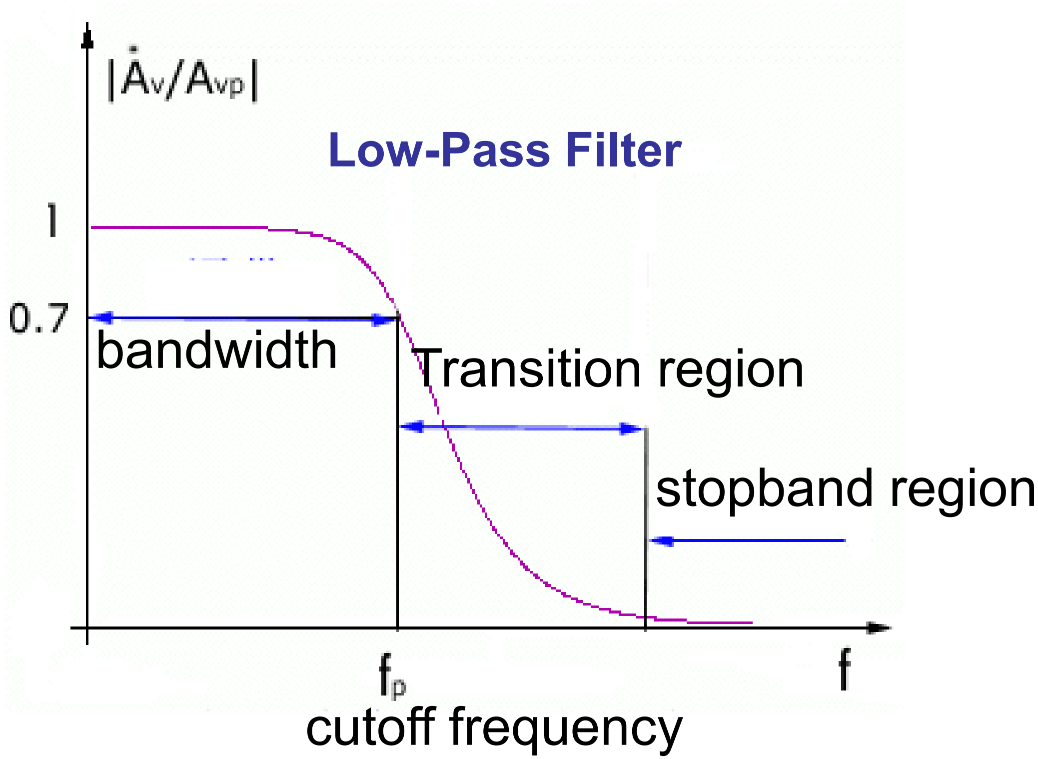
Basic Filter Responses



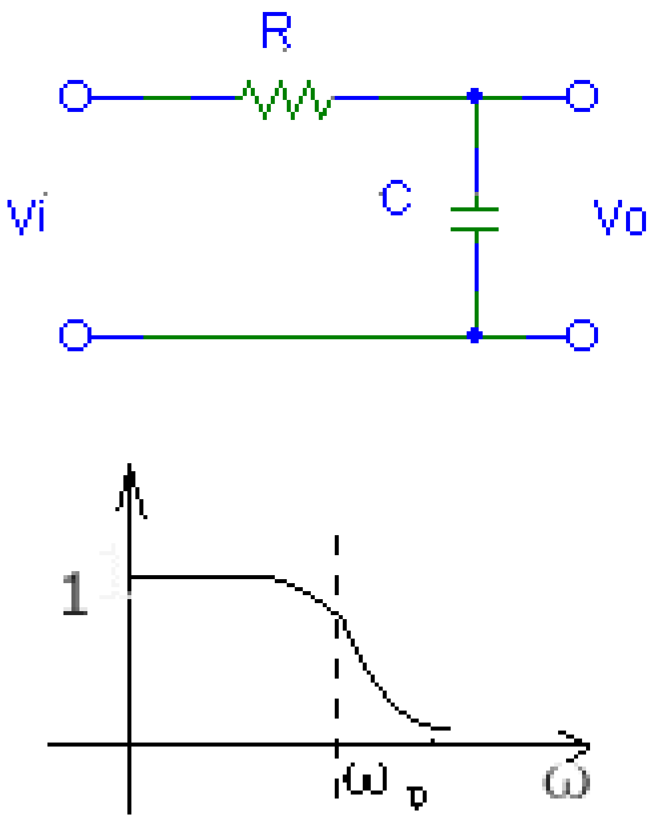
voltage gain

$$A(s) = \frac{v_O(s)}{v_i(s)}$$

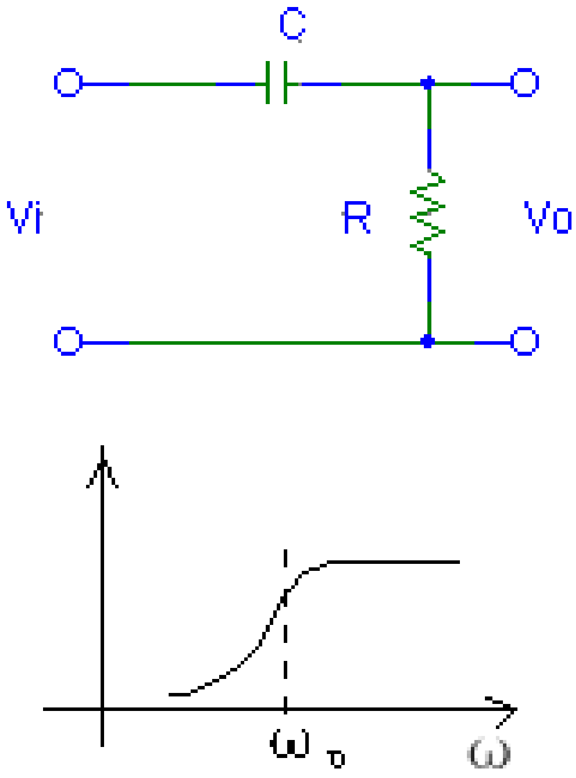
$$S = j\omega \qquad \dot{A}(j\omega) = \frac{\dot{V}_o(j\omega)}{\dot{V}_i(j\omega)} = |A(j\omega)| \angle \varphi(j\omega)$$



Basic Passive Filter Responses



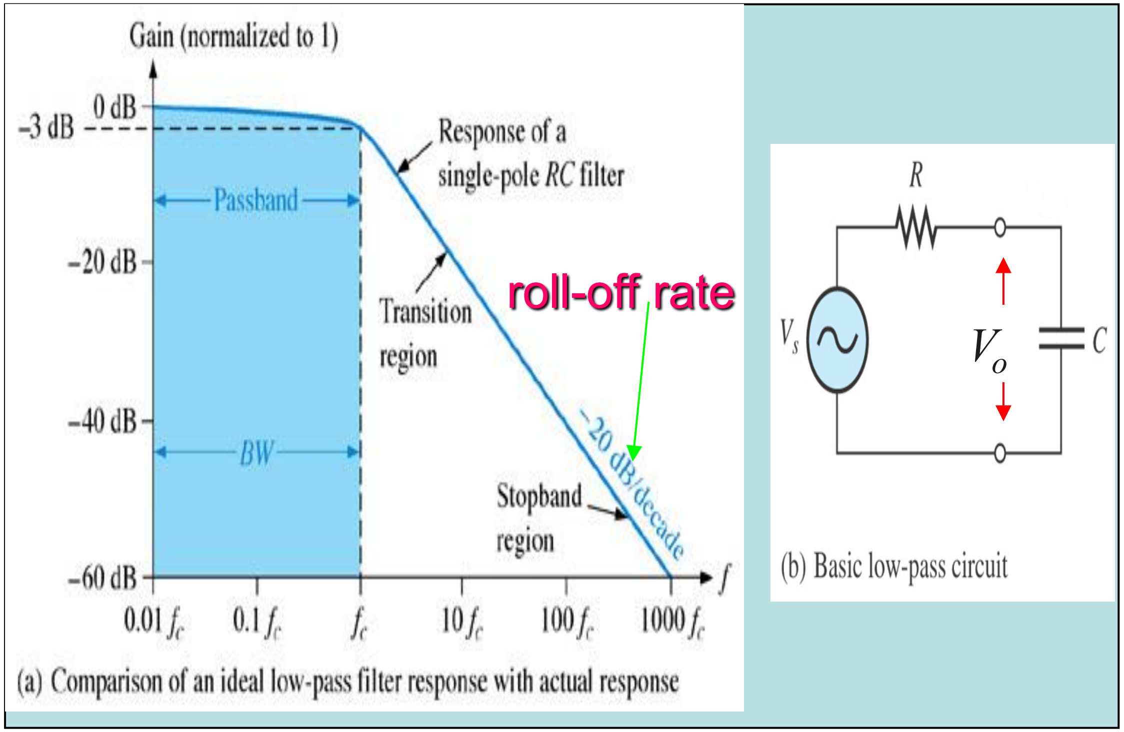
Low Pass Filter



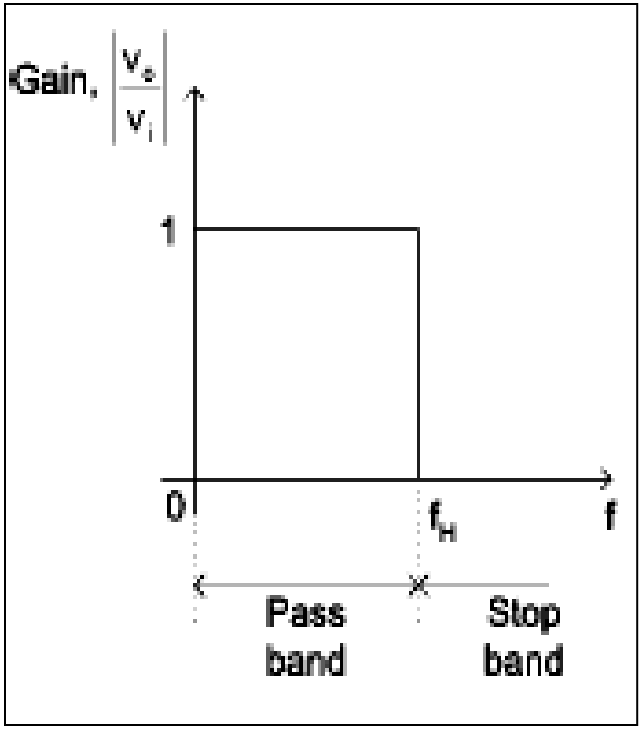
High Pass Filter

Low pass Filter Responses

- A Low-Pass Filter is a filter that passes frequencies from 0Hz to critical frequency, f_c and significantly attenuates all other frequencies.



Actual response

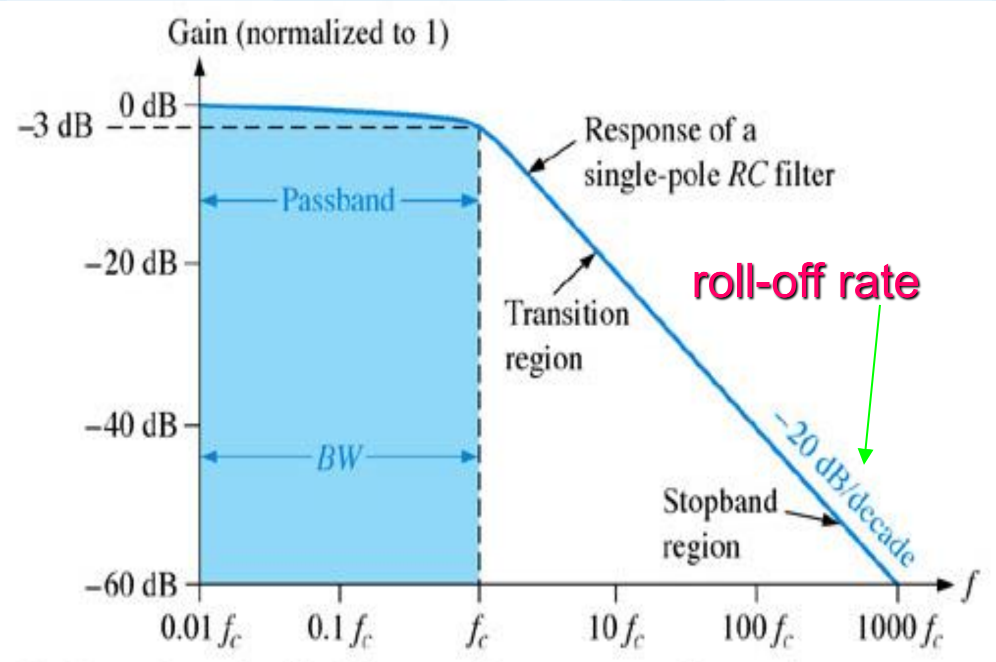


Ideal response

- Ideally, the response drops abruptly at the critical frequency, f_H

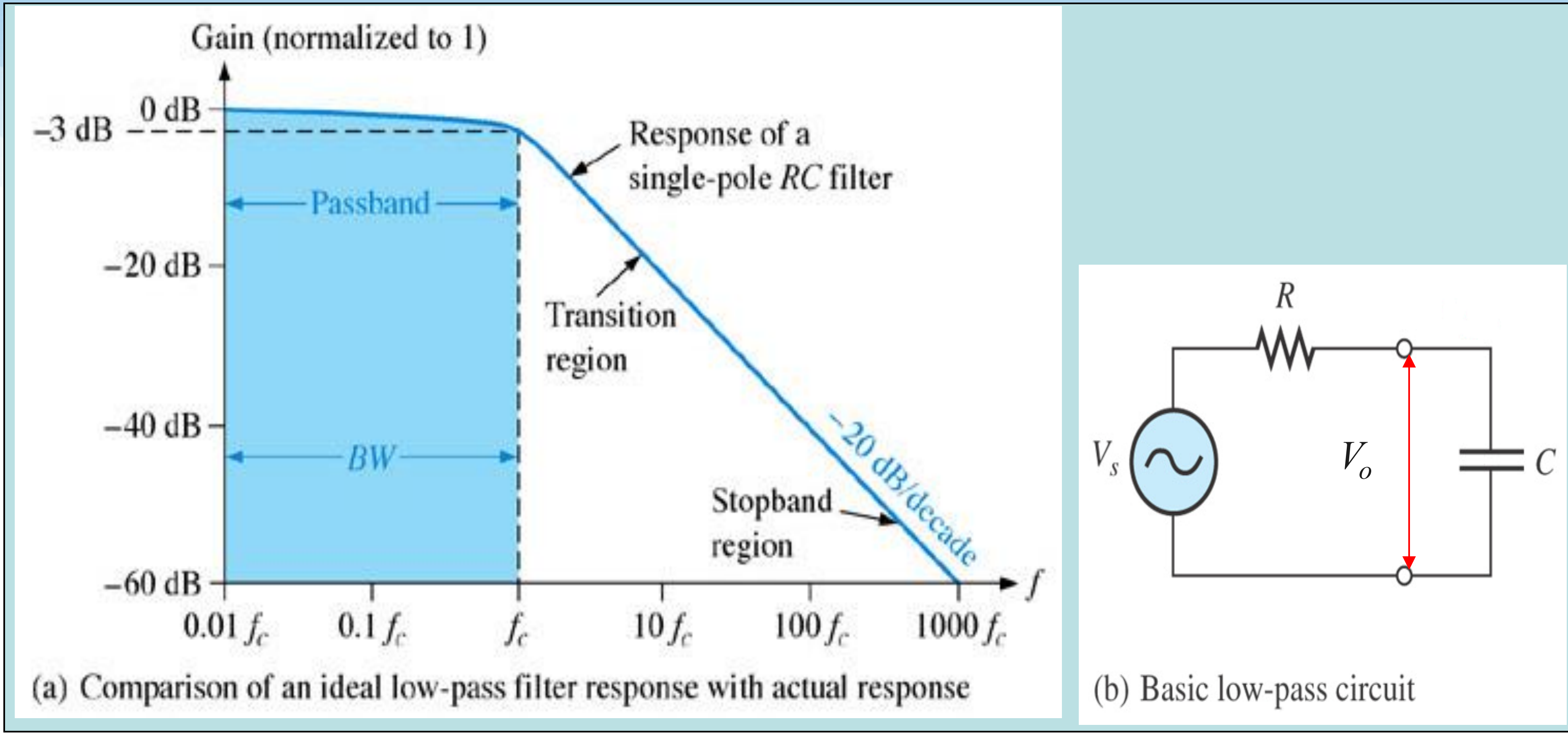
- **Pass band** of a filter is the range of frequencies that are allowed to pass through the filter with minimum attenuation (usually defined as less than -3 dB of attenuation).

- **Transition region** shows the area where the fall-off occurs.



- **Stop band** is the range of frequencies that have the most attenuation.

- **Critical frequency**, f_c, (also called the cutoff frequency) defines the end of the pass band and normally specified at the point where the response drops – 3 dB (70.7%) from the pass band response.

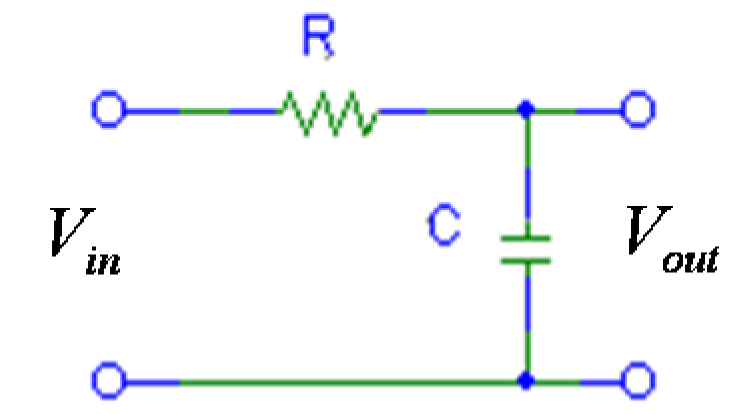


(a) Comparison of an ideal low-pass filter response with actual response

(b) Basic low-pass circuit

- At low frequencies, X_C is very high and the capacitor circuit can be considered as open circuit. Under this condition, $V_o = V_{in}$ or $A_V = 1$ (unity).
- At very high frequencies, X_C is very low and the V_o is small as compared with V_{in}. Hence the gain falls and drops off gradually as the frequency is increased.

Transfer function of the passive Low pass Filter



$$V_{out} = \frac{V_{in}\left(\frac{1}{j\omega C}\right)}{\left(R + \frac{1}{j\omega C}\right)}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{(Rj\omega C + 1)}$$

Take, $RC = \frac{1}{\omega_c}$ $\quad \omega_c = \frac{1}{RC}$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$$

→ **Transfer Function of Low Pass Filter**

When, $\omega << \omega_c, \ \frac{\omega}{\omega_c} = 0$ $\quad \left|\frac{V_{out}}{V_{in}}\right| = 1 = 0dB$

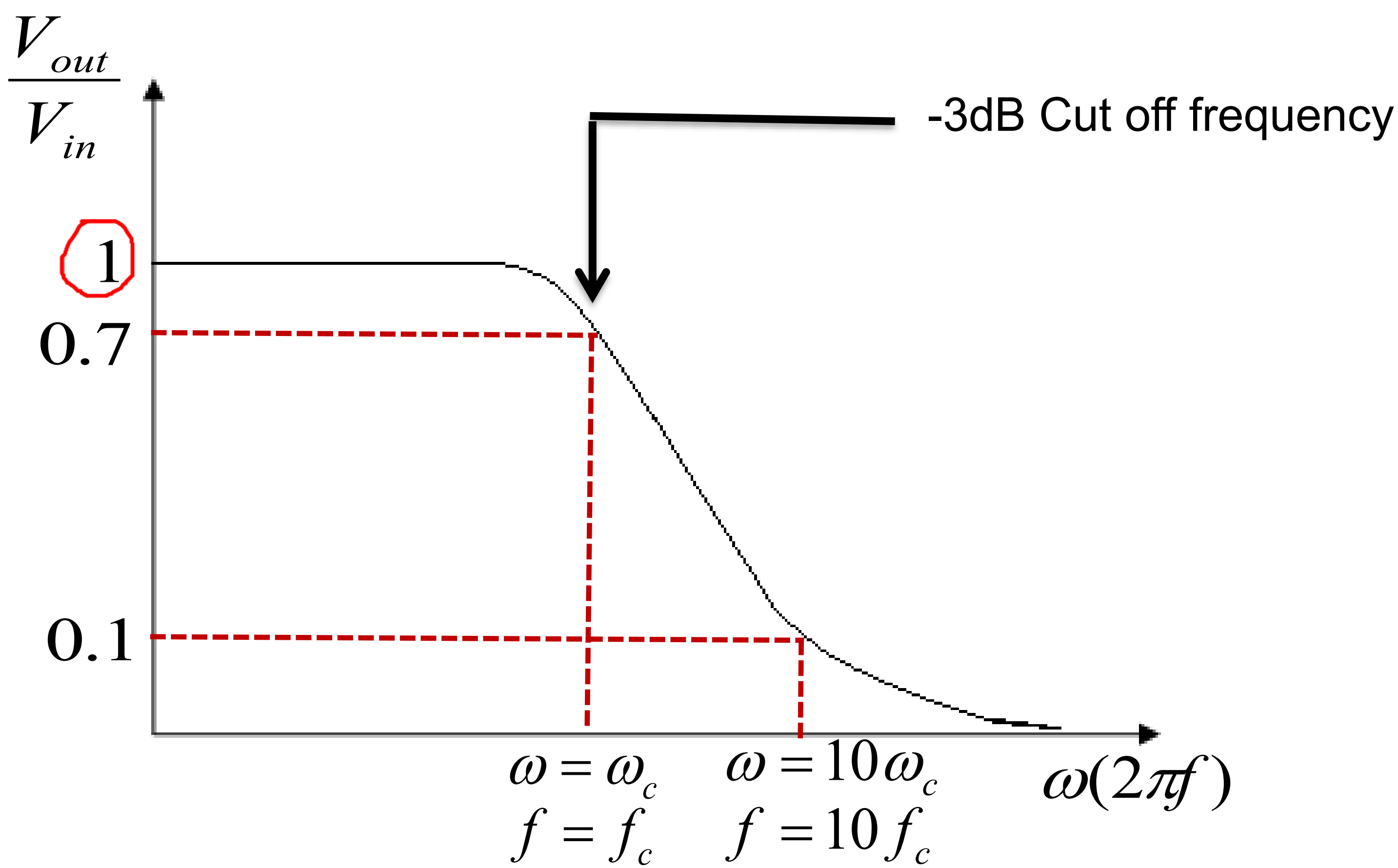
When, $\omega = \omega_c, \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}}$ for, $f = f_c$

$$20\log\left|\frac{V_{out}}{V_{in}}\right| = 20\log\left|\frac{1}{\sqrt{2}}\right| = 20\times(-0.15) = -3dB$$

When, $\omega >> \omega_c, 1+\frac{\omega}{\omega_c} \approx \frac{\omega}{\omega_c}$

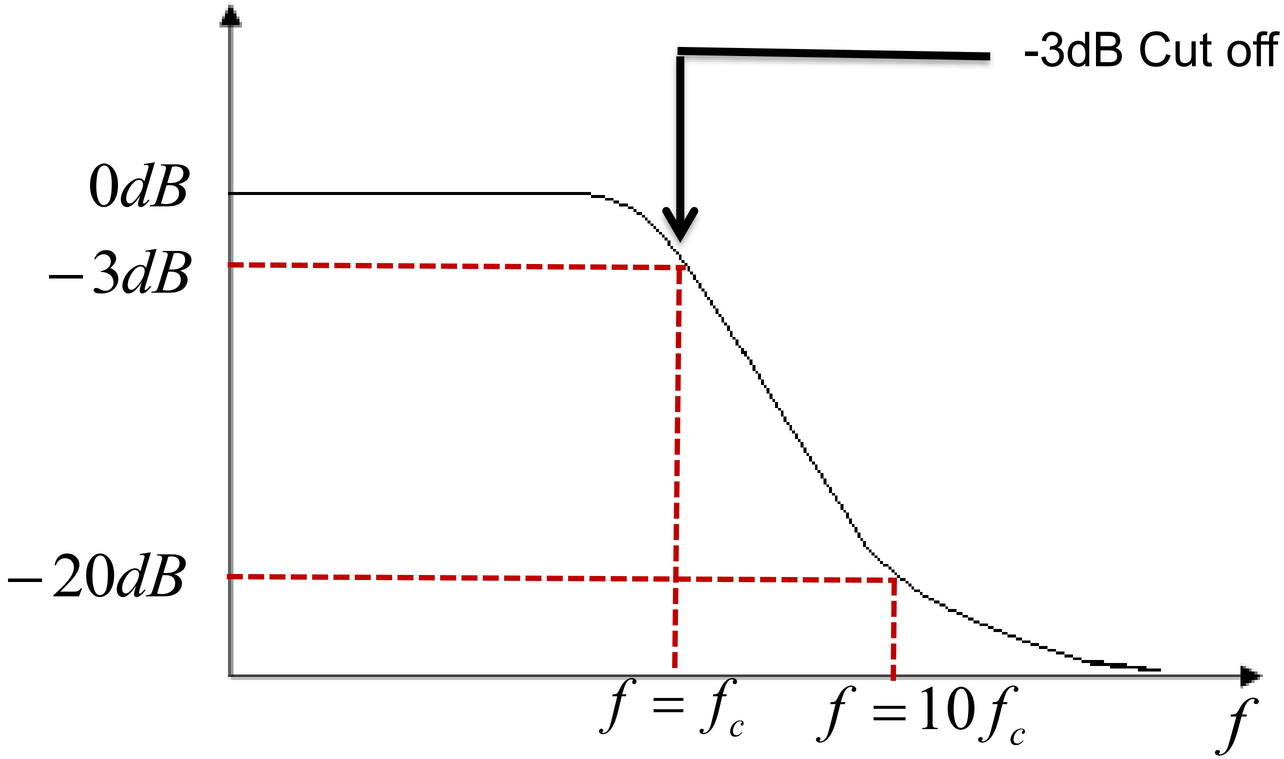
Therefore, $\left|\frac{V_{out}}{V_{in}}\right| \approx \frac{\omega_c}{\omega} = 20\log\left|\frac{\omega_c}{\omega}\right| dB$

Voltage Gain Vs frequency response Graph



Voltage Gain (dB) Vs frequency response Graph

$$20\log(\frac{V_{out}}{V_{in}}) \rightarrow Gain(dB)$$



- The **bandwidth** of an **ideal** low-pass filter is equal to f_c:

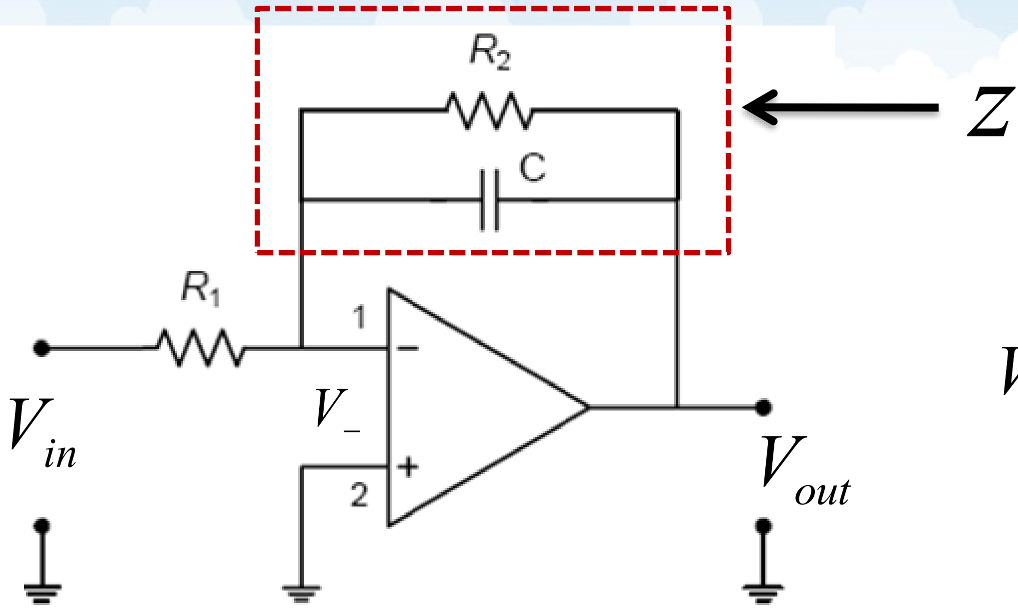
$$BW = f_c$$

- The critical frequency of a low-pass RC filter occurs when **$X_C = R$** and can be calculated using the formula below:

$$\omega_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC}$$

Transfer function of the Active Low pass Filter



$$V_- = \frac{ZV_{in} + R_1 V_{out}}{R_1 + Z}$$

$$V_- = V_+ = 0 \qquad \frac{V_{out}}{V_{in}} = \frac{-Z}{R_1}$$

$$Z = \frac{R_2 \left(\frac{1}{j\omega C}\right)}{\left(R_2 + \frac{1}{j\omega C}\right)}$$

$$Z = \frac{R_2}{(1 + jR_2 \omega C)}$$

$$\frac{V_{out}}{V_{in}} = \frac{-R_2}{R_1(1 + jR_2 \omega C)}$$

Take, $\omega_c = \frac{1}{R_2 C}, G_o = \frac{R_2}{R_1}$

$$\frac{V_{out}}{V_{in}} = \frac{-G_o}{\left(1 + j\left(\frac{\omega}{\omega_c}\right)\right)}$$

G_o = DC Gain (Low frequency Gain)

ω_c = 3dB Cut off frequency

When, $\omega \ll \omega_c, \dfrac{\omega}{\omega_c} = 0, \quad \left|\dfrac{V_{out}}{V_{in}}\right| = G_o, Gain(dB) = 20\log\left|\dfrac{R_2}{R_1}\right|$

When, $\omega = \omega_c, \left|\dfrac{V_{out}}{V_{in}}\right| = \dfrac{G_o}{\sqrt{2}}$ for, $f = f_c$

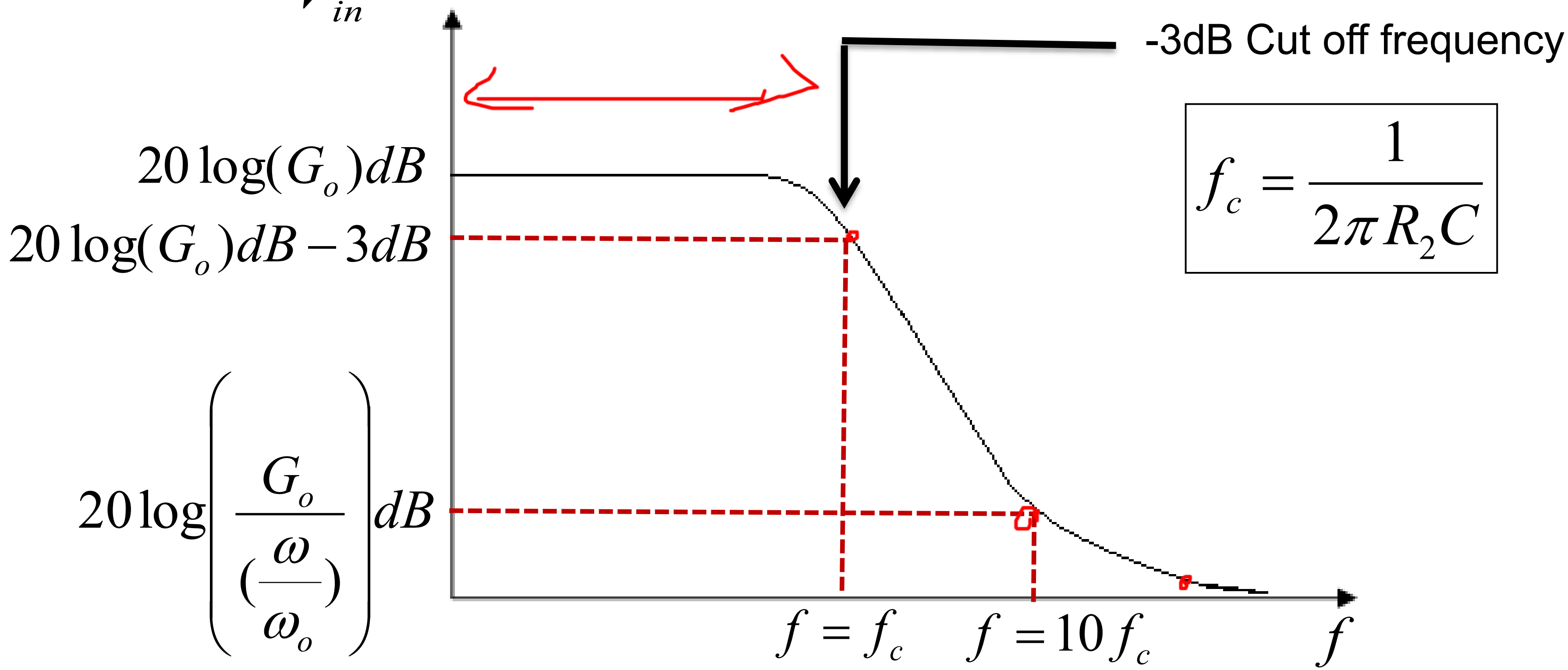
$$Gain(dB) = 20\log\left|\frac{G_o}{\sqrt{2}}\right| = 20\log|G_o|(dB) - 3dB$$

When, $\omega \gg \omega_c, \ 1 + \dfrac{\omega}{\omega_c} \approx \dfrac{\omega}{\omega_c}$

Therefore, $\left|\dfrac{V_{out}}{V_{in}}\right| = \dfrac{G_o}{(\dfrac{\omega}{\omega_c})}, Gain(dB) = 20\log\left|\dfrac{G_o}{(\dfrac{\omega}{\omega_c})}\right|$

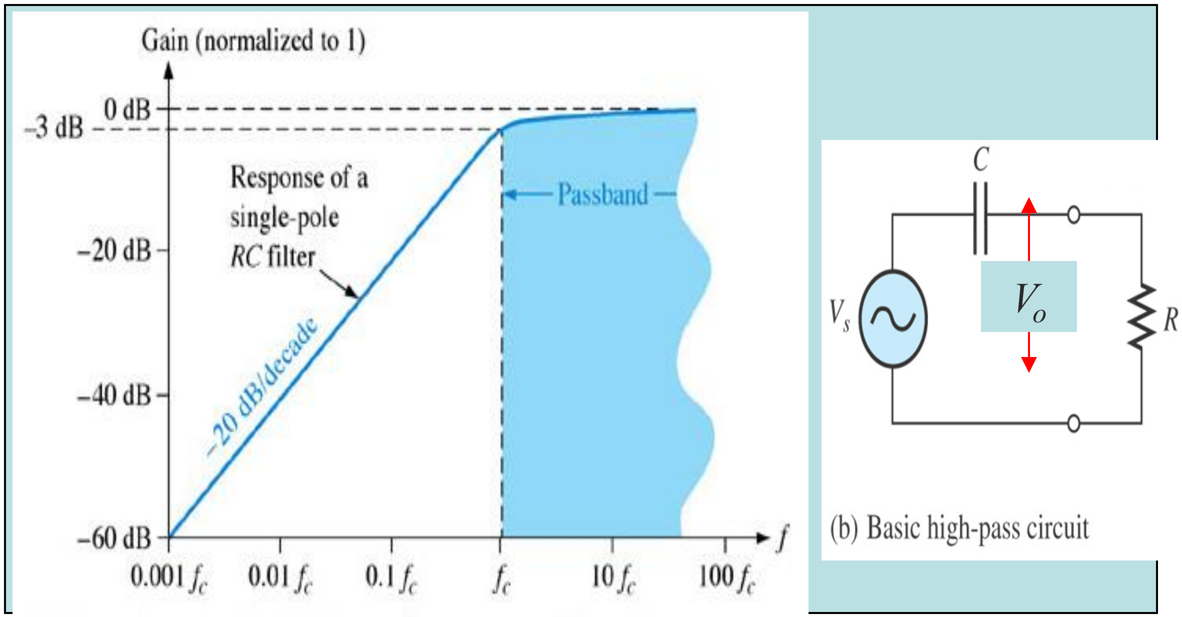
Voltage Gain (dB) Vs frequency response Graph

$$20\log\left(\frac{V_{out}}{V_{in}}\right) \rightarrow Gain(dB)$$



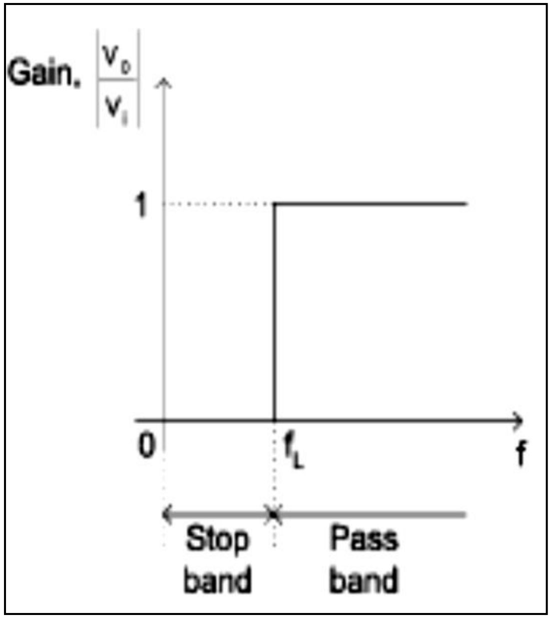
High pass Filter Responses

- A **high-pass filter** is a filter that significantly attenuates or rejects all frequencies **below** f_c and passes all frequencies **above** f_c.
- The pass band of a high-pass filter is all frequencies above the critical frequency.



(a) Comparison of an ideal high-pass filter response with actual response

Actual response



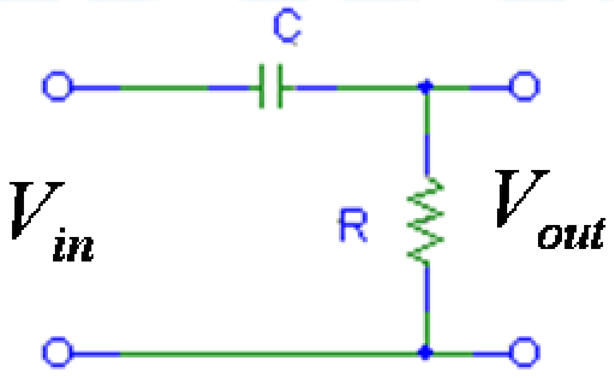
Ideal response

- Ideally, the response rises abruptly at the critical frequency, f_L

- The critical frequency of a high-pass RC filter occurs when **X_C = R** and can be calculated using the formula below:

$$f_c = \frac{1}{2\pi RC}$$

Transfer function of the passive High pass Filter



$$V_{out} = \frac{V_{in} R}{\left(R + \frac{1}{j\omega C}\right)}$$

$$\frac{V_{out}}{V_{in}} = \frac{Rj\omega C}{(Rj\omega C + 1)}$$

Take, $RC = \frac{1}{\omega_c}$ $\quad \omega_c = \frac{1}{RC}$

Transfer Function:

$$\left(\frac{V_{out}}{V_{in}}\right) = \frac{j(\frac{\omega}{\omega_c})}{\left(1 + j(\frac{\omega}{\omega_c})\right)}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{(\frac{\omega}{\omega_c})}{\sqrt{\left(1 + (\frac{\omega}{\omega_c})^2\right)}}$$

When, $\omega >> \omega_c$, $1+(\frac{\omega}{\omega_c})^2 \approx (\frac{\omega}{\omega_c})$ $\therefore \left|\frac{V_{out}}{V_{in}}\right| = 1 = 0dB$

When, $\omega = \omega_c$, $\therefore \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}}$, $20\log\left|\frac{V_{out}}{V_{in}}\right| = 20\log\left|\frac{1}{\sqrt{2}}\right| = -3dB$

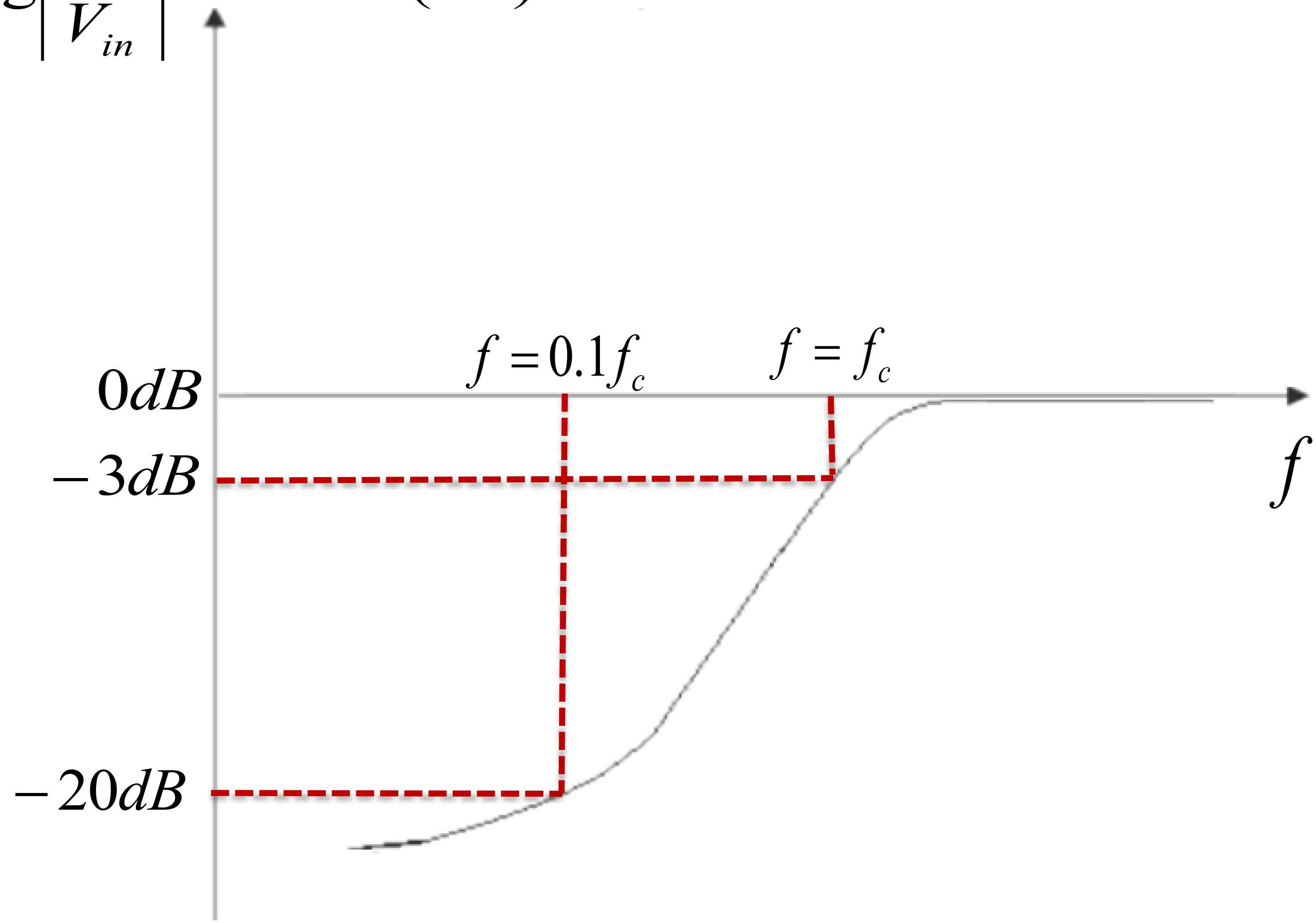
When, $\omega << \omega_c$, $1+(\frac{\omega}{\omega_c})^2 \approx 1$, $\left|\frac{V_{out}}{V_{in}}\right| \approx (\frac{\omega}{\omega_c})$

$$20\log\left|\frac{V_{out}}{V_{in}}\right| = 20\log\left|\frac{\omega}{\omega_c}\right|(dB)$$

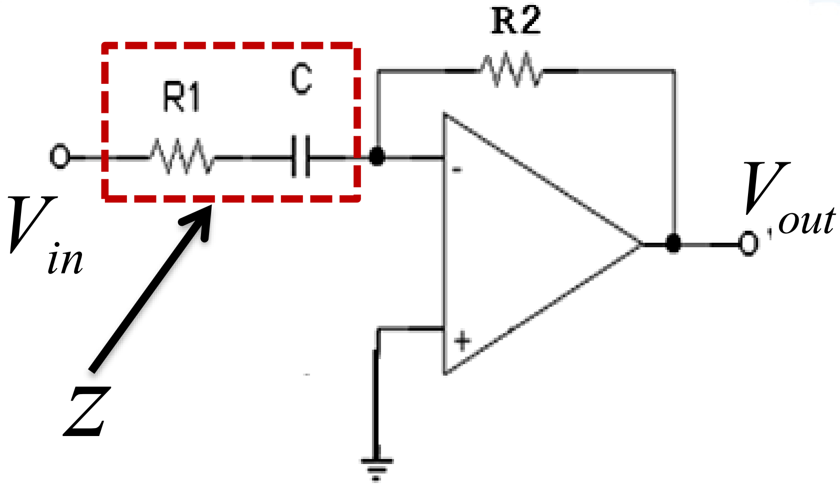
If , $\omega_c = 10\omega$, $20\log\left|\frac{V_{out}}{V_{in}}\right| = 20\log\left|\frac{1}{10_c}\right| = -20(dB)$

Voltage Gain (dB) Vs frequency Graph

$$20\log\left|\frac{V_{out}}{V_{in}}\right| \rightarrow Gain(dB)$$



Transfer function of the Active High pass Filter



$$V_- = \frac{ZV_{out} + R_1 V_{in}}{R_2 + Z}$$

$$V_- = V_+ = 0 \qquad \frac{V_{out}}{V_{in}} = \frac{-R_2}{Z}$$

$$Z = (R_1 + \frac{1}{j\omega C})$$

$$\frac{V_{out}}{V_{in}} = \frac{-jR_2\omega C}{(1 + jR_1\omega C)}$$

Take, $\omega_c = \dfrac{1}{R_1 C}, G_o = \dfrac{R_2}{R_1}$

$$\frac{V_{out}}{V_{in}} = \frac{-G_o(\frac{\omega}{\omega_c})}{(1 + j(\frac{\omega}{\omega_c}))}$$

G_o = DC Gain (High frequency Gain)

ω_c = 3dB Cut off frequency

When, $\omega \ll \omega_c, \quad 1+(\frac{\omega}{\omega_c})^2 \approx 1,$

$$\left|\frac{V_{out}}{V_{in}}\right| \approx G_o(\frac{\omega}{\omega_c}), Gain(dB) = 20\log\left|G_o(\frac{\omega}{\omega_c})\right|$$

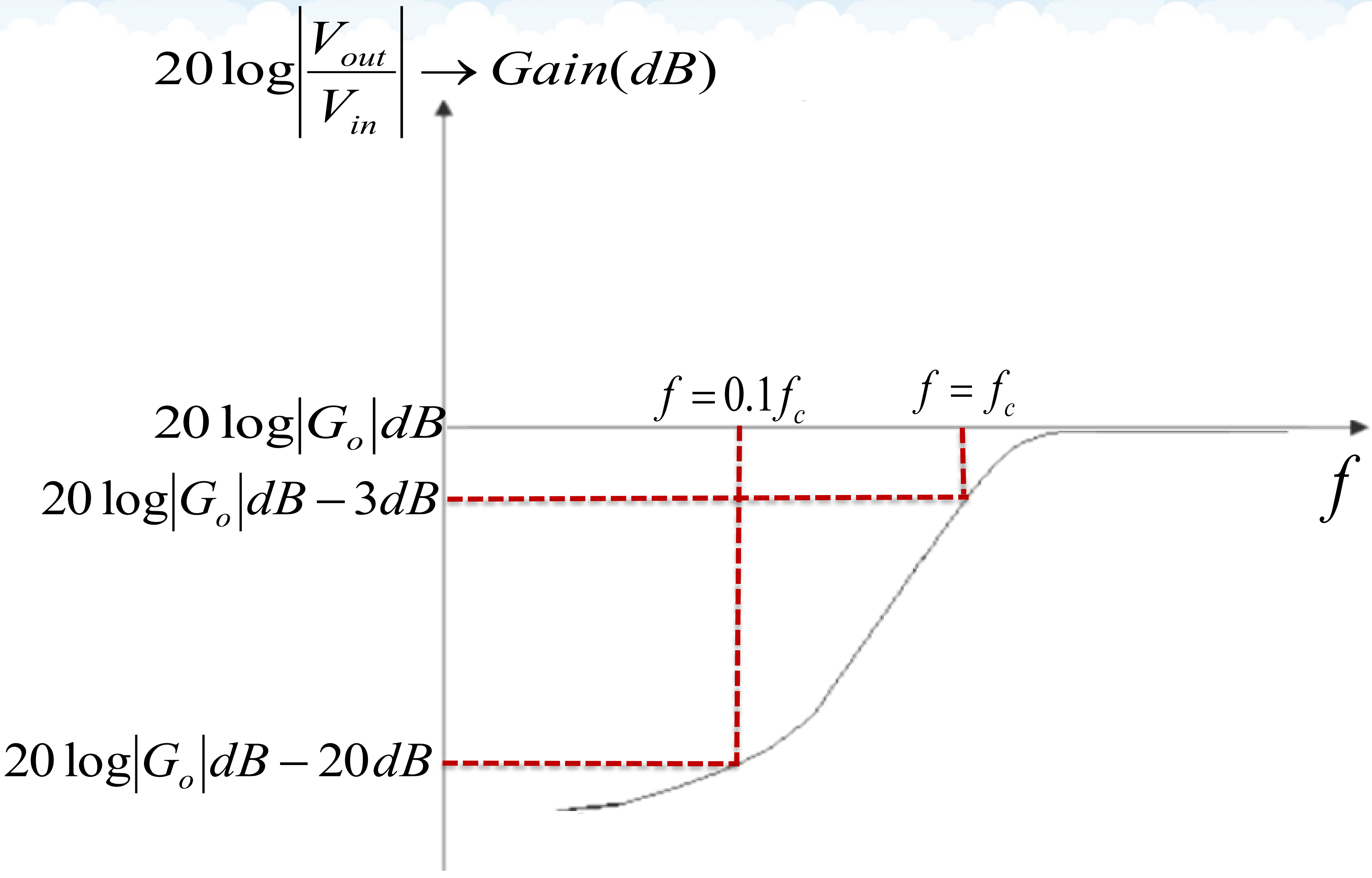
When, $\omega = \omega_c, \quad \left|\frac{V_{out}}{V_{in}}\right| = \frac{G_o}{\sqrt{2}}$ for, $f = f_c$

$$Gain(dB) = 20\log\left|\frac{G_o}{\sqrt{2}}\right| = 20\log|G_o|(dB) - 3dB$$

When, $\omega \gg \omega_c, \quad 1+\frac{\omega}{\omega_c} \approx \frac{\omega}{\omega_c}$

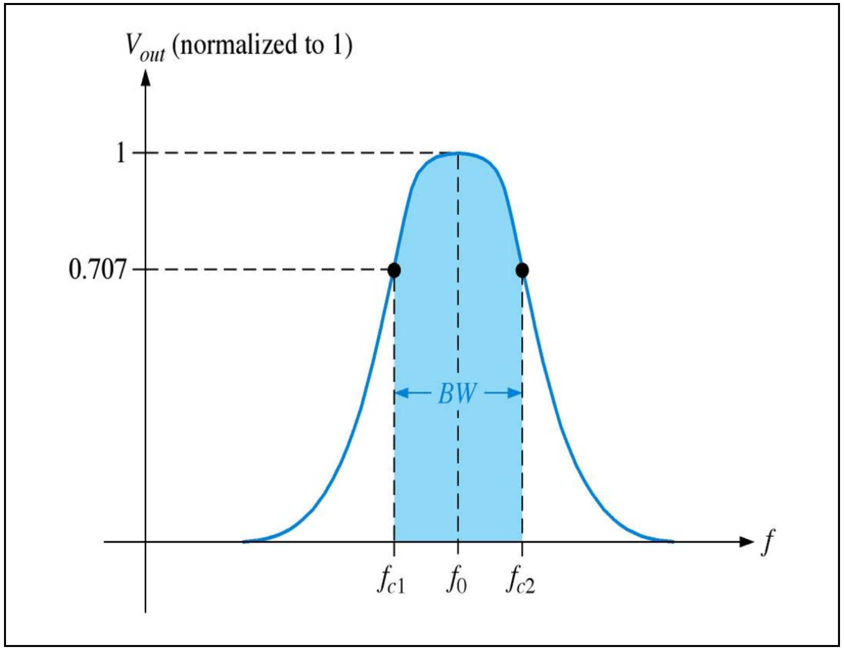
Therefore, $$\left|\frac{V_{out}}{V_{in}}\right| = G_o, Gain(dB) = 20\log\left|\frac{R_2}{R_1}\right|$$

Voltage Gain (dB) Vs frequency Graph

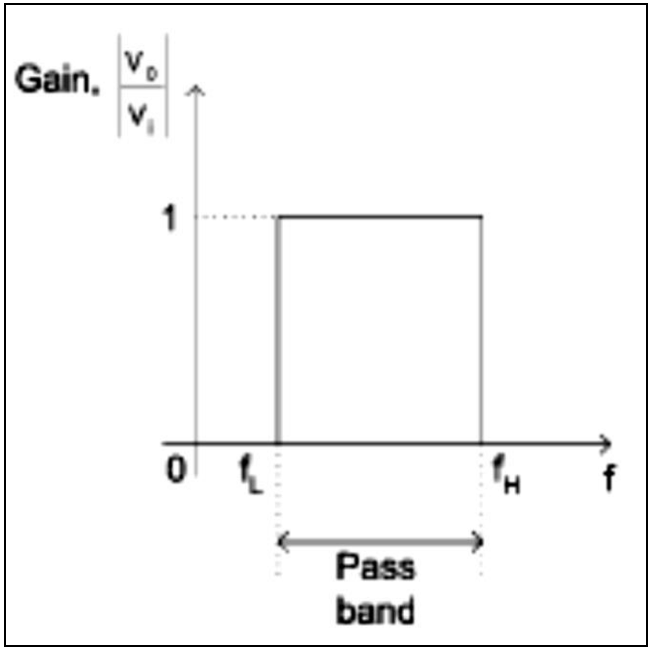


Band pass Filter Responses

- A band-pass filter passes all signals lying within a band between a lower-frequency limit and upper-frequency limit and essentially rejects all other frequencies that are outside this specified band.

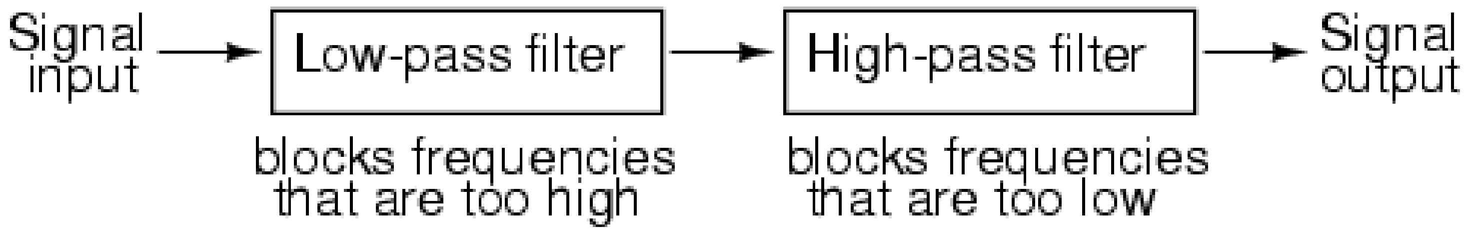


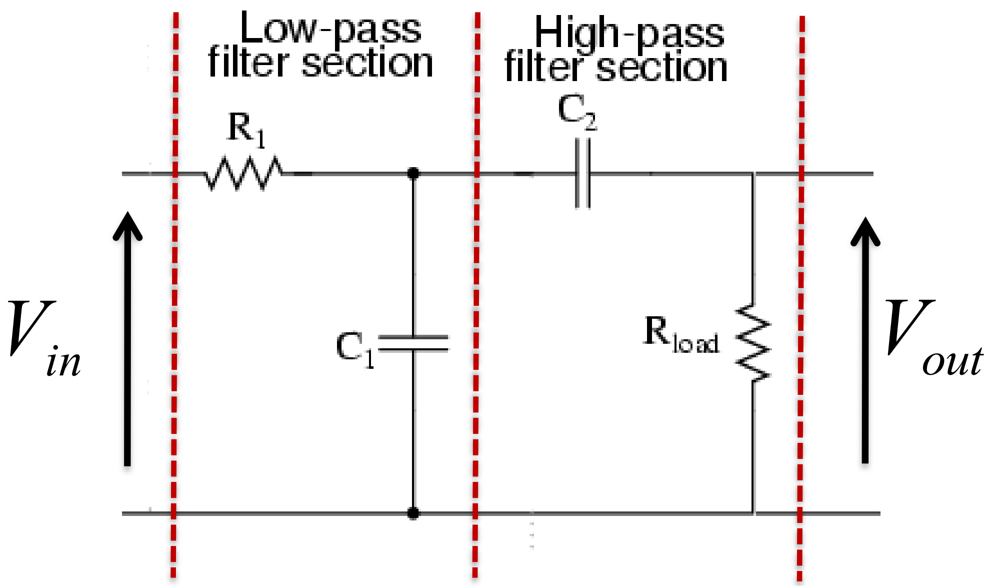
Actual response

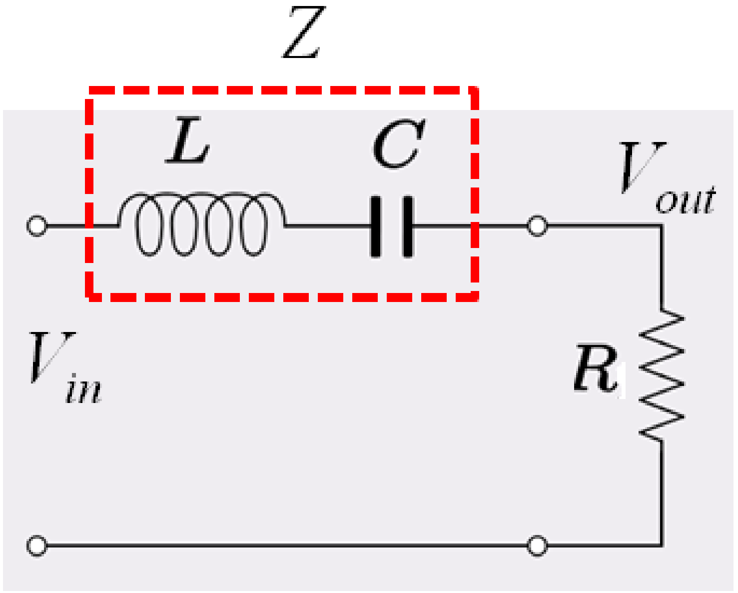


Ideal response

Block Diagram of Band pass Filter







RC Band pass filter

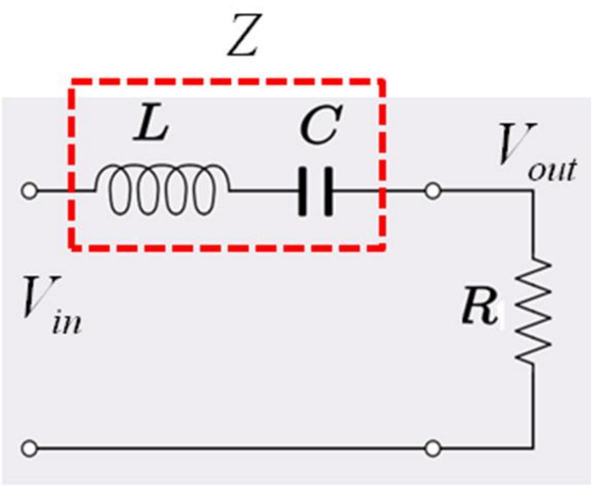
RLC Band pass filter

Transfer function for the RLC Band pass filter

$$V_{out} = \frac{RV_{in}}{(R+Z)} \qquad Z = (j\omega L + \frac{1}{j\omega C}) = j(\omega L - \frac{1}{\omega C})$$

$$\frac{V_{out}}{V_{in}} = \frac{R}{\left(R + j(\omega L - \frac{1}{\omega C})\right)} = \frac{1}{\left(1 + j(\frac{\omega L}{R} - \frac{1}{\omega RC})\right)}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{\left(1 + (\frac{\omega L}{R} - \frac{1}{\omega RC})^2\right)}}$$



When, $\omega = \omega_0, \ (\frac{\omega_0 L}{R} - \frac{1}{\omega_0 RC})^2 = 0 \rightarrow \left|\frac{V_{out}}{V_{in}}\right|_{max} = 1$

$$\left(\frac{\omega_0 L}{R} - \frac{1}{\omega_0 RC}\right)^2 = 0, \qquad \omega_0{}^2 LC = 1, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$f_0 =$ Center frequency

For 3dB cut off frequency, $\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}}$

Therefore, $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\left(1 + \left(\frac{\omega_c L}{R} - \frac{1}{\omega_c RC}\right)^2\right)}}$

$$\left(\frac{\omega_c L}{R} - \frac{1}{\omega_c RC}\right) = \pm 1$$

By solving this equation can obtain upper and lower 3dB cut off frequencies

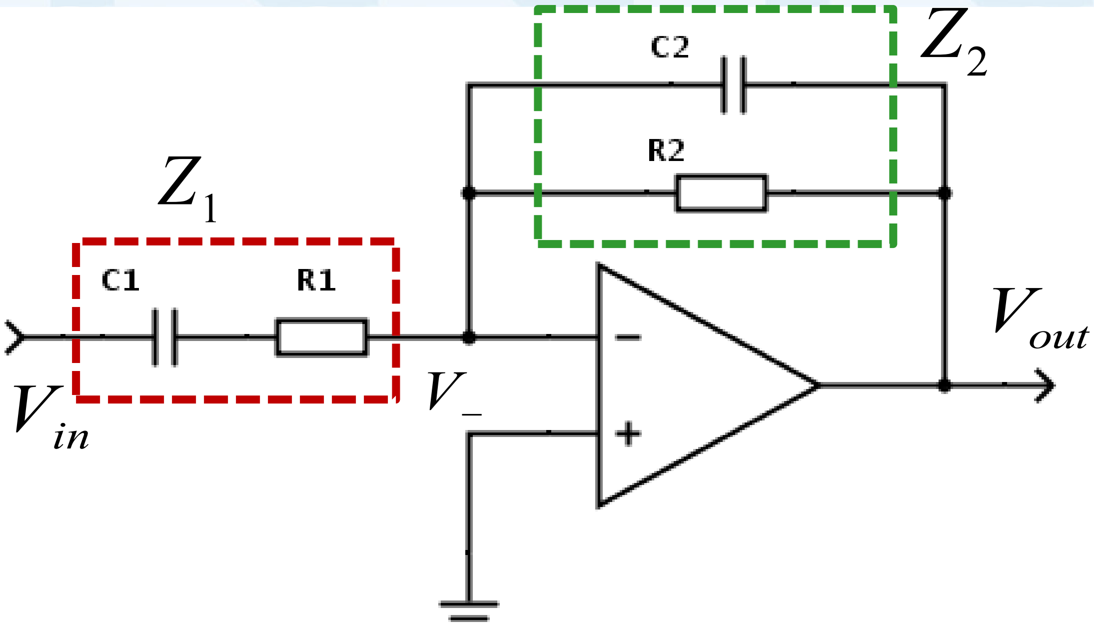
$$\omega_l = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$ (Lower cut off frequency)

$$\omega_u = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$ (Upper cut off frequency)

$$\omega_0 = \sqrt{\omega_l \times \omega_u} = \frac{1}{\sqrt{LC}}$$ (Center frequency)

$$BW = f_u - f_l$$ (Band Width)

Band pass Active filter



$$Z_1 = R_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = \frac{R_2}{(1 + j\omega R_2 C_2)}$$

$$\frac{V_{out}}{V_{in}} = \frac{-Z_2}{Z_1} = \frac{j\omega C_1 R_2}{(1 + j\omega R_2 C_2)(1 + j\omega R_1 C_1)}$$

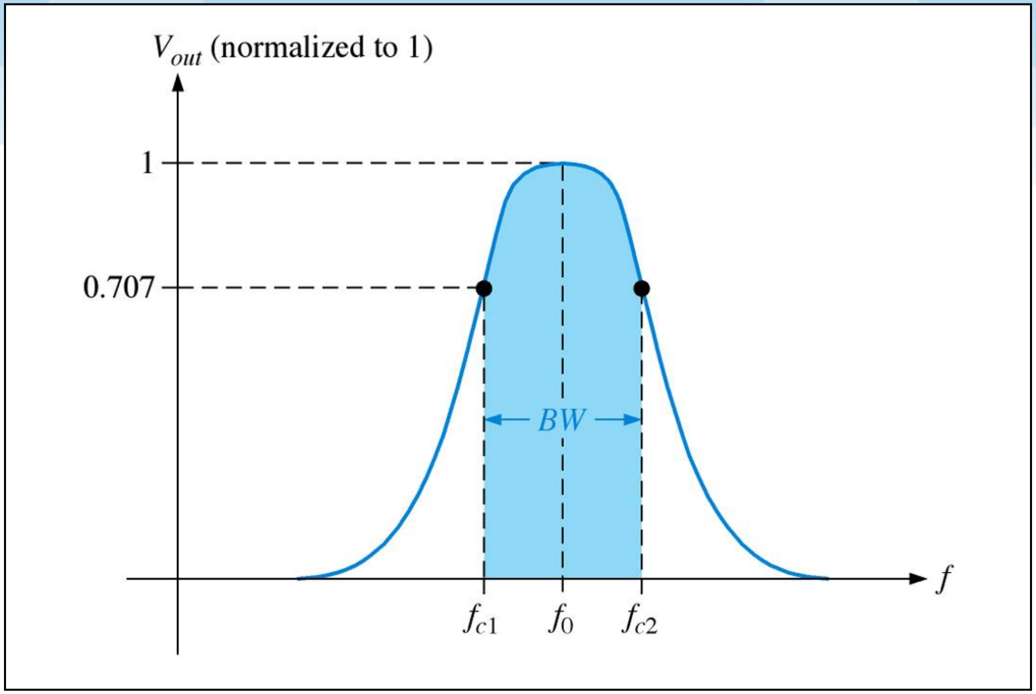
Take, $\omega_{c1} = \dfrac{1}{R_1 C_1}, \quad \omega_{c2} = \dfrac{1}{R_2 C_2}, \quad G_o = \dfrac{-R_2}{R_1}$

Transfer function

$$\frac{V_{out}}{V_{in}} = \frac{-G_o j(\frac{\omega}{\omega_{c1}})}{\left(1 + j(\frac{\omega}{\omega_{c1}})\right)\left(1 + j(\frac{\omega}{\omega_{c2}})\right)}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{G_o(\frac{\omega}{\omega_{c1}})}{\sqrt{\left(1 - \frac{\omega^2}{\omega_{c1}\omega_{c1}})\right)^2 + \left((\frac{\omega}{\omega_{c1}}) + (\frac{\omega}{\omega_{c2}})\right)^2}}$$

$$f_0 = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$ **(Center frequency)**



$$BW = f_{c2} - f_{c1}$$

$$f_o = \sqrt{f_{c1} f_{c2}}$$

- The bandwidth (BW) is defined as the difference between the upper critical frequency (f_{c2}) and the lower critical frequency (f_{c1}).
- The frequency about which the pass band is centered is called the ***center frequency***, f_o and defined as the geometric mean of the critical frequencies.

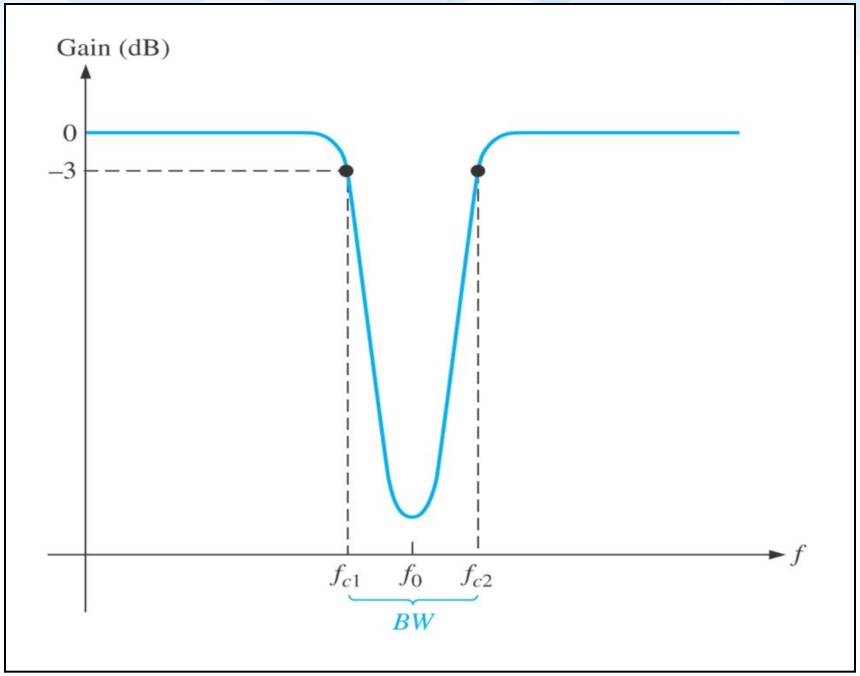
- The ***quality factor (Q)*** of a band-pass filter is the ratio of the center frequency to the bandwidth.

$$Q = \frac{f_o}{BW}$$

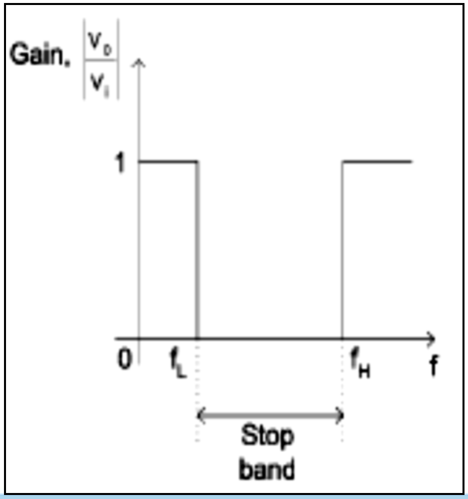
- The higher value of Q, the narrower the bandwidth and the better the selectivity for a given value of f_o.
- (Q>10) as a narrow-band or (Q<10) as a wide-band
- The quality factor (Q) can also be expressed in terms of the damping factor (DF) of the filter as :

$$Q = \frac{1}{DF}$$

Band Stop Filter Responses



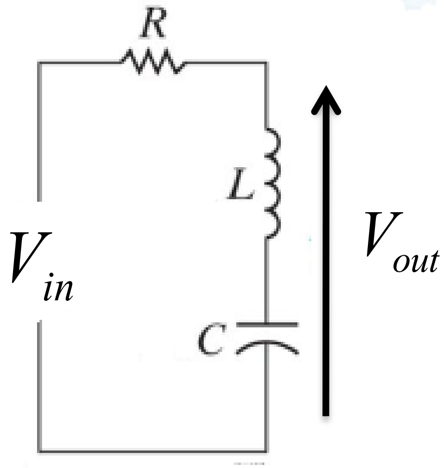
Actual response



Ideal response

- **Band-stop filter** is a filter which its operation is **opposite** to that of the band-pass filter because the frequencies **within** the bandwidth are **rejected**, and the frequencies above f_{c1} and f_{c2} are **passed**.
- For the band-stop filter, the **bandwidth** is a band of frequencies between the 3 dB points, just as in the case of the band-pass filter response.

Transfer function for the RLC Band Reject filter



$$\frac{V_{out}}{V_{in}} = \frac{j(\omega L - \frac{1}{\omega C})}{\left(R + j(\omega L - \frac{1}{\omega C})\right)}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{(\omega L - \frac{1}{\omega C})}{\sqrt{\left(R^2 + (\omega L - \frac{1}{\omega C})^2\right)}}$$

When, $\omega = \omega_0, \quad (\omega_0 L - \frac{1}{\omega_0 C})^2 = 0 \rightarrow \left|\frac{V_{out}}{V_{in}}\right|_{\min} = 0$

$$\omega_0^{\,2} LC = 1, \;\; \omega_0 = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$f_0 =$ Center frequency

For 3dB cut off frequency, $\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}}$

Therefore,

$$\frac{1}{\sqrt{2}} = \frac{(\omega L - \frac{1}{\omega C})}{\sqrt{\left(R^2 + (\omega L - \frac{1}{\omega C})^2\right)}}$$

$$R^2 = (\omega L - \frac{1}{\omega C})^2 \rightarrow (\omega L - \frac{1}{\omega C})(\frac{1}{R}) = \pm 1$$

By solving this equation can obtain upper and lower 3dB cut off frequencies

$$\omega_l = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

(Lower cut off frequency)

$$\omega_u = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

(Upper cut off frequency)

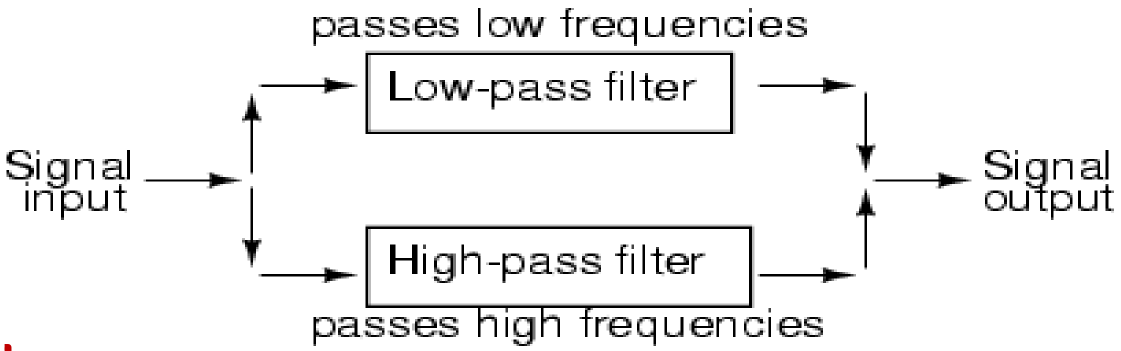
$$\omega_0 = \sqrt{\omega_l \times \omega_u} = \frac{1}{\sqrt{LC}}$$

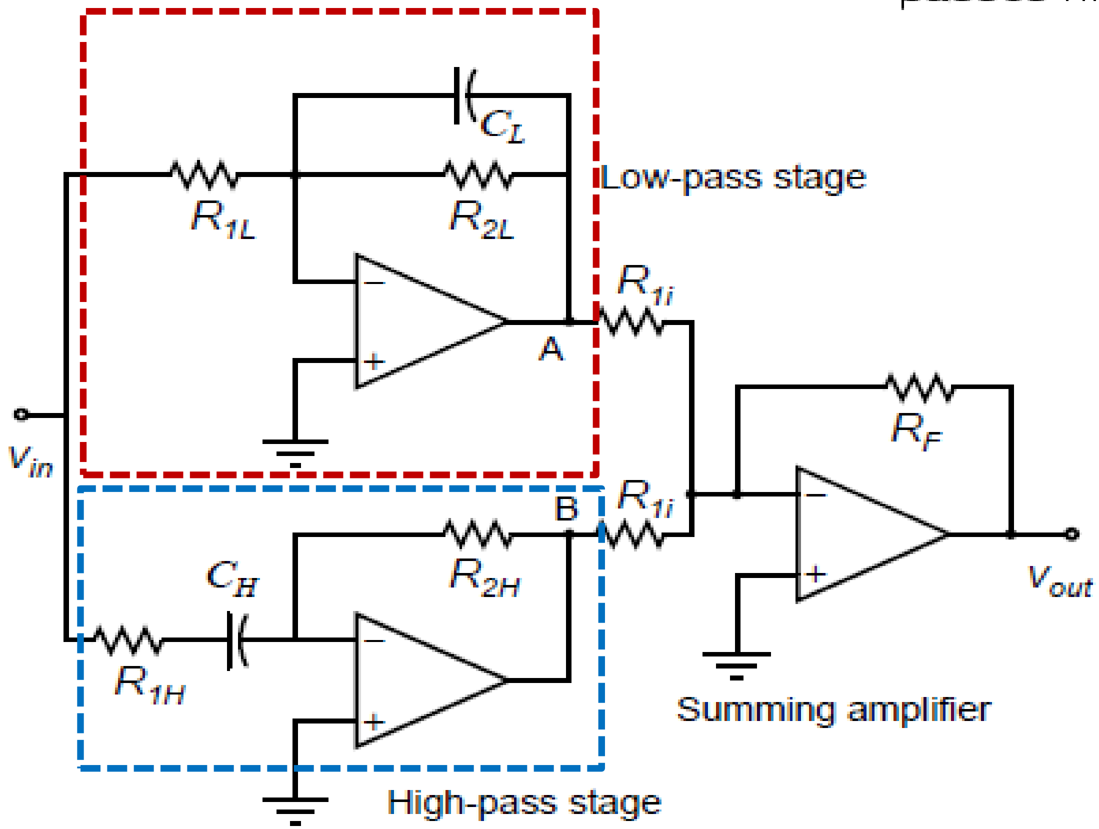
(Center frequency)

$$BW = f_u - f_l$$

(Band Width)

Band Stop Active Filter



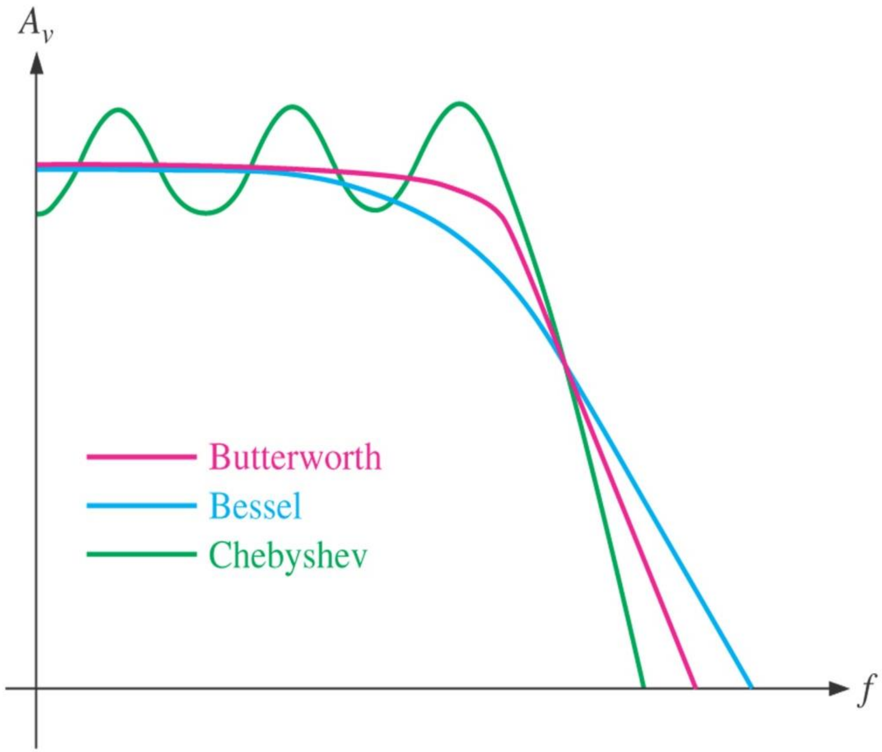


Filter Response Characteristics

- **There are 3 characteristics of filter response**

1. Butterworth characteristic
2. Chebyshev characteristic
3. Bessel characteristic.

- Each of the characteristics is identified by the shape of the response curve



Butterworth Characteristic

- Very flat amplitude, $A_{v(dB)}$, response in the passband.

- Role-off rate is *20dB*/decade/pole.

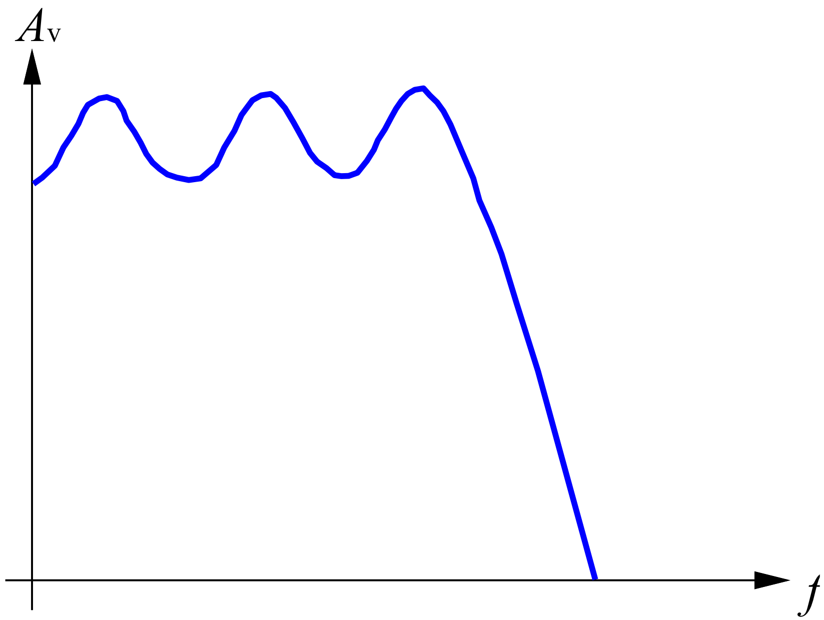
- Phase response is not linear.

- Used when all frequencies in the passband must have the same gain.

- Often referred to as a *maximally flat response*.

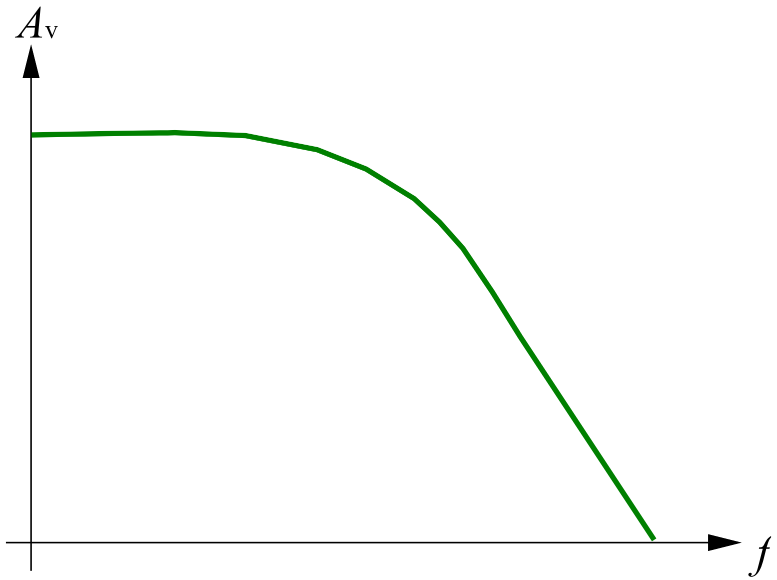
Chebyshev Characteristic

- Overshoot or ripples in the passband.
- Role-off rate greater than *20dB*/decade/pole.
- Phase response is not linear - worse than Butterworth.
- Used when a rapid roll-off is required.



Bessel Characteristic

- Flat response in the pass band.
- Role-off rate less than *20dB*/decade/pole.
- Phase response is linear.

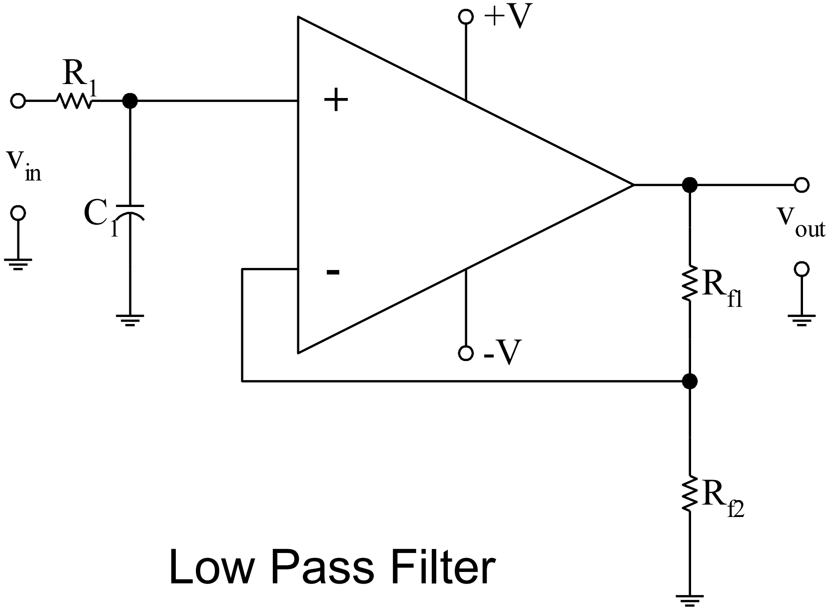


- Used for filtering pulse waveforms without distorting the shape of the waveform.

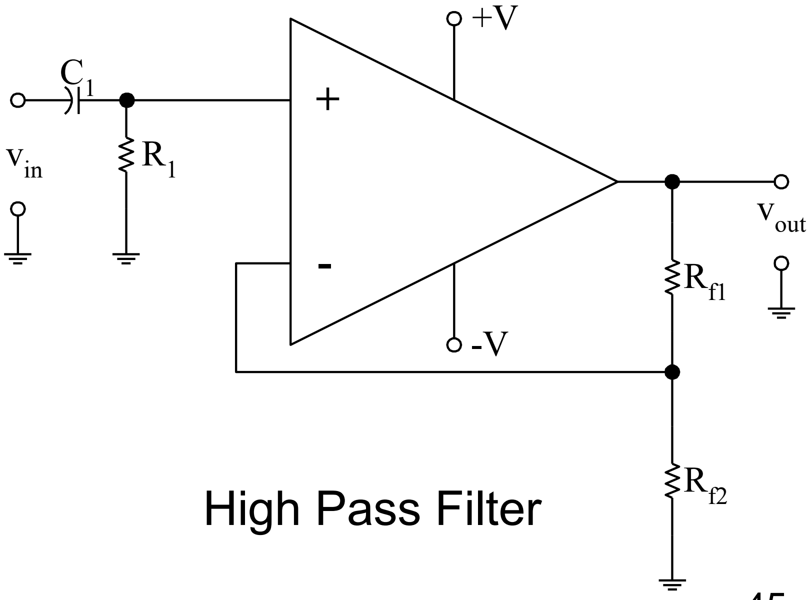
Pole of the Filter

- A ***pole*** is nothing more than an *RC* circuit
- **n-pole** filter → contains **n-RC** circuit.

Eg: Single-Pole Low/High-Pass Filter



Low Pass Filter



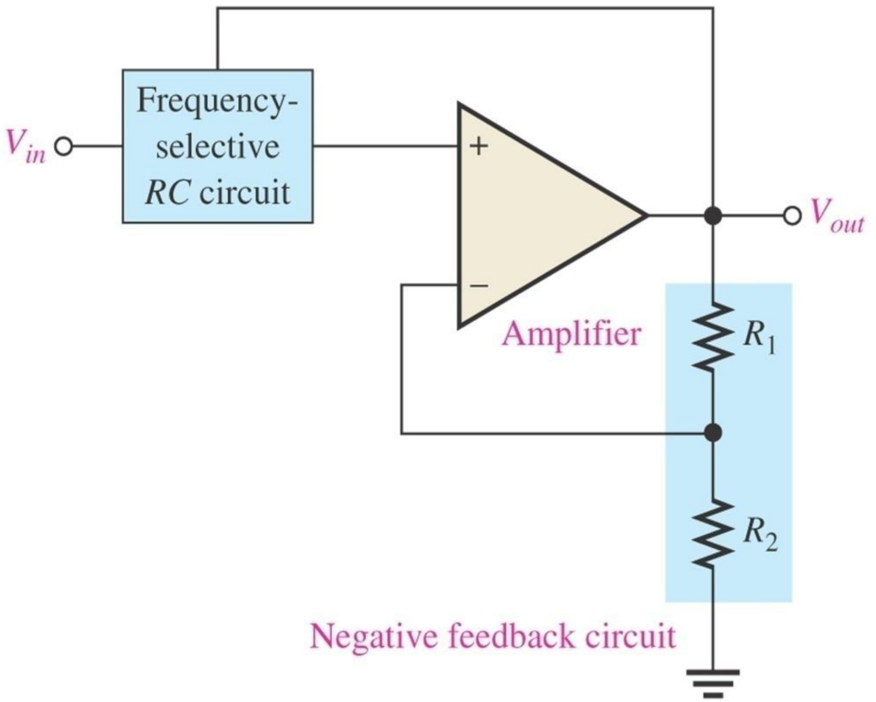
High Pass Filter

Damping Factor

- The **damping factor (DF)** of an active filter determines which response characteristic the filter exhibits.

- This active filter consists of **an amplifier, a negative feedback circuit** and **RC circuit**.
- The amplifier and feedback are connected in a **non-inverting configuration**.
- DF is determined by the negative feedback and defined as :

$$DF = 2 - \frac{R_1}{R_2}$$



General diagram of active filter

- The value of DF required to produce a desired response characteristics depends on **order** (number of poles) of the filter.
- A pole (single pole) is simply **one resistor** and **one capacitor**.
- The **more poles** filter has, the faster its roll-off rate