PHY 359 2.0 / ASP 487 2.0
Telecommunication

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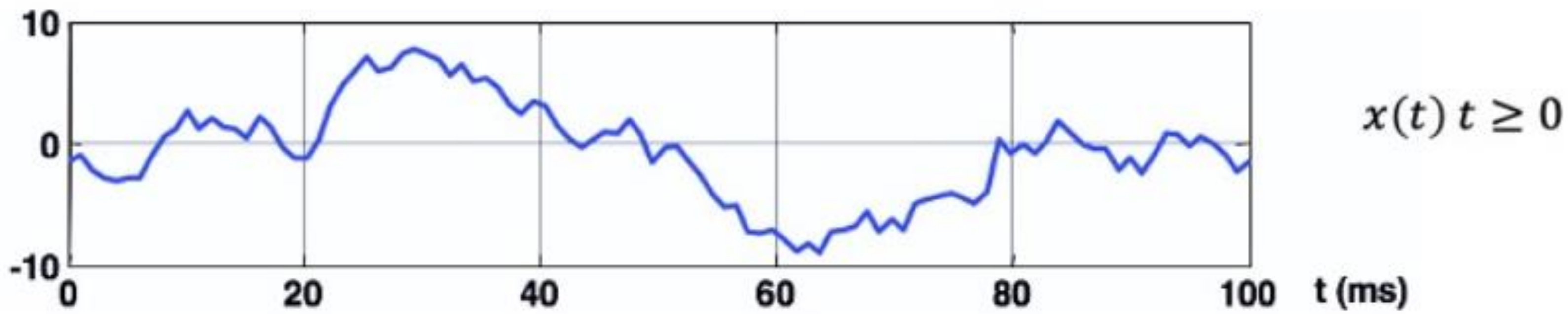
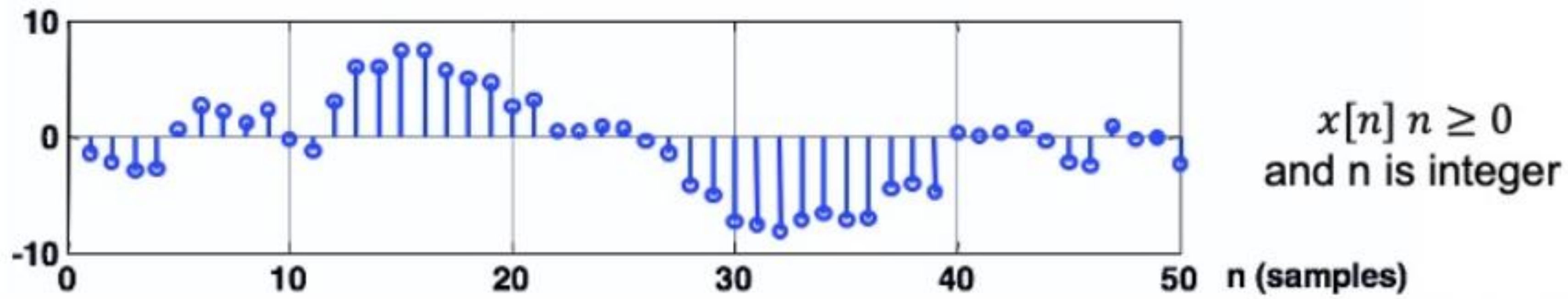
Signals and Signal Space

Continuous and discrete-time signals

Discrete time signals

- A continuous-time signal can be converted to a discrete-time signal by sampling it at a certain frequency.
- The **sampling theorem** determines the necessary sampling frequency.

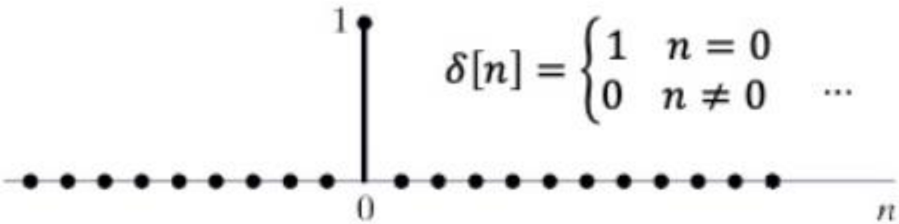




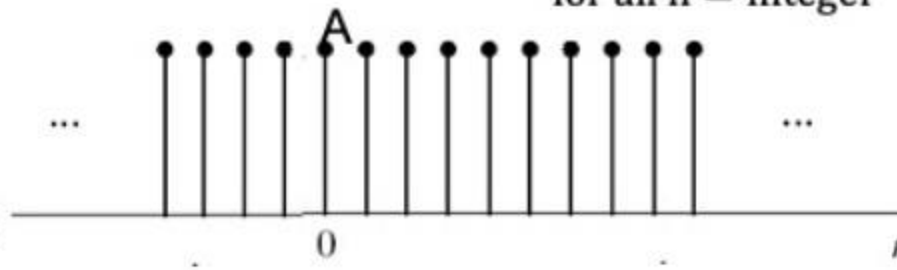
Basic Discrete signals

Impulse signal

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

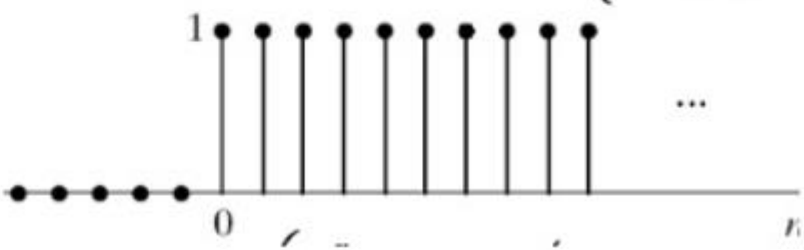


$$x[n] = A,$$
for all n = integer



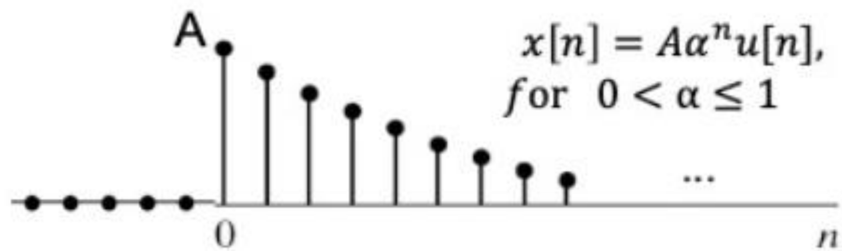
Unit step signal

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

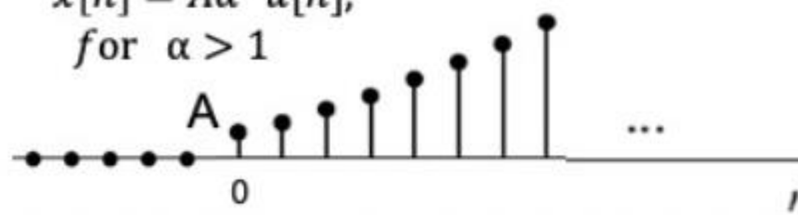


Causal exponential signals

$$x[n] = A\alpha^n u[n],$$
$for \quad 0 < \alpha \leq 1$



$$x[n] = A\alpha^n u[n],$$
$for \quad \alpha > 1$



Basic Discrete signals

❖ Unit impulse

Represented by the discrete **delta function**. When n = 0, $\delta(0) = 1$, otherwise $\delta(n) = 0$.

❖ DC voltage

The DC voltage is sampled, and the signal is represented as $x[n] = A$, where A is the voltage value. This is therefore a sequence of samples: {A, A, A, A, …}.

❖ Unit step signal

Represented by the sum of lots of **unit impulses**, at each sampling points for $n \geq 0$. For $n < 0$, $x[n] = 0$.

Basic Discrete signals

❖ Exponential signal

The signal is exponential rise or fall depending on the value of the constant a. If $0 < \alpha \leq 1$, x[n] is an exponent decaying signal. If $\alpha > 1$, x[n] is an exponent growing signal.

x(t) - represents a signal in continuous time,

x[n] - indicates a signal in sequence of numbers.

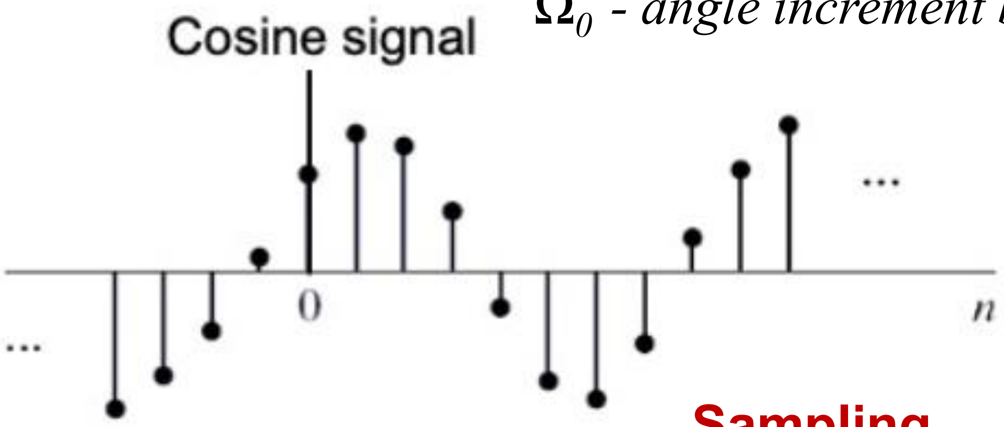
n - sample number (from 0)

x[n] - magnitude of sample n

Discrete Sinusoidal signals

$x[n] = A\cos(\Omega_0 n + \phi)$ *for n = integer*

Ω_0 *- angle increment between samples in radian per sample*



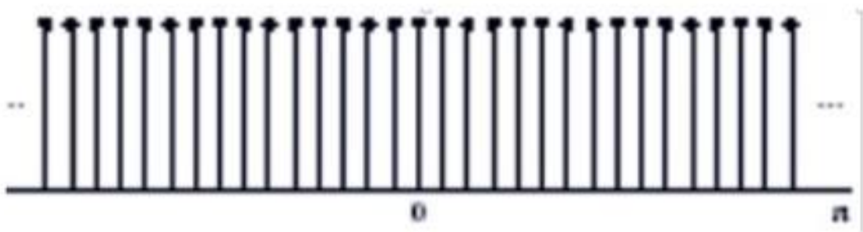
$X(t) = A\cos(\omega_0 t + \phi)$ **Sampling** → $x[n] = A\cos(\Omega_0 n + \phi)$

- For continuous-time signals, **"*t*"** is the continuous parameter. Therefore, there are an infinite number of values for ***x(t)***. In contrast, ***x[n]*** is only defined for integer values of ***n***.
- In continuous time, the **angular frequency** ω_0 which has a unit of **rad/sec**. In discrete time we use the **angular phase** increment between samples Ω_0, which has a unit of **rad/sample**. (Ω_0 the discrete frequency of the signal.)
- The relationship between ω_0 and Ω_0 is: $\Omega_0 = \omega_0 \times T_s$, where T_s is the sampling period = $1/f_s$.

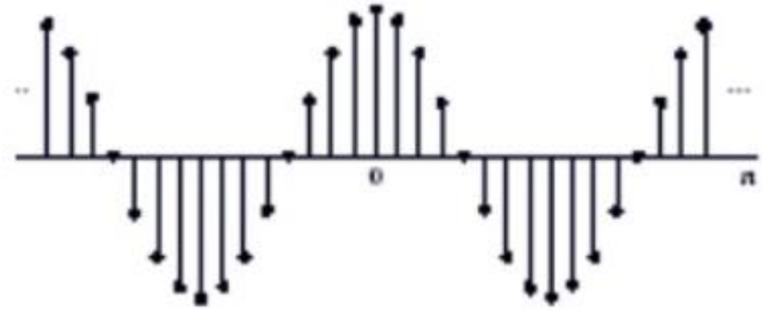
Discrete Sinusoidal signals

$x[n] = A\cos(\Omega_0 n + \phi)$ for n = integer

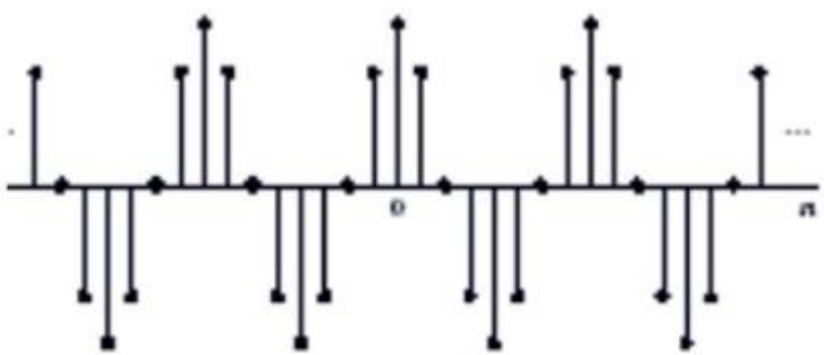
Ω_0 is in rad/sample



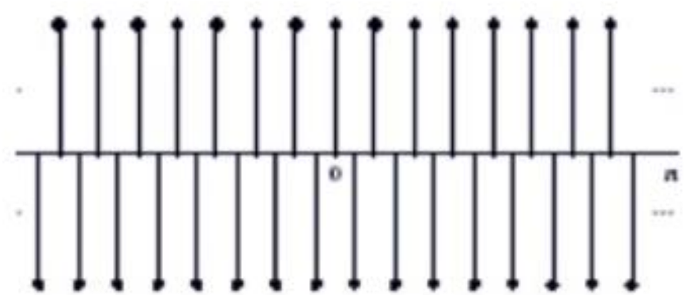
(a) When $\Omega_0 = 0$ *or* $x[n]$ is DC



(b) When $\Omega_0 = \frac{\pi}{8}$
i.e. 16 sample/cycle



(c) When $\Omega_0 = \frac{\pi}{4}$
i.e. 8 sample/cycle



(d) When $\Omega_0 = \pi$
i.e. 2 sample/cycle

Operations on Discrete signals

The following four fundamental operations can be applied to discrete signals as the sequence of sample values.

1. Sum of two signals: $s[n] = x[n] + y[n]$
2. Product of two signals: $p[n] = x[n] \, . \, y[n]$
3. Amplification of a signals: $y[n] = \alpha \, . \, x[n]$
4. Delaying a signal by k samples: $y[n] = x[n - k]$

- The most important operation, **Y[n] = x[n - k]** (Delay operator)

  Here y[n] is the x[n] sequence delayed by k sample periods.

Introduction

Analogue to Digital Converter (ADC)

An ADC Circuit produces a series of binary numbers which represents the value of an analogue input signal at regular intervals of time.

ADC Parameters

Range

The range of an ADC is the range of voltages the ADC can convert to a Digital number.

A typical ADC may have ranges,

Ex: 0V – 5V, 0V – 10V, -5V – 5V

ADC Parameters

Resolution

The Number of bits in the converter is capable of handling.
An **‘n’** bit ADC with a **0V to ‘x’** Volt range divides the range in to $\mathbf{2^n}$ steps. Therefore,

The lowest detectable voltage variation (Resolution) = $\frac{\mathbf{x}}{\mathbf{2^n}}$

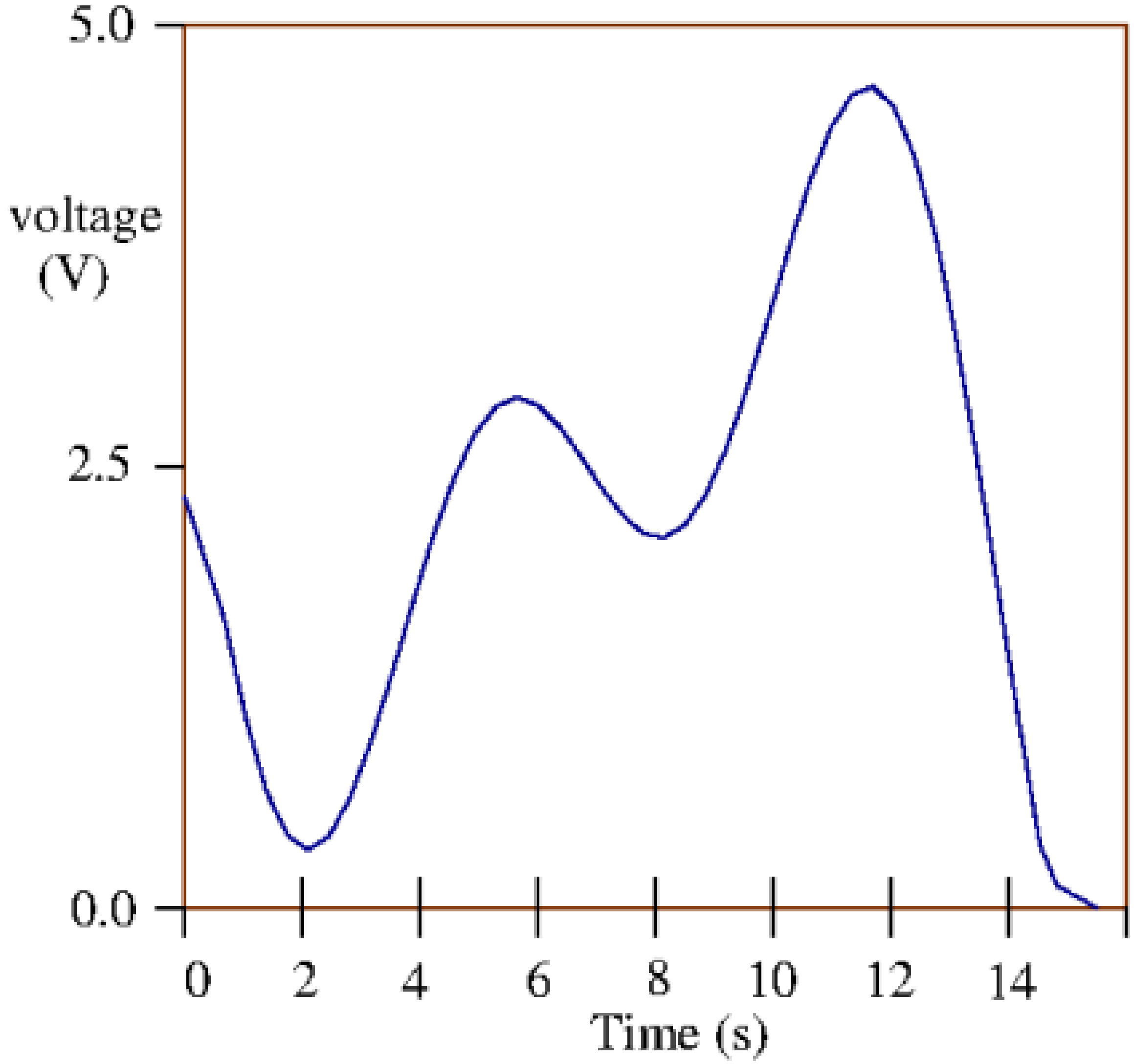
Ex: An 10bit ADC with a 5V range can detect a variation of $\frac{\mathbf{5}}{\mathbf{2^{10}}}$ **= $\mathbf{1024}$ = 4.88mV**

ADC Parameters

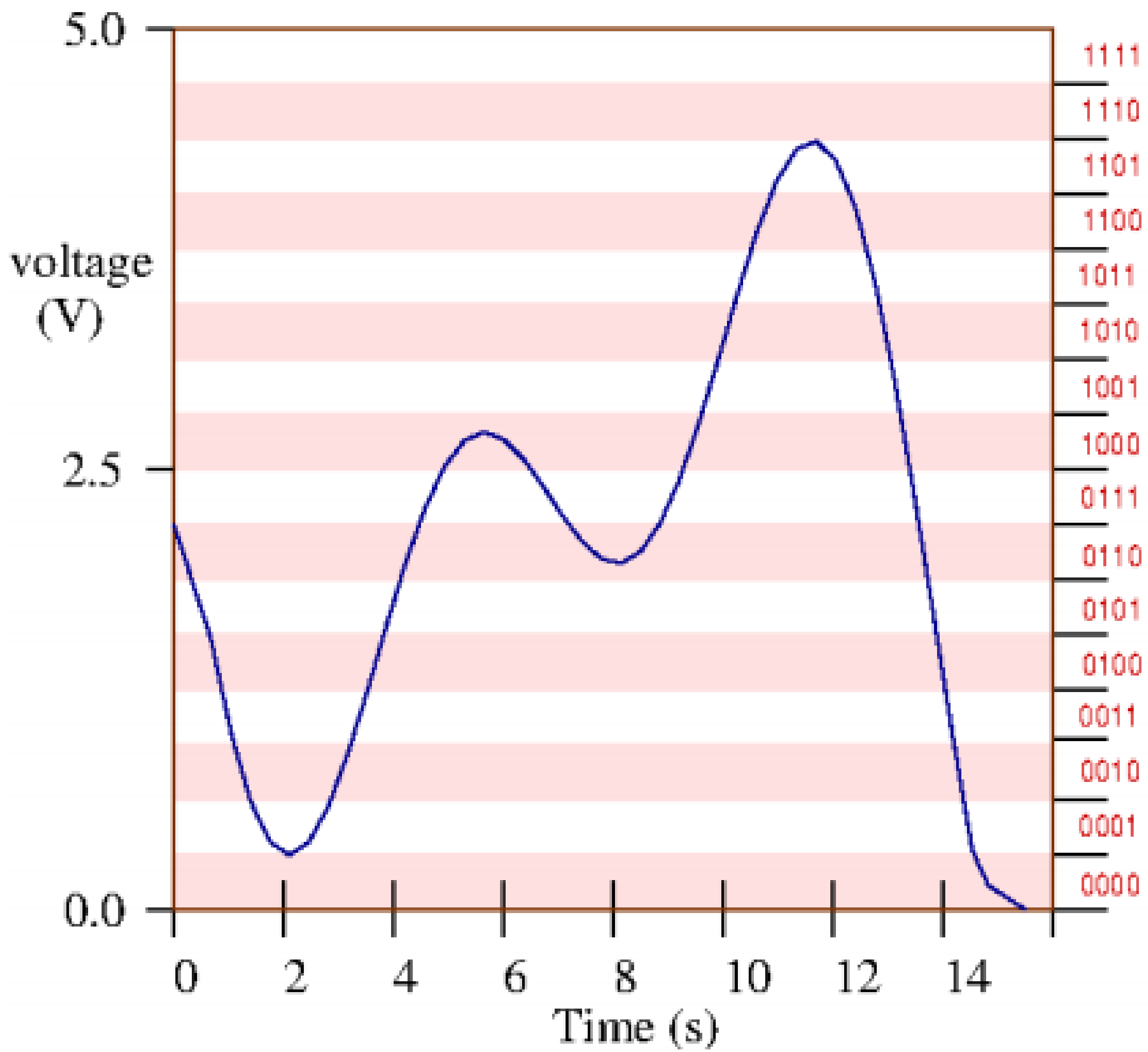
Conversion time

- The speed of an ADC is normally specified as the time taken for a single conversion.
- Conversion of typical ADCs range for ms to ns,
- Fast converters are required for digitalizing fast varying signal (ex: sound).
  (Ex: For digitalizing high quality music a 12m or 16 bit ADC with a speed of few μs is required.)
- For human voice an 8bit ADC with 100μs speed is sufficient.
- The conversion time required for a given application can be determine using **sampling theorem**.

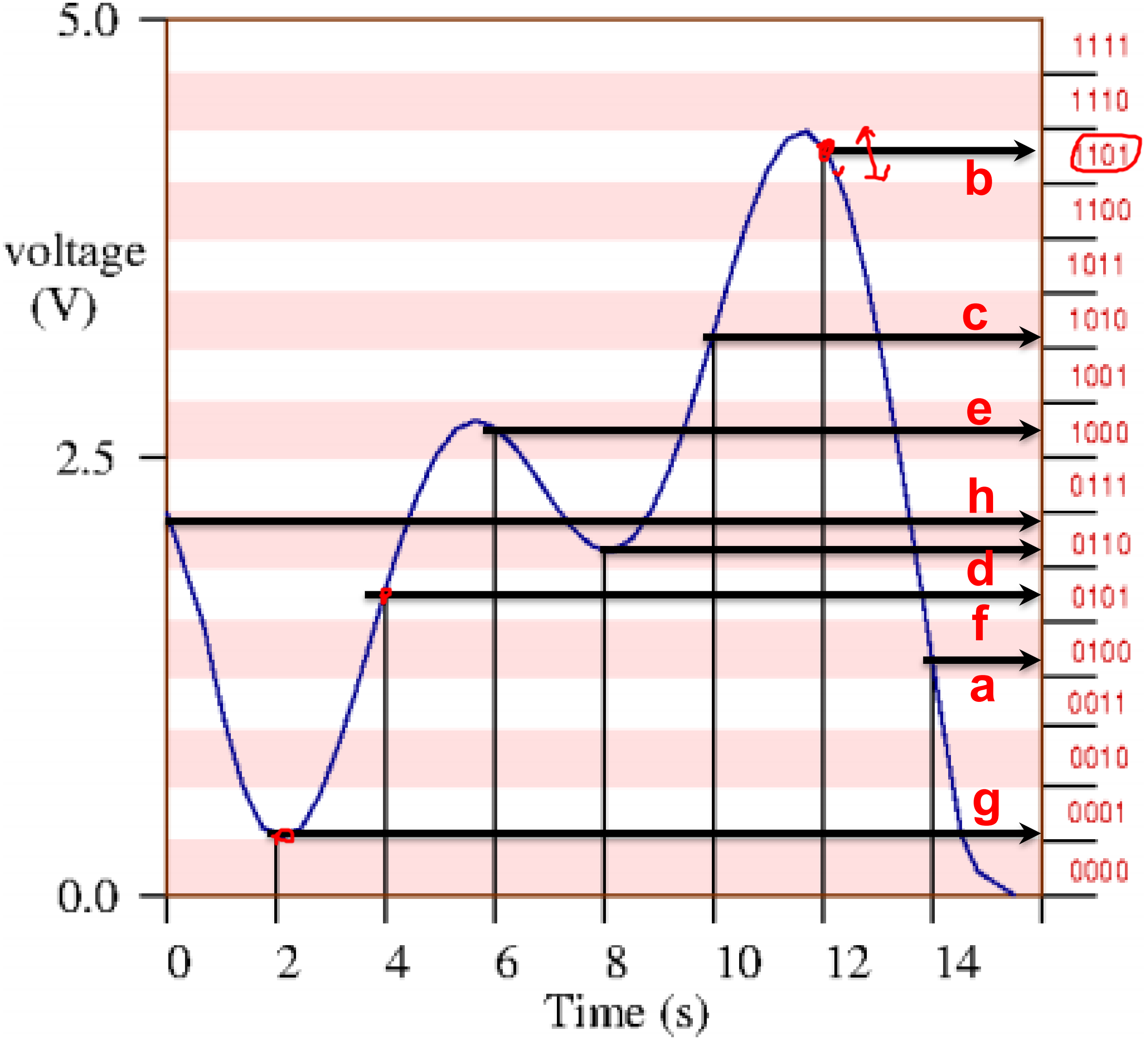
Signal Sampling and Quantization



Sampling and Quantization

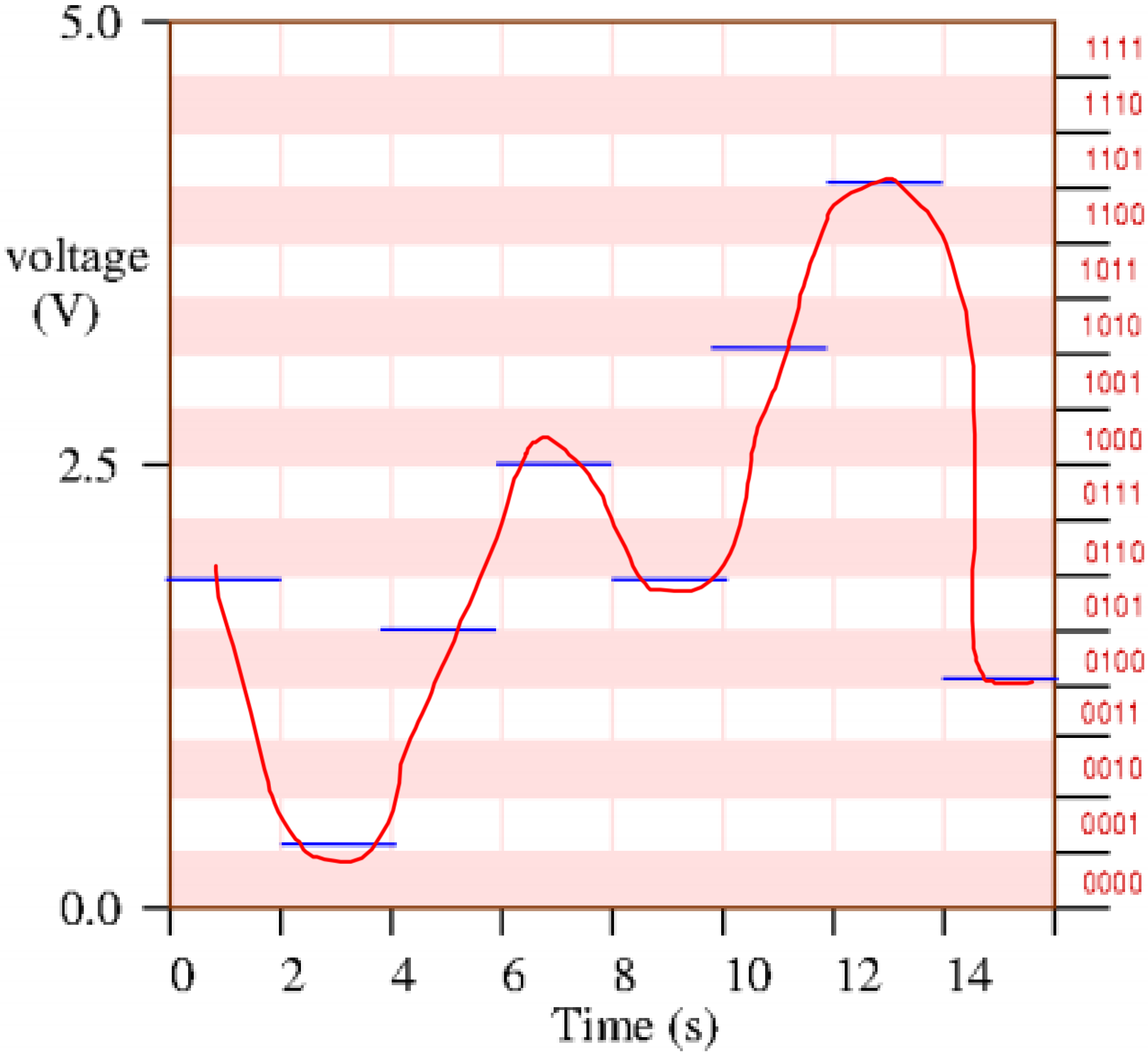


Sampling and Quantization - ADC



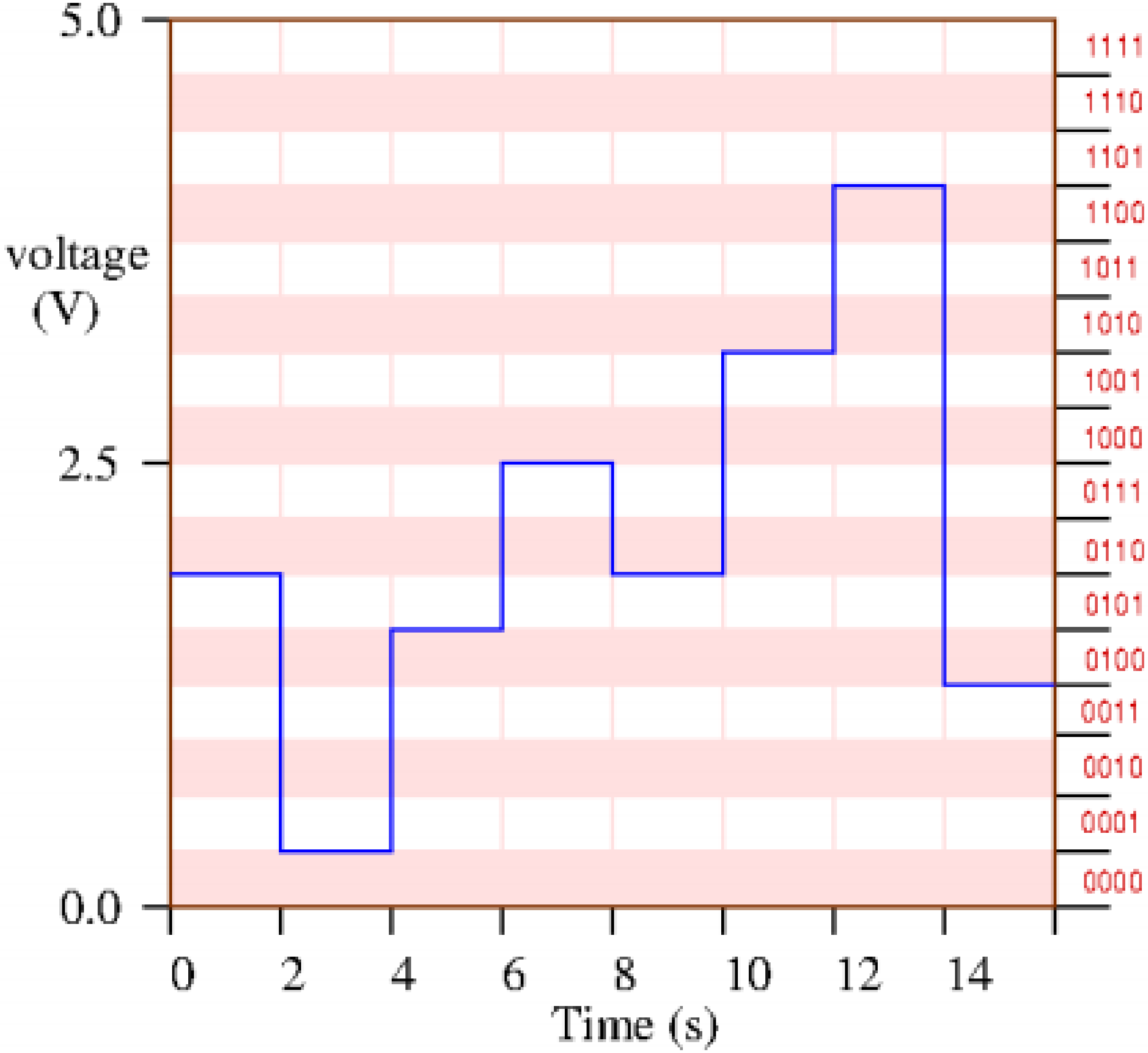
0110 - h
0001 - g
0101 - f
1000 - e
0110 - d
1010 - c
1101 - b
0100 - a

Sampling and Quantization - DAC



0110
0001
0101
1000
0110
1010
1101
0100

Digital to Analogue Conversion



0110
0001
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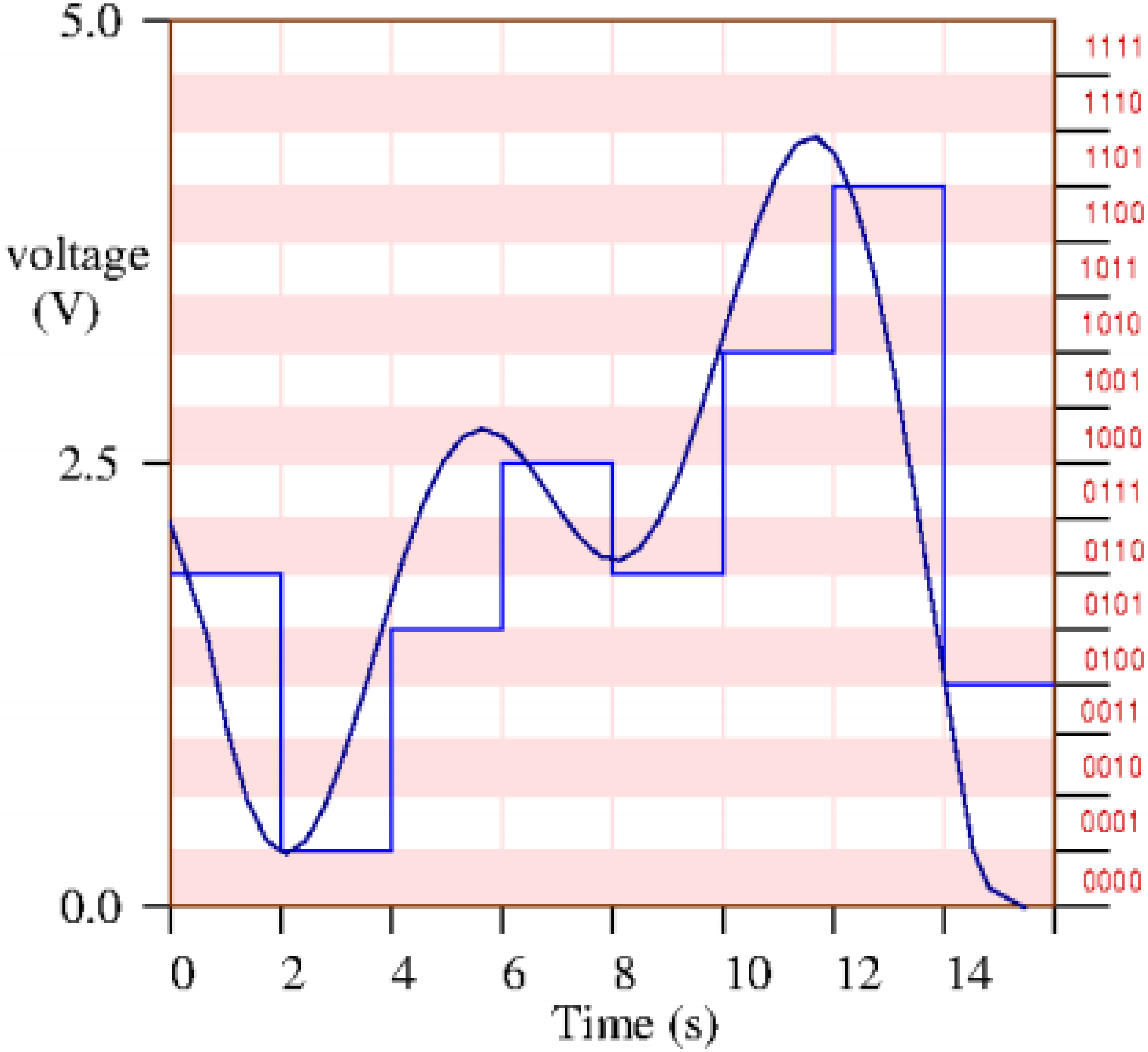
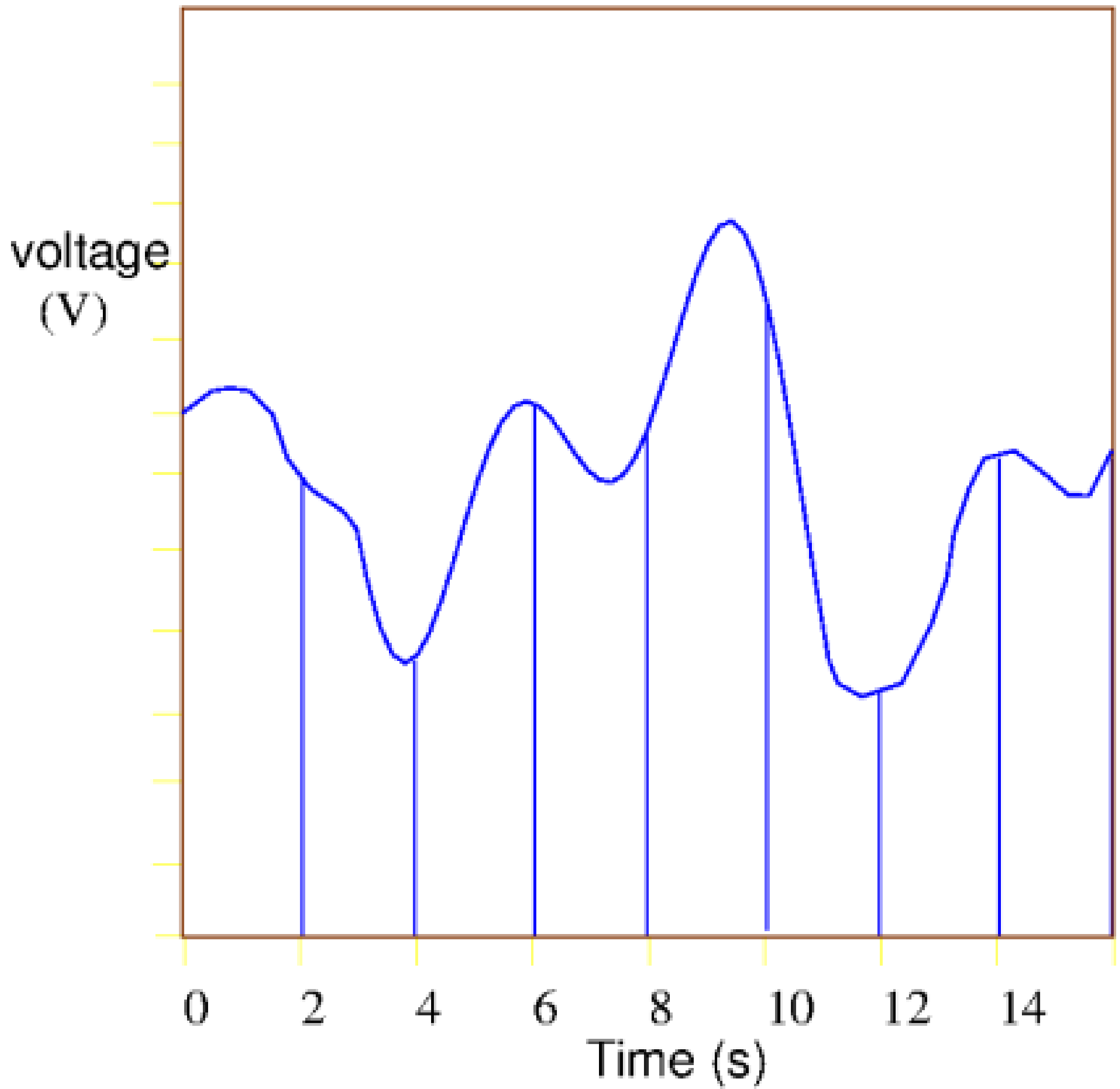
1010
1101
0100

Digital to Analogue Conversion

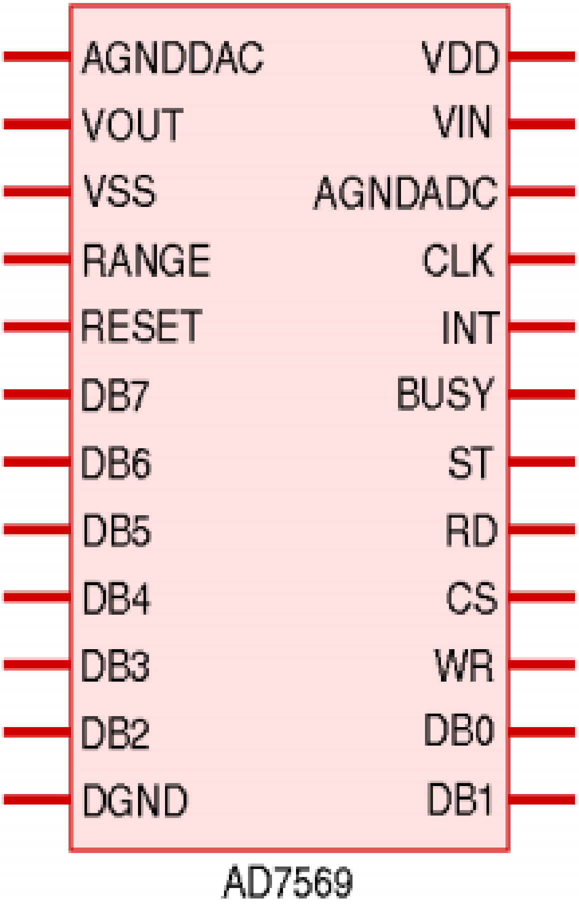


0110
0001
0101
1000
0110
1010
1101
0100

Sampling Rate



Example - AD7569



Includes an ADC and a DAC

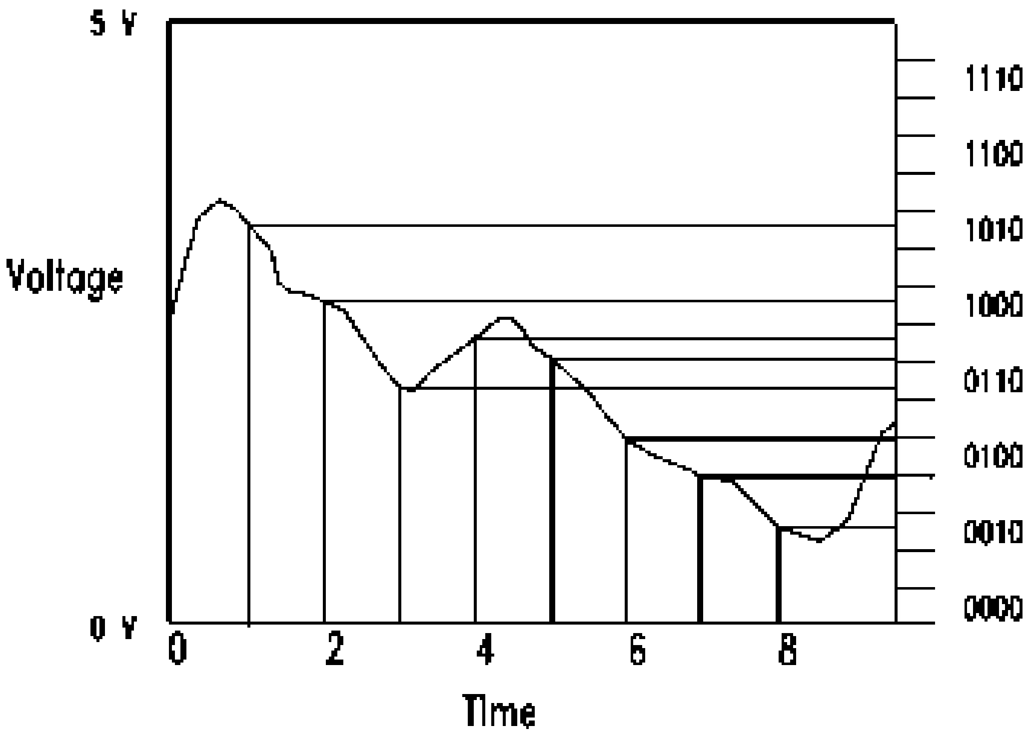
Ranges: 0 to 5V, 0 to 2.5 V
-2.5V to 2.5V

Resolution: 8-bits

Conversion time: 2 microseconds

Sampling Theorem

How can we decide a proper sampling rate for a given signal?



Sampling

- The other important inherent feature of the ADC is its discrete nature in time. When a signal is digitalized, only samples of the signal amplitude at finite time intervals are available.
- For example the continuous signal shown in the figure above is represented by a series of samples taken at regular time intervals. This process is called sampling.
- Due to the process of sampling information about the signal in between sampling point are lost. This is called sampling error.

Sampling Theorem

- According to the above discussion one may conclude that an analogue signal can never be accurately represented in digital form.

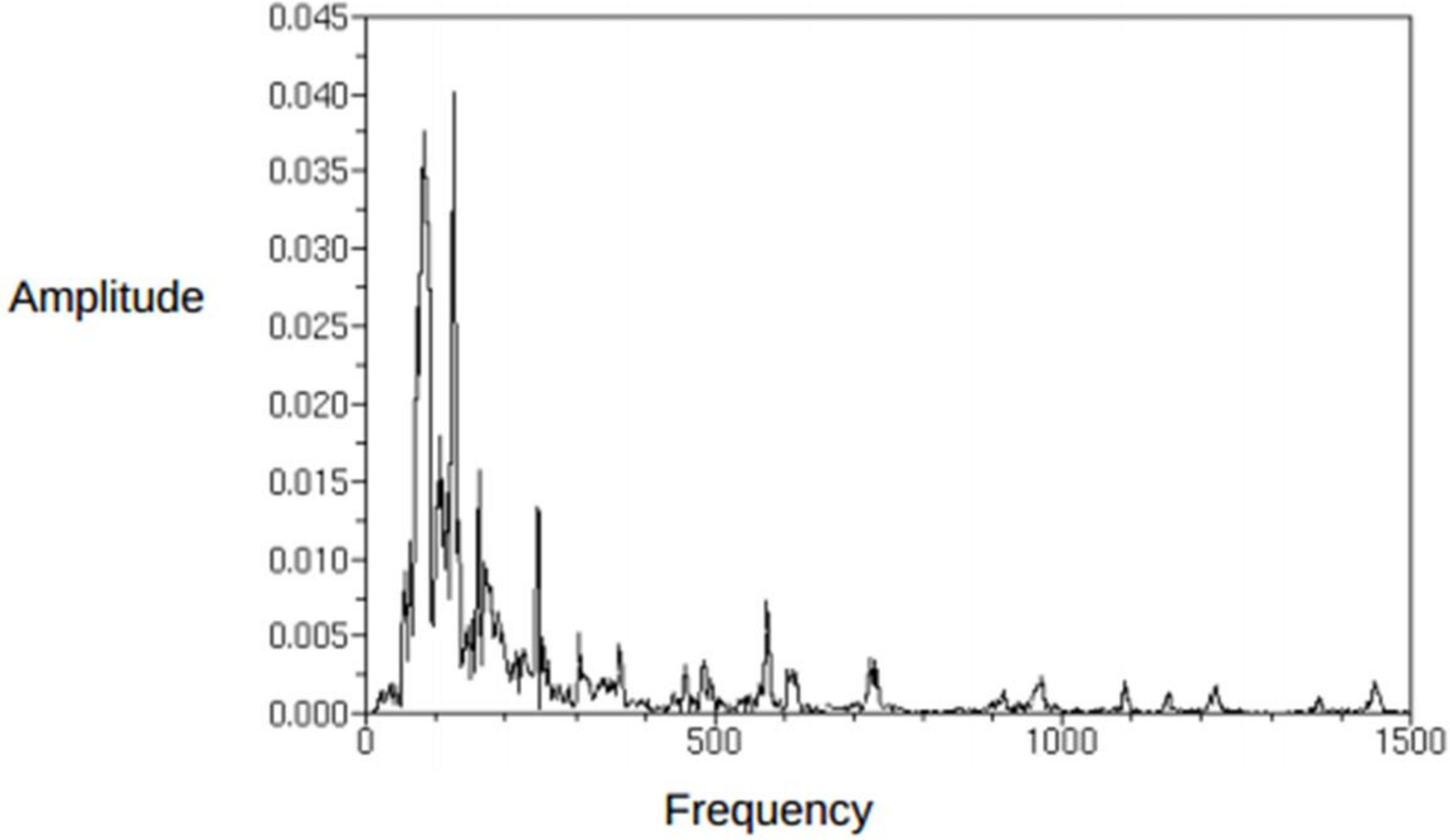
- However, according to the sampling theorem it is possible to digitize a signal without loosing information if the speed of the ADC is chosen properly.

Sampling Theorem (Nyquist Theorem)

A signal that contains frequencies up to a maximum of f_{max} can be adequately represented by samples taken at a rate greater than or equal to $2f_{max}$.

It means,
it can be adequately represented by samples taken at time intervals less than $\frac{1}{2f_{max}}$

Example: Frequency spectrum of a signal

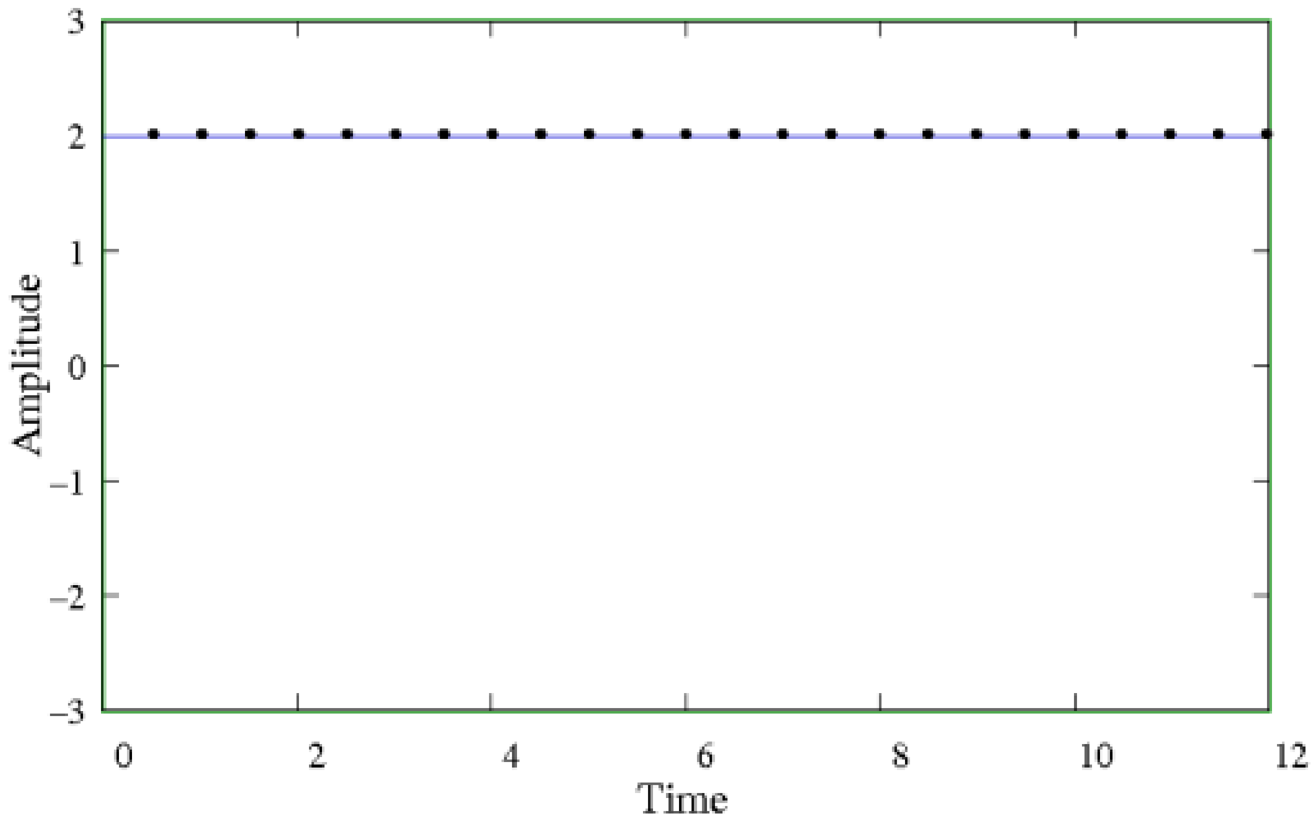


Sampling Theorem

A continuous signal can be properly sampled only if it does not contain frequency components above one-half of the sampling rate.

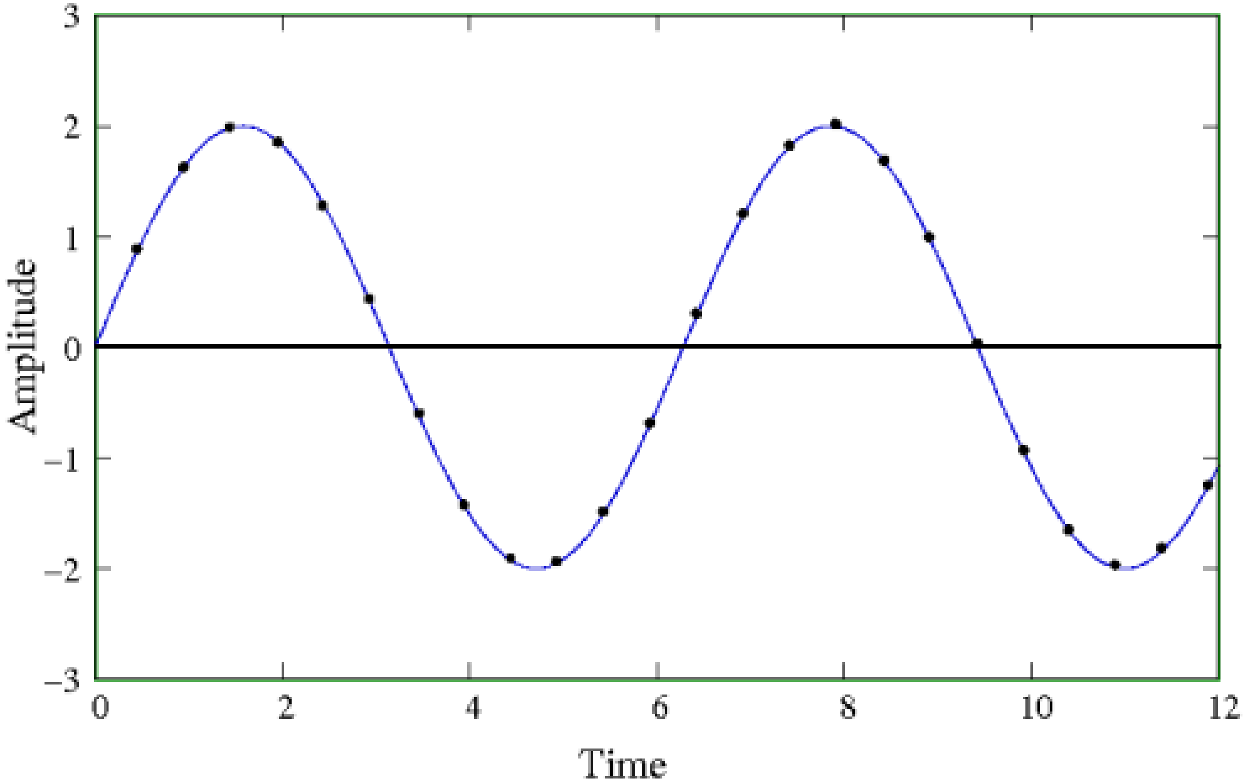
If frequency components above one-half of the sampling rate are present, aliasing will result.

Sampling Theorem



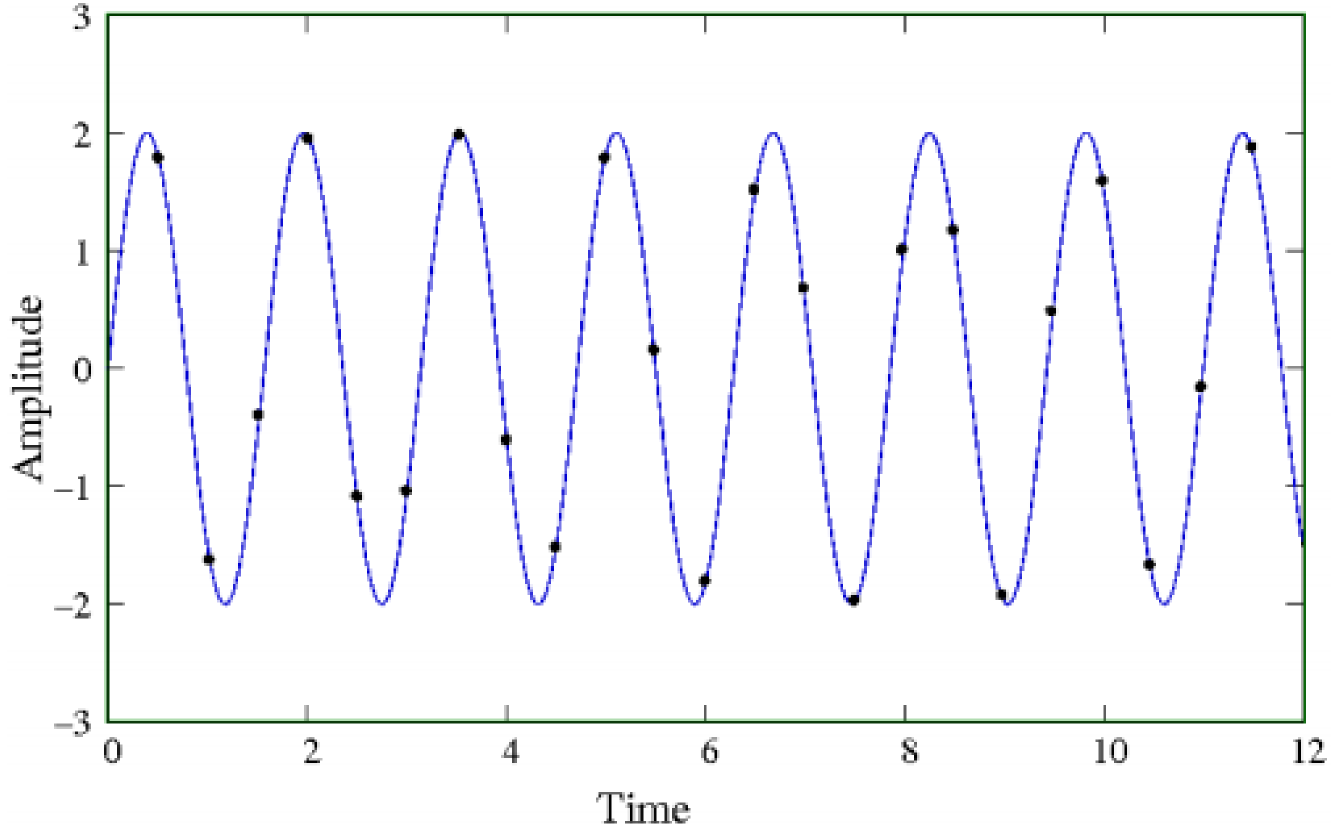
Sampling a constant voltage: one sample is enough

Sampling Theorem



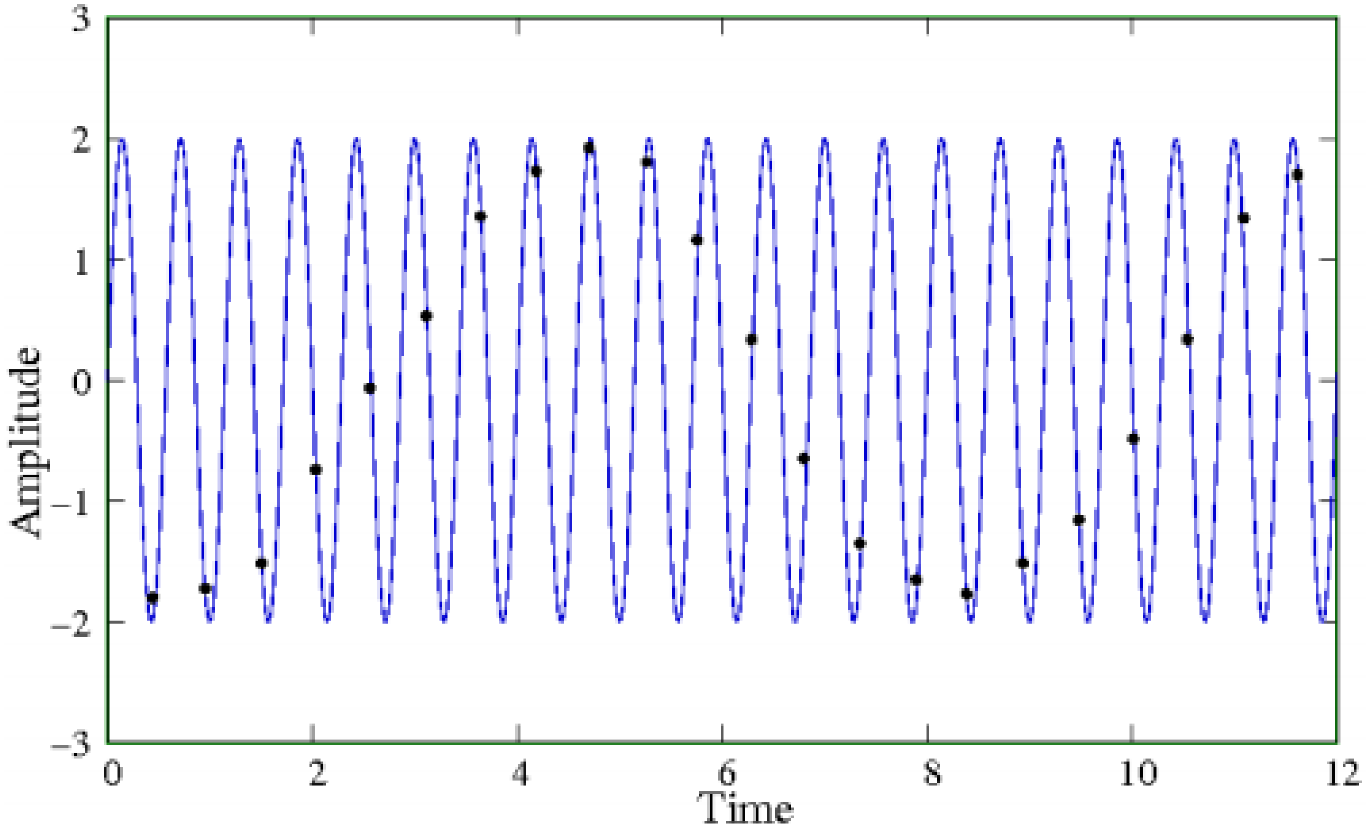
Sampling a sine wave: $f_{sample} \simeq 13 \times f_{signal}$

Sampling Theorem



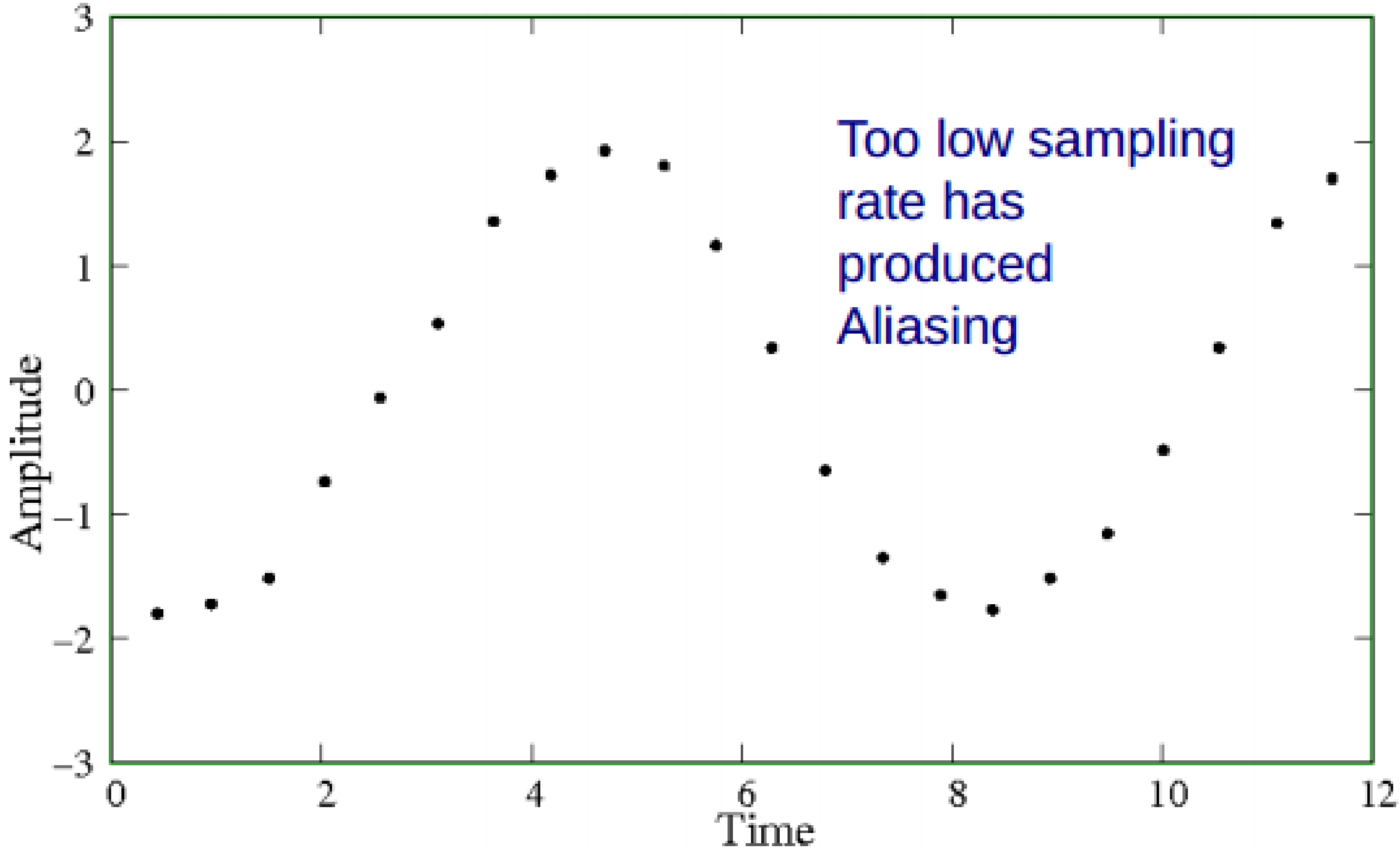
Sampling a sine wave: $f_{\text{sample}} \simeq 4 \times f_{\text{signal}}$

Sampling Theorem



Sampling a sine wave $f_{sample} \simeq 0.8 \times f_{signal}$

Sampling Theorem



Sampling a sine wave $f_{sample} \simeq 0.8 \times f_{signal}$

Aliasing

If sampling time $\mathbf{t_s}$ is used for digitizing a signal that contains frequency components up to $\mathbf{f_{max}}$ and if $\mathbf{t_s > \frac{1}{2f_{max}}}$ then one will have to face the problem called aliasing.

Consider a signal given by

$$a(t) = \cos(2\pi f_0 t)$$

If this is sampled at a sampling interval $t_s > \frac{1}{2f_0}$

we can write $$f_0 = \frac{1}{2t_s} + f$$

Then, $$a(t) = \cos\left[2\pi\left(\frac{1}{2t_s} + f\right)t\right]$$

$$a(t) = \cos\left[2\pi\left(\frac{1}{2t_s} + f\right)t\right]$$

If this signal is sampled at $\mathbf{t_s}$ time intervals, the $\mathbf{m^{th}}$ sample will be taken at $\mathbf{t = (m+1)t_s}$

Now, the *m*+1 th sample will be

$$\begin{aligned} a_{m+1} &= \cos\left[2\pi\left(\frac{1}{2t_s} + f\right)mt_s\right] \\ &= \cos\left(m\pi + 2\pi f m t_s\right) \\ &= \cos(2\pi f m t_s). \end{aligned}$$

If we sampled $b(t) = \cos(2\pi f t)$ at the same rate,

still the m+1 th sample will be $b_{m+1} = \cos(2\pi f m t_s)$

- In order to avoid aliasing, frequencies above the Nyquist frequency must be filtered using a low pass filter.
- Such a filter is called an anti-aliasing filter.

Problems:

What are the suitable sample rates for digitizing human speech?

And music?

What should be the cut-off frequencies of the anti aliasing filters used in the above cases?

Humans can here from 20 Hz to about 20000 Hz.

Therefore, when music is digitized, sampling time must be less than 1/40,000 seconds.

i.e. more than 40000 samples per second.
In CDs, 44100 is used (44.1kHz).

Next → Electronic Filters