# Special Theory of Relativity





#### 13<sup>th</sup> Lecture

## Brief Introduction to General Theory of Relativity

Soon after the Special Theory was published in 1905, Einstein turned his attention to phenomena that occurred when observers were not restricted to movement with constant relative velocities (i.e. with zero acceleration) but with varying velocities (i.e. with accelerations not zero). The results of his reasoning embody The **General Theory of Relativity, which was** presented in 1916.

#### Brief Introduction to General Theory of Relativity



### The Concept of Gravity : Space & Time Curves



The principle of equivalence at a single point in space effects of gravitation and accelerated motion are equivalent and cannot be distinguished from each other.



## What is curve space-time?

Newton's Gravity Concept

Newton's law of universal gravitation is usually stated as that every particle attracts every other particle in the universe



## What is curve space-time?

Einstein's Gravity Concept

The key difference between Einstein and Newton gravity is that Einstein gravity describes that gravity is a curvature in a 4-dimensional space-time fabric proportional to object masses, whereas



Another result of the General Theory is the effect of a gravitational mass on time. The prediction is that all time processes will be slower on a large mass than on a small mass, or that

Time will move more slowly on a relatively larger planet, such as Jupiter, than on Earth.

(PTO)

Although a clock which runs at a certain rate on the Earth will run slower on Jupiter, it will run even slower on the Sun.

Indeed, Einstein found that a second of time on the Sun should correspond to 1.000002 Earth seconds.

It is not surprising to find that a flying bullet or arrow is pulled towards the Earth. These have weight (even while in the flight), but most people are surprised to find that a light beam also has a weight. This did not surprise the scientists of the day, however, because one of the theories of the light says that the light is made up of small particles are called photons which travel at a velocity of  $3x(10^8)$  m/s. These were considered to have mass, and it was reasoned that if they did, when light falls on a surface these photons would exert a pressure on the surface similar to the pattern of falling rain-drops on a roof.

This effect has been observed and is known as radiation pressure. The pressure is very small, and for the Sun's rays on the Earth it is only about a tenth of an ounce per acre  $(7x10^{-6}) N/m^{2})$ , giving a total of but 160tons (1.62x10<sup>5</sup> kg) for the entire surface of the Earth. Fortunately, gravitational attraction of the Sun on the Earth is many times stronger, so that we are not pushed off into space by the radiation pressure of the Sun's rays.

Brief Introduction to General Theory of Relativity

### **Radiation pressure :**

Radiation pressure is the **pressure** associated with the interaction of electromagnetic radiation on any given surface. In other words Radiation pressure is defined as force per unit area exerted by electromagnetic waves.



#### Brief Introduction to General Theory of Relativity



To test the prediction of the General Theory that light is deflected by a gravitational field, actually we would have to 'weigh a beam of light'. It isn't possible to catch a lot of photons and pile them on a scale, as we could with bullets, since no one has yet succeed in building a photon trap. (Moreover, scientists today believe that the rest mass of the photon is zero!)

(Moreover, scientists today believe that the rest mass of the photon is zero!) So the photons must be weighted while in flight. This is not difficult to do in theory, since if a beam of light is affected by a gravitational field, its path will be curved, and this is easy to determine, providing the curvature is sufficient. But if a beam of light is not affected by a gravitational field, its path through it will be a straight line, which is also easy to determine.

The beam of light must, of course, come from a star. The procedure for weighing is illustrated by the following figure. The initial position of the star A, whose light is to be weighted, is shown in figure.



(a) Incoming light beam is straight.

There are no intervening gravitational masses, so the star's light travels in a straight line from the star to the observer on the Earth. Later on, the earth has travelled sufficiently far in its orbit so that the sun comes between the earth and the star in such a position that the light from the star just grazes the sun's surface on its way to the observer on the earth as in the following figure.



(b) Incoming light beam is bend sun's mass

But here a difficulty is encountered, for if the star's light is just grazing the sun's surface the observer will not be able to see the star, since the sun's light will be too bright. The only solution is to observe the star's light grazing the sun during a total eclipse of the sun during a total eclipse of the sun when the moon blots out the sun's light completely, as illustrated. For this reason Einstein suggested that this effect be looked for **PTO**) during a total solar eclipse.

Since the deflection of the star's light, while grazing the sun is so slight, precise photographic techniques are necessary. The actual procedure consists of photographing the star while in position (a), showing its position relative to its neighboring stars, and then again at a later time during a total solar eclipse, as in (b). In this later position, it will appear to the observer (and his camera) that the star is at the apparent position A'. A picture taken at this position is compared with that of position (a). The comparison should show that the star appears to have moved, provided that the Sun's mass has deflected the star's light as predicted by the General Theory. In particular, Einstein predicted that the deflection would be 1.74 seconds of arc for a light beam grazing the sun's surface.

The most favorable total eclipse of the Sun after the General Theory was presented in 1916 occurred on 29th May 1919. This eclipse was particularly favorable for the test because the earth and the Sun happen to be lined up with a patch of bright stars at the end of May every year, and so were a number of stars to choose from during this eclipse. Accordingly, to British astronomical expeditions were fitted out. One, under A. C. Crommelin, went to Sobral in Northen Brazil, while the other, under A. S. Eddington, went to the Gulf of Guinea. Both groups photographed a number of stars, and upon their return to England the photographic plates ware developed and compared with pictures taken when the Sun was not in the same stars' vicinity.

The Sobral group found that their stars had moved on average of 1.98 seconds of arc, and the principle group that theirs had moved 1.6 seconds of arc. This nearness to the 1.74 seconds of arc predicted by Einstein was sufficient to verify the effect. Since then, more than ten different results have been reported which also confirm the prediction.

It is interesting to speculate how massive a star would have to be so that its gravitational attraction is strong enough to prevent any of the star's light from leaving the star. It can be shown that for a star of the same radius as the Sun this would occur if its mass were approximately 400,000 times the Sun's mass. If such stars existed, we would never be able to see them, regardless of how close they were or how brilliantly they shone. PTO

Using the Newton law of gravitation, a value can also be obtained for the bending of a light ray. This turns out to be exactly one half of the value given by the General Theory of Relativity; and so far the Sun the value would be 0.87 seconds of arc. None of the experimental determination to date has been of this order of magnitude all being larger and within a reasonable range of the Einstein value. This again points up the slight difference between the Newton law and the Einstein Theory.

To measure this slight difference, literally speaking, we would have to put a clock on the Sun, synchronize it with one just like it on the Earth, and then periodically compare the two. With the difference in time rates indicated, the Sun clock would be one second behind the Earth clock after 500,000 seconds, or after just under six days. Of course, we have no way of putting a clock on the Sun; but we do not have to, since we have many 'atomic clocks' there already.

#### Atomic clock

An atomic clock is a clock device that uses an electronic transition frequency in the microwave, optical, or ultraviolet region of the electromagnetic spectrum of atoms as a frequency standard for its timekeeping element.

Atomic clocks are the most accurate time and frequency standards known, and are used as primary standards for international time distribution services, to control the wave frequency of television broadcasts, and in global navigation satellite systems such as GPS.



The light from the sun is caused by many different types of vibrating atoms, the frequencies of these vibrations can be determined experimentally, from which the times per vibration can be computed. The frequencies and corresponding times per vibration also can be measured for the same atoms vibrating on Earth. These can then be compared the former. As we saw previously, if the frequencies of vibration of the atoms in the Sun are less than those for the same atoms on Earth, it means that the times per vibrations have increased, or that time itself is slowed down on the Sun.

Since the prediction was that the frequencies of the Sun's light would be decreased, the frequencies were expected to be shifted towards the red end of the visible spectrum, because the frequency of the colour red is lower than that of the other colours in the spectrum.





To differentiate this particular red shift from others effects which also produce a red shift, it is referred to as the relativistic or Einstein shift.

The Einstein shift was first looked for in the Sun. Unfortunately, the expected shift was so small that it was barely within the limits of measurement, so that these early attempts did not confirm the effect conclusively.

Since then, a class of stars called the white dwarfs has been used to detect the shift. These white dwarfs are small in comparison with most stars, but extremely dense. In particular,  $\beta$  Sirius (The companion of Sirius, the Dog Star, which is really a double star) has a diameter about 3% that of the Sun's, but its density is more than 25,000 times greater. On such a star a pint of the nuclear fluid making up the star would weigh about 18 tons! You can expect that life would be slower on such a star, where a person would be so crushed by his own weight that he couldn't even more! PTO

Since the predicted frequency shift for the star  $\beta$ Sirius is over thirty times that expected for the Sun, this star was used by Adams in 1925 in his attempt to find the predicted effect. He found a frequency shift towards the red end of the spectrum of the expected amount. This constituted proof that a strong gravitational field does slow down time processes, as the General Theory predicted.

The idea of representing any physical event as a point in terms of the four dimensional space (x, y, z, t) was first conceived by Poincare in 1904. However, it is developed in its full glory by Minkowski in 1908.

Consider the motion of a particle thrown vertically upwards with an initial velocity v. It will reach a height of  $H=v^2/(2g)$ , where its velocity will become zero and it will start descending. If we consider (x, y, z, t) as our four dimensional space, then the motion of the projectile is a two dimensional motion with coordinates (z, t), where z is the vertical axis. The graph of z *vs. t* will have the form as shown in the following figure.



The some results plotted with ordinate and abscissa interchanged will look as in the following figure.



Now consider the motion of a two stage rocket. In a two stage rocket the lift- off is attained by the firing of the first stage which sustains it to attain a desired elevation. At that elevation all the fuel in the first stage is burnt out and the stage physically separates and falls back to the ground.

At the instant the first stage separates, the second stage is ignited and the rocket takes-off with increased speed. The motion of such a rocket is illustrated by the (z, t) diagram in the following figure.



However, in plotting this diagram the rocket is treated as a point particle.

This lines *a* and *b* in the above figure are known as world lines.

Consider the case of a photon (corpuscle of light) travelling in a (x, t) plane. Its motion will be given by,

$$x = \pm ct = \pm T$$

depending upon whether it is travelling in the  $\pm x$  or -x direction. Both these paths are incorporated in a single equation on  $x^2 = T^2$ . Both these lines divide the space-time plot into four zones as in the following figure. Equations  $x = \pm T$  can be interpreted in terms of  $\pm x$  as well as  $\pm T$ , thus giving the four zones.



Light cones for past, present and future events...



Light cones for past, present and future events...

The four- dimensional plot of the same situation would provide light cones in the +T and -T directions, and the remaining two zones have the same property, and can be grouped into one (continuous) zone, which we call the third zone.

Let us now consider the events taking place in two inertial frames in 'future' and 'past' zones separately.

I. Future zone :

Consider two inertial frames S and S' moving relative to each other with a uniform velocity v. Then space-time event in S' given by has the value,

$$x'^{2} - T'^{2} = \gamma^{2} \left( x - \frac{v}{c} T \right)^{2} - \gamma^{2} \left( T - \frac{v}{c} x \right)^{2}$$
$$= \gamma^{2} \left( 1 - \frac{v^{2}}{c^{2}} \right) (x^{2} - T^{2})$$
$$= (x^{2} - T^{2})$$

Similarly, starting for the events in , we obtain

$$x^2 - T^2 = x^{1^2} - T^{1^2}$$

Thus,  $(x^2 - T^2)$  is an invariant quantity; it has the same value in two inertial frames, under Lorentz Transformation. Another important point to note is that in 'future' zones. T > 0 and x < T, i.e.  $(x^2 - T^2)$  is always negative (and so is  $x'^2 - T'^2$ ).

#### II. Past zone :

In this case also the situation is same in the 'future' zone case, except that, here, T < 0.

#### **III.** Third zone :

Here there is no restriction on T, and  $(x^2 - T^2)$  is positive.

#### Thus,

| i.   | For future zone | $(x^2 - T^2) < 0$ | ; $T > 0$      |
|------|-----------------|-------------------|----------------|
| ii.  | For past zone   | $(x^2 - T^2) < 0$ | ; <i>T</i> < 0 |
| iii. | For third zone  | $(x^2 - T^2) > 0$ |                |

In four dimensional case the results are,

| i.   | For future cone             | $(x^2 + y^2 + z^2 - T^2) < 0$ | ; <i>T</i> > 0 |
|------|-----------------------------|-------------------------------|----------------|
| ii.  | For past cone               | $(x^2 + y^2 + z^2 - T^2) < 0$ | ; <i>T</i> < 0 |
| iii. | For remaining (third) space | $(x^2 + y^2 + z^2 - T^2) > 0$ |                |

We now physically interpret the above figure as follows. Consider events represented by points p1, p2, p3, p4 and p5 (in the following diagram) in the two dimensional (x, t) space. From O it is possible to make contact with future (T >0) event p1 with speed less than c (the velocity of light), or with p2 with speed equal to *c* but never with p3 or p4. Similarly, from past (T < 0) point p6 it is possible to make contact with O (and not with p1 or p2 directly) with speed less than c, or from point p5 with speed equal to c, but, again, never with points p3 or p4. In other words, the events represented by points p3 and p4 have no communication possible with past or future, and have no causal relationship with O.



Connection between events occurring in light cones



Connection between events occurring in light cones

Similar interpretation exists for event-points in four – dimensional space. Because of these properties, the **future** cone is also called the time-like cone, while the **past** cone is called the light-like cone, and the remainder four dimensional space is called space-like. It is usual to say that the **past cone** is the light-cone for the event O.









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## Einstein and Eddington (2008) More at IMDbPro »

TV Movie - 90 min - Drama | History

7.3

Your rating: \*\*\*\*\*\*\*\* 10/10 ©

Ratings: 7.3/10 from 1,797 users Reviews: 11 user | 1 critic

A look at the evolution of Albert Einstein's theory of relativity, and Einstein's relationship with British scientist Sir Arthur Eddington, the first physicist to understand his ideas.

Director: Philip Martin Writer: Peter Moffat Stars: David Tennant, Richard McCabe and Patrick Kennedy



#### Storyline

Sir Arthur Eddington is a renowned physicist at Cambridge University and an expert in the measurement of the physical world. He along with all of his colleagues are also avowed Newtonians. Sir Oliver Lodge suggests that he read a new thesis put forward by a German-Swiss scientist named Albert Einstein who is suggesting that Sir Isaac Newton may have got it wrong. The expectation is that Einstein's theories will be disproven but Eddington admits that his General Theory of Relativity has merit. These are turbulent times as England and Germany are at war and Eddington's own loyalty is called into question when, as a Quaker, he refuses to fight. In the end, Eddington develops a series of tests to either prove or disprove Einstein's theories. For his part, Einstein has his own struggles during this period: the breakdown of his marriage, his integration into the university in Berlin and his own strident pacifism that led him to oppose German militarism and the First World War... *Written by garykmcd* 



#### Thank You !