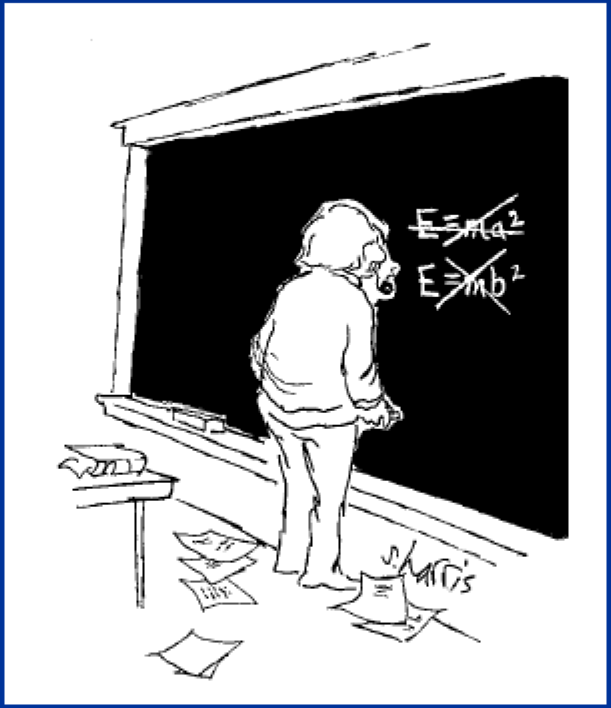
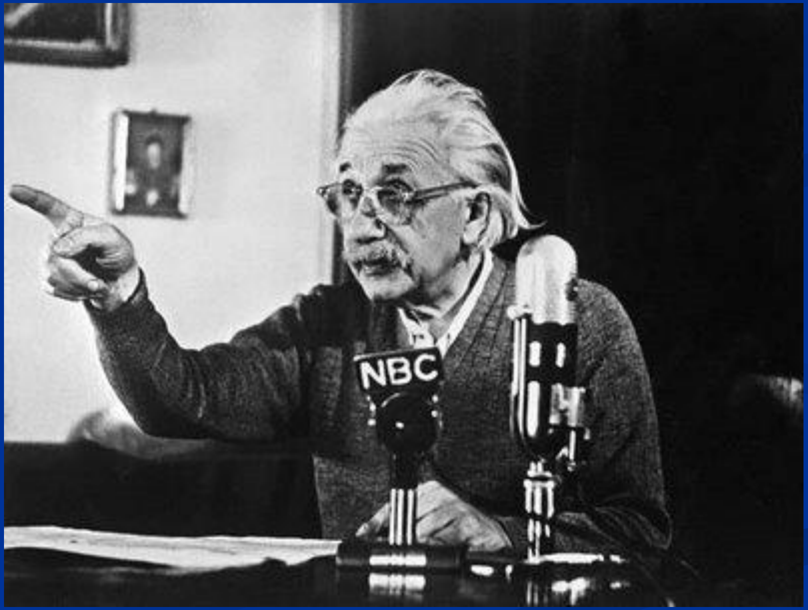
Special Theory of Relativity





13th Lecture

2019

UNIVERSITY OF SRI JAYEWARDENEPURA
FACULTY OF APPLIED SCIENCES

B. Sc. General Degree Second Year Second Semester Course Unit Examination

December, 2019

DEPARTMENT OF PHYSICS

PHY 207 1.0 / PHY 257 1.0 / PHY 302 1.0 / PHY 327 1.0

- Special Theory of Relativity

Time : One hour

No of Questions : 04

No of Pages : 02

Total marks : 100

Answer all questions

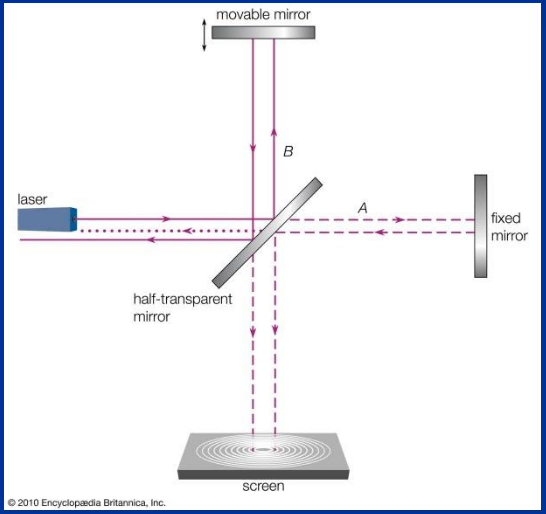
Assume, velocity of Light (c) = 3×10^8 ms^{-1}

01. Write short notes on,

(i) Michelson Morley Experiment and

(ii) Twin Paradox

in special theory of relativity (STR). **(25 Marks)**



Michelson-Morley experiment, an attempt to detect the velocity of Earth with respect to the hypothetical luminiferous ether, a medium in space proposed to carry light waves. First performed in Germany in 1880–81 by the physicist A.A. Michelson, the test was later refined in 1887 by Michelson and Edward W. Morley in the United States.

The procedure depended on a Michelson interferometer, a sensitive optical device that compares the optical path lengths for light moving in two mutually perpendicular directions. Michelson reasoned that, if the speed of light were constant with respect to the proposed ether through which Earth was moving, that motion could be detected by comparing the speed of light in the direction of Earth's motion and the speed of light at right angles to Earth's motion. No difference was found. This null result seriously discredited the ether theories and ultimately led to the proposal by Albert Einstein in 1905 that the speed of light is a universal constant.

01. Write short notes on,

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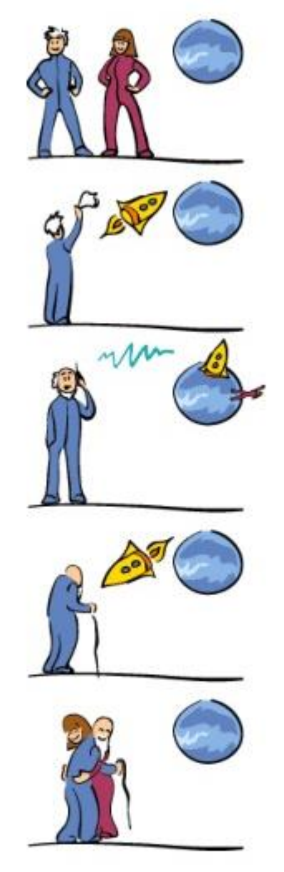
(ii) **Twin Paradox**

in special theory of relativity (STR). **(25 Marks)**



In physics, the twin paradox is a thought experiment in special relativity involving identical twins, one of whom makes a journey into space in a high-speed rocket and returns home to find that the twin who remained on Earth has aged more. This result appears puzzling because each twin sees the other twin as moving, and so, as a consequence of an incorrect and naive application of time dilation and the principle of relativity, each should paradoxically find the other to have aged less.

02. **(i)** Write down **two** main Einstein's Postulates in Special Theory of Relativity (STR).

(ii) Obtain the following relativistic time equation, starting from the above Postulates in STR.

$$t_2 = t_1 \frac{1}{\sqrt{1 - v^2/c^2}}$$ (Symbols have their usual meanings)

(iii) The first human trip to the Moon took about three days (*approximately* 3×10^5 *seconds*) each way. The distance from the Earth to the Moon is roughly $4 \times 10^8\ m$.

(a) **Find** the velocity of the space ship.

(b) When they returned, **how much** younger were the astronauts than their twin brother who remained on the Earth ?

(25 Marks)

02. **(i)** Write down **two** main Einstein's Postulates in Special Theory of Relativity (STR).

Theory!

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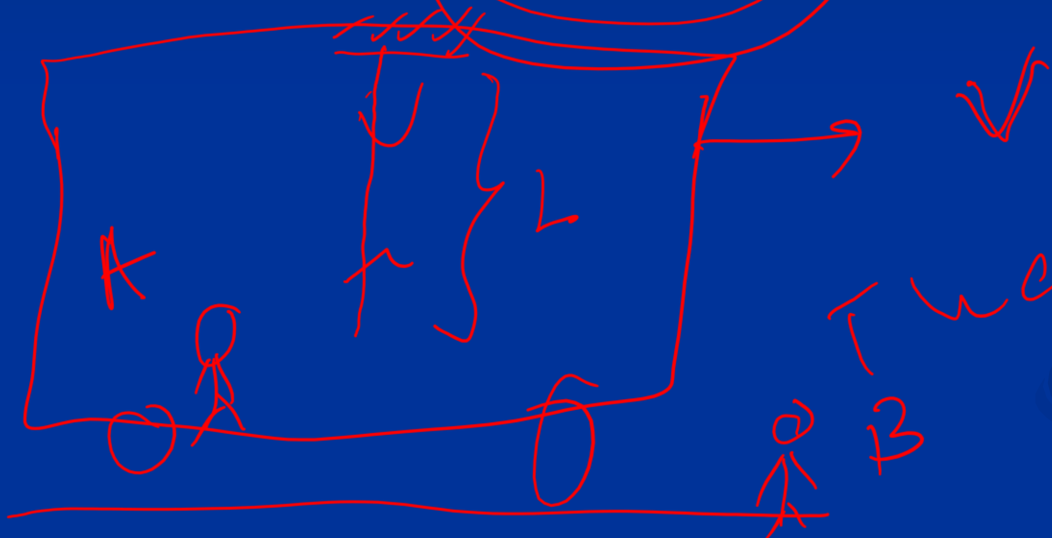
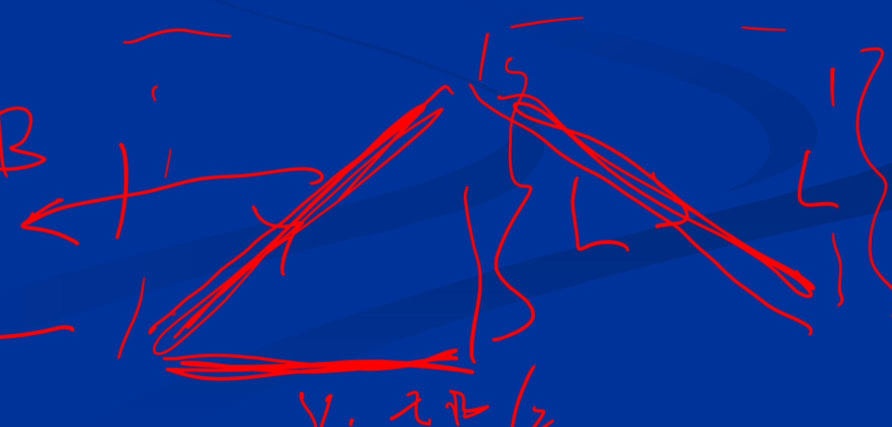
$$t_2 = t_1 \frac{1}{\sqrt{1 - v^2/c^2}} \quad \text{(Symbols have their usual meanings)}$$



A:

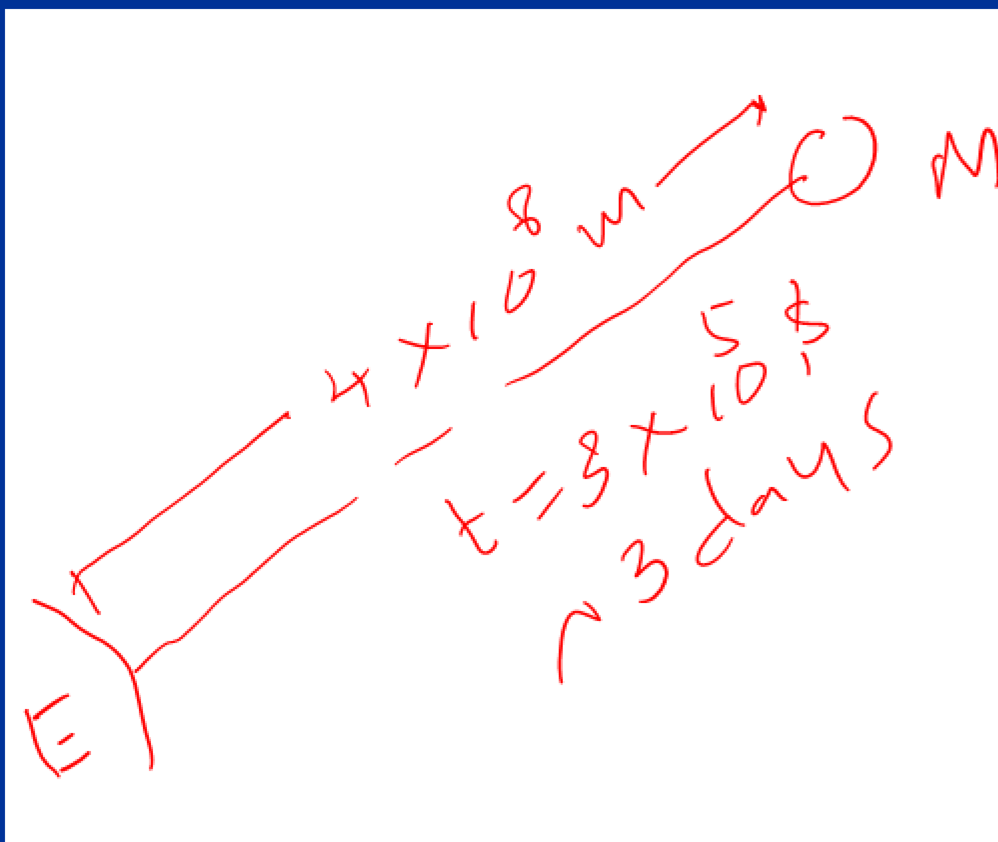
$$t_A = \frac{2L}{c}$$



$$t_B = \frac{2(\quad}{c}$$

(iii) The first human trip to the Moon took about three days (*approximately* $3\text{x}10^5$ *seconds*) each way. The distance from the Earth to the Moon is roughly $4 \text{ x } 10^8\ m$.

(a) **Find** the velocity of the space ship.



Velocity of the ship

$$v = \frac{distance}{time}$$

$$v = \frac{4\times10^8 m}{3\times10^5 s}$$

$$v = \ldots\ldots\ldots ms^{-1}$$

(iii) The first human trip to the Moon took about three days (*approximately* 3×10^5 *seconds*) each way. The distance from the Earth to the Moon is roughly $4 \times 10^8\ m$.

(a) **Find** the velocity of the space ship.

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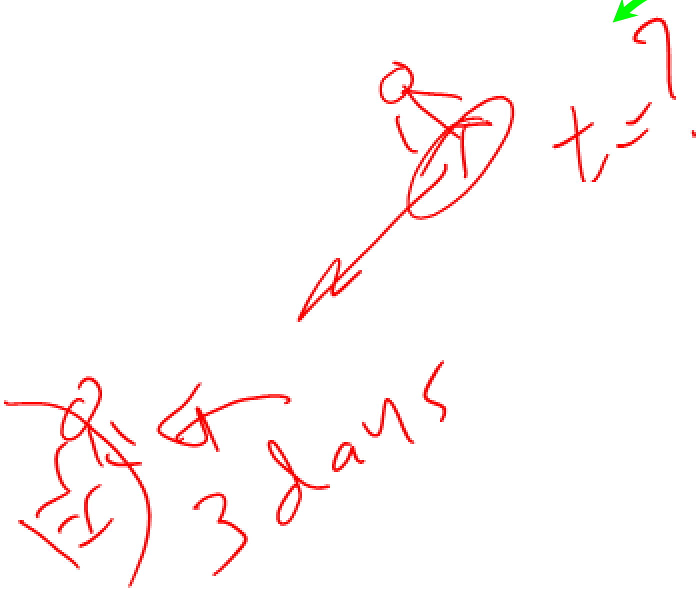


Using relativistic time equation :

$$t_2 = t_1 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t_1 = t$ $\qquad$ $t_2 = 3\ days$

$v = known$

03. **(i)** Derive an expression for the length contraction ($l_2 = l_1\sqrt{1 - v^2/c^2}$) starting from the relativistic time equation (Symbols have their usual meanings).

(ii) A rod of length **20 cm** is held at an angle of 45^o to the horizontal. It's now projected with a velocity of **0.9c** along the horizontal such that the rod always keeps the same angle of 45^o during the motion. What will be the length of the rod as seen by,

(a) an observer stationary on the ground ?

(b) an observer moving with the rod ?

(25 Marks)

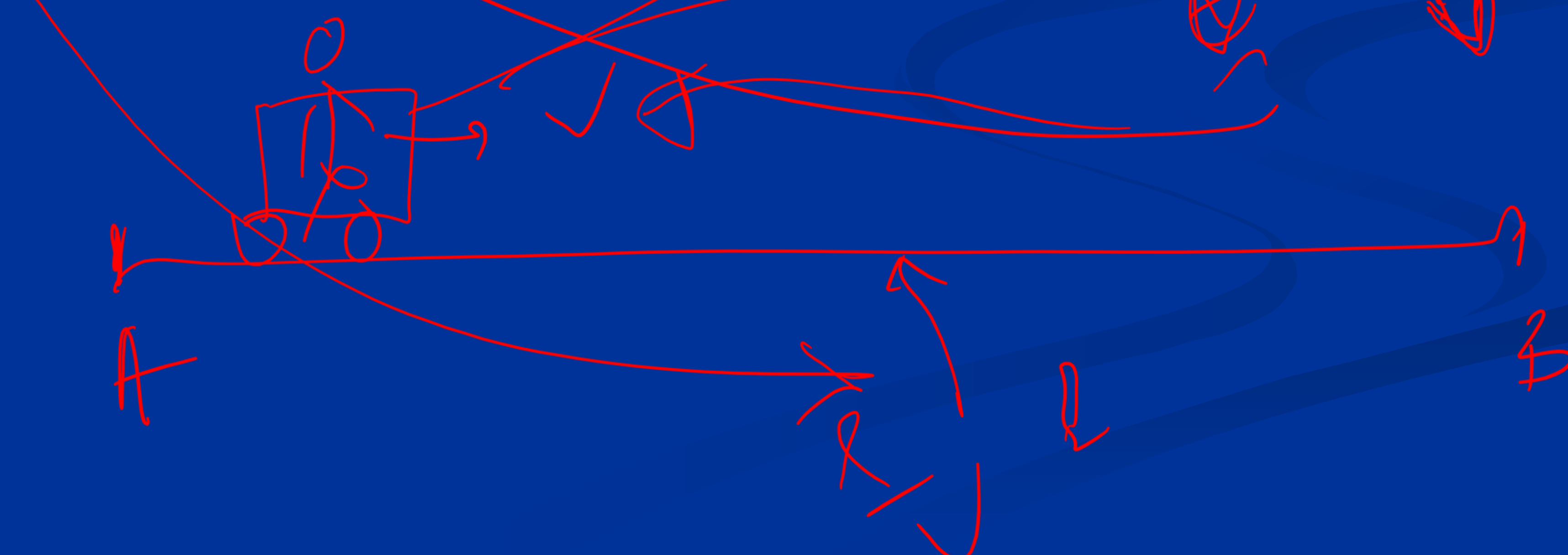
03. **(i)** Derive an expression for the length contraction ($l_2 = l_1\sqrt{1 - v^2/c^2}$) starting from the relativistic time equation (Symbols have their usual meanings).

$t_2 = \frac{t_1}{\sqrt{1 - v^2/c^2}}$

Them!

$v = \frac{s}{t}$

$t = \frac{L}{v}$



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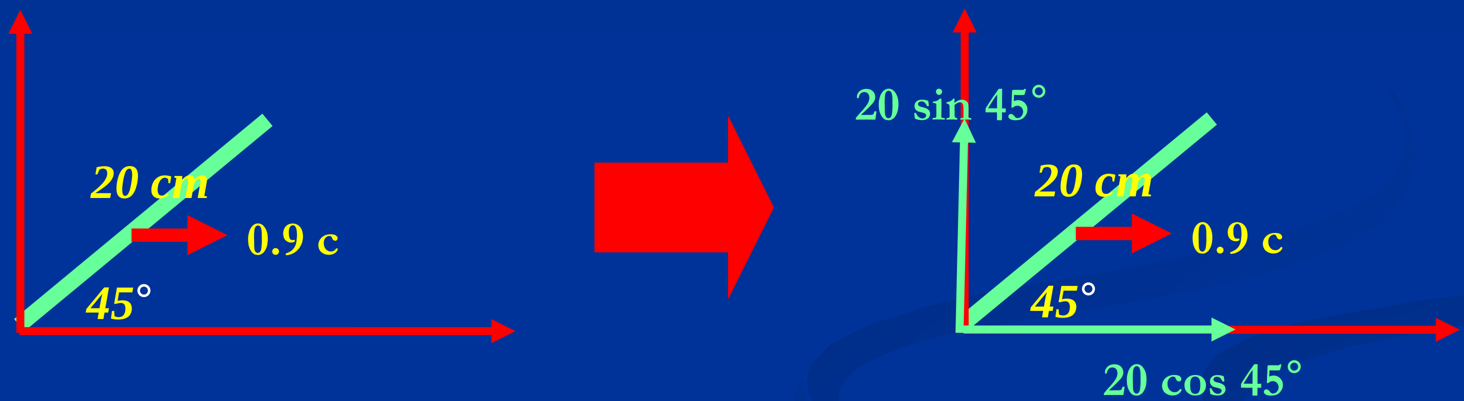

$$l^1 = l\sqrt{1-\frac{v^2}{c^2}}$$

$$l_x^{\ 1} = 20\cos 45°\sqrt{1-\frac{(0.9c)^2}{c^2}}$$

$$l_y^{\ 1} = 20\sin 45°$$

Length as seen by Sta; Obs; = $l^1 = \sqrt{l_x^{1\,2} + l_y^{1\,2}}$

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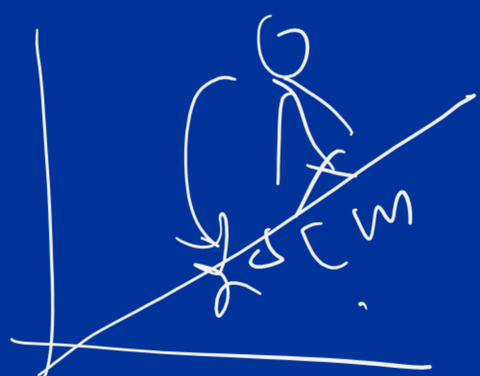

04. **(i)** What is the **Doppler Effect** in Relativity for a moving light source ?
You are given the following mathematical equation for the Doppler effect,

$f_o = \dfrac{f_s}{\gamma\,(1-\beta\ \cos\theta)}$. Where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = \dfrac{v}{c}$ and other symbols have their usual meanings.

(ii) A **blue** coloured vehicle appears as a **purple** coloured vehicle to a stationary observer on the Earth due to its speed.

(a) Find the velocity of the vehicle. (Wavelengths of **blue** and **purple** light are 450 *nm* and 400 *nm* respectively.)

(b) Is the above incident **practically possible** ? Briefly explain your answer.

(25 Marks)

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Theory !

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derive

(ii) A **blue** coloured vehicle appears as a **purple** coloured vehicle to a stationary observer on the Earth due to its speed.

(a) Find the velocity of the vehicle. (Wavelengths of **blue** and **purple** light are 450 *nm* and 400 *nm* respectively.)

Source frequency = true colour of the vehicle

f_s = *frequency of the blue colour*

For E-M Waves :

$$v = f\lambda$$

$$f = \frac{c}{\lambda}$$

$$f_s = \frac{3\times10^8\ ms^{-1}}{450\times10^{-9}\ m}$$

$$f_s = 6.67\times10^{14}\ Hz$$

Observed frequency = appeared colour of the vehicle

f_o = *frequency of the purple colour*

$$f_o = \frac{3\times10^8\ ms^{-1}}{400\times10^{-9}\ m}$$

$$f_o = 7.50\times10^{14}\ Hz$$

$$f_o = 7.50\times10^{14}\ Hz \quad > \quad f_s = 6.67\times10^{14}\ Hz$$

That means, $f_o > f_s$

Then, the frequency appears to increase!

$$f_o = f_s\sqrt{\frac{1+\beta}{1-\beta}}$$

(ii) A **blue** coloured vehicle appears as a **purple** coloured vehicle to a stationary observer on the Earth due to its speed.

(a) Find the velocity of the vehicle. (Wavelengths of **blue** and **purple** light are 450 *nm* and 400 *nm* respectively.)

$$f_o = f_s \sqrt{\frac{1+\beta}{1-\beta}}$$

$$7.5\times10^{14} = 6.67\times10^{14} \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\beta = 0.12$$

$$\frac{v}{c} = 0.12$$

$$v = 0.12c$$

$$v = 3.6\times10^{7}\ ms^{-1}$$

(b) Is the above incident **practically possible** ? Briefly explain your answer.

$v \propto k c$

$k < 1$

$v_{max} = 14 \times 330\ m s^{-1}$ (Supersonic)

$\ll c$

Impossible

2018

UNIVERSITY OF SRI JAYEWARDANEPURA

B. Sc. General Degree Second Year Course Unit Examination – Oct/Nov, 2018.

PHY 207 1.0 / PHY 257 1.0 / PHY 302 1.0 / PHY 327 1.0

- Special Theory of Relativity

Time : One hour

Answer all questions

Assume, velocity of Light (c) = $3 \times 10^{8}\ ms^{-1}$

01. Write down the **two** main Einstein's Postulates in Special Theory of Relativity (STR).

Obtain the following relativistic time equation, starting from the above Postulates in STR.

$$t^1 = \gamma\, t \;, \quad \text{where, } \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \text{; (Symbols have their usual meanings)}$$

An alpha particle and a beta particle, which are created in a particle accelerator, travel a total distance of 10.0 m between two detectors in 50 ns and 40 ns respectively, as measured in the laboratory frame.

(a) What is the lifetime of the alpha particle as measured in its own frame?

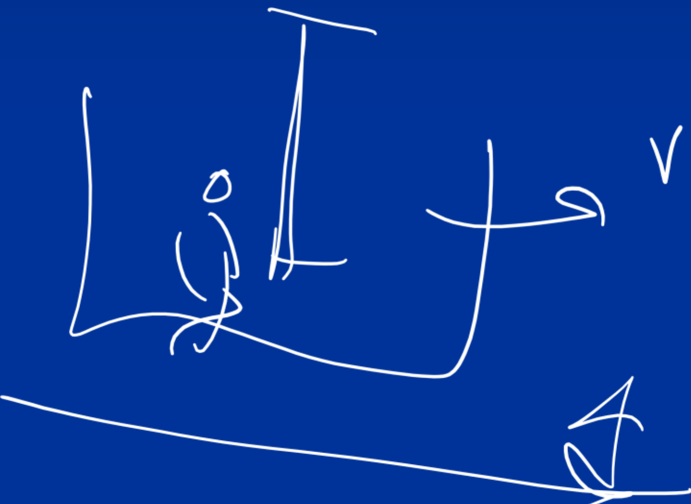
(b) What is the lifetime of the alpha particle as measured in the frame of the beta particle?

<u>01.</u> Write down the **two** main Einstein's Postulates in Special Theory of Relativity (STR).

Them !

Obtain the following relativistic time equation, starting from the above Postulates in STR.

$$t^1 = \gamma\, t\,, \quad \text{where, } \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}; \text{ (Symbols have their usual meanings)}$$



Then

$$t' = t \cdot \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$t' = \gamma\, t \;;\; \gamma ;$$

An alpha particle and a beta particle, which are created in a particle accelerator, travel a total distance of 10.0 m between two detectors in 50 ns and 40 ns respectively, as measured in the laboratory frame.

(*a*) What is the lifetime of the alpha particle as measured in its own frame?

$$v = \frac{distance}{time}$$

$$v_\alpha = \frac{10\ m}{50 \times 10^{-9}\ s}$$

$$v_\beta = \frac{10\ m}{40 \times 10^{-9}\ s}$$

Using relativistic time equation :

$$t_2 = t_1 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_1 = t$$

$$t_2 = 50\ ns$$

$$v = v_\alpha$$

An alpha particle and a beta particle, which are created in a particle accelerator, travel a total distance of 10.0 m between two detectors in 50 ns and 40 ns respectively, as measured in the laboratory frame.

(*a*) What is the lifetime of the alpha particle as measured in its own frame?

(*b*) What is the lifetime of the alpha particle as measured in the frame of the beta particle?

V_α $\quad$ V_β $\quad$ α $\quad$ $V(\alpha,\beta)$

$$t' = t \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$V(\alpha,\beta) = \frac{V_\alpha - V_\beta}{1 - \frac{V_\alpha V_\beta}{c^2}}$$

<u>02.</u> Derive an expression for the length contraction ($l_2 = l_1\sqrt{1-v^2/c^2}$) starting from the relativistic time equation (Symbols have their usual meanings).

A star known as Alfa-Centauri is about 4.0 light years (1 light year = 9.4608 x

10^{15} 1015m) distant from the Earth. If suppose a rocket from the Earth is to reach it in five years, how fast would it have to go?

What is the length of the trip (from the Earth to Alpha-Centauri) according to an observer in the rocket?

<u>02.</u> Derive an expression for the length contraction ($l_2 = l_1\sqrt{1 - v^2/c^2}$) starting from the relativistic time equation (Symbols have their usual meanings).

Theory !

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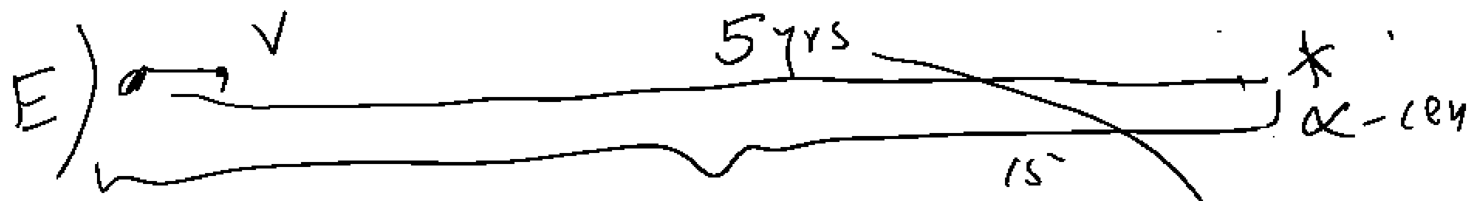
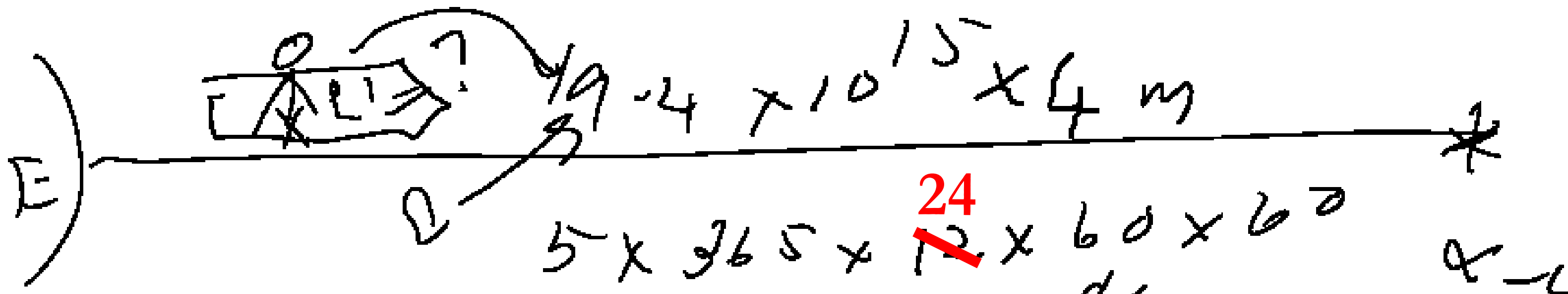


9.4608×10^{15} m $\times 4$

$$V = \frac{s}{t}$$

$= \text{-- -- ——}$ m/yrs

What is the length of the trip (from the Earth to Alpha-Centauri) according to an observer in the rocket?



$$V = .6c$$

$$\ell' = \ell\sqrt{1 - \frac{v^2}{c^2}}$$

$$= 9.4 \times 10^{15} \times 4\sqrt{1 - \frac{V^2}{c^2}}$$

03. **Derive** the equation,

$$E^2 - p^2 c^2 = m_o^2 c^4 ,$$

starting from the Einstein's energy equation, $E = m c^2$. (Symbols have their usual meanings)

Hence, **obtain** the equation,

$$m = \gamma m_o , \quad \text{where, } \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} ;$$

for **mass variation** in relativistic dynamics. (Symbols have their usual meanings)

A proton is accelerated to a velocity 0.95c by using a particle accelerator. Rest mass of the proton is $1.67 \mathrm{x} 10^{-27}$ kg. **Calculate** the mass of the moving proton.

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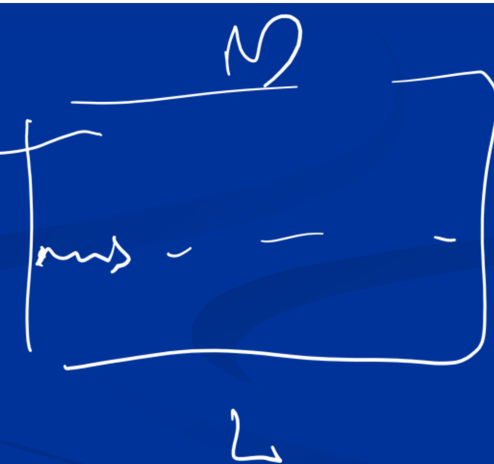
Theory of V



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$$E^2 - p^2 c^2 = m_o^2 c^4$$

$$E = mc^2$$

$$p = mv$$

$$m = m_o \cdot \frac{1}{\sqrt{1 - v^2/c^2}}$$

A proton is accelerated to a velocity 0.95c by using a particle accelerator. Rest mass of the proton is 1.67×10^{-27} kg. **Calculate** the mass of the moving proton.

P

$v = 0.95c$

$m_0 = 1.67 \times 0^{-27}$ kg

$$m = m_0 \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$m = 1.67 \times 10^{-27} \frac{1}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}}$$

$$= \checkmark > 1.67 \times 10^{-27}$$

04. What is meant by the **Doppler Effect** in Relativity for a moving light source?

You are given the following mathematical equation for the Doppler effect,

$$f_o = \frac{f_s}{\gamma\left(1-\beta\ \cos\theta\right)}.$$ Where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$ and other symbols have their usual meanings.

A spacecraft moves towards the Earth with a constant velocity $c/2$ as viewed from the Earth's frame. The spacecraft emits light of wave length λ as measured in its own frame. The wave length of the light as seen by an observer on the Earth is 6000 $\overset{o}{A}$. (1 $\overset{o}{A} = 10^{-10}\ m$)

Find the value of λ.

<u>04.</u> What is meant by the **Doppler Effect** in Relativity for a moving light source?

Theory)

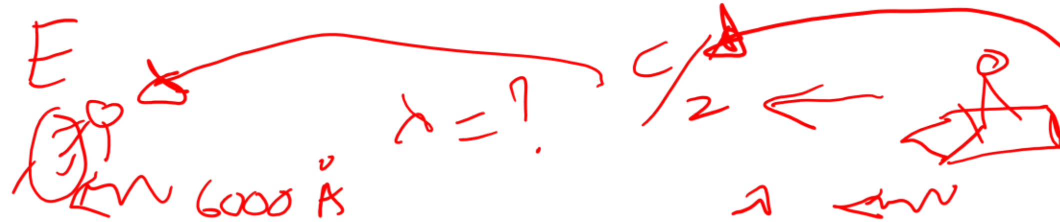
You are given the following mathematical equation for the Doppler effect,

$$f_o = \frac{f_s}{\gamma\left(1-\beta\ \cos\theta\right)}.$$ Where $\gamma = 1\Big/\sqrt{1-\beta^2}$, $\beta = \dfrac{v}{c}$ and other symbols have their usual meanings.

not derived

A spacecraft moves towards the Earth with a constant velocity $c/2$ as viewed from the Earth's frame. The spacecraft emits light of wave length λ as measured in its own frame. The wave length of the light as seen by an observer on the Earth is $6000\ \mathring{A}$. ($1\ \mathring{A} = 10^{-10}\ m$)

Find the value of λ.



$c = f\lambda$

$$f_o = f_s \cdot \sqrt{\frac{1+\beta}{1-\beta}}, \quad \beta = \frac{v}{c}$$

$$\frac{c}{6000\ \mathring{A}} = \frac{c}{\lambda}$$

$\lambda = \sim\ \mathring{A}$

Thank You !