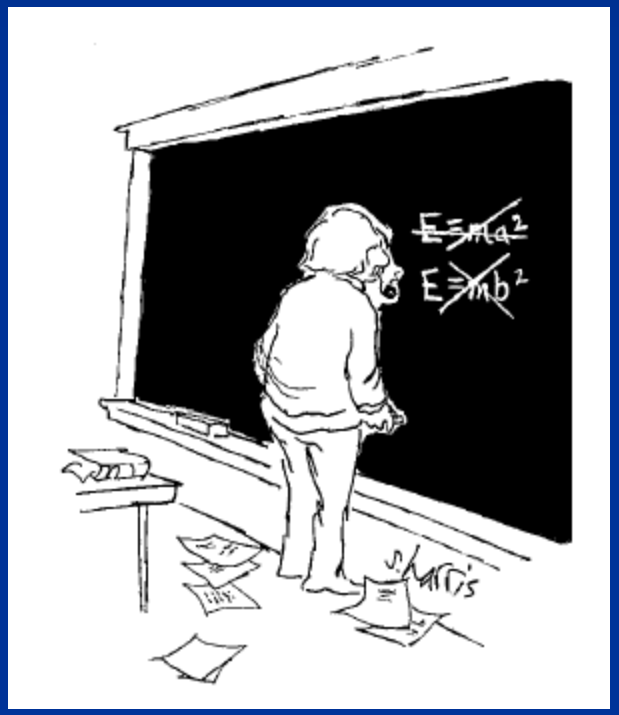
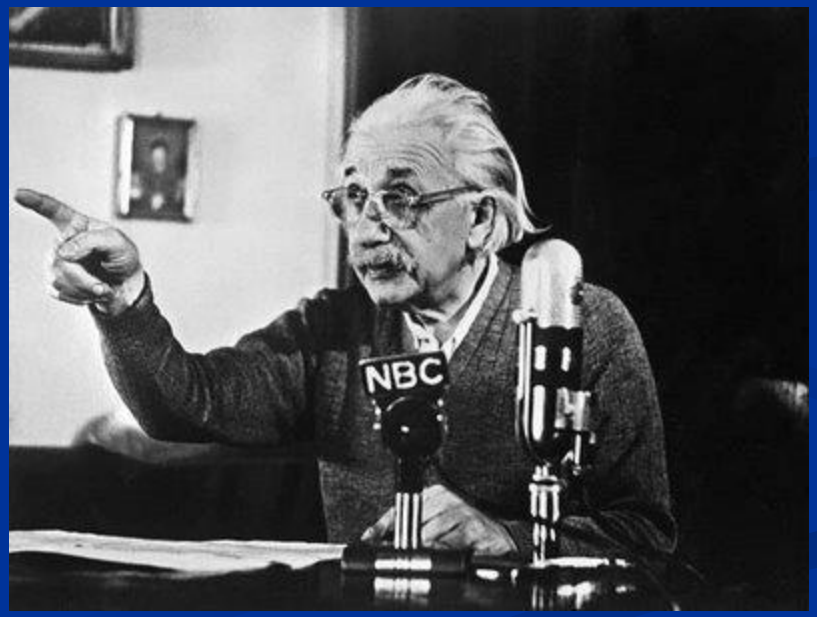
Special Theory of Relativity





12th Lecture

2024

UNIVERSITY OF SRI JAYEWARDENEPURA
FACULTY OF APPLIED SCIENCES

Bachelor of Science Degree Second Year Second Semester Course Unit Examination – February/March 2024

DEPARTMENT OF PHYSICS

PHY 207 1.0 Special Theory of Relativity

Time: One (01) hour **No. of questions: 02** **No. of pages: 02** **Total marks: 100**

Instructions:

1. **Answer ALL questions**

Assume the velocity of light (c) = $3 \times 10^8\ ms^{-1}$ and all the symbols have their usual meaning.

1. (a) i. State the two main postulates of the Special Theory of Relativity. (06 marks)

ii. Lorentz transformation for two coordinate systems S' and **S**, where S' is moving with a constant velocity v relative to the system S along the positive x-axis is given below,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y' = y, \quad z' = z, \quad \text{and } t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Obtain the inverse Lorentz transformation from the above equations. (08 marks)

iii. Use the results from question (ii) and show that the space-time line element, $ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$ is invariant under Lorentz transformation of coordinates.
(Hint: Consider an event (x_1, y_1, z_1, t_1) in S_1 inertial frame and the corresponding event (x_2, y_2, z_2, t_2) observed by S_2 inertial frame.) (12 marks)

(b) i. Derive the relativistic length contraction equation. (10 marks)

ii. A rigid bar of length $\boldsymbol{L_0}$ = 1.5 m is at rest relative to the frame $\boldsymbol{S'}$. $\boldsymbol{S'}$ is moving with a constant velocity of 0.98c relative to the frame $\boldsymbol{S}$ along the positive x direction. If the bar makes an angle $\boldsymbol{\theta} = 45^0$ with respect to the $\boldsymbol{x'}$ axis, what is the length $\boldsymbol{L}$ of the bar relative to $\boldsymbol{S}$? (Assume that the bar is lying only in the $\boldsymbol{x'y'}$ plane and discard the $\boldsymbol{z'}$ coordinate.) (12 marks)

iii. A π-meson has a mean lifetime of $2 \times 10^{-8} s$ when measured at rest. If it is moving with a speed of 0.99c, use the following relativistic time dilation equation to find how far it goes before decaying into another particle.

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$; where t_0 is the proper time. (12 marks)

(Total: 60 marks)

2. (a) i. Obtain the relativistic velocity transformation equation given below using the Lorentz coordinate transformation.

$$u = \frac{u' + v}{1 + \frac{vu'}{c^2}} \text{ ; where } u = \frac{dx}{dt} \text{ and } u' = \frac{dx'}{dt'}.$$

(08 marks)

ii. A man on the moon observes two spaceships coming toward him from opposite directions at speeds of 0.8c and 0.9c respectively. What is the relative speed of the two spaceships as measured by an observer on either one of the spaceships? (08 marks)

iii. Following is the relativistic Doppler effect equation for observed frequency from a directly receding source.

$$f_{observed} = f_{source}\sqrt{\frac{1-\beta}{1+\beta}} \text{ ; where } \beta = \frac{v}{c}.$$

Use the above relation to calculate how fast a galaxy must be receding from Earth, if an absorption line found at wavelength 550 nm from that galaxy appears to be red-shifted to 700 nm for an observer on Earth. (08 marks)

(b) i. The Following expression gives the total energy ***E*** of a particle in terms of its relativistic momentum ***P*** and rest mass m_0.

$$E^2 = m_0^2c^4 + P^2c^2.$$

Creating an electron and a positron pair from a high-energy photon is an example of the Pair-Production. Show that the given Pair-production will not occur in free space, but it requires the presence of a heavy nucleus using the above relationship. (10 marks)

ii. Except Pair-production, list another two experimental evidences that proves the Special Theory of Relativity. (06 marks)

(Total: 40 marks)

$$E^2 = (pc)^2 + (m_0 c^2)^2$$

$m_0 \leftarrow$ rest mass.

For the photon,

$$E^2 = (p_{photon}\, c)^2$$

$$E = p_{photon}\, c$$

Positron + electron.

$$E = \sqrt{(p_{electron}\, c)^2 + (m_0 c^2)^2} + \sqrt{(p_{posi..}\, c)^2 + (m_0 c^2)^2}$$

Energy Conservation

$$p_{photon}\, c = \sqrt{(pc_{elect..})^2 + (m_0 c^2)^2} + \sqrt{(p_{positron}\, c)^2 + (m_0 c^2)^2}$$

Obviously,

$$\Rightarrow p_{photon}\, c > p_{electron}\, c + p_{positron}\, c$$

$$p_{photon} > p_{electron} + p_{positron}$$

Momentum will not be conserved without recoil of an extra mass.

2023

UNIVERSITY OF SRI JAYEWARDANEPURA - FACULTY OF APPLIED SCIENCES

B. Sc. General Degree Second Year Second Semester Course Unit Examination – March/April, 2023

DEPARTMENT OF PHYSICS

PHY 207 1.0 - Special Theory of Relativity

Time : One hour; No of Questions : 04; No of Pages : 02 & Total marks : 100

Answer all questions

Assume, velocity of light (c) = 3×10^8 ms^{-1}

01. Write down the **two** main Einstein's Postulates in Special Theory of Relativity (STR).

THEORY !!!

Obtain the following relativistic time equation, starting from the above postulates in STR.

$t^1 = \gamma\, t$, where, $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$; (symbols have their usual meanings).

THEORY !!!

Your starship passes the Earth with a relative speed of 0.9c. After traveling 10.0 years (your time), you stop turn, and then travel back to the Earth with the same relative speed. The trip back takes another 10.0 years (your time).

How long does the round trip take according to measurements made on Earth? (Neglect any effects due to the accelerations involved with stopping, turning, and getting back up to speed.)

(25 Marks)

Your starship passes the Earth with a relative speed of 0.9c. After traveling 10.0 years (your time), you stop turn, and then travel back to the Earth with the same relative speed. The trip back takes another 10.0 years (your time).

How long does the round trip take according to measurements made on Earth? (Neglect any effects due to the accelerations involved with stopping, turning, and getting back up to speed.)

02. Derive an expression for the length contraction ($l_2 = l_1\sqrt{1-v^2/c^2}$) starting from the relativistic time equation (Symbols have their usual meanings).

THEORY !!!

A meter stick in frame S^1 makes an angle of $30°$ with the x^1 axis. If that frame moves parallel to the x axis of frame S with speed 0.9 c relative to frame S, what is the length of the stick as measured from S ?

(25 Marks)

A meter stick in frame S^1 makes an angle of $30°$ with the x^1 axis. If that frame moves parallel to the x axis of frame S with speed $0.9\,c$ relative to frame S, what is the length of the stick as measured from S ?

03. Galaxy A is reported to be receding from us with a speed of 0.35c. Galaxy B, located in precisely the opposite direction, is also found to be receding from us at this same speed.

What multiple of c gives the recessional speed an observer on Galaxy A would find for

(a) our galaxy and

(b) Galaxy B ?

{You may assume that the Lorentz velocity transformation equation for the above case takes the following form;

$$U_x^1 = \frac{U_x - v}{1 - \frac{v}{c^2}U_x}$$

. Where symbols have their usual meanings.}

(25 Marks)

04. A sodium light source moves in a horizontal circle at a constant speed of $0.1c$ while emitting light at the proper wavelength of $\lambda_0 = 589$ nm. Wavelength λ is measured for that light by a detector fixed at the center of the circle.

What is the wavelength shift $\lambda - \lambda_0$?

{You may assume that the relationship between the observed frequency and the source frequency for the above case takes the following form;

$$f_o = \frac{f_s}{\gamma\left(1-\beta\ \cos\theta\right)}.$$

Where, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = \frac{v}{c}$ *and other symbols have their usual meanings.}*

(25 Marks)

Thank You !