Space & Atmospheric Physics

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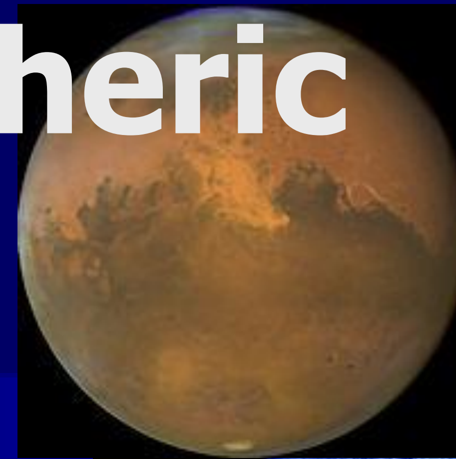
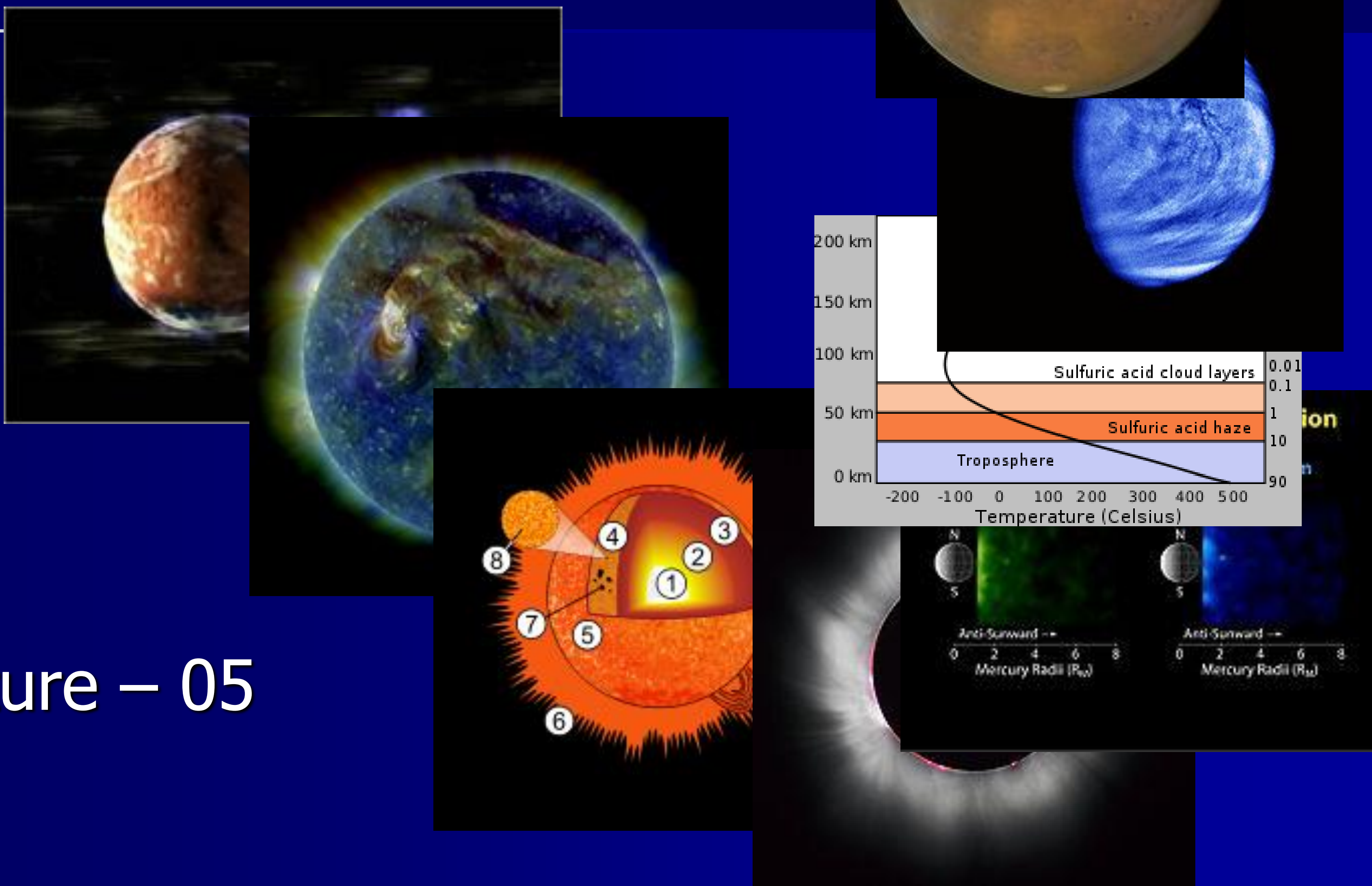


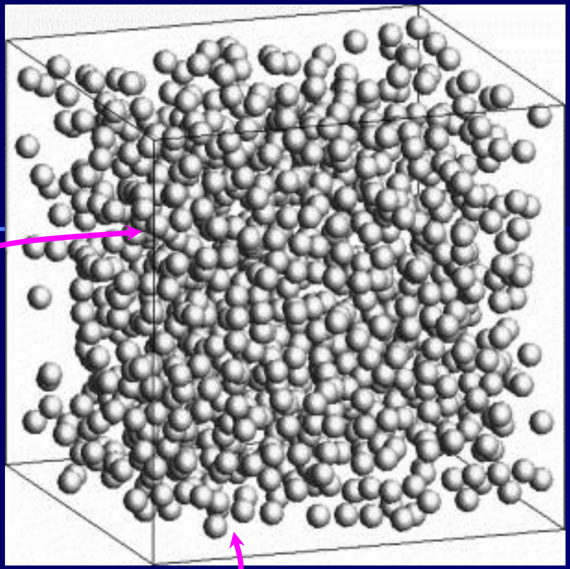


Lecture – 05

Earth Atmosphere

Retaining of Gases in the Earth
Major / Minor constituents
Barometric Equation
Scale Height
Atmospheric Regions
Temperature Profiles
Retaining of Gases
Number Density Profiles

Density of the Atoms



Assume there are *r* atoms in this volume

Masses of the atoms are:

$$m_1, m_2, m_3, \ldots, m_r$$

Number densities of those atoms are:

$$N_1, N_2, N_3, \ldots, N_r$$

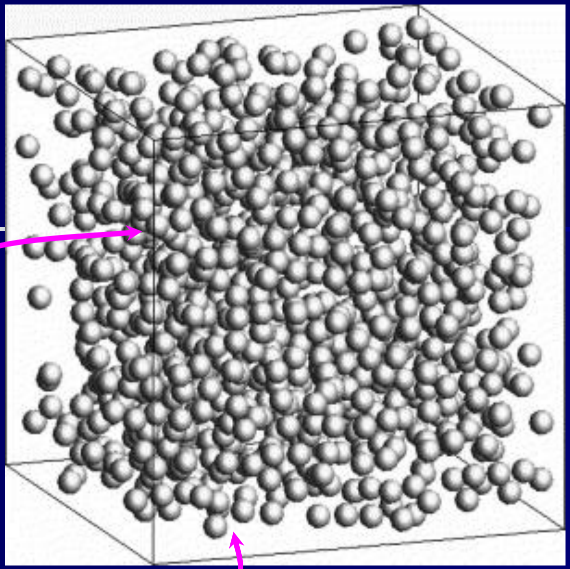
Total Mass of the atoms in the above volume:

$$m_1.N_1 + m_2.N_2, + m_3.N_3, \ldots, + m_r.N_r$$

(This is called the **density** because we consider the unit volume)

Total Molecular Number density: $N = N_1 + N_2, + N_3, \ldots, + N_r$

Density of the Atoms



Atoms, *r*

Volume, 1 m³

Mean Molecular mass : $\overline{m}$

$$\overline{m} = \frac{Total\ \ Mass}{Total\ \ Molecular\ \ Number\ \ Density}$$

$$\overline{m} = \frac{m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r}{N_1 + N_2 + N_3 + ... + N_r}$$

$$\overline{m} = \frac{m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r}{N}$$

Total Mass per unit volume

$$N.\overline{m} = m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r$$

Density

Density of the Atoms

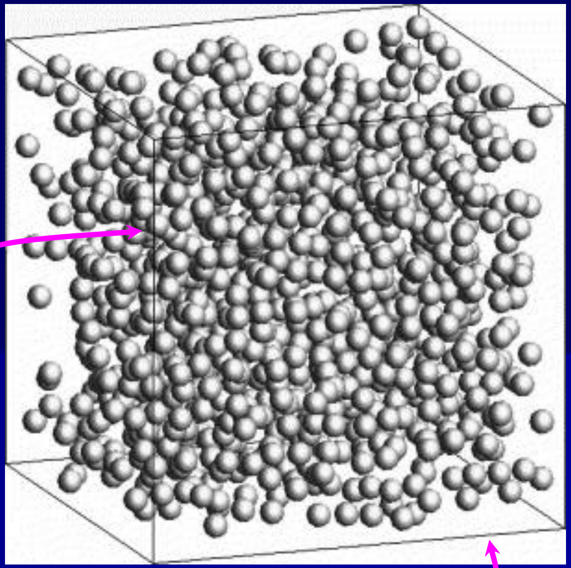
Mean Molecular Number Density

Density

$$\rho = N \times \overline{m}$$

Total Molecular Number Density

Atoms, *r*



Volume, 1 m³

For the Ideal Gas

$$PV = nRT$$

Number of molecules per volume, V

$$PV = \frac{NV}{N_o} RT$$

Avogadro Number (Number of molecules in a molecular weight)

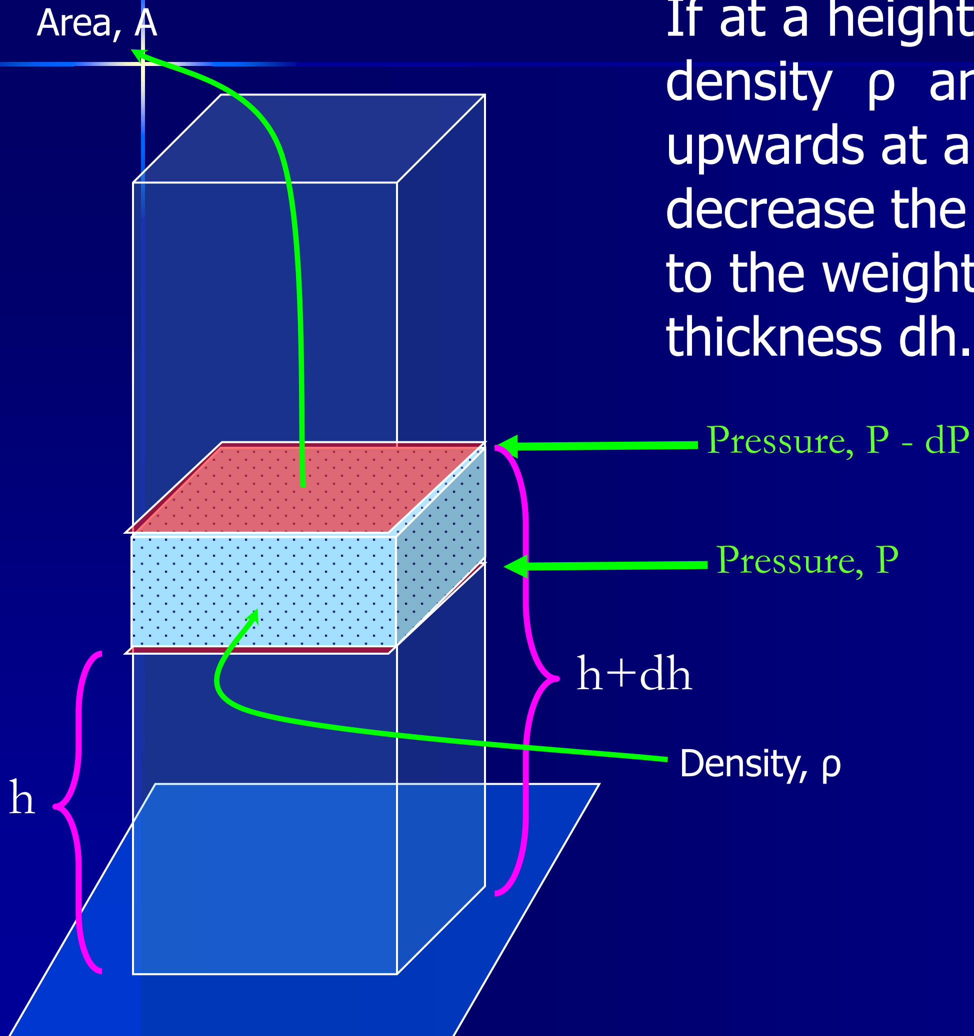
Boltzmann Constant

$$P = NkT$$

Where,

$$k = \frac{R}{N_o}$$

Pressure Profile



3-D View

The pressure at the Earth's surface (or at higher levels) is a result of the weight of the overlying atmosphere [force per unit area]. If at a height of h the atmosphere has density ρ and pressure P then moving upwards at an infinitesimally small dh will decrease the pressure by amount dP equal to the weight of the layer of atmosphere of thickness dh.

Pressure of the Lower Layer =

Pressure of the higher Layer +

$$\frac{\text{Weight of the air molecules in the selected part}}{\text{Cross area of the selected part}}$$

$$P = P - dP + \frac{A.dh.\rho.g}{A}$$

Pressure Profile

$$\frac{dP}{P} = -\frac{\overline{m}g}{kT}dh$$

The Pressure at height h can be written as:

$$P(h) = P_o\, e^{\frac{-\overline{m}g}{kT}h}$$

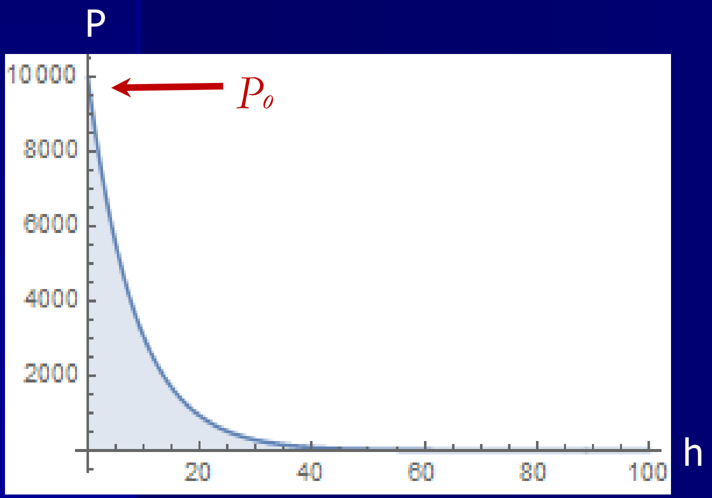
This is the general formula as the Pressure at height; This translate as the pressure decreasing exponentially with height !

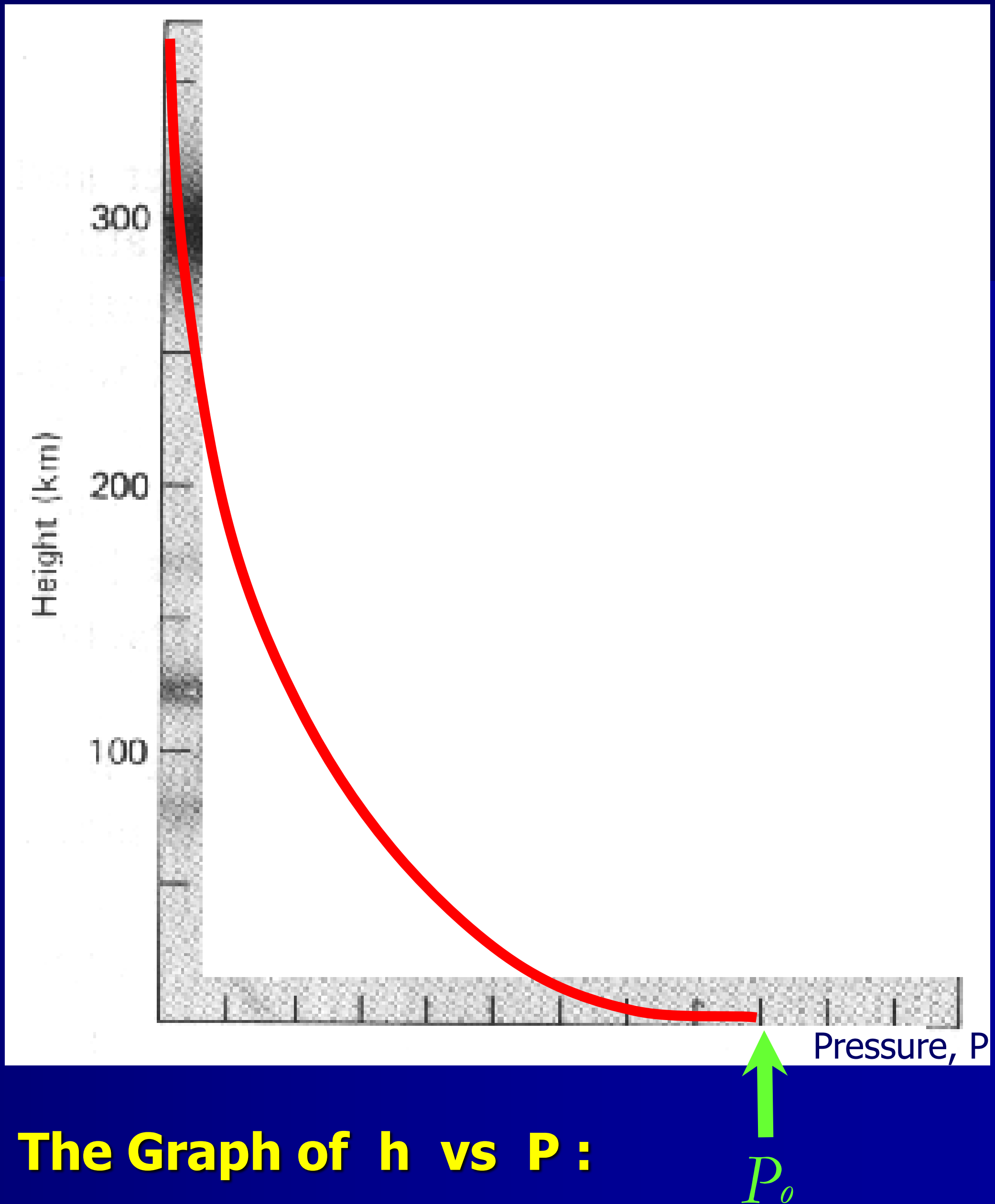
If h=0 then P=Po (1); That means Po is the pressure at h=0 level or The Ground Level.

Also $\frac{-\overline{m}g}{kT}h$ is independent of the units. That means $\frac{kT}{\overline{m}g}$ is also a some height !

The Graph of P vs h :

$$P(h) = P_o \, e^{\frac{-\overline{m}g}{kT}h}$$





The Graph of h vs P :

The Graph of h vs P :

Practical Values

Percent sea level pressure	Altitude (km)
100	0
50	5.6
10	16.2
1	31.2
0.1	48.1
0.01	65.1
0.001	79.2
0.00003	100

Theoretical Values...

```
In[166]:=
po = 100;
m = 5*10^(-26);
g = 10;
k = 1.4*10^(-23);
t = 300;
ph = 100;
hH = (k*t)/(m*g)/
Solve[ph == po * Ex
Out[172]= 8.4
Out[173]= {{h → 0.}}
```

```
po = 100;
m = 5*10^(-26);
g = 10;
k = 1.4*10^(-23);
t = 300;
ph = 50;
hH = (k*t)/(m*g)/1000
Solve[ph == po * Exp[-h/hH], h]
8.4
{{h → 5.82244}}
```

```
po = 100;
m = 5*10^(-26);
g = 10;
k = 1.4*10^(-23);
t = 300;
ph = 10;
hH = (k*t)/(m*g)/100
Solve[ph == po * Exp[-]
8.4
{{h → 19.3417}}
```

```
po = 100;
m = 5*10^(-26);
g = 10;
k = 1.4*10^(-23);
t = 300;
ph = 1;
hH = (k*t)/(m*g)/1000
Solve[ph == po * Exp[-h/hH], h]
8.4
{{h → 38.6834}}
```

Scale Height (H)

$$H = \frac{kT}{\overline{m}g}$$

where:

- k = Boltzmann constant = 1.38 x 10^{-23} J·K^{-1}
- T = mean planetary surface temperature in kelvins
- $\overline{m}$ = mean molecular mass of dry air (units kg)
- g = acceleration due to gravity on planetary surface (m/s^2)

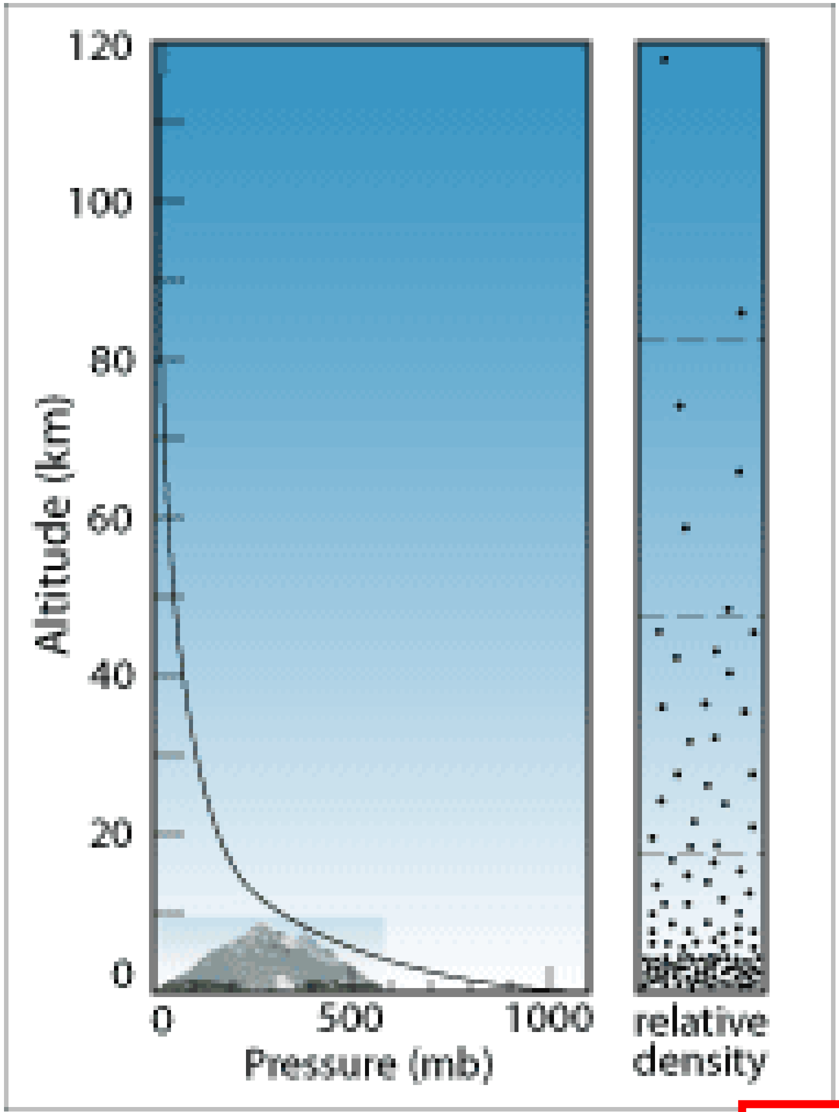
$$H = \frac{(1.4\times10^{-23})\times(300)}{(5.0\times10^{-26})\times(10)}$$

$$H = 8.4km$$

Theoretically this *H* is a constant. But practically this *H* is not a constant. Because, the values of "**mean molecular mass**", "**acceleration due to gravity**" and "**mean planetary surface temperature**" are changing with respect to height from the Earth surface.

The Graph of Scale Heights vs P :

Height	Pressure	
H	Po / e	0.36 Po
2 H	Po / e^2	0.13 Po
3 H	Po / e^3	0.04 Po
4 H	Po / e^4	0.01 Po
5 H	Po / e^5	0.006 Po
………	…….	
n H	Po / e^n	



Pressure and density decrease rapidly with altitude.

Scale Height of the Earth, H

Temp, *T* vs Sca Hght, *H*

T (K)	H (m)
290	8500
273	8000
260	7610
210	6000

bars		millibars		atmospheres		millimeters of mercury
1.013 bar	=	1013 mb	=	1 atm	=	760 mm Hg

Correspondence of atmospheric measurement units.

	Height (km)	Pressure	
6 x 1	6	Po / 2	Po / 2^1
6 x 2	12	Po / 4	Po / 2^2
6 x 3	18	Po / 8	Po / 2^3
6 x 4	24	Po / 16	Po / 2^4
6 x 5	30	Po / 32	Po / 2^5
	………	…….	
	6 n	Po / 2^n	

Molecular Number Density

Using the Pressure Equation : $$P(h) = P_o\, e^{\frac{-h}{H}}$$

Where, $H = 8.4km$

For the Ideal Gas

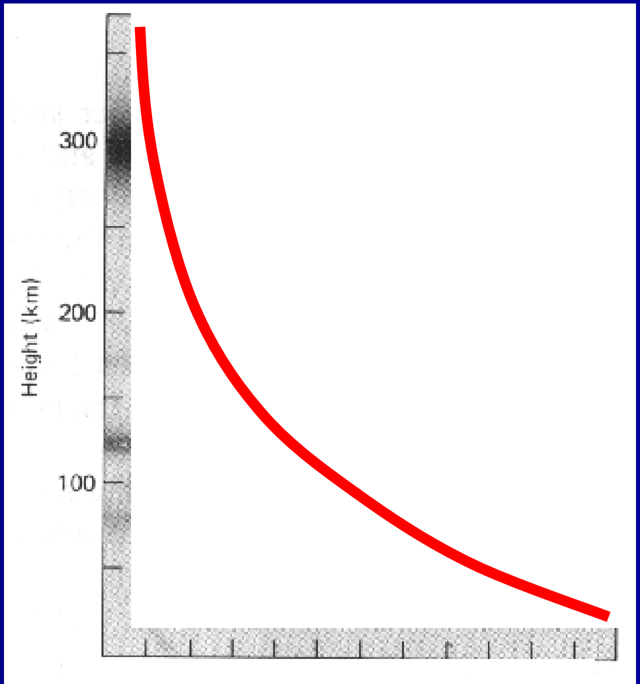
$$PV = nRT$$

$$P = NkT$$

$$N = \frac{P}{kT}$$

$$N(h) = \frac{P(h)}{kT}$$ **&** $$No = \frac{Po}{kT}$$

$$N(h) = N_o\, e^{-\frac{h}{H}}$$



Molecular Number Density

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$

If h = H,

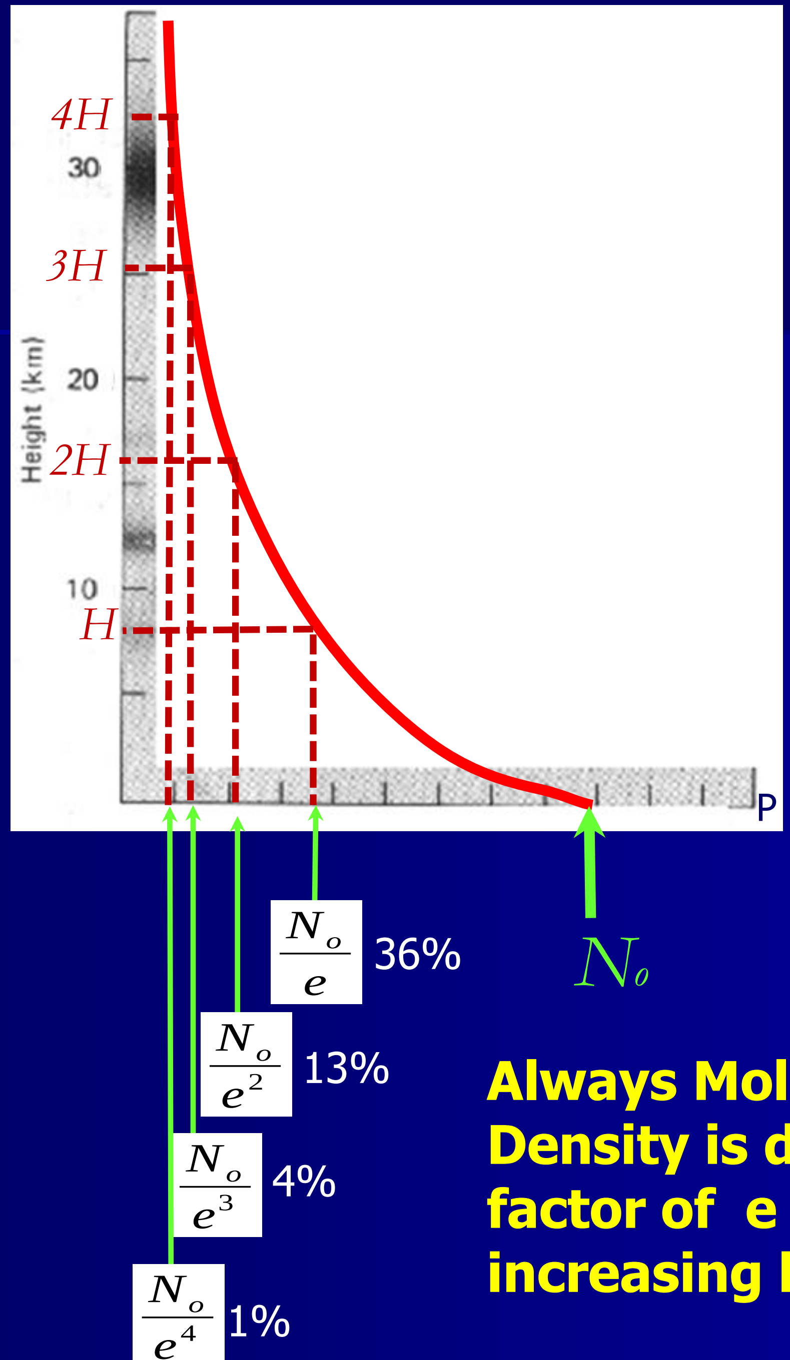
$$N(H) = N_o e^{\frac{-H}{H}}$$

$$\Rightarrow N(H) = \frac{N_o}{e}$$

$$\Rightarrow 0.36 N_o$$

Height	Mol Num Den	
H	No / e	0.36 No
2 H	No / e^2	0.13 No
3 H	No / e^3	0.04 No
4 H	No / e^4	0.01 No
5 H	No / e^5	0.006 No
....	....	
n H	No / e^n	

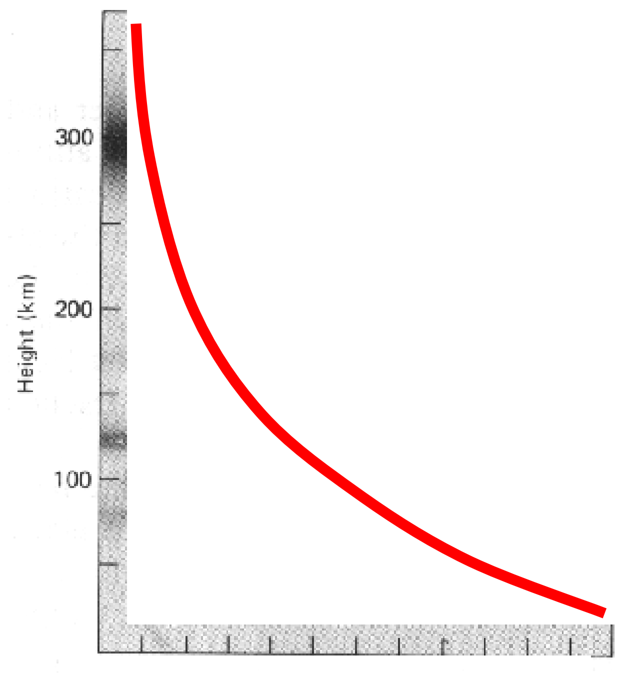
The Graph of H vs N :



Always Molecular Number Density is decreasing by a factor of e when height is increasing by a multiplies of H

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$



Molecular Number Density

Eg:
At which height from the surface of the Earth, which you can expect the Molecular Number Density which is half of that of the initial value of the Molecular Number Density ?

If N(h) = No/2 when h=h,

$$\frac{N_o}{2} = N_o e^{\frac{-h}{H}}$$

$$h = \sim 6km$$

	Height (km)	Pressure	
6 x 1	6	No / 2	No / 2^1
6 x 2	12	No / 4	No / 2^2
6 x 3	18	No / 8	No / 2^3
6 x 4	24	No / 16	No / 2^4
6 x 5	30	No / 32	No / 2^5
	………	…….	
	6 n	No / 2^n	

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$

If h=6 km Then N(h) = ?, $\Rightarrow$ $N = \frac{N_o}{2}$

If h=60 km Then N(h) = ?, $\Rightarrow$ $N = \frac{N_o}{2^{10}} =\sim \frac{N_o}{1000}$

If h=600 km Then N(h) = ?, $\Rightarrow$ $N = \frac{N_o}{2^{100}} =\sim \frac{N_o}{10^{30}}$

That means at 600 km height, the Molecular Number Density is (1/(10^30)) from its initial value.

Consider Linear Distance ;
At 600 km height, the Molecular Linear Distance is (1/(10^30))^(1/3) = (1/(10^10)) from its initial value.

$$= \left(\frac{1}{10^{30}}\right)^{\frac{1}{3}}$$

Molecular Number Density

$$= \left(\frac{1}{10^{30}} \right)^{\frac{1}{3}}$$

Linear Distance of the molecules = **Mean Free Path** ;

This is "Separation between two atoms"

Mean Free Path on the ground level $= 6.0 \times 10^{-8}$ m

Mean Free Path at altitude 600 km height from the ground level
:

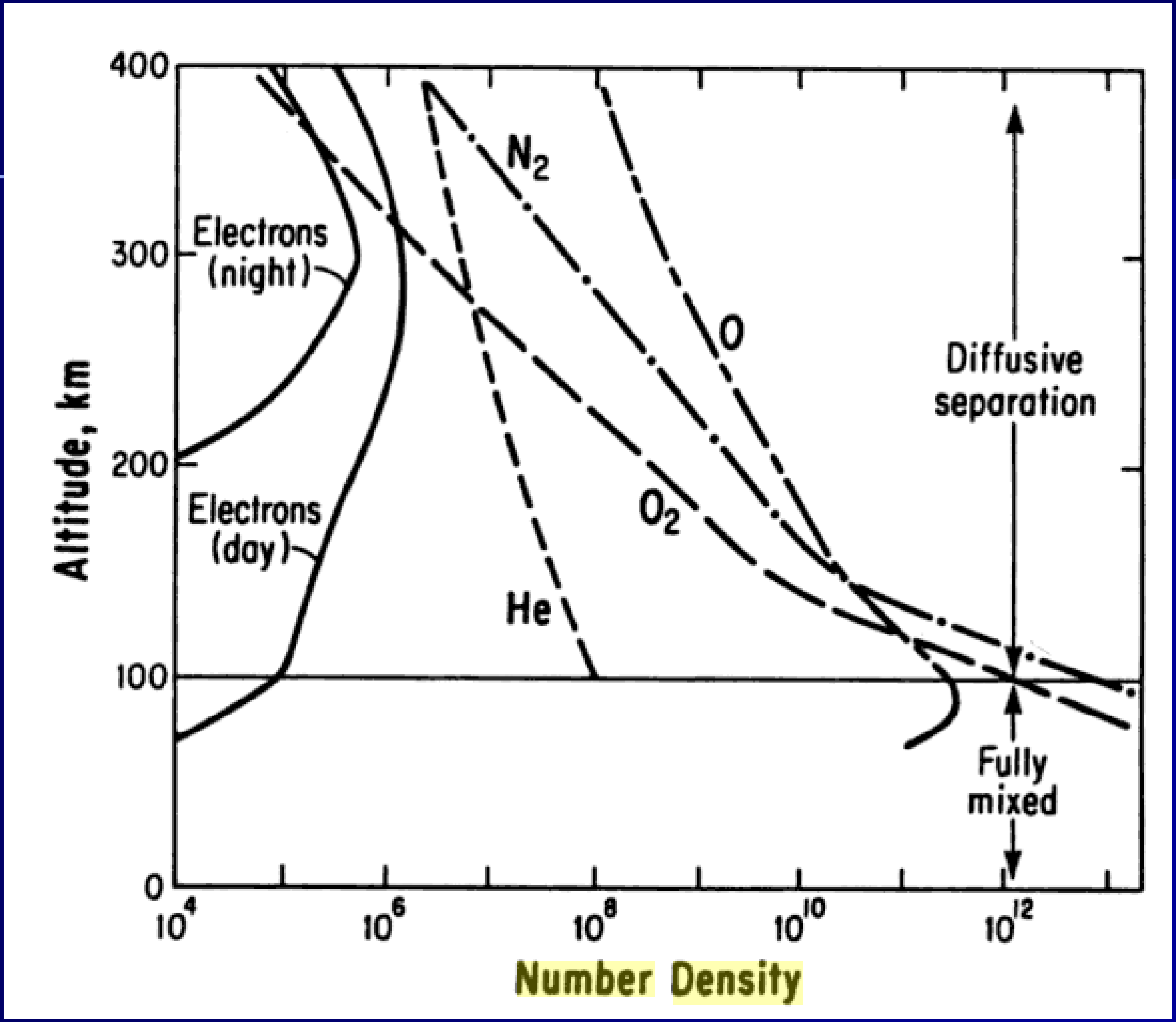
$$= 6 \times 10^{-8} \times \left(10^{30}\right)^{\frac{1}{3}}$$

$$= 6 \times 10^{-8} \times 10^{10}$$

$$= 600\,m$$

That means the **gap between two atoms** on that 600 km height (altitude) from the ground level is very high ! At that level there is no mean "**The gas**", because the **mean free path is very high** (600 m)

Molecular Number Density



Density

Using the Molecular Number Density Equation :

$$N(h) = N_o e^{-\frac{h}{H}}$$

Where, $H = 8.4km$

Mean Molecular Number Density

Density

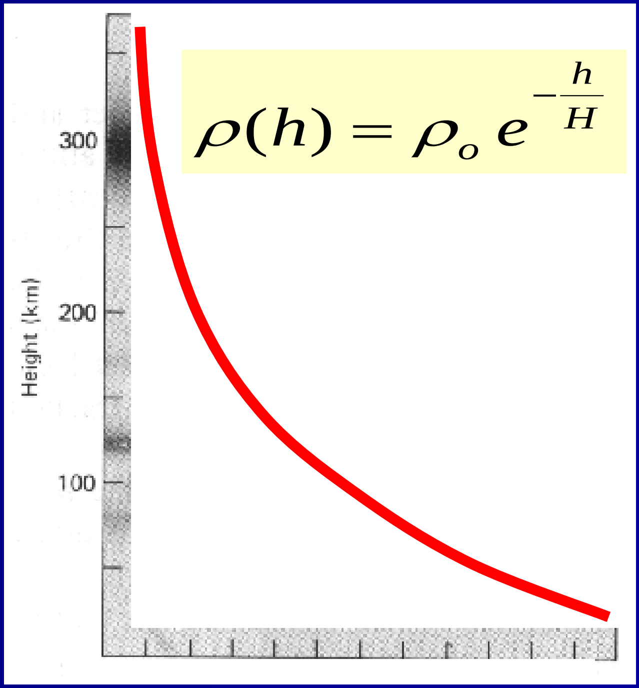
$$\rho = N \times \overline{m}$$

Total Molecular Number Density

$$\rho(h) = N(h) \times \overline{m}$$

&

$$\rho_o = N_o \times \overline{m}$$

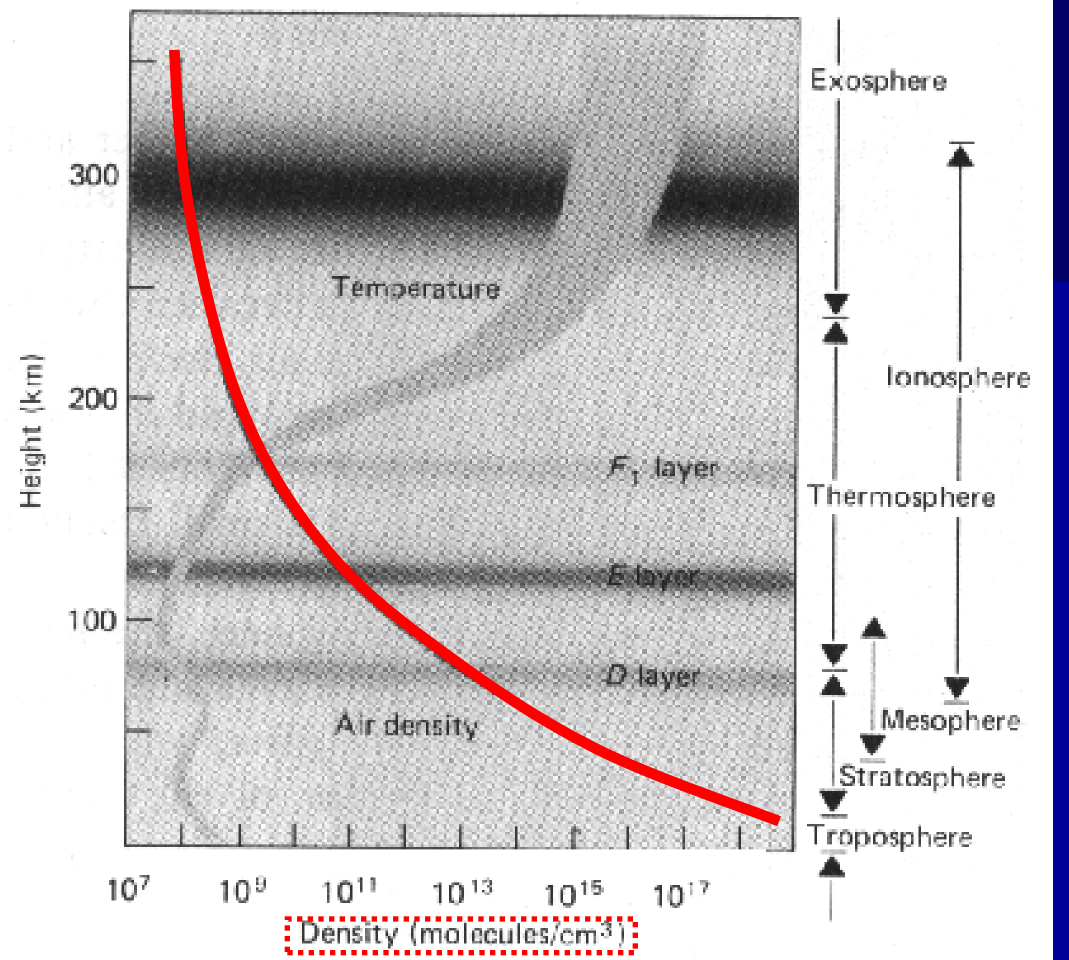


Density

Density

$$\rho(h) = \rho_o \, e^{-\frac{h}{H}}$$

Where, $H = 8.4km$



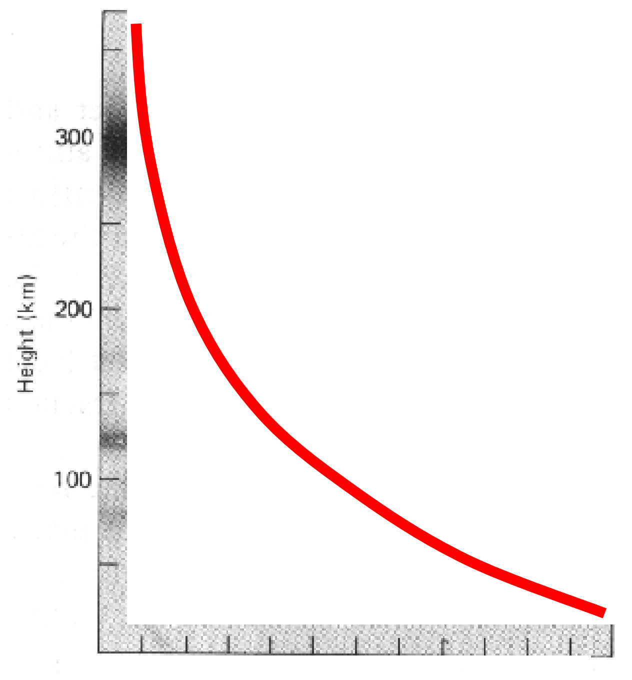
Variations in the density of Earth's neutral atmosphere with altitude

If h = H,

$$\rho(H) = \rho_o \, e^{\frac{-H}{H}} \Rightarrow \rho(H) = \frac{\rho_o}{e} \Rightarrow 0.36\rho_o$$

Density

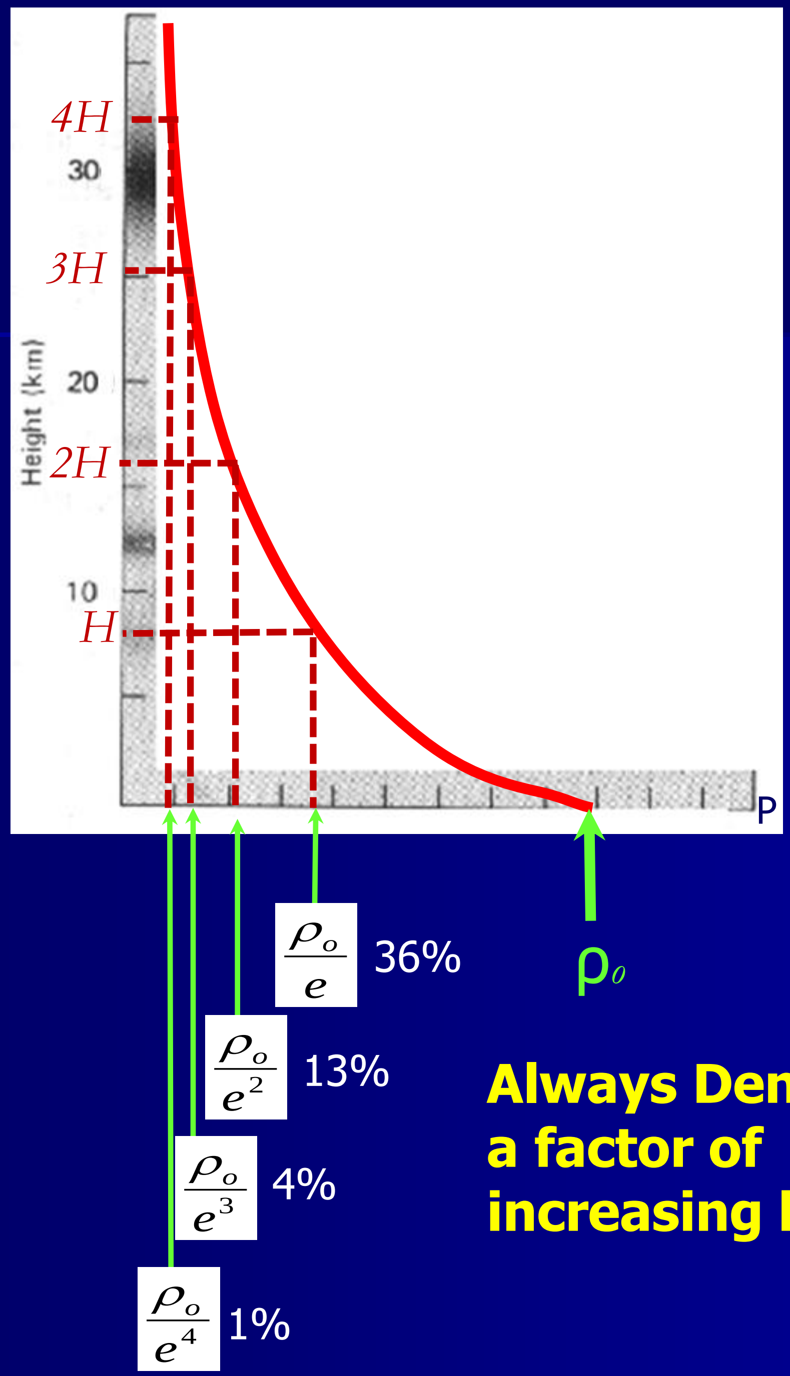
$$\rho(h) = \rho_o\, e^{-\frac{h}{H}}$$



Density

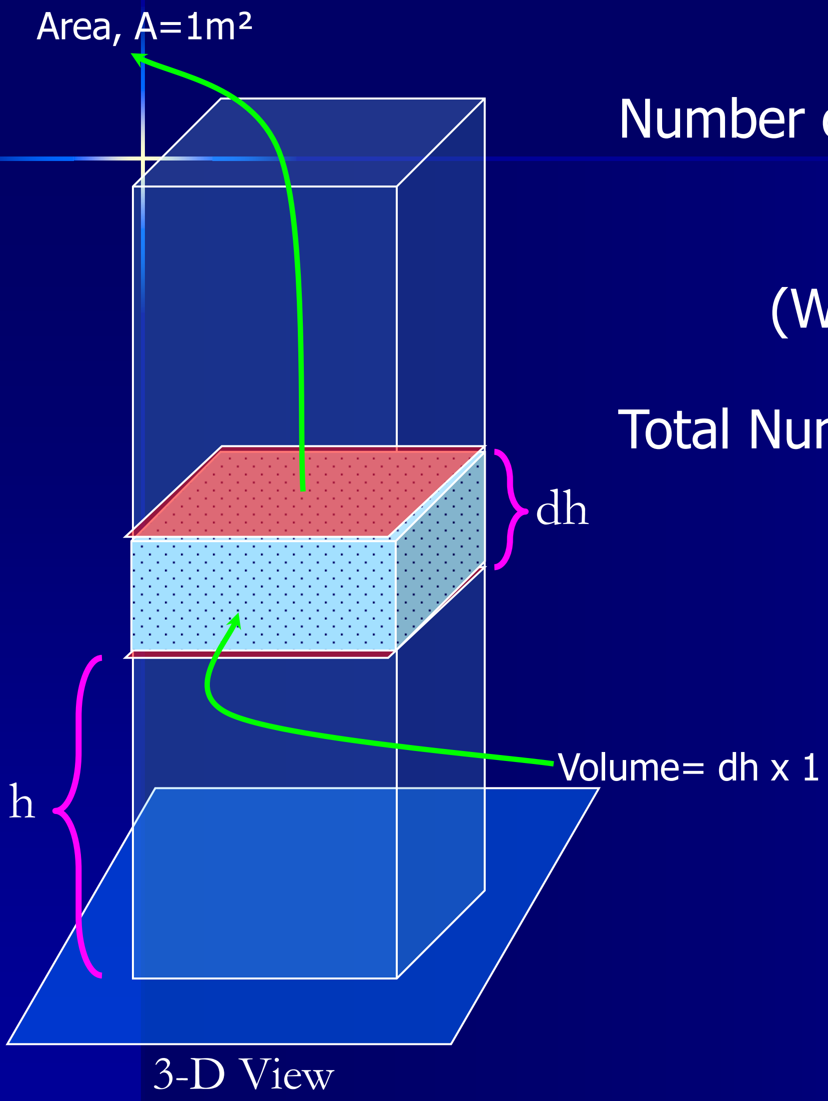
Height	Mol Num Den	
H	ρo / e	0.36 ρo
2 H	ρo / e^2	0.13 ρo
3 H	ρo / e^3	0.04 ρo
4 H	ρo / e^4	0.01 ρo
5 H	ρo / e^5	0.006 ρo
………	…….	
n H	ρo / e^n	

The Graph of H vs ρ :



Always Density is decreasing by a factor of e when height is increasing by a multiplies of H

Total Number of Molecules from Earth Surface to altitude h :



Number of molecules in a selected part =

$$N \times dh \times 1$$

(Where N is the molecular number density)

Total Number of molecules from h=h to h=∞

$$\int_{h=h}^{\infty} N.dh$$

Where,

$$N(H) = N_o e^{\frac{-h}{H}}$$

$$\int_{h=h}^{\infty} N_o e^{\frac{-h}{H}} .dh$$

Total Number of Molecules from Earth Surface to altitude h :

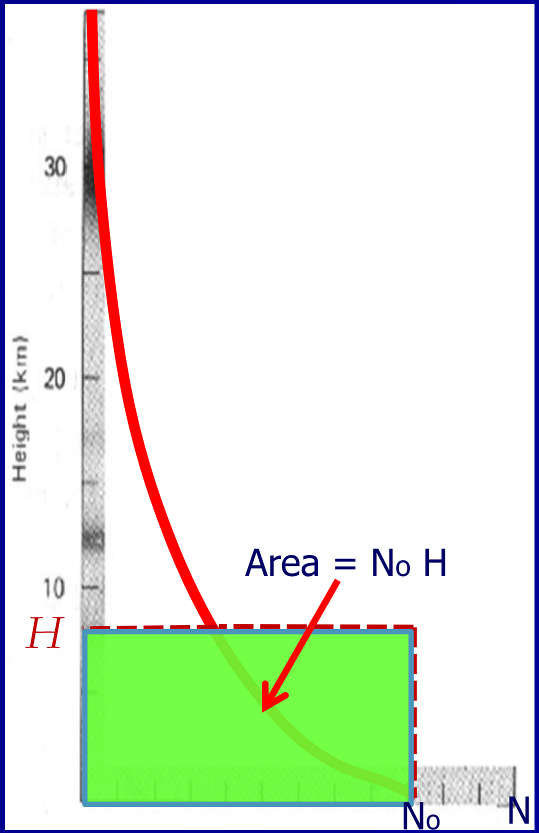
$$N_{\substack{Total \\ h\rightarrow\infty}} = N_o H \; e^{-\frac{h}{H}}$$

Case I :

$$N_{\substack{Total \\ 0\rightarrow\infty}} = N_o H$$

That means, if the molecular number density of the atmosphere of the Earth varies **linearly** without varying **exponentially**, the atmosphere of the Earth will diminish after ~8.4 km (a scale height).

This gives to us another definition for the Scale Height !



Total Number of Molecules from Earth Surface to altitude h :

$$N_{\underset{h\to\infty}{Total}} = N_o H \; e^{-\frac{h}{H}}$$

Case II :

$$\frac{N_{\underset{h\to\infty}{Total}}}{N_{\underset{0\to\infty}{Total}}} = \frac{N_o H \; e^{-h/H}}{N_o H} = e^{-h/H}$$

Fraction of the Number of Molecules from the specific height h.

If h=H km Then RATIO = ?, ➡ $\left(e^{-h/H}\right)_{h\to H} = e^{-H/H} = e^{-1}$

~ 40 %

60 % of the total molecules exist bellow H (8.4 km) !

Total Number of Molecules from Earth Surface to altitude h :

If h=2H km Then RATIO = ?, $\left(e^{-h/H}\right)_{h\to 2H} = e^{-2H/H} = e^{-2}$

~ 15 %

85 % of the total molecules exist bellow 2H (16.8 km) !

If h=3H km Then RATIO = ?, $\left(e^{-h/H}\right)_{h\to 3H} = e^{-3H/H} = e^{-3}$

~ 5 %

95 % of the total molecules exist bellow 3H (16.8 km) !

h (km)		N (h →∞) / N (0 →∞)	% below h
H	08.4	36.78	63.21
2 H	16.8	13.53	86.46
3 H	25.2	4.97	95.02
4 H	36.6	1.83	98.16
5 H	42.0	0.67	99.32
6 H	50.4	0.24	99.75
7 H	58.8	0.09	99.90
8 H	67.2	0.03	99.96
9 H	75.6	0.01	99.98
10 H	84.0	0.004	99.995

Sketch the size of the Earth's Atmosphere

20 cm straight line

This is the size of the Earth's Atmosphere

If we assume the earth to be an Orange which has a radius of 20 cm; then the peel (rind) of the orange is like the atmosphere of the Earth!

1 mm thick line

Earth Atmosphere

Retaining of Gases in the Earth
Major / Minor constituents
Barometric Equation
Scale Height
Atmospheric Regions
Temperature Profiles
Retaining of Gases
Number Density Profiles


Thank You !
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