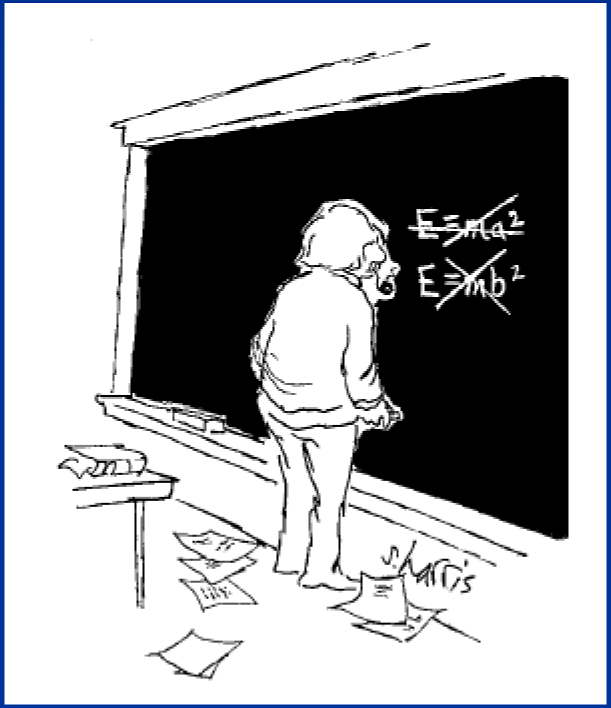
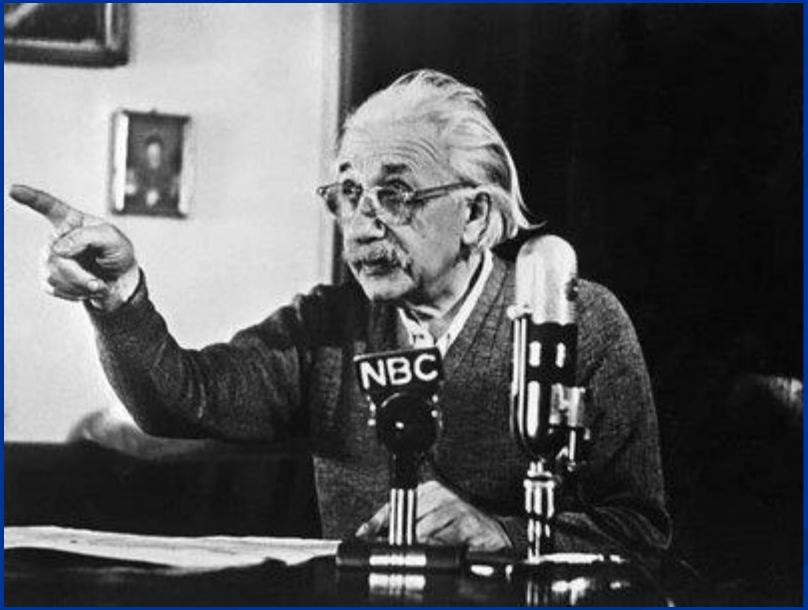
Special Theory of Relativity





12th Lecture

UNIVERSITY OF SRI JAYEWARDANEPURA - FACULTY OF APPLIED SCIENCES

B. Sc. General Degree Second Year Second Semester Course Unit Examination - **April/May, 2022**

DEPARTMENT OF PHYSICS

PHY 207 1.0 / PHY 257 1.0 / PHY 302 1.0 / PHY 327 1.0 - Special Theory of Relativity

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Time : One hour; No of Questions : 04; No of Pages : 02 & Total marks : 100

Answer all questions

Assume, velocity of light (c) = 3×10^8 ms^{-1}

01. Write down the **two** main Einstein's Postulates in Special Theory of Relativity (STR).

Obtain the following relativistic time equation, starting from the above postulates in STR.

$$t^1 = \gamma\, t \;, \quad \text{where, } \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$; (symbols have their usual meanings).

The mean lifetime of stationary muons is measured to be 2.20 ms. The mean lifetime of high-speed muons in a burst of cosmic rays observed from Earth is measured to be 16.0 ms. What is the speed of these cosmic-ray muons relative to Earth?

(25 Marks)

Write down the **two** main Einstein's Postulates in Special Theory of Relativity (STR).

Obtain the following relativistic time equation, starting from the above postulates in STR.

$$t^1 = \gamma\, t \ , \quad \text{where, } \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \ ; \ \text{(symbols have their usual meanings).}$$

The mean lifetime of stationary muons is measured to be 2.20 ms. The mean lifetime of high-speed muons in a burst of cosmic rays observed from Earth is measured to be 16.0 ms. What is the speed of these cosmic-ray muons relative to Earth?

$$t_2 = t_1 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Untitled-1 * - Wolfram Mathematica 12.1

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```
In[4]:= t1 = 2.2
        t2 = 16.0
        Solve[t2 == t1 * (1 / Sqrt[1 - v^2 / c^2]), v]

Out[4]= 2.2

Out[5]= 16.

Out[6]= {{v → -0.990502 c}, {v → 0.990502 c}}
```

02. Derive an expression for the length contraction ($l_2 = l_1\sqrt{1-{v^2}/{c^2}}$) starting from the relativistic time equation (Symbols have their usual meanings).

A rod lies parallel to the x axis of reference frame S, moving along this axis at a speed of 0.6 c. Its rest length is 2.0 m. What will be its measured length in frame S?

(25 Marks)

Derive an expression for the length contraction ($l_2 = l_1\sqrt{1-{v^2}/{c^2}}$) starting from the relativistic time equation (Symbols have their usual meanings).

A rod lies parallel to the x axis of reference frame S, moving along this axis at a speed of 0.6 c. Its rest length is 2.0 m. What will be its measured length in frame S?

$$l^1 = l\sqrt{1-\frac{v^2}{c^2}}$$

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```
In[1]:= l = 2.0
        v = 0.6 c
        ld = l * Sqrt[1 - (v^2 / c^2)]

Out[1]= 2.

Out[2]= 0.6 c

Out[3]= 1.6
```

03.

Let us assume two objects **A** and **B** are moving in an opposite direction to each other with constant velocities $\mathbf{V_A}$ and $\mathbf{V_B}$ respectively. **Find the relative velocity** of B with respect to A, $V_{(B,A)}$ starting from the Lorentz velocity transformation equation.

A particle moves along the x^1 axis of frame S^1 with velocity 0.40 c. Frame S^1 moves with velocity 0.60 c with respect to frame S. What is the velocity of the particle with respect to frame S?

{You may assume that the Lorentz velocity transformation equation for the above case takes the following form;

$$U_x^1 = \frac{U_x - v}{1 - \frac{v}{c^2}U_x}.$$ *Where symbols have their usual meanings.}*

(25 Marks)

Let us assume two objects **A** and **B** are moving in an opposite direction to each other with constant velocities **V_A** and **V_B** respectively. **Find the relative velocity** of B with respect to A, $V_{(B,A)}$ starting from the Lorentz velocity transformation equation.

Relative Motion for the two bodies in Relativity

Let us assume two objects are moving in an opposite direction to each other,

Using Lorentz transformation equations; $u_x' = \dfrac{u_x - v}{1 - \frac{v}{c^2} u_x}$

For this example; $U_x^1 = V_A$, $U_x = -V_B$ and $v = V_{(B,A)}$

Direction of B is opposite to the A

$$U_x^1 = \frac{U_x - v}{1 - \frac{U_x v}{c^2}} \Rightarrow V_A = \frac{(-V_B) - v}{1 - \frac{(-V_B)v}{c^2}} \Rightarrow -v = \frac{V_A + V_B}{1 + \frac{V_A V_B}{c^2}}$$

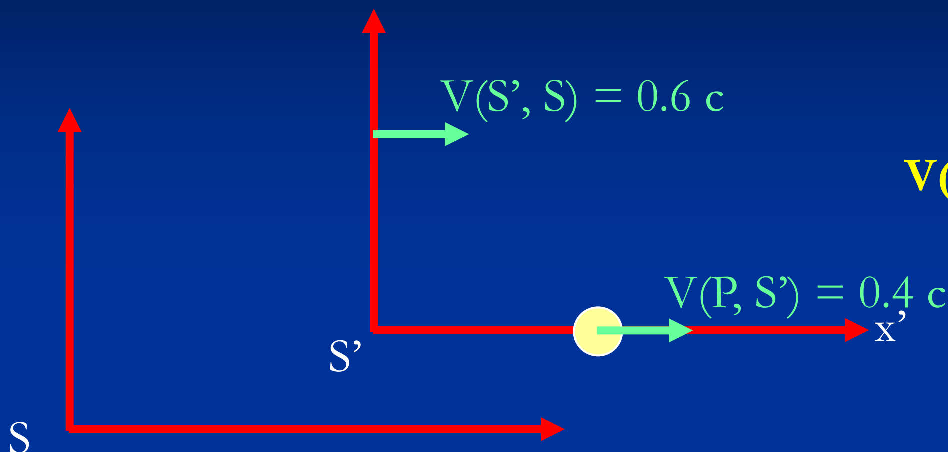
$$-v = \frac{V_A + V_B}{1 + \frac{V_A V_B}{c^2}}$$

This v denotes $V(B, A)$. $V(B, A)$ has a negative value. ∴ The direction of $V(B, A)$ should be the opposite direction. ∴ $V(A, B)$ is +ve;

$$v = V_{(A,B)} = \frac{V_A + V_B}{1 + \frac{V_A V_B}{c^2}}$$

Or $v = V_{(B,A)}$

A particle moves along the x^1 axis of frame S^1 with velocity 0.40 c. Frame S^1 moves with velocity 0.60 c with respect to frame S. What is the velocity of the particle with respect to frame S?



V(P, S) = ???

$$v = V_{(P,S)} = \frac{V_{P,S'} + V_{S',S}}{1 + \dfrac{V_{P,S'} V_{S',S}}{c^2}}$$

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```
In[4]:= vpsd = 0.4 C
        vsds = 0.6 C
        v = (vpsd + vsds) / (1 + ((vsds * vpsd) / C^2))

Out[4]= 0.4 C

Out[5]= 0.6 C

Out[6]= 0.806452 C
```

04. A spaceship, moving away from Earth at a speed of 0.9 c, reports back by transmitting a signal at a frequency (measured in the spaceship frame) of 100 MHz. **To what frequency** must Earth receivers be tuned to receive the report?

{You may assume that the relationship between the observed frequency and the source frequency for the above case takes the following form;

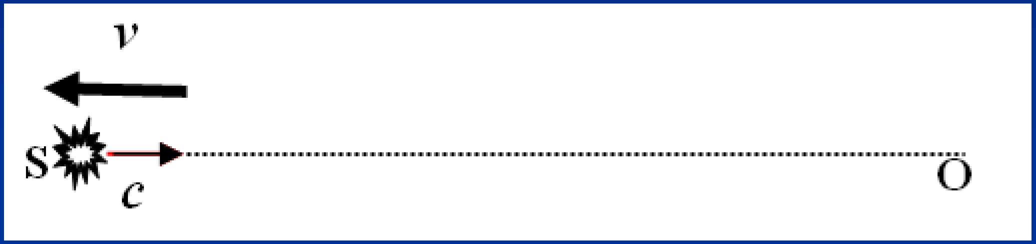
$$f_o = \frac{f_s}{\gamma\,(1-\beta\ \cos\theta)}.$$

Where, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = \frac{v}{c}$ *and other symbols have their usual meanings.}*

(25 Marks)

A spaceship, moving away from Earth at a speed of 0.9 c, reports back by transmitting a signal at a frequency (measured in the spaceship frame) of 100 MHz. **To what frequency** must Earth receivers be tuned to receive the report?

(b) If the source is receding directly from the observer:



Then, $\theta = \pi$ and $\cos\theta = -1$ ➡ $f_o = f_s \dfrac{1}{\gamma(1+\beta)}$ where, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

$\beta = \dfrac{v}{c}$

➡ $$f_o = f_s \sqrt{\frac{1-\beta}{1+\beta}}$$

$\beta = \dfrac{0.9c}{c}$ $\quad f_s = 100 MHz$ ➡ $f_o = f_s \sqrt{\dfrac{1-\beta}{1+\beta}} = ?$

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March, 2021

DEPARTMENT OF PHYSICS

PHY 207 1.0 / PHY 257 1.0 / PHY 302 1.0 / PHY 327 1.0 - Special Theory of Relativity

Time : One hour

No of Questions : 04

No of Pages : 02

Total marks : 60

Answer all questions

Assume, velocity of Light (c) = 3×10^8 ms^{-1}

01. Particle X, which is created in a particle accelerator, travels a total distance of *100.0 m* between two detectors in *410 ns* as measured in the laboratory frame before decaying into other particles.

What is the lifetime of the particle X as measured in its own frame.

(15 Marks)

Velocity of the particle X w.r.t lab frame :

$$v = \frac{dis\tan ce}{time}$$

$$v = \frac{100\,m}{410\,ns}$$

$$v = 2.44 \times 10^{8}\ ms^{-1}$$

Using relativistic time equation :

$$t_2 = t_1 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t_1 = ?$

$t_2 = 410 \times 10^{-9}\ s$

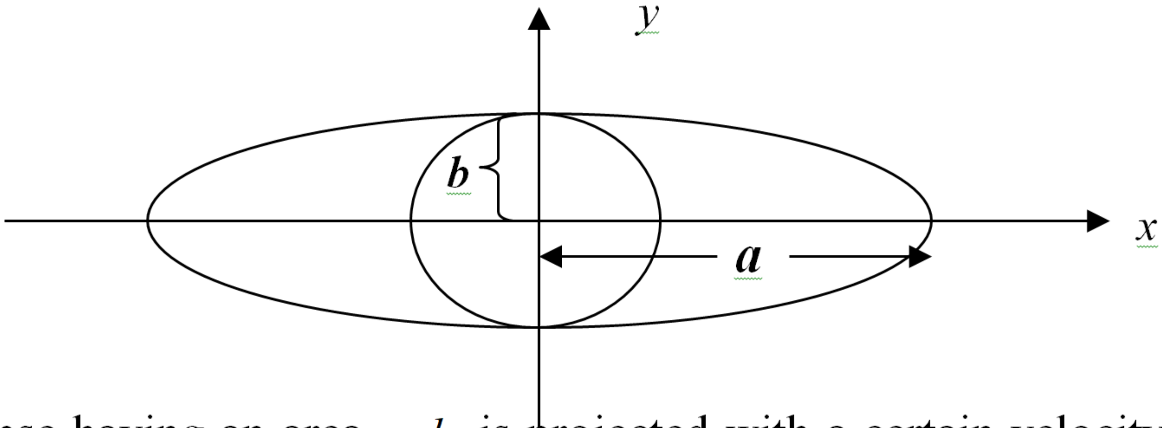
$v = 2.44 \times 10^{8}\ ms^{-1}$

$$t_1 = t_2 \sqrt{1 - \frac{v^2}{c^2}}$$

$$t_1 = 410 \sqrt{1 - \frac{(2.44 \times 10^{8})^2}{(3.0 \times 10^{8})^2}}$$

$$t_1 = 238\,ns$$

02.



An ellipse having an area πab is projected with a certain velocity. It was observed that the ellipse appears as a circle of area πb^2. Determine the velocity of projection of the ellipse. (Where, $a \rangle b$.)

(15 Marks)

$$l^1 = l\sqrt{1-\frac{v^2}{c^2}}$$

$l = a$ and $l' = b$ ➡ $b = a\sqrt{1-\frac{v^2}{c^2}}$ ➡ $b^2 = a^2(1-\frac{v^2}{c^2})$

➡ $\frac{v^2}{c^2}a^2 = a^2 - b^2$ ➡ $v^2 = c^2\frac{a^2-b^2}{a^2}$ ➡ $v = \frac{c}{a}\sqrt{a^2-b^2}$

03. Let A be the twin on the earth and B be the twin in the ship in the <u>twin paradox episode</u>. Comment on the following statement using your knowledge of special theory of relativity.

" The twin B can go to the future, but <u>can not</u> go to the past "

(15 Marks)

"The twin B can go to the FUTURE, but can not go to the PAST"

What is FUTURE ?

What is PAST ?

"The twin B can go to the FUTURE; but not his own FUTURE and its other one's FUTURE !!!

"The twin B can not go to the PAST"; according to the Relativity (STR) time can not negative !!!

04. A spacecraft moves towards the Earth with velocity $c/2$ as viewed from Earth's frame. The spacecraft emits light of wave length λ as measured in its own frame. The wave length of light as seen by an observer on the Earth is $6000\ \mathring{A}$. ($1\ \mathring{A} = 10^{-10}$ m)

Find the value of λ.

{You may assume that the relationship between the observed frequency and the source frequency for the above case takes the following form;

$$f_o = \frac{f_s}{\gamma\ (1-\beta\ \cos\theta)}.$$

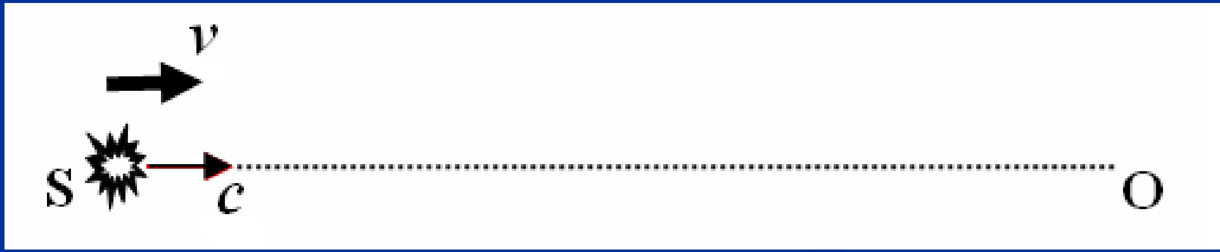
Where, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = \dfrac{v}{c}$ *and other symbols have their usual meanings.}*

(15 Marks)

A spacecraft moves towards the Earth with velocity $c/2$ as viewed from Earth's frame. The spacecraft emits light of wave length λ as measured in its own frame. The wave length of light as seen by an observer on the Earth is 6000 $\mathring{A}$. (1 $\mathring{A}$ = 10^{-10} m)

Find the value of λ.

(a) If the source is directly approaching the observer:



Then, $\boxed{\theta = 0}$ and $\boxed{\cos\theta = 1}$ ➡ $\boxed{f_o = f_s \dfrac{1}{\gamma(1-\beta)}}$ where, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

$\beta = \dfrac{v}{c}$

➡ $$f_o = f_s \sqrt{\frac{1+\beta}{1-\beta}}$$

A spacecraft moves towards the Earth with velocity $c/2$ as viewed from Earth's frame. The spacecraft emits light of wave length λ as measured in its own frame. The wave length of light as seen by an observer on the Earth is 6000 $\overset{o}{A}$. (1 $\overset{o}{A}$ = 10^{-10} m)

Find the value of λ.

$$f_o = f_s \sqrt{\frac{1+\beta}{1-\beta}}$$

$\beta = \frac{c/2}{c}$ & $f_s = (\lambda)$ and $f_o = (6000\,\overset{o}{A})$

$$C = f\lambda$$

$f_s = \frac{C}{\lambda}$ and $f_o = \frac{C}{6000\,\overset{o}{A}}$

$$\frac{C}{6000\,\overset{o}{A}} = \frac{C}{\lambda\,\overset{o}{A}} \sqrt{\frac{1-\beta}{1+\beta}} \rightarrow \lambda \Rightarrow$$ can be calculated !!!

Thank You !