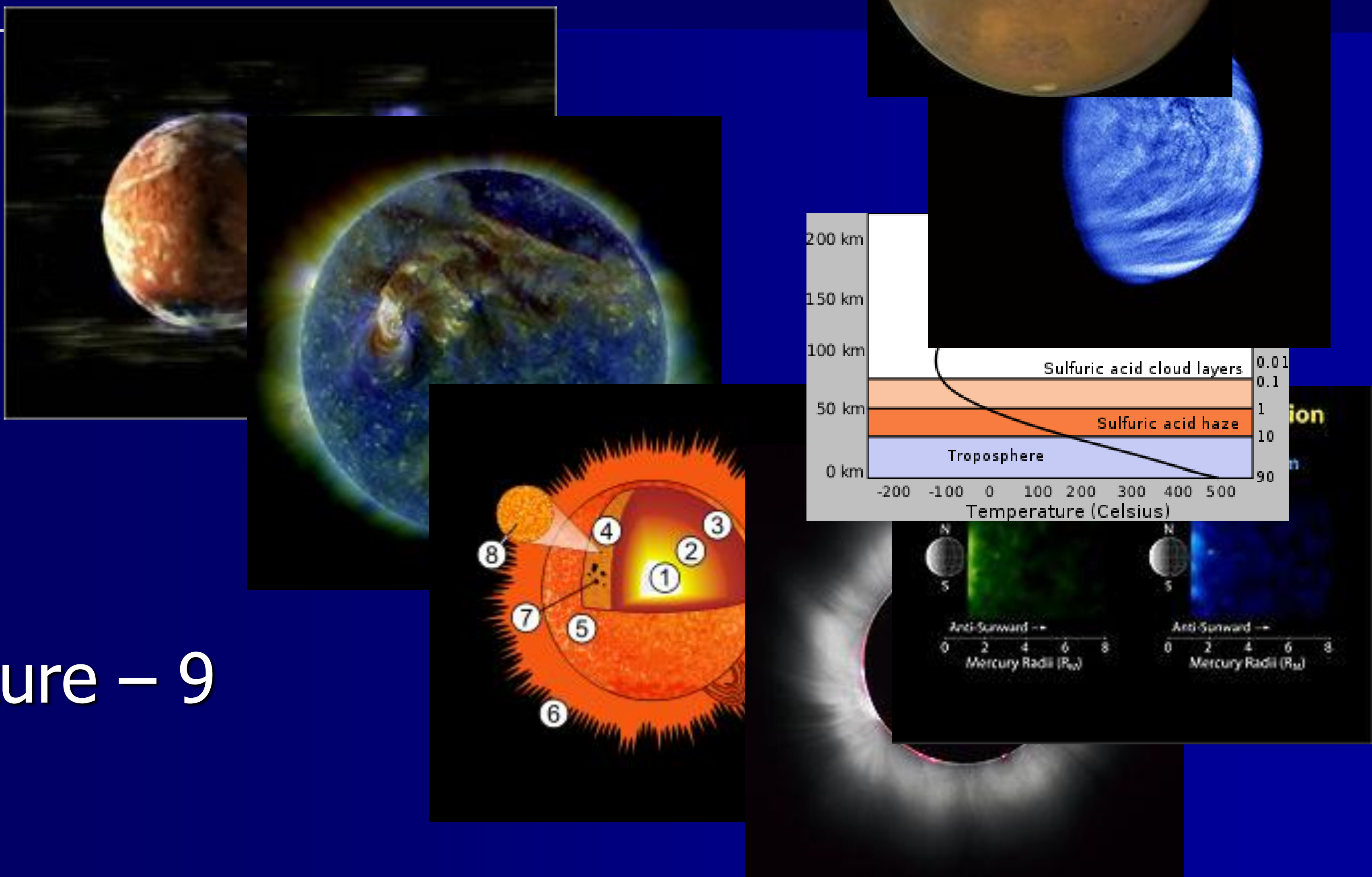
Space Physics

Space & Atmospheric Physics



Lecture – 9

Penetration Depth

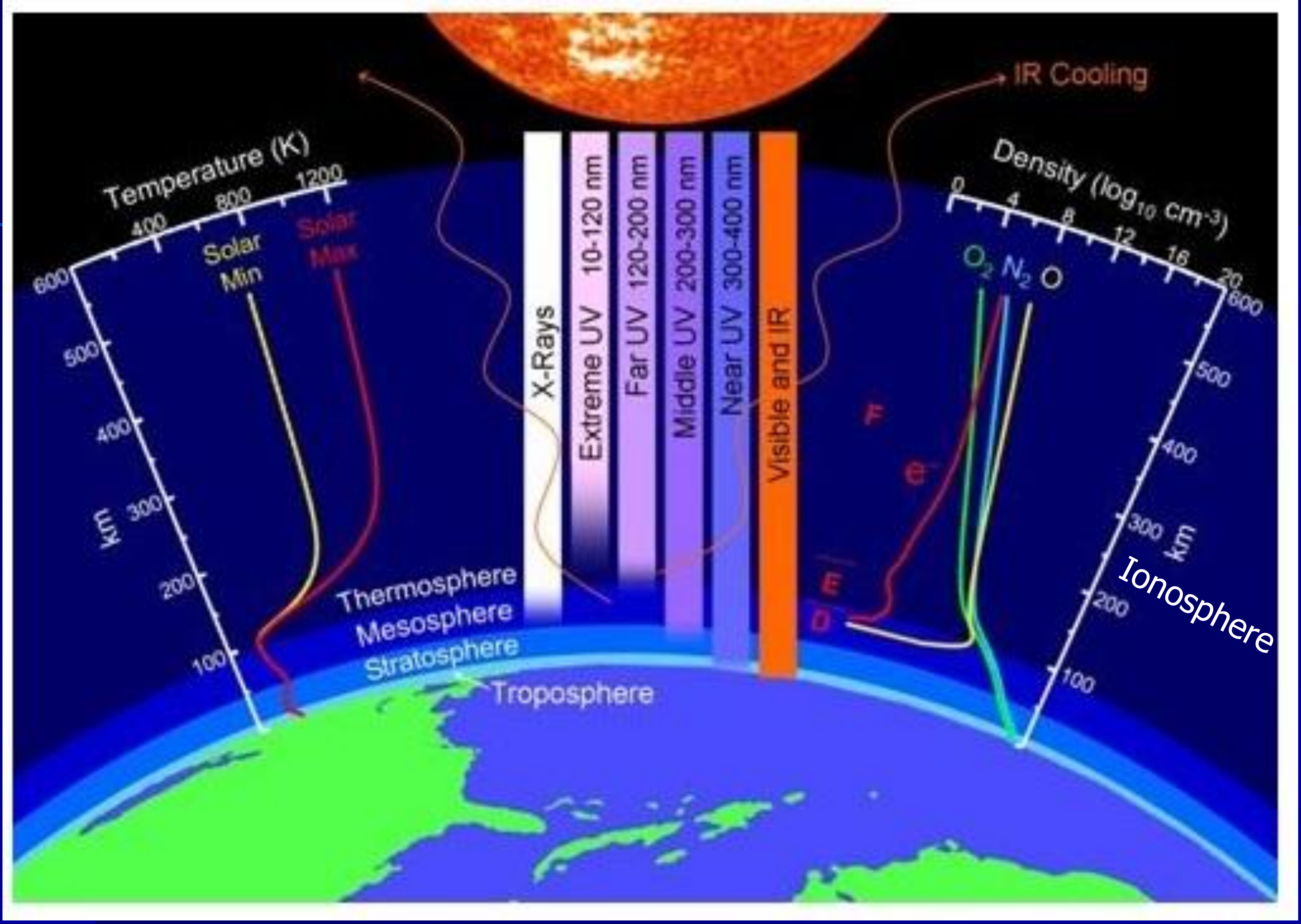
Penetration Depth is defined as the depth at which the intensity of the radiation in the atmosphere falls to 1/e (~37%) of its original value of the surface.

The equation of the intensity;

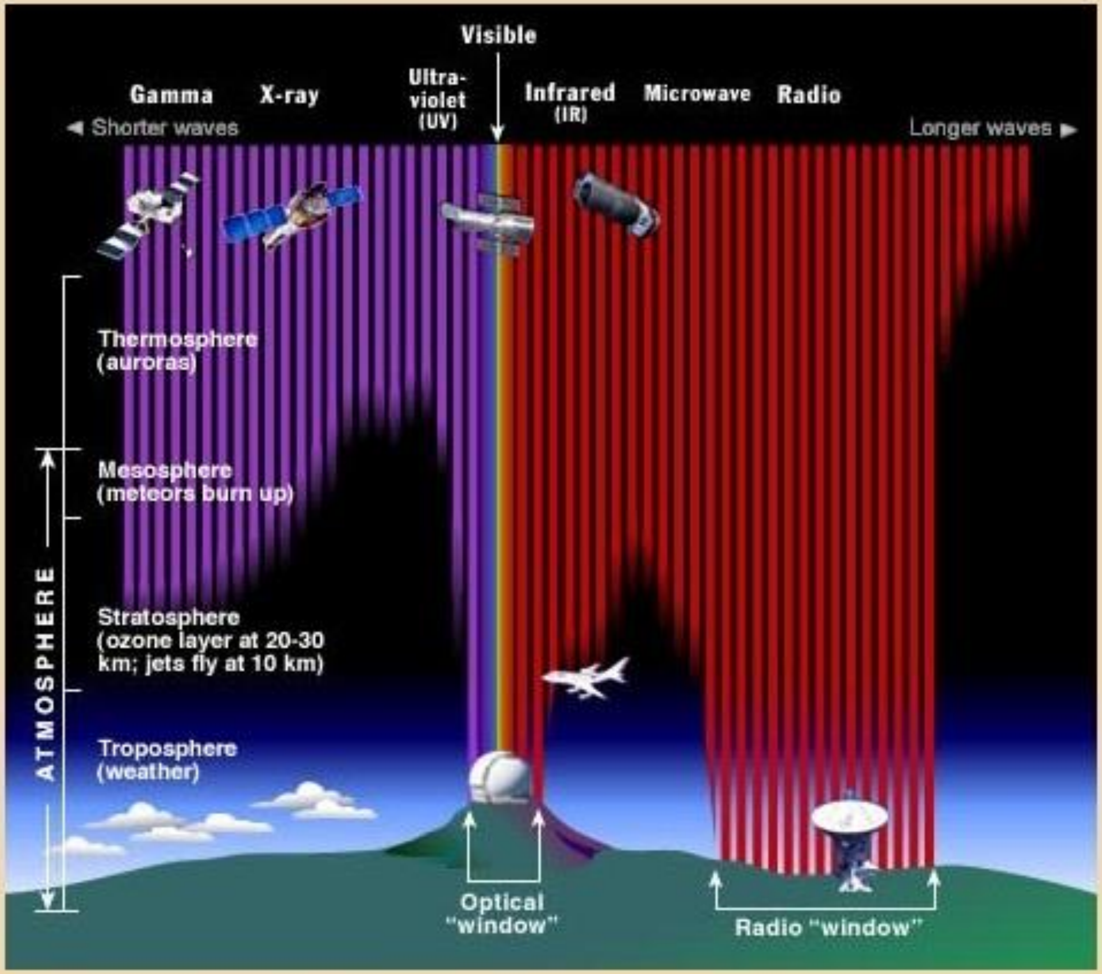
$$I(h) = I(0)\ e^{-\alpha\ h}$$

Where ***alpha*** is some constant.

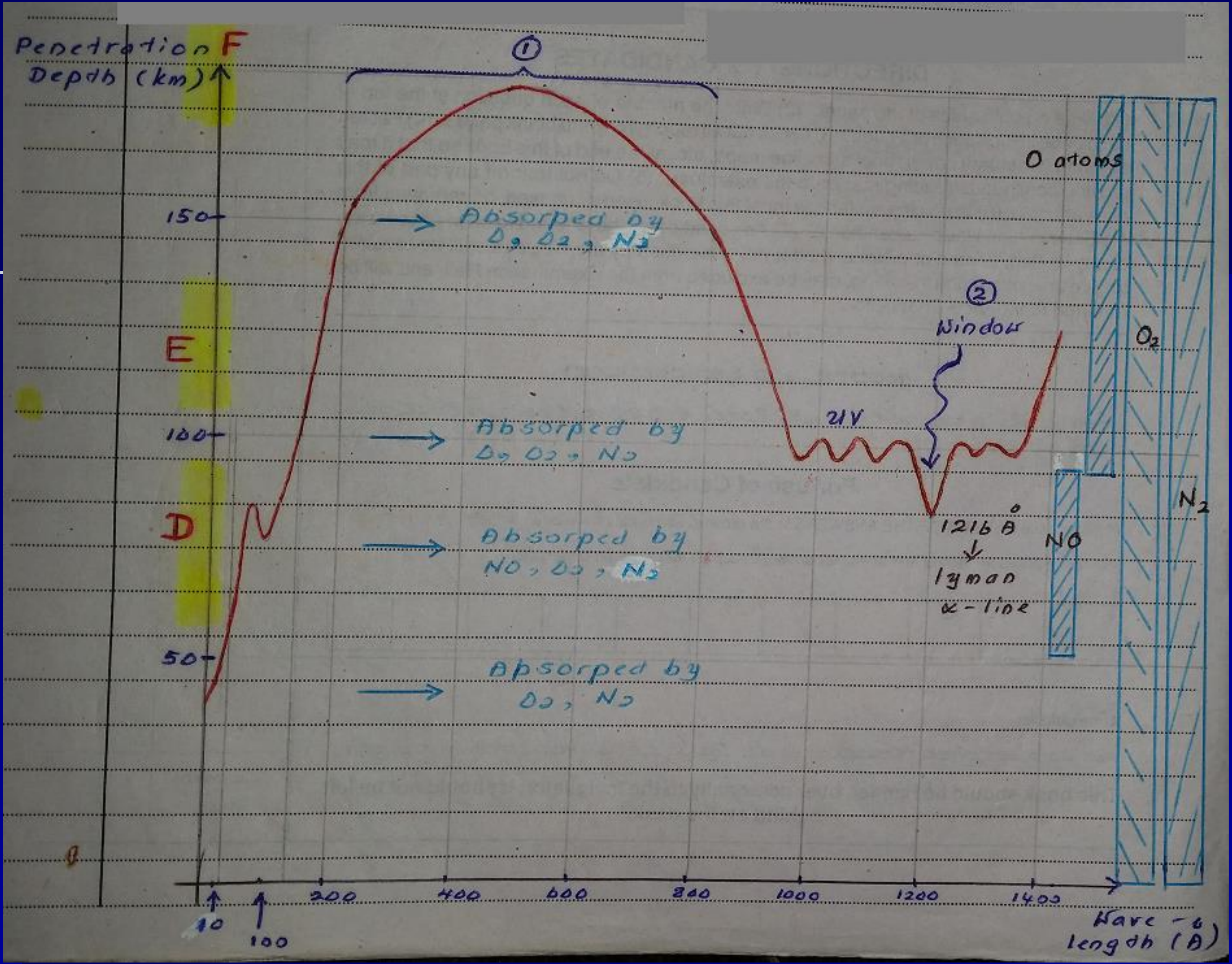
Penetration Depth = $1/\alpha$


IR Cooling
Temperature (K)
400
800
1200
600
500
400
300
200
100
km
Solar Min
Solar Max
X-Rays
Extreme UV 10-120 nm
Far UV 120-200 nm
Middle UV 200-300 nm
Near UV 300-400 nm
Visible and IR
Density (log10 cm-3)
0
4
8
12
16
20
O2 N2 O
F
e-
E
D
Ionosphere
Thermosphere
Mesosphere
Stratosphere
Troposphere

Solar EM Radiation Penetration into Earth's Atmosphere



Various wavelengths of solar EM radiation penetrate Earth's atmosphere to various depths. Fortunately for us, all of the high energy X-rays and most UV is filtered out long before it reaches the ground. Much of the infrared radiation is also absorbed by our atmosphere far above our heads. Most radio waves do make it to the ground, along with a narrow "window" of IR, UV, and visible light frequencies. **Credit:** Image courtesy STCI/JHU/NASA.



The graph of Penetration Depth vs wave-length of the Radiation comes from the Sun

Regular and Irregular Variations of the Ionosphere

The ionosphere we have described up to now and the numerical values we have given refer to an average, or typical as some people prefer to call it, **ionosphere**.
In practice these values vary by more than an order of magnitude with **time** and **location**.
some of these changes follow a known pattern, whereas others come and go on an irregular basis.

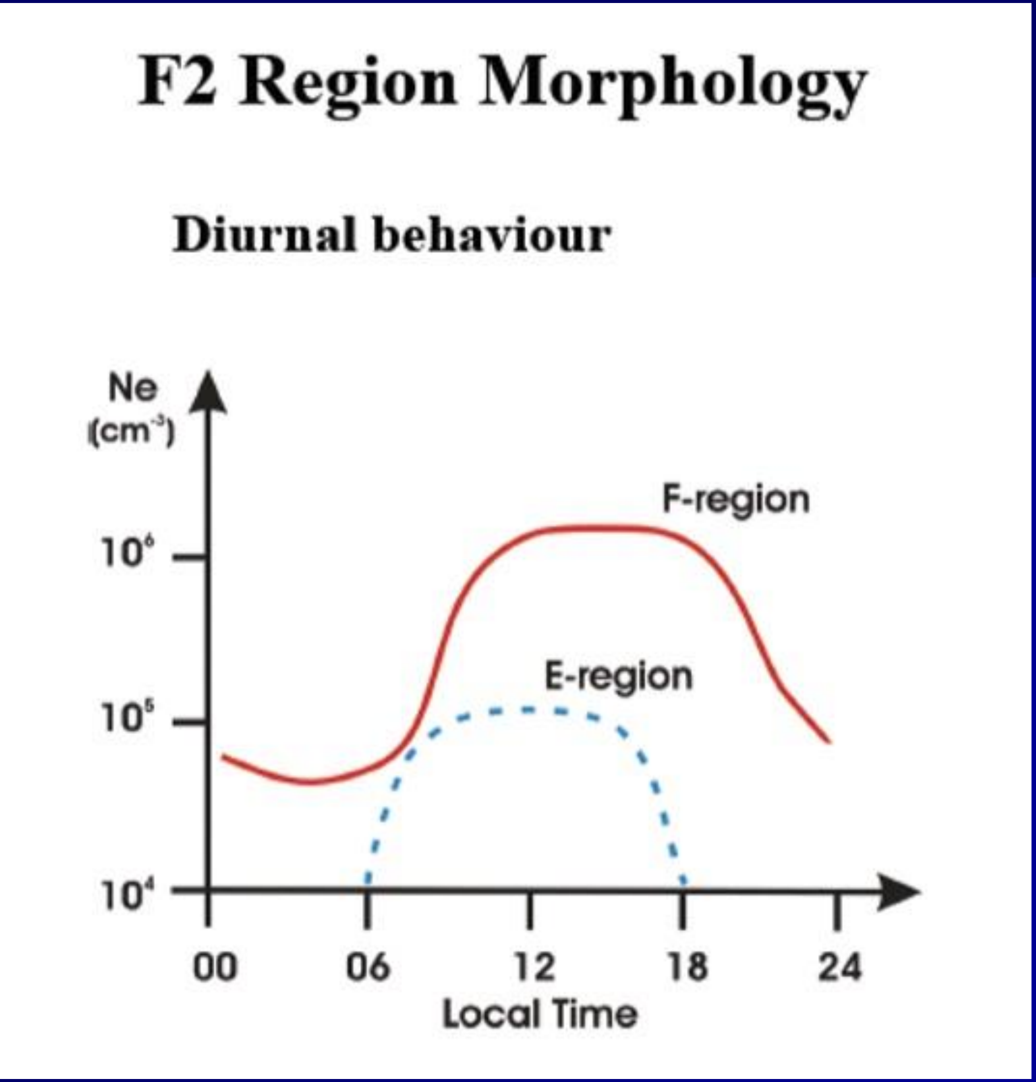
Regular Variations of the Ionosphere

- **The Latitudinal Dependence**

The latitudinal dependence of the ionospheric parameters, **mainly due to the change of the solar zenith angle with latitude**, but also **due to the change in the dip angle of the Earth's magnetic field**. There is also a **small longitudinal variation** because the **Earth's Magnetic Field various with longitude** along any given geographic latitude. The N_m (Molecular Number Density - electrons) can easily **vary by an order of magnitude from the polar to the equatorial regions**.

- **The Diurnal Variation**

The diurnal variation of the ionosphere which includes the peaking of the electron density usually in the early afternoon, the **sharp changes near sunrise and unset,** and the **disappearance of the lower layers during the night**. The N_m can again vary by an order of magnitude between night and day.



The Diurnal Behaviour of the E- Region and F-Region

Regular Variations of the Ionosphere

- **The Seasonal Variation**

The seasonal variation, which is also due to the change in the average zenith angle of the Sun as we move **between the summer and winter solstices** (සූර්ය නිවෘත්තිය).

- **The 27 Day Cycle**

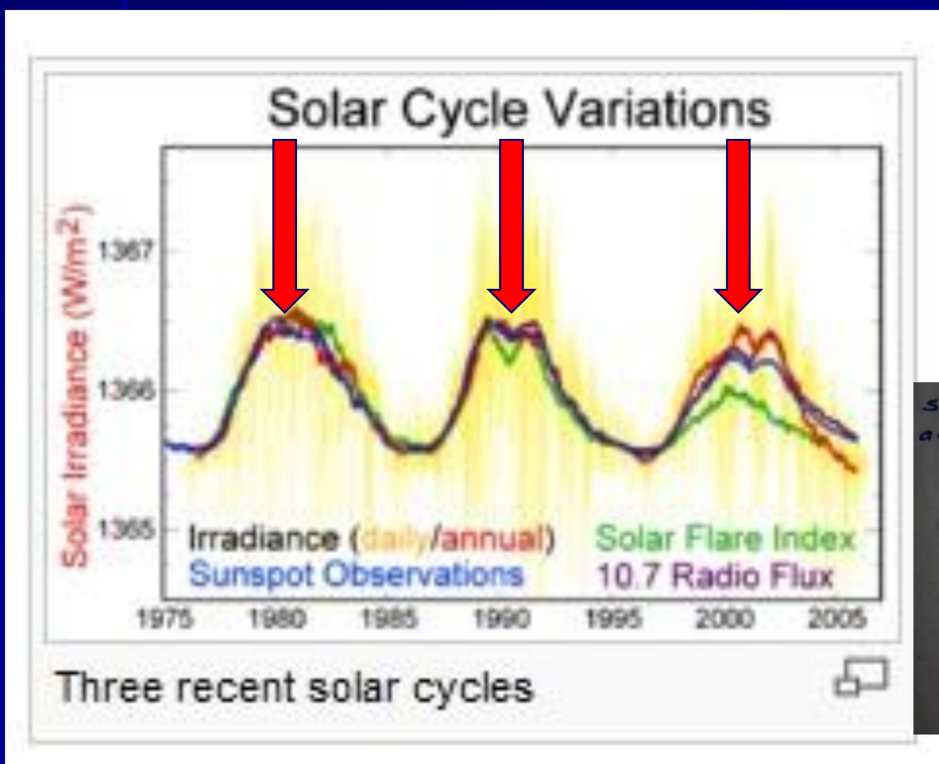
The 27 day cycle due to the intrinsic (true) rotation of the Sun. This cycle is especially noticeable during periods of **high solar activity when a very activity region** might last for more than one rotation of the Sun.

Active regions also have a tendency (willingness) to form in the same general area of other past active regions so that there is often a long lasting longitudinal asymmetry of activity on the Sun.

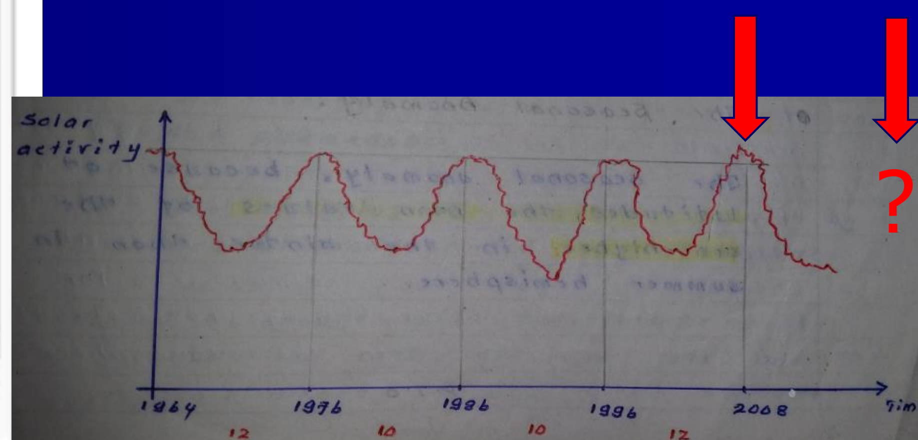
Regular Variations of the Ionosphere

- **The 11 year Solar Cycle**

The 11 year solar cycle, which represents the fairly regular increase and decrease of the solar activity and therefore of the ionizing radiation from the Sun with a period of approximately **11.1 years** (may be 11.2 years). The last solar maximum occurred in 2008 !



Three recent solar cycles



Solar Cycles						
Cycle	Started	Finished	Duration (years)	Maximum (monthly SSN (Smoothed Sunspot Number))[4]	Minimum (monthly SSN; end of cycle)[5][6]	Spotless Days (end of cycle)[7][8][9]
Solar cycle 1	March 1755	June 1766	11.3	86.5	11.2	
Solar cycle 2	June 1766	June 1775	9.0	115.8	7.2	
Solar cycle 3	June 1775	September 1784	9.3	158.5	9.5	
Solar cycle 4	September 1784	May 1798	13.7	141.1	3.2	
Solar cycle 5	May 1798	December 1810	12.6	49.2	0.0	
Solar cycle 6	December 1810	May 1823	12.4	48.7	0.1	
Solar cycle 7	May 1823	November 1833	10.5	71.5	7.3	
Solar cycle 8	November 1833	July 1843	9.8	146.9	10.6	
Solar cycle 9	July 1843	December 1855	12.4	131.9	3.2	~654
Solar cycle 10	December 1855	March 1867	11.3	97.3	5.2	~406
Solar cycle 11	March 1867	December 1878	11.8	140.3	2.2	~1028
Solar cycle 12	December 1878	March 1890	11.3	74.6	5.0	~736
Solar cycle 13	March 1890	February 1902	11.9	87.9 (Jan 1894)	2.7	~938
Solar cycle 14	February 1902	August 1913	11.5	64.2 (Feb 1906)	1.5	~1019
Solar cycle 15	August 1913	August 1923	10.0	105.4 (Aug 1917)	5.6	534
Solar cycle 16	August 1923	September 1933	10.1	78.1 (Apr 1928)	3.5	568
Solar cycle 17	September 1933	February 1944	10.4	119.2 (Apr 1937)	7.7	269
Solar cycle 18	February 1944	April 1954	10.2	151.8 (May 1947)	3.4	446
Solar cycle 19	April 1954	October 1964	10.5	201.3 (Mar 1958)	9.6	227
Solar cycle 20	October 1964	June 1976	11.7	110.6 (Nov 1968)	12.2	272
Solar cycle 21	June 1976	September 1986	10.3	164.5 (Dec 1979)	12.3	273
Solar cycle 22	September 1986	May 1996	9.7	158.5 (Jul 1989)	8.0	309
Solar cycle 23	May 1996	December 2008 [10]	12.6	120.8 (Mar 2000)	1.7	820 (through Jan 15, 2011) [11]
Solar cycle 24	December 2008 [10]					
Mean			11.1	114.1	5.8	

Regular Variations of the Ionosphere

- **The 11 year Solar Cycle**

The fact that all these variations follow a rather well-prescribed pattern does not necessarily mean that these patterns follow the predictions of the simple Chapman layer theory. According to the Chapman theory, for example, the highest F_0, F_2 and the lowest h_m must occur when the Sun reaches the smallest zenith angle, which naturally occurs at noon. The Chapman theory also predicts lower critical frequencies at higher latitudes and for the same latitude lower critical frequencies in the winder hemisphere.

All the variations of the ionosphere that do not follow the predictions of the Chapman Theory came to be known as **anomalies** and over the years many anomalies of this kind have been reported and diseased in the literature.

Thus we have:

- **The Equatorial or Geomagnetic Anomaly**
- **The Seasonal Anomaly**
- **The December Anomaly**
- **The Diurnal Anomaly**

Regular Variations of the Ionosphere

- **The 11 year Solar Cycle**

  - **The Equatorial or Geomagnetic Anomaly**

The equatorial or geomagnetic anomaly, because the $f_0 f_2$ varies with the geomagnetic rather than with the geographic latitude plus the fact that the noon values of the $F_0 F_2$ show a decrease along the geomagnetic equator at the equinoxes (clinic).

  - **The Seasonal Anomaly**

The seasonal anomaly, because at mid-latitudes the noon values of the $F_0 F_2$ are higher in the winter **than** in the summer hemisphere.

Regular Variations of the Ionosphere

- **The 11 year Solar Cycle**

  - **The December Anomaly**

    The December anomaly, because on a worldwide basis the $F_0 F_2$ values of the ionosphere are in general higher around December.

  - **The Diurnal Anomaly**

    The diurnal anomaly, because the diurnal variation of the $F_0 F_2$ is not always symmetric around the local noon.

    This phenomenon is especially pronounced at mid-latitudes during the summer months when the evening and early night values of the $F_0 F_2$ approach and often exceed the corresponding noon value. The segment of high critical frequencies around noontime which is missing from the diurnal plot of the $F_0 F_2$ has been given the descriptive name **midday bite out**.

Regular Variations of the Ionosphere

- **The 11 year Solar Cycle**
  - **The Diurnal Anomaly**

People have tried to account for these so called anomalies by including effects that were neglected by the simple Chapman theory.

Some of the most important ones are:

1. Ambipolar diffusion in the presence of the Earth's magnetic field.
2. The coupling between the ionosphere and the plasmasphere.
3. The dragging of ionospheric plasma by neutral winds in the upper atmosphere.
4. The change with temperature of the **production rate** the **loss rate** and the **scale height** at any given attitude.

Irregular Variations of the Ionosphere

Besides the different anomalies which we have discussed above, the ionosphere shows also the following **structural irregularities**.

- **The Sporadic - E**

The sporadic-E, which is the frequent formation of a thin layer (1-5 km) of excess ionization at an attitude of about 110 km. The electron density of this layer can exceed by more than a factor of two the ambient electron density of the E-region.

The sporadic-E has been studied extensively both from the theoretical and the experimental point of view, but still there is no general agreement on the cause of this phenomenon. According to one of the more widely discussed theories, the appearance of the sporadic-E is due to **strong shear winds which often develop near the maximum of the E-layer.**

Irregular Variations of the Ionosphere

- **The Spread F**

  Ionograms occasionally show a large spread in the equivalent height from which the F-region echoes are returned. This time spread, which is much broader than the time width of the transmitted radio pulses, is produced either by a blobby structure of the F-region which causes in depth multiple scattering, or by a wavy structure of the F-region which permits the reflection of the radio waves by curved surface at different distance from the vertical. This phenomenon might last sometimes for several hours and is usually a good indication of disturbed conditions in the ionosphere.

- **The Ionospheric Irregularities**

  The ionospheric irregularities, which represent local perturbations (කැළඹීම්) by a few percent in the electron density of the ionosphere. These irregularities are often elongated (long) along the lines of the Earth's Magnetic Field and their dimensions are of the order of 1 to 10 km.

Irregular Variations of the Ionosphere

- **Travelling Ionospheric Disturbances**

These are large size perturbations of the electron density extending sometimes over 1000 km. They have been observed to travel with speeds of the order of 300 ms^{-1} over large distances and occasionally to make a full circle around the globe. The mechanism causing these large scale disturbances is not well understood. One possible suggestion is that they are produced by the sudden precipitation (running down) of a large number of energetic particles either in the polar regions or in the vicinity of a magnetic anomaly.

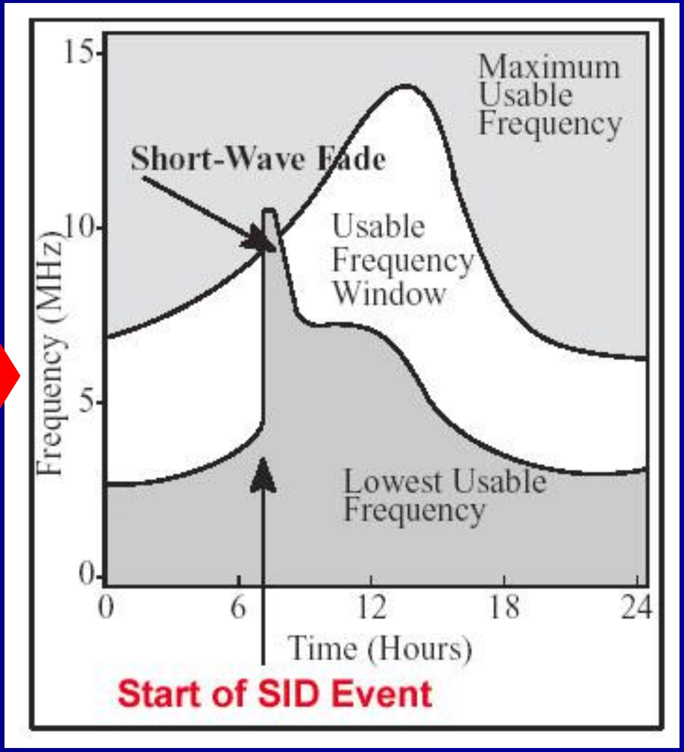
- **The Mid-Latitude Trough**

This is a minimum in the electron densities of the ionosphere which develops primarily during the night time at a geomagnetic latitude (dip latitude) of approximately 60 degrees.

Irregular Variations of the Ionosphere

- **Sudden Ionospheric Disturbances (S I D)**

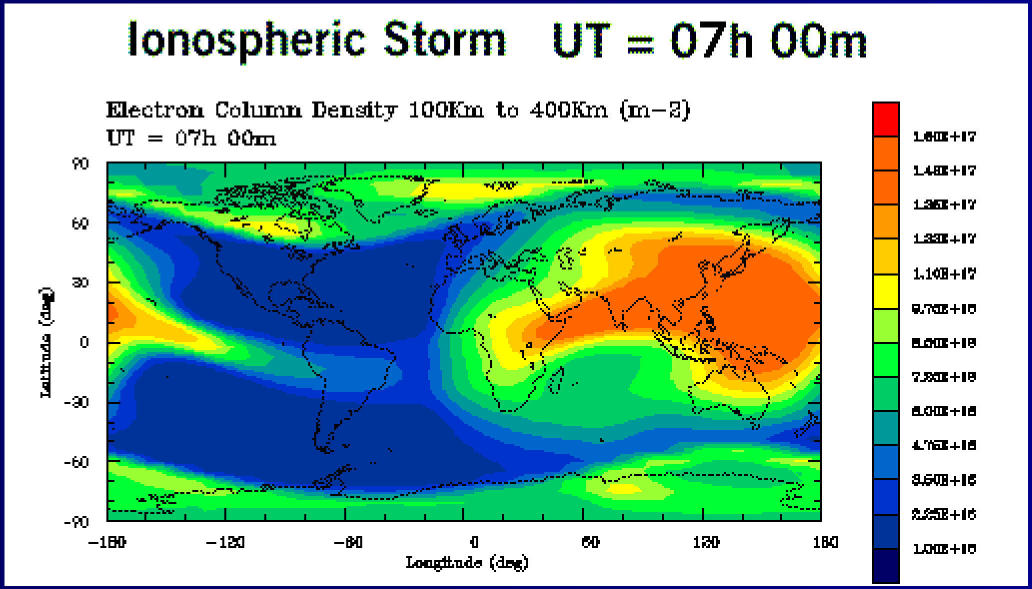
These are caused by the enhanced ultra-violet and X-ray radiation from the Sun during solar flare events. They occur only in the Sun-lit side of the Earth and they last, like the solar flares, from a few minutes to about one hour.



Irregular Variations of the Ionosphere

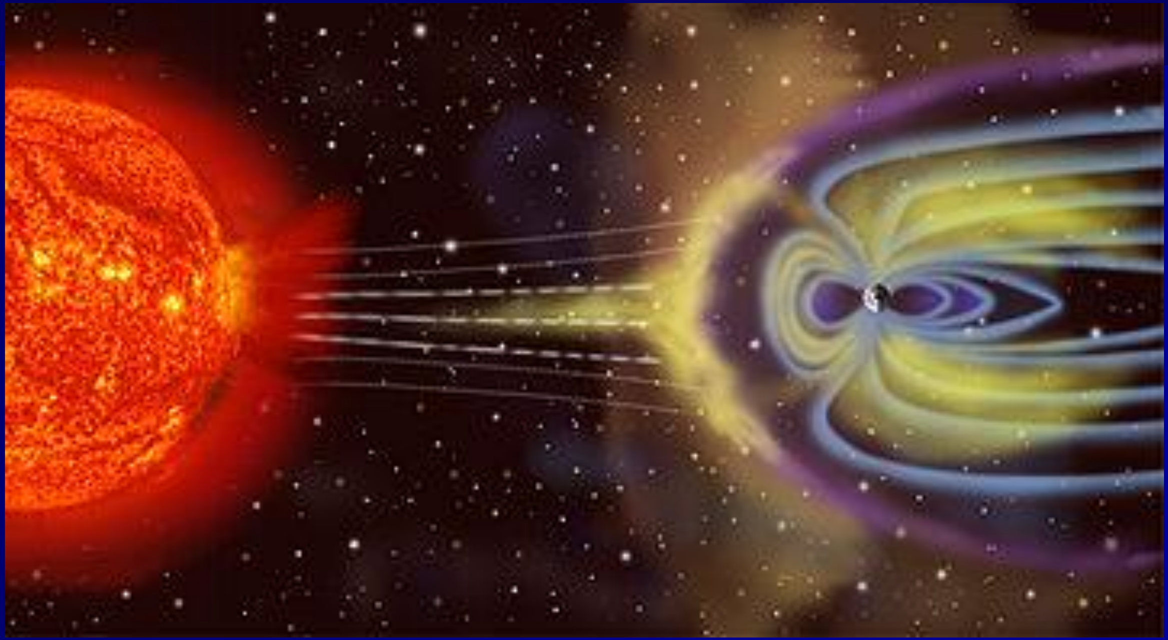
- **Ionospheric Storms**

These are closely associated with geomagnetic storms and can last from one to four days affecting the ionosphere over the entire globe. Observations of the unusual behavior of the ionosphere during these storms have been made and continue to be made by many groups around the world.



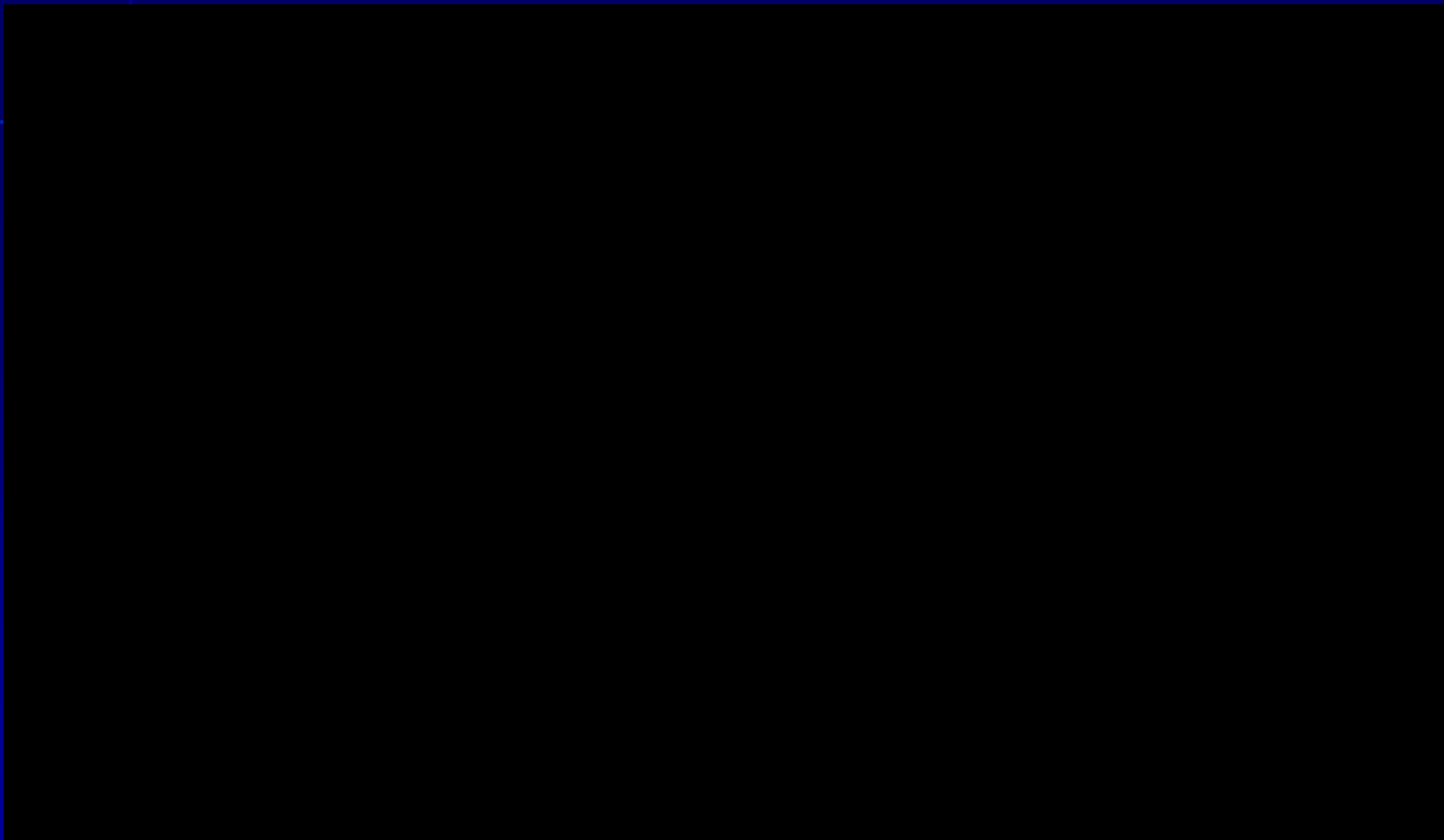
Many diurnal, seasonal and latitudinal storm effects have been discovered and serious efforts have been made for their theoretical interpretation.

The Magnetosphere



The Earth's Magnetic Fields
The Dipole Magnetic Field
Motion of charged particles in a Dipole Magnetic Field
The Radiation Belts
The boundary and the tail of the Magnetosphere

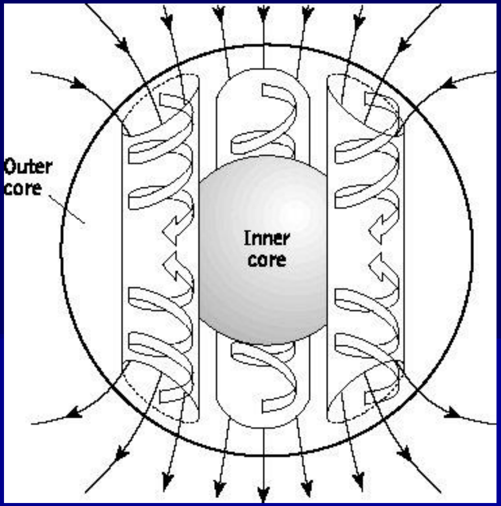
The Earth's Magnetosphere



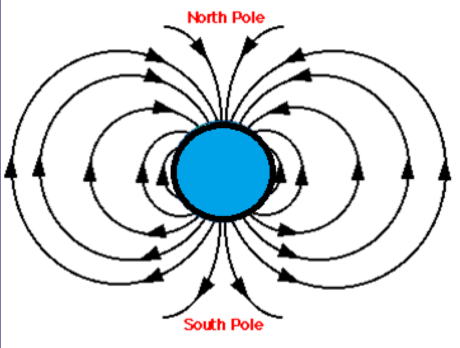
NASA ScienceCasts : Earth's Magnetosphere

The Earth's Magnetic Field

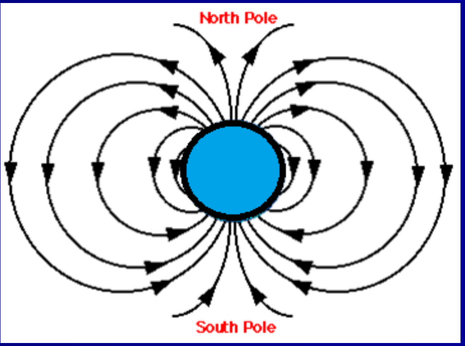
Present theories believe that the Earth's magnetic field arises (appears) from electric currents flowing in the molten metallic core of the planet, which has a radius approximately one-half the radius of the Earth



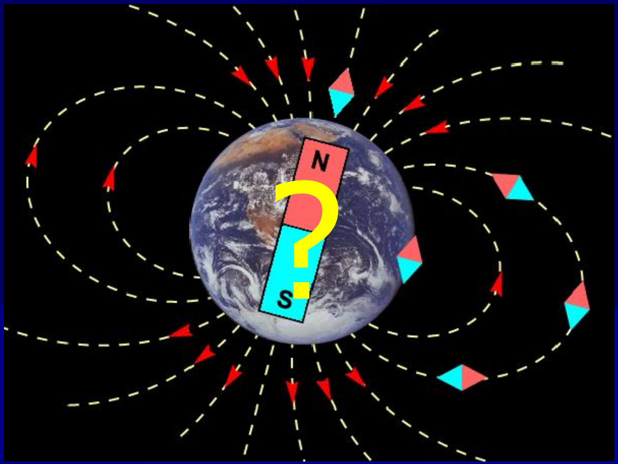
The currents are attributed to a dynamo mechanism operating inside the core. Recent discoveries suggest that the strength and orientation of the terrestrial magnetic field have changed considerably over geological periods. There is also strong evidence that the Earth's magnetic field has reversed its direction several time during the life time of our planet.



Million of years

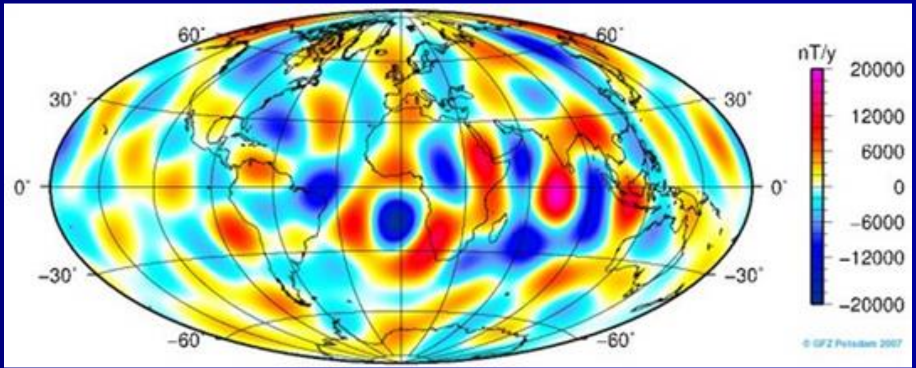


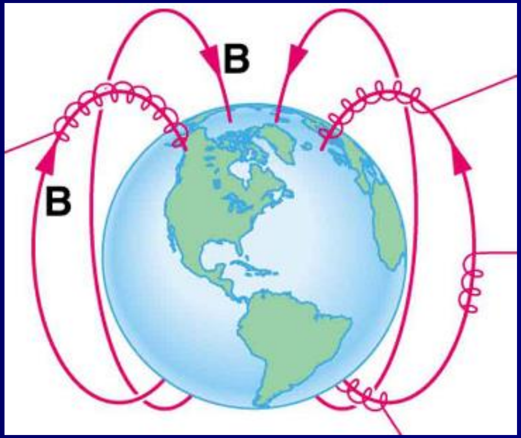
The Earth's Magnetic Field



The Earth's magnetic field resembles a di-pole magnetic field. Large scale regional departures from the dipole field are called geomagnetic anomalies and are attributed to irregular or eddies in the dynamo current system.

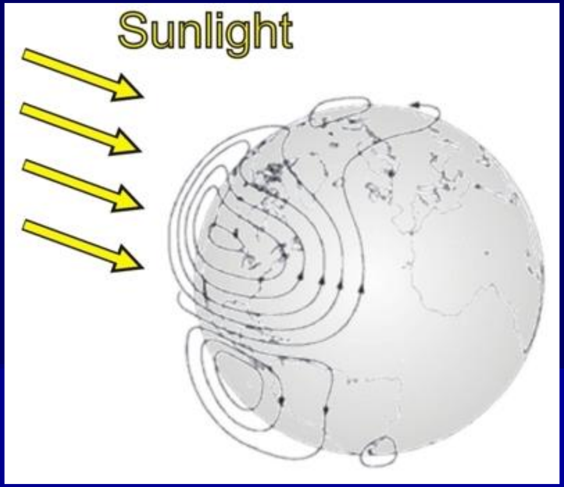
There are also smaller size anomalies due to local mineral deposits which are called surface magnetic anomalies and are helpful in locating these deposits of ferromagnetic materials.



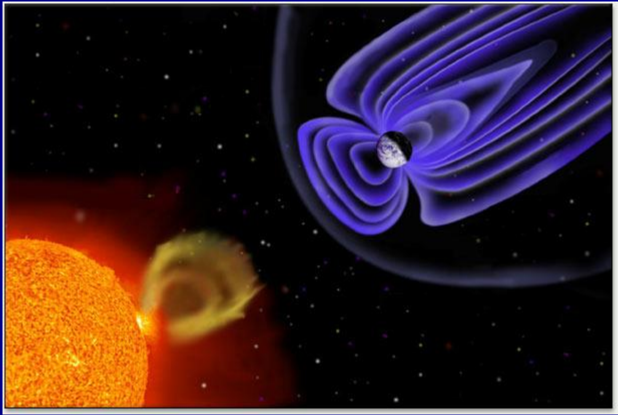


A very small components (less than 0.1%) of the terrestrial magnetic field is due to currents of charge particles in the outer atmosphere of the Earth.

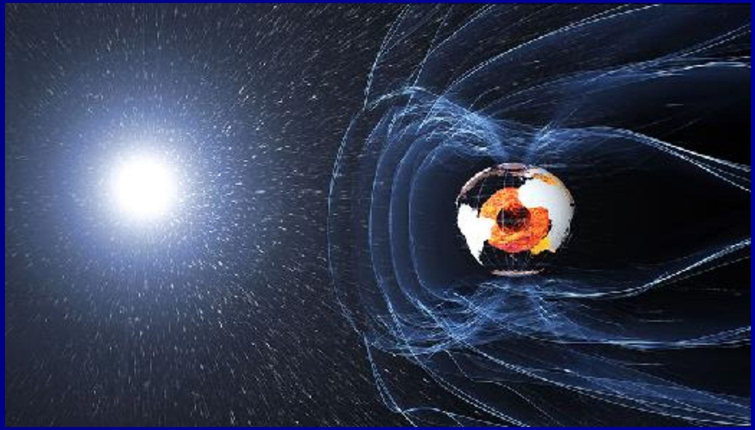
The Earth's Magnetic Field



The sq-currents, eg: which circulates between the Sunlit and the dark hemisphere, is responsible for the small, regular diurnal (daytime) variations of the magnetic field observed on the ground.



After a large solar flare, the enhanced flux of energetic particles from the Sun can produce strong ring currents of the charged particles around the Earth that can produce large fluctuations (occasionally as high as 3%) of the terrestrial magnetic field. These magnetic disturbances, which might last for several days, are described by the different indices of geomagnetic activity.

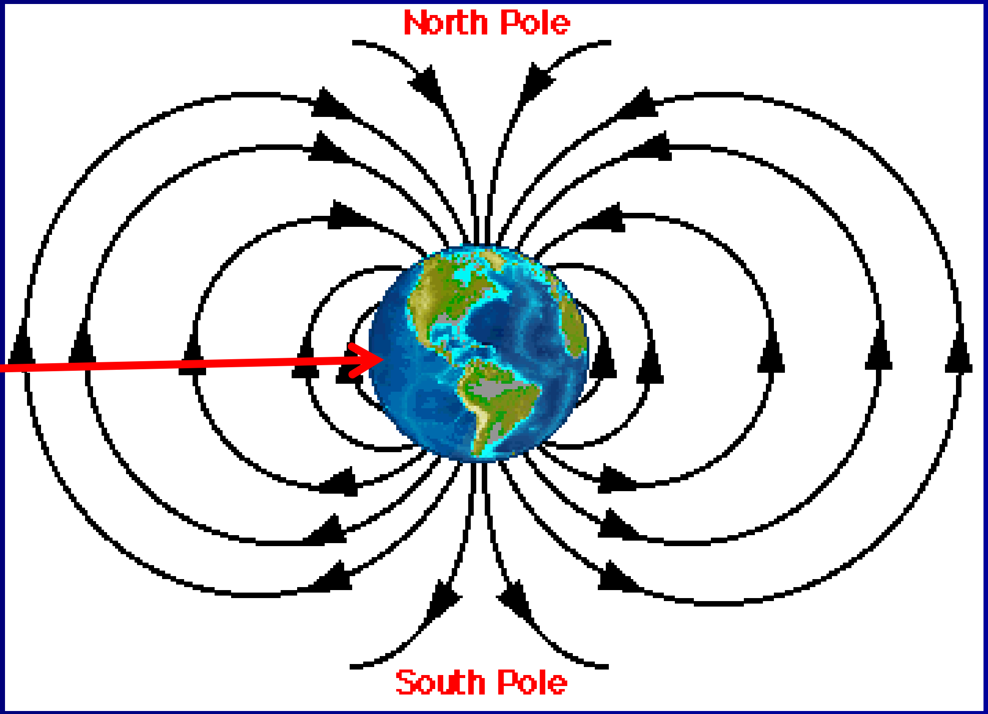


The Earth's Magnetic Field

The magnetic field of the Earth can be represented to a good approximation, by a **dipole-field** with a magnetic moment $M = 8.05 \pm 0.02 \times 10^{25}$ Gauss cm^3. The intensity of the field at the equator is ~0.3 Gauss and at poles ~0.6 Gauss.

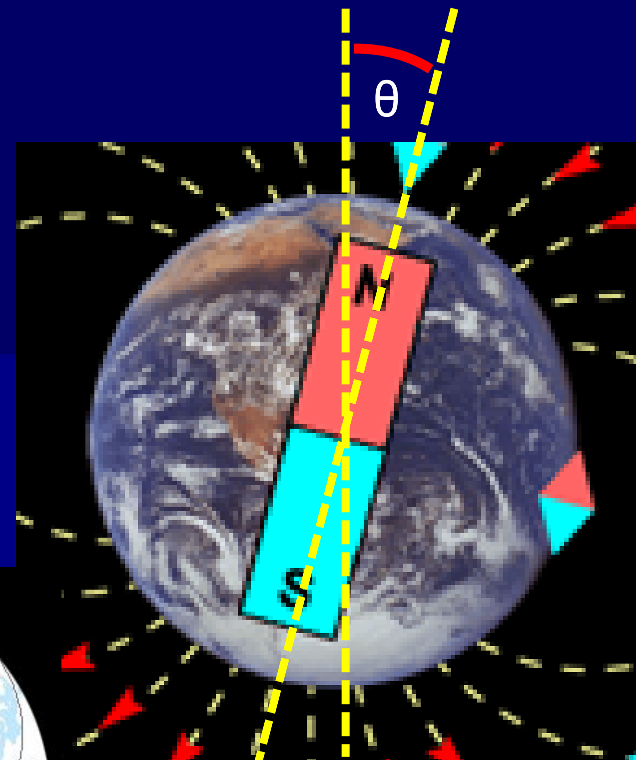
$$1\ \text{Gauss (G)} = \sim 1 \times 10^{-4}\ \text{Tesla (T)}$$

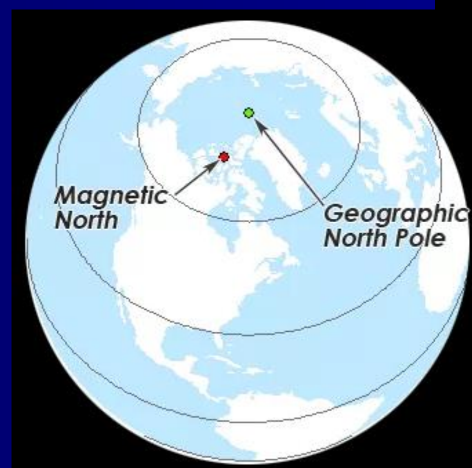
The Earth magnetic field intensity at the equator ~40,000 nT.



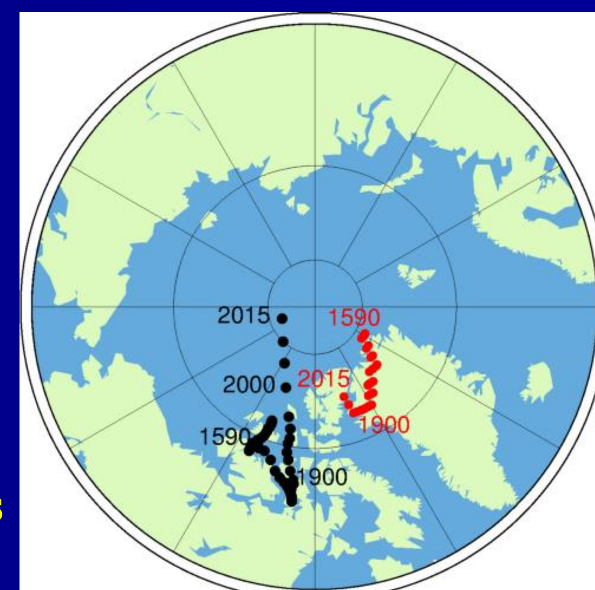
The Earth's Magnetic Field

The centered dipole (its axis passes through the center of the Earth) which fits best the Earth's magnetic field has its axis directed along the line (79°N, 290°E) to (79°S, 110°E). These are referred to respectively as the **north geomagnetic pole** and the **south geomagnetic pole**.





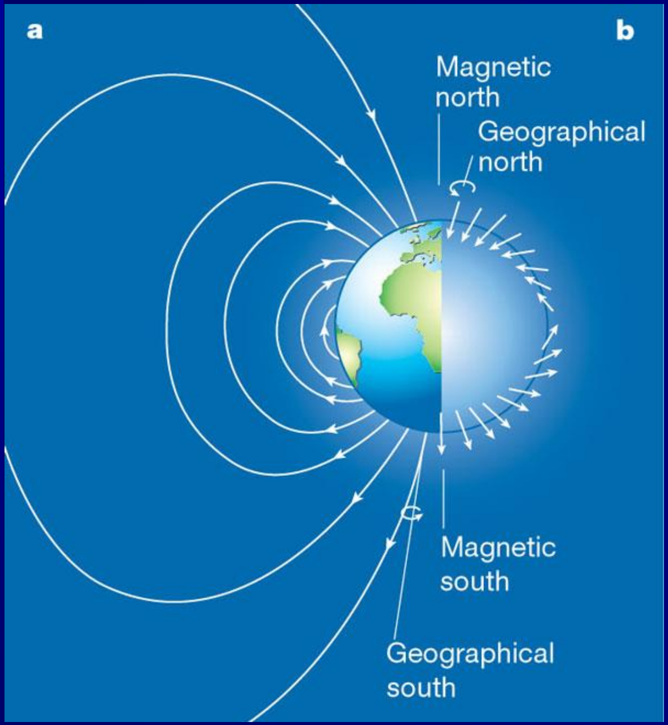
The **actual magnetic poles** are asymmetrically (unsymmetrically) located at (73°N, 262°E) to (68°S, 145°E).

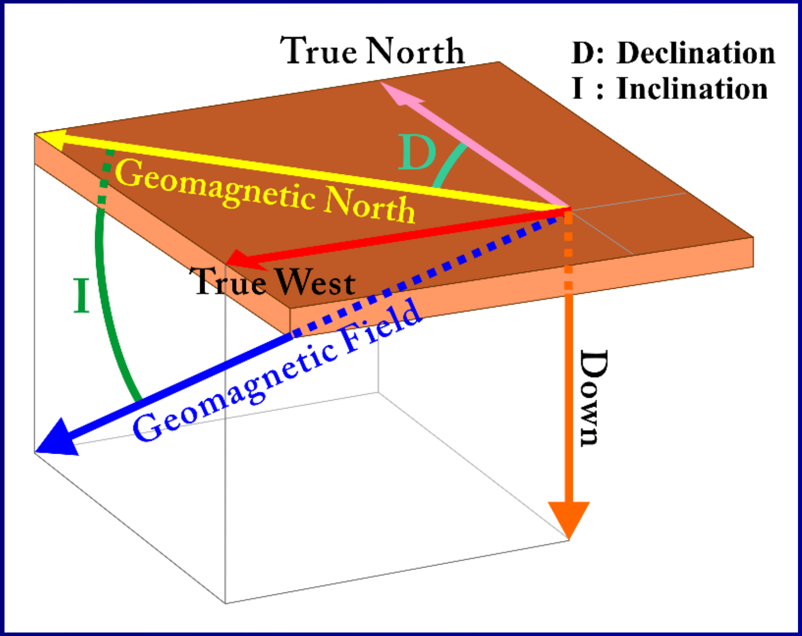


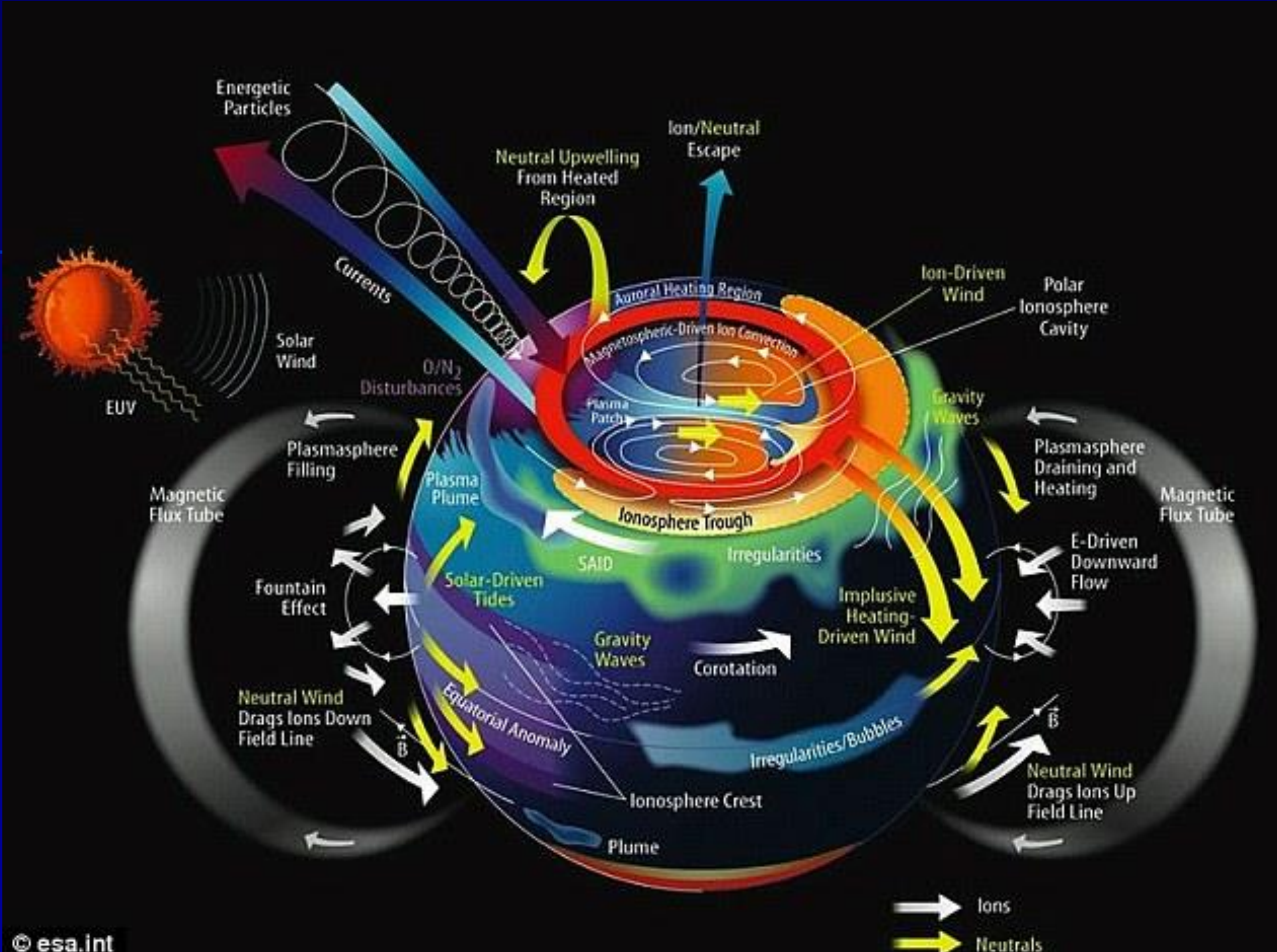
The **Earth magnetic poles** are changing with time !

The Earth's Magnetic Field

The coordinates of the dipole field are called the geomagnetic latitude and the geomagnetic longitude. The angle the magnetic field makes with the horizontal is called the **inclination** or **dip** of the magnetic field, and the angle its horizontal component makes with the local geographic meridian (Earth Centre line from North Pole to South Pole) is called the **declination** (drop) of the magnetic field.

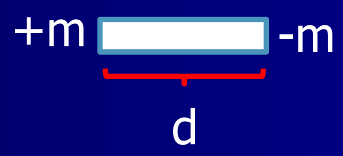




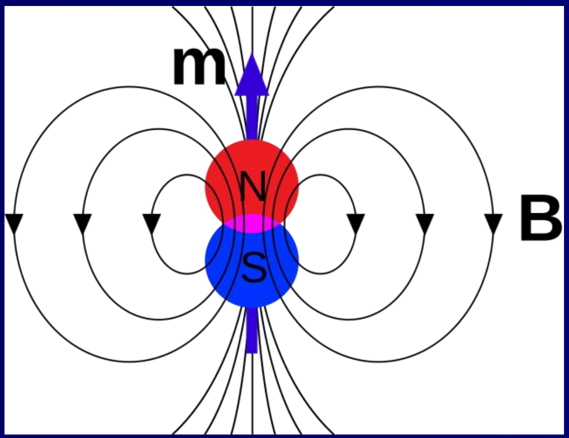

Energetic Particles
Ion/Neutral Escape
Neutral Upwelling From Heated Region
Currents
Auroral Heating Region
Magnetospheric-Driven Ion Convection
Ion-Driven Wind
Polar Ionosphere Cavity
Solar Wind
EUV
O/N$_2$ Disturbances
Plasma Patch
Gravity Waves
Plasmasphere Filling
Plasmasphere Draining and Heating
Magnetic Flux Tube
Plasma Plume
Ionosphere Trough
Magnetic Flux Tube
SAID
Irregularities
E-Driven Downward Flow
Fountain Effect
Solar-Driven Tides
Implusive Heating-Driven Wind
Gravity Waves
Corotation
Neutral Wind Drags Ions Down Field Line
Equatorial Anomaly
Irregularities/Bubbles
Neutral Wind Drags Ions Up Field Line
Ionosphere Crest
Plume
Ions
Neutrals

The Dipole Magnetic Field

Magnetic Dipole Moment



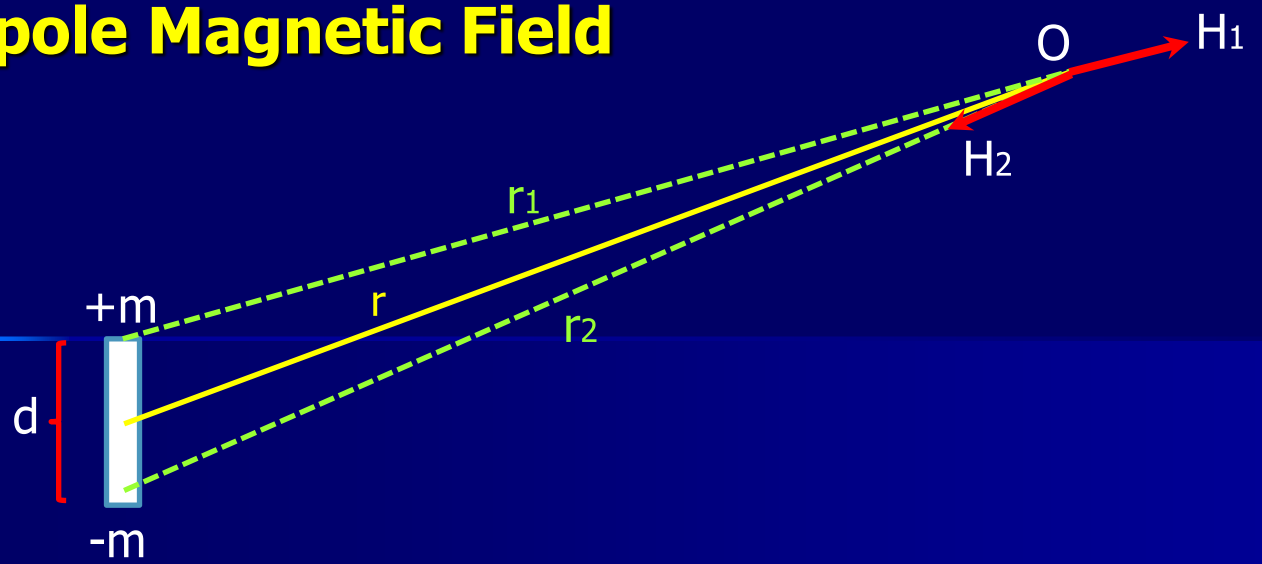
A measure of the magnetic strength of a magnet is set with its axes perpendicular to the magnetic field



Magnetic Dipole Moment, M

$$M = m \times d$$

The Dipole Magnetic Field

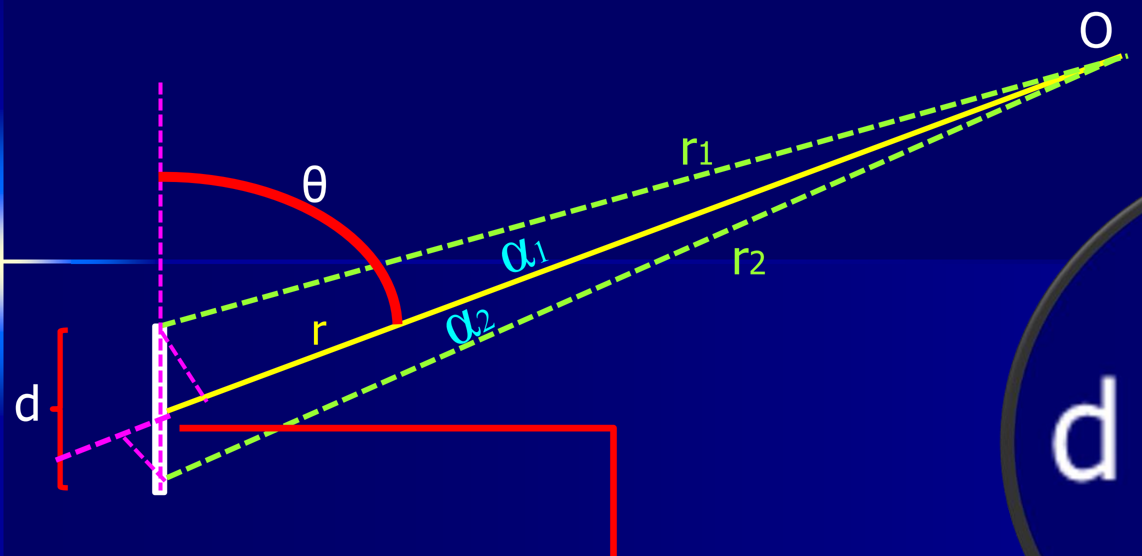


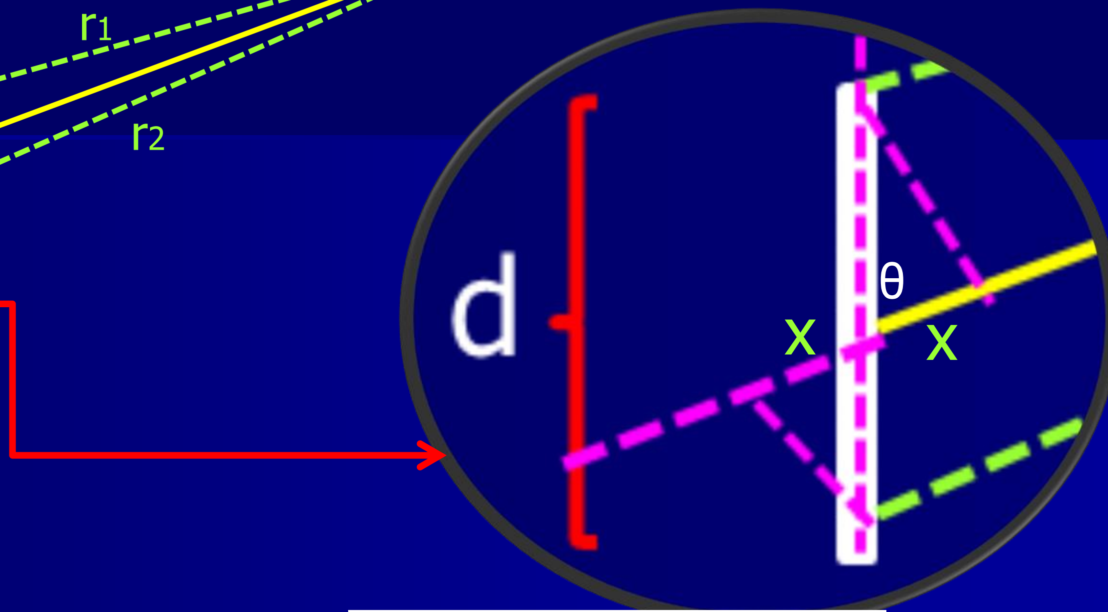
Let us consider the magnetic field at a point O, at distance r_1 and r_2 from the two poles of a magnetic dipole. Let m be the strength of each of the two poles and d the distance that separate them. The product m d is called the Magnetic Dipole Moment (M).

The magnetic field at O will have the two components,

$$H_1 = \frac{\mu_o}{4\pi} \cdot \frac{m}{r_1^2}$$ and $$H_2 = \frac{\mu_o}{4\pi} \cdot \frac{m}{r_2^2}$$

The Dipole Magnetic Field





$$r = r_1 \cos\alpha_1 + \frac{d}{2}\cos\theta$$

$$\Rightarrow \quad r_1 = \frac{1}{\cos\alpha_1}\left(r - \frac{d}{2}\cos\theta\right)$$

$$\Rightarrow \quad \cos\alpha_1 = \frac{1}{r_1}\left(r - \frac{d}{2}\cos\theta\right)$$

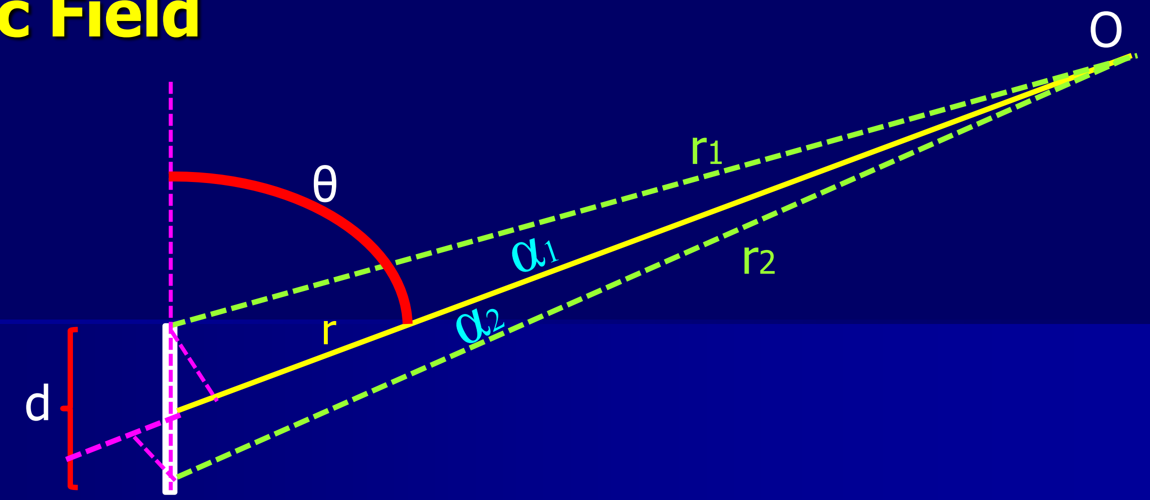
$$r = r_2 \cos\alpha_2 - \frac{d}{2}\cos\theta$$

$$\Rightarrow \quad r_2 = \frac{1}{\cos\alpha_2}\left(r + \frac{d}{2}\cos\theta\right)$$

$$\Rightarrow \quad \cos\alpha_2 = \frac{1}{r_2}\left(r + \frac{d}{2}\cos\theta\right)$$

(As seen from the above diagram, the distance r of the point O from the center of the dipole is given by the above expressions)

The Dipole Magnetic Field



Using the law of Sine for the above diagram :

$$\frac{\sin\alpha_1}{d/2} = \frac{\sin\theta}{r_1} \quad \text{and} \quad \frac{\sin\alpha_2}{d/2} = \frac{\sin\theta}{r_2}$$

$$\Rightarrow \frac{d}{2}\sin\theta = r_1 \sin\alpha_1 \quad \text{and} \quad \frac{d}{2}\sin\theta = r_2 \sin\alpha_2$$

$$\Rightarrow \frac{d}{2}\sin\theta = r_1 \sin\alpha_1 = r_2 \sin\alpha_2$$

$$\Rightarrow \sin\alpha_1 = \frac{d\sin\theta}{2r_1} \quad \text{and} \quad \sin\alpha_2 = \frac{d\sin\theta}{2r_2}$$

The Dipole Magnetic Field

When $r_1 \approx r_2 \gg d$, the angles α_1 and α_2 tend to zero and the respective cosines to unity.

i.e.; $d \ll r_1, r_2$ and $r_1 \approx r_2$

$\sin\alpha_1 \to 0$ and $\sin\alpha_2 \to 0$

$\cos\alpha_1 \to 1$ and $\cos\alpha_2 \to 1$ When, $\alpha_1 \to 0$ and $\alpha_2 \to 0$

$$r_2^3 - r_1^3 = \left(\frac{1}{\cos\alpha_2}\left(r + \frac{d}{2}\cos\theta\right)\right)^3 - \left(\frac{1}{\cos\alpha_1}\left(r - \frac{d}{2}\cos\theta\right)\right)^3$$

• • • •

$$r_2^3 - r_1^3 = 3\ r^2 d\cos\theta$$

$$r_1^3 + r_2^3 = \left(\frac{1}{\cos\alpha_1}\left(r - \frac{d}{2}\cos\theta\right)\right)^3 + \left(\frac{1}{\cos\alpha_2}\left(r + \frac{d}{2}\cos\theta\right)\right)^3$$

• • • •

$$r_2^3 + r_1^3 = 2\ r^3$$

The Dipole Magnetic Field

When $r_1 \approx r_2 \gg d$, the angles α_1 and α_2 tend to zero and the respective cosines to unity.

i.e.; $d \ll r_1, r_2$ and $r_1 \approx r_2$

$\sin\alpha_1 \to 0$ and $\sin\alpha_2 \to 0$

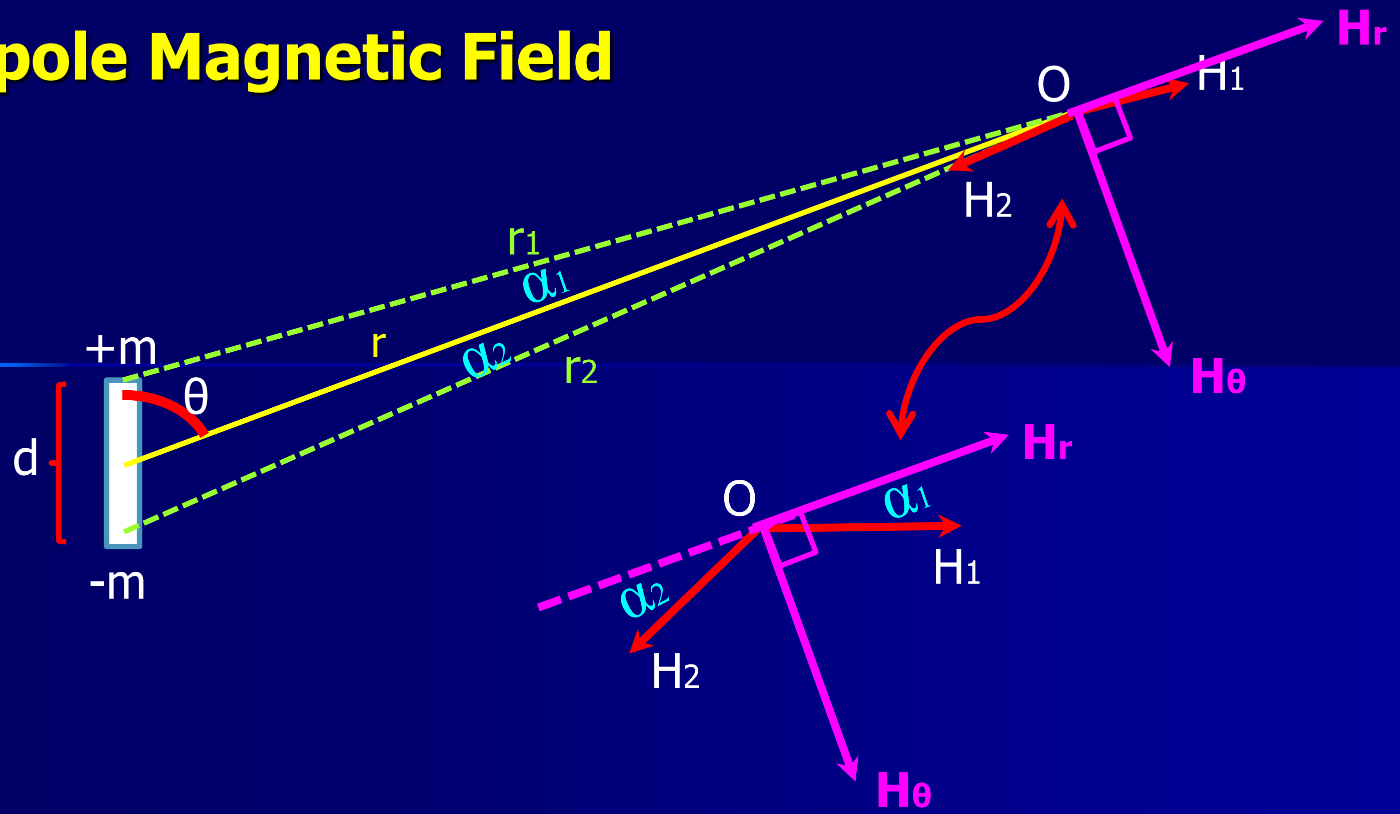
$\cos\alpha_1 \to 1$ and $\cos\alpha_2 \to 1$ When, $\alpha_1 \to 0$ and $\alpha_2 \to 0$

$$r_1^3.r_2^3 = \left(\frac{1}{\cos\alpha_1}\left(r - \frac{d}{2}\cos\theta\right)\right)^3 \times \left(\frac{1}{\cos\alpha_2}\left(r + \frac{d}{2}\cos\theta\right)\right)^3$$

• • • •

$$r_1^3.r_2^3 = r^6$$

The Dipole Magnetic Field



The radial component **Hr** of the magnetic field at a point O a large distance from the dipole is given by :

$$H_r = H_1 \cos\alpha_1 - H_2 \cos\alpha_2$$

And the tangential component **Hθ** of the magnetic field at a point O is given by :

$$H_\theta = H_1 \sin\alpha_1 + H_2 \sin\alpha_2$$

$$H_r = H_1 \cos\alpha_1 - H_2 \cos\alpha_2$$

$$H_r = \left(\frac{\mu_o}{4\pi}\cdot\frac{m}{r_1^2}\right)\left(\frac{1}{r_1}\left(r - \frac{d}{2}\cos\theta\right)\right) - \left(\frac{\mu_o}{4\pi}\cdot\frac{m}{r_2^2}\right)\left(\frac{1}{r_2}\left(r + \frac{d}{2}\cos\theta\right)\right)$$

....

$$H_r = \frac{\mu_o}{4\pi}\cdot\frac{2M}{r^3}\cos\theta$$

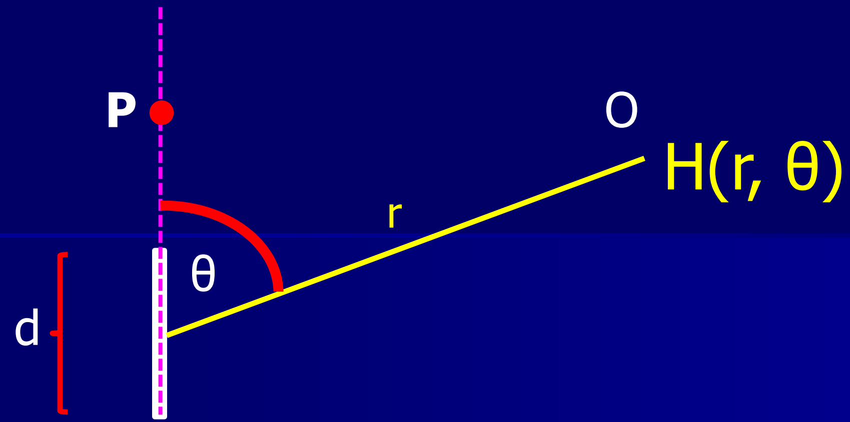
$$H_\theta = H_1 \sin\alpha_1 + H_2 \sin\alpha_2$$

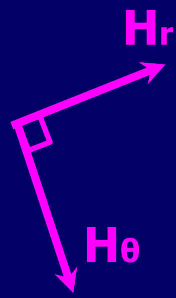
$$H_\theta = \left(\frac{\mu_o}{4\pi}\cdot\frac{m}{r_1^2}\right)\left(\frac{d\sin\theta}{2r_1}\right) + \left(\frac{\mu_o}{4\pi}\cdot\frac{m}{r_2^2}\right)\left(\frac{d\sin\theta}{2r_2}\right)$$

....

$$H_\theta = \frac{\mu_o}{4\pi}\cdot\frac{M}{r^3}\sin\theta$$

The Dipole Magnetic Field





$$H_r = \frac{\mu_o}{4\pi} \cdot \frac{2M}{r^3} \cos\theta$$

$$H_\theta = \frac{\mu_o}{4\pi} \cdot \frac{M}{r^3} \sin\theta$$

The line connecting the two poles of the dipole defines the axis of the north and south magnetic poles. Therefore, the angle θ represents the **geomagnetic co-latitude**. At the poles, where **θ=0**, the magnetic field **H_P** is all in the **radial direction** and is given by the expression,

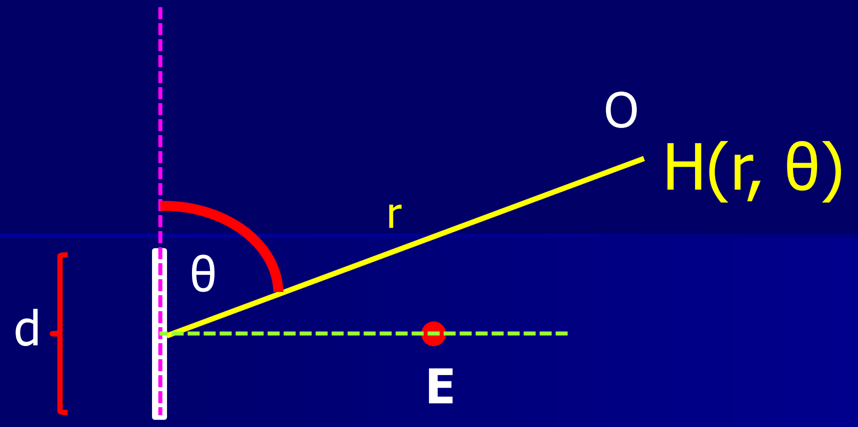
$$H_P = H_r \Big|_{\theta=0}$$

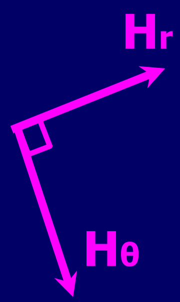
Because,

$$H_\theta \Big|_{\theta=0} \rightarrow 0$$

$$\therefore \quad H_P = \frac{\mu_o}{4\pi} \cdot \frac{2M}{r^3}$$

The Dipole Magnetic Field





$$H_r = \frac{\mu_o}{4\pi} \cdot \frac{2M}{r^3} \cos\theta$$

$$H_\theta = \frac{\mu_o}{4\pi} \cdot \frac{M}{r^3} \sin\theta$$

While at the equator, where **θ=90°,** the magnetic field **H_E** is entirely in the **tangential direction** and is given by the expression,

$$H_E = H_\theta \Big|_{\theta=90^o}$$

Because,

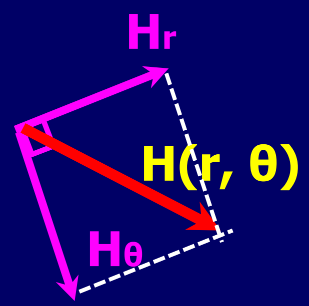
$$H_r \Big|_{\theta=90^o} \rightarrow 0$$

$$\therefore \quad H_E = \frac{\mu_o}{4\pi} \cdot \frac{M}{r^3}$$

Thus the magnetic field at the poles has twice the intensity of the magnetic field at the equator !

i.e.; **$H_P = 2\,H_E$**

The Dipole Magnetic Field



$$H_r = \frac{\mu_o}{4\pi} \cdot \frac{2M}{r^3} \cos\theta$$

$$H_\theta = \frac{\mu_o}{4\pi} \cdot \frac{M}{r^3} \sin\theta$$

Total Magnetic Field

Now we can compute the intensity of the total magnetic field at any **geomagnetic co-latitude, θ** from radial and the tangential components we have already obtained,

$$H = \left[H_r^2 + H_\theta^2\right]^{1/2}$$

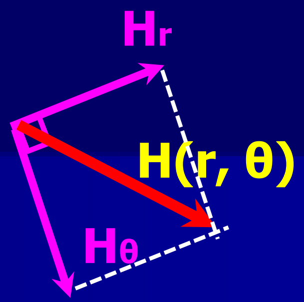
$$\Rightarrow \quad H = \left[\left(\frac{\mu_o}{4\pi} \cdot \frac{2M}{r^3} \cos\theta\right)^2 + \left(\frac{\mu_o}{4\pi} \cdot \frac{M}{r^3} \sin\theta\right)^2\right]^{1/2}$$

. . . .

$$\Rightarrow \quad H(r,\theta) = \frac{\mu_o}{4\pi} \frac{M}{r^3} \left[1 + 3\cos^2\theta\right]^{1/2}$$

The Dipole Magnetic Field

Total Magnetic Field



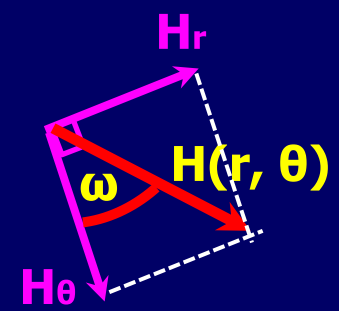
$$H(r,\theta)=\frac{\mu_o}{4\pi}\frac{M}{r^3}\left[1+3\cos^2\theta\right]^{1/2}$$

The above equation shows that the intensity of the dipole magnetic field decreases with distance as the third power (r^3) of the radial distance and for the same r varies, as we have seen already, by a factor of 2 from the poles to the equator. Using the above derived equations, we can express the total magnetic field at any given value of **θ** in terms of the equatorial magnetic field at the same radial distance.

$$H(r,\theta)=H_E\left[1+3\cos^2\theta\right]^{1/2}$$

Where, $H_E=\frac{\mu_o}{4\pi}\frac{M}{r^3}$

The Dipole Magnetic Field



$$H_r = \frac{\mu_o}{4\pi} \cdot \frac{2M}{r^3} \cos\theta$$

$$H_\theta = \frac{\mu_o}{4\pi} \cdot \frac{M}{r^3} \sin\theta$$

Total Magnetic Field

From the radial (vertical) and the tangential (horizontal) components of the magnetic field, we can also find the **inclination angle**, **ω** (dip) of the field.

$$\tan\omega = \frac{H_r}{H_\theta} \Rightarrow \tan\omega = \frac{\frac{\mu_o}{4\pi} \cdot \frac{2M}{r^3} \cos\theta}{\frac{\mu_o}{4\pi} \cdot \frac{M}{r^3} \sin\theta}$$

$$\Rightarrow \tan\omega = \frac{2\cos\theta}{\sin\theta}$$

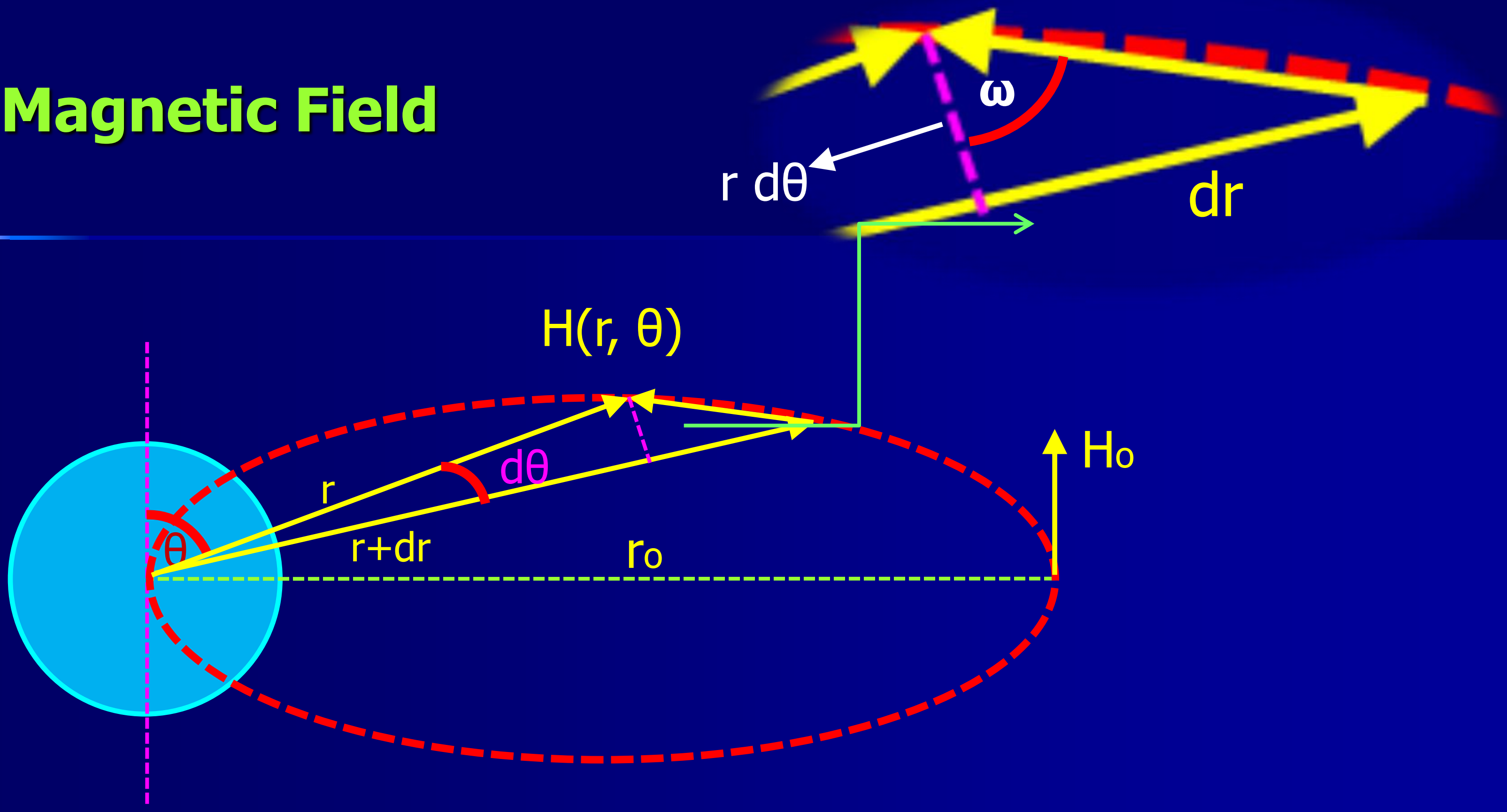
$$\Rightarrow \tan\omega = 2\cot\theta$$

$$\Rightarrow \omega = \omega(\theta) \neq \omega(r)$$

The above equation shows that the dip of the magnetic field is independent of the **radial distance** and therefore, the **magnetic field at any altitude above a given station will always be parallel to the magnetic field on the ground**.

The Dipole Magnetic Field

Total Magnetic Field



An imaginary line to which the **magnetic field is always tangential** is called a **line of force** or a **field line**. For such a line as seen from the above figure we have,

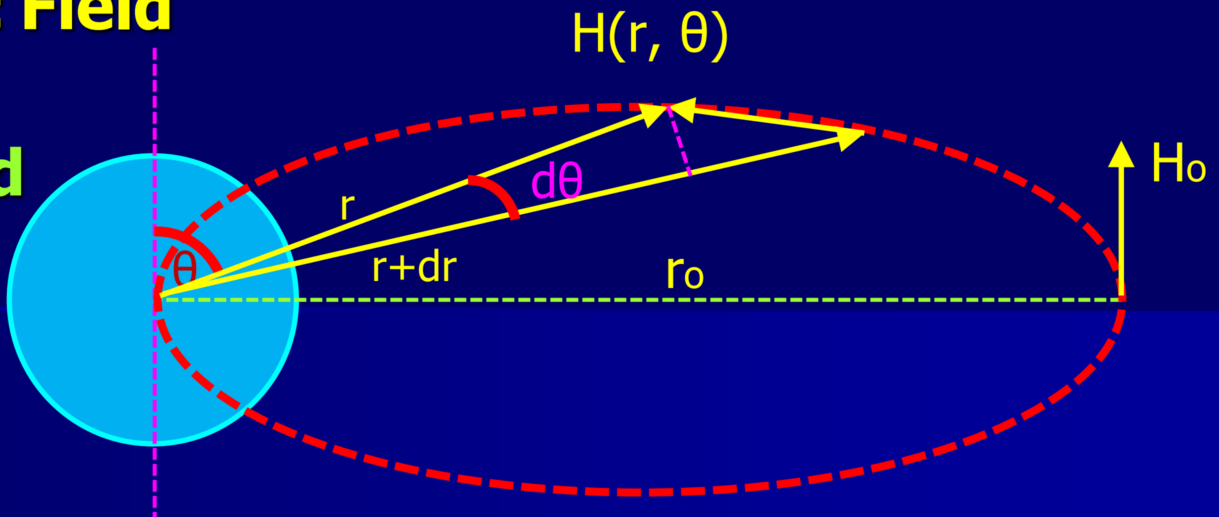
$$\tan\omega = \frac{dr}{rd\theta}$$

$$\therefore dr = rd\theta \tan\omega$$

$$\therefore dr = rd\theta .2\cot\theta$$

The Dipole Magnetic Field

Total Magnetic Field



$$\therefore dr = rd\theta.2\cot\theta$$

$$\int \frac{dr}{r} = 2\int \frac{\cos\theta}{\sin\theta} d\theta + c_1$$

$$\ln(r) = 2\ln(\sin\theta) + \ln(c)$$

$$\ln(r) = \ln(\sin^2\theta) + \ln(c)$$

$$\ln(r) = \ln(c.\sin^2\theta)$$

$$r = c.\sin^2\theta$$

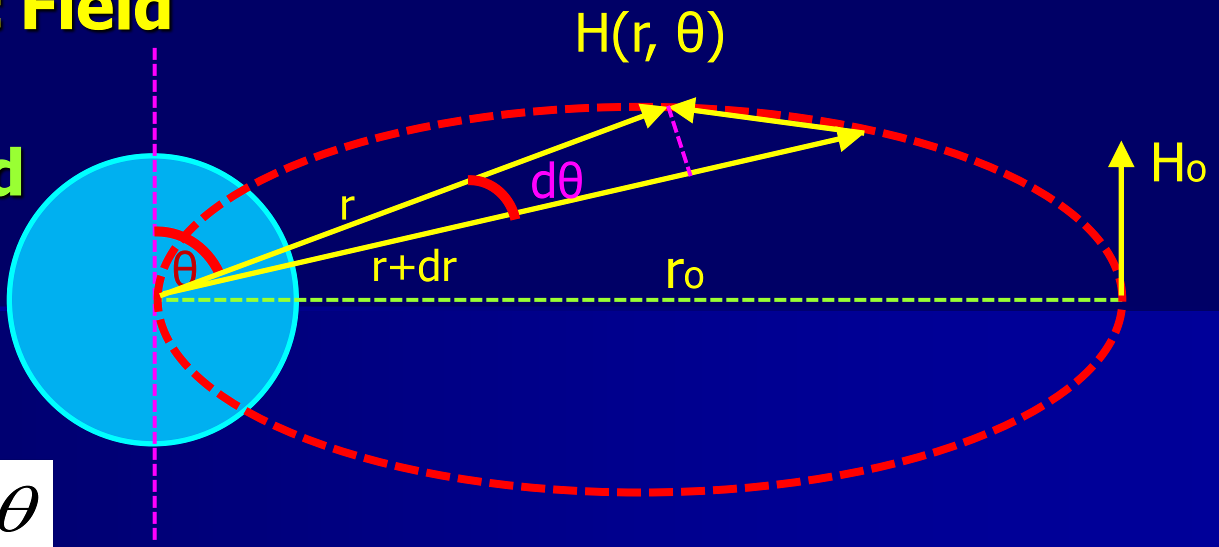
Boundary Conditions :

If $\theta = \pi/2$ and $r = r_o$ then $c = r_o$

$$r = r_o \sin^2\theta$$

The Dipole Magnetic Field

Total Magnetic Field



$$r = r_o \sin^2\theta$$

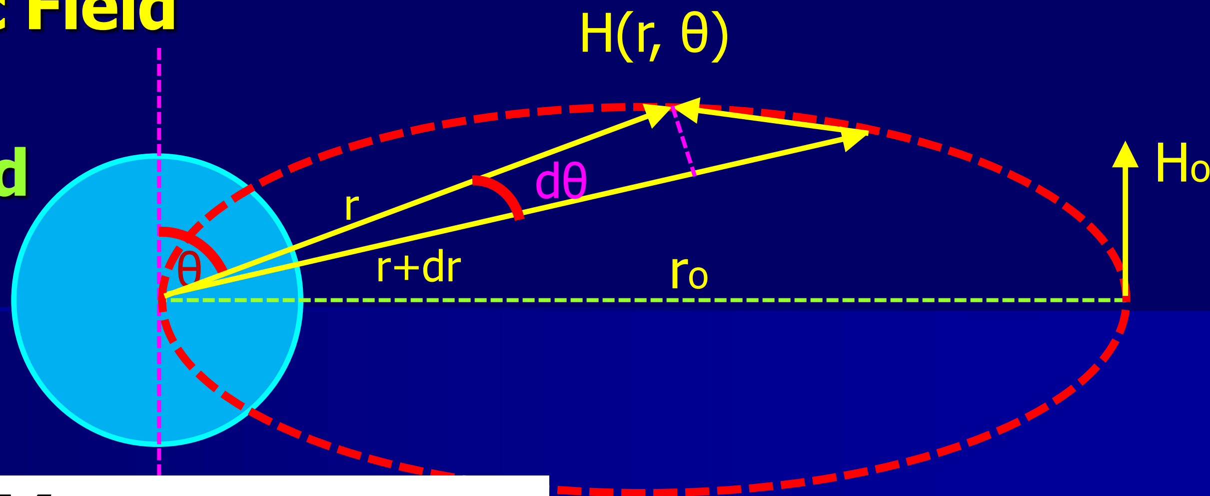
Where **r_o** is the radial distance of the field line at the equator. The above equation is a very important relation because it gives the geometry of the field lines of the dipole field. The magnetic field at the equatorial crossing of a given field line is usually denoted by **H_o** and it is very important parameter because by knowing H_o we can determine the magnetic field at any point along this field line.

Using $$H(r,\theta) = \frac{\mu_o}{4\pi}\frac{M}{r^3}\left[1+3\cos^2\theta\right]^{1/2}$$ and $$r = r_o \sin^2\theta$$

$$r^3 = r_o^3 \sin^6\theta$$

The Dipole Magnetic Field

Total Magnetic Field



$$H(r,\theta) = \frac{\mu_o}{4\pi} \frac{M}{(r_o^{\,3} \sin^6\theta)} [1 + 3\cos^2\theta]^{1/2}$$

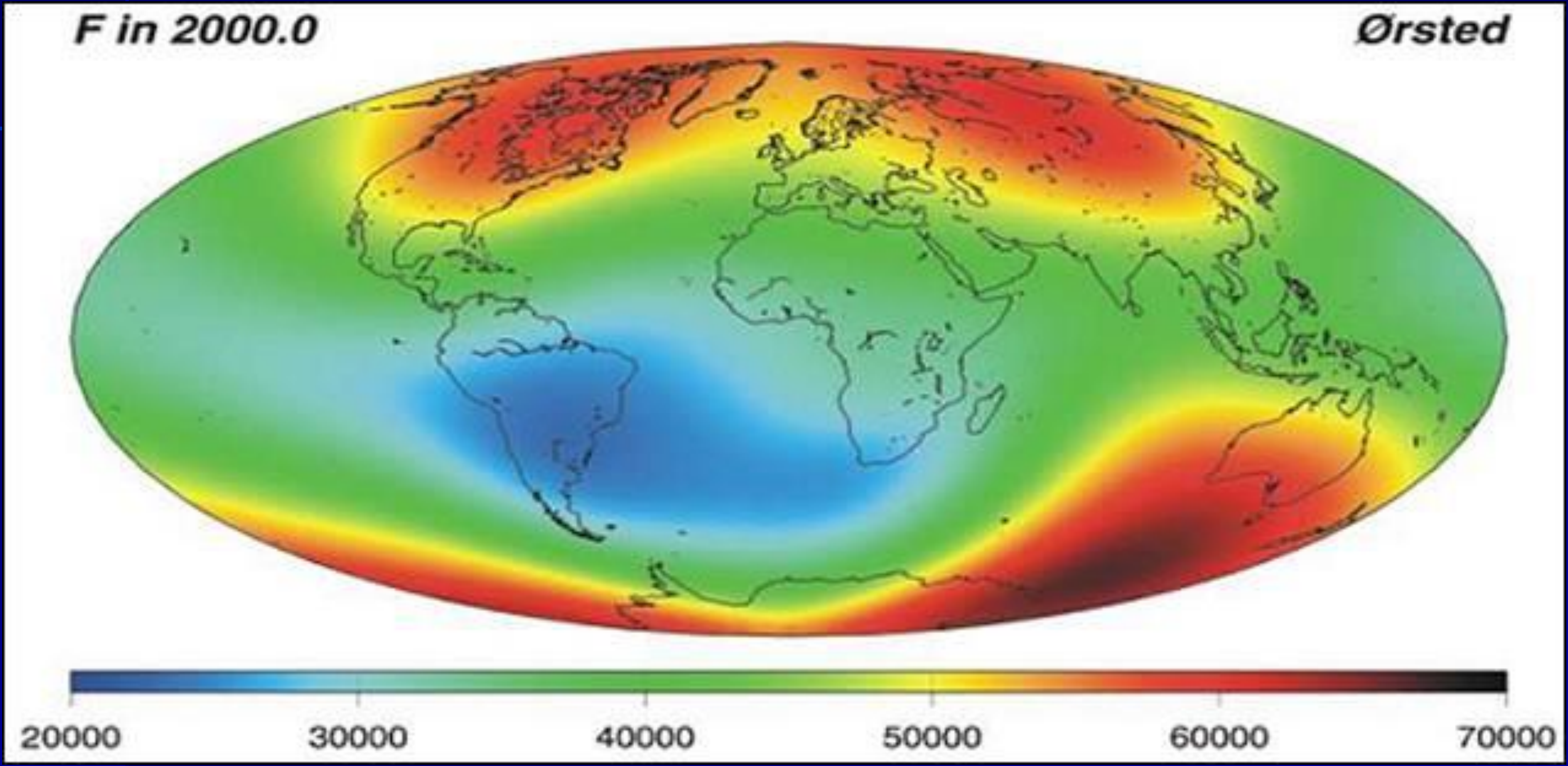
$$H(r,\theta) = \frac{\mu_o}{4\pi} \frac{M}{r_o^{\,3}} \frac{[1 + 3\cos^2\theta]^{1/2}}{\sin^6\theta}$$

$$H(r,\theta) = H_o \frac{[1 + 3\cos^2\theta]^{1/2}}{\sin^6\theta}$$

Where, $H_o = \frac{\mu_o}{4\pi} \frac{M}{r_o^{\,3}}$

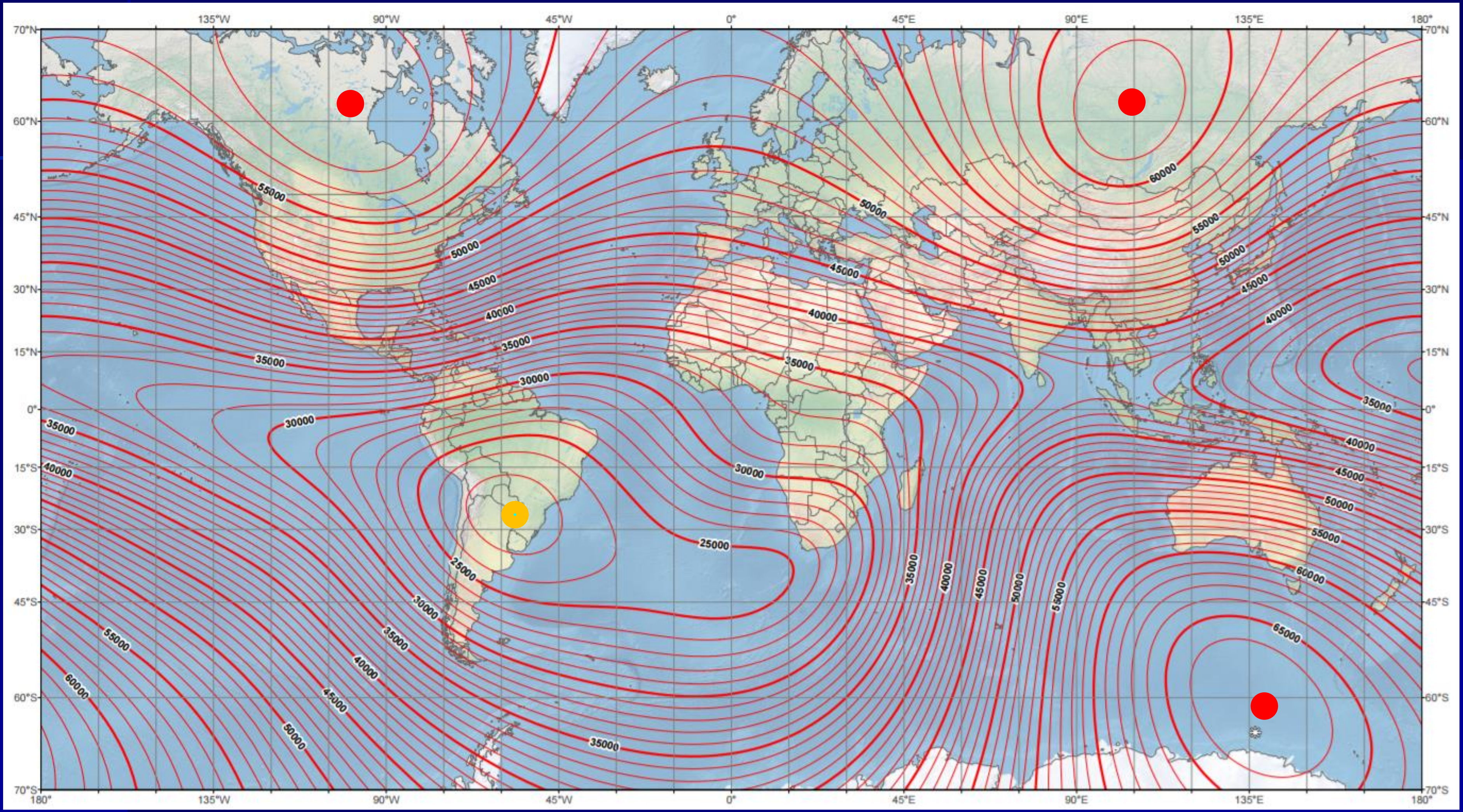
The above equation is probably the most useful expression in the **Mathematical description of the dipole magnetic field** !

The Earth Magnetic Field



The Earth's magnetic field ranges between approximately ~25,000 nT and ~65,000 nT (0.25–0.65 G).

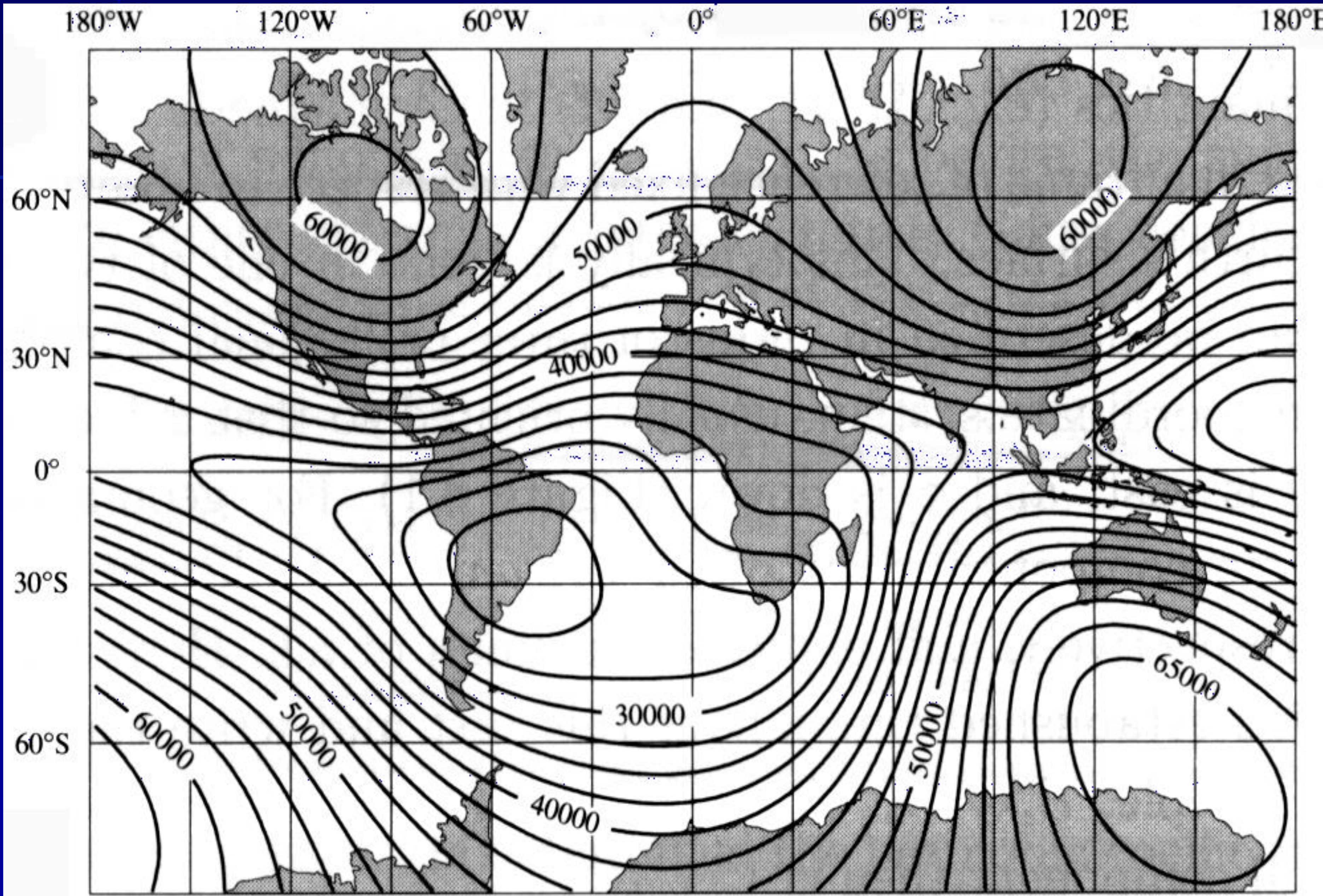
The Earth Magnetic Field



Min -

Max

Earth magnetic Field

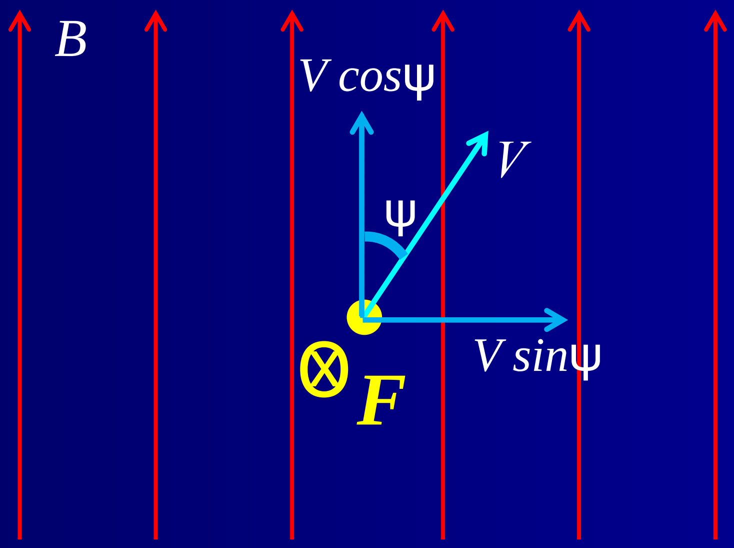


The Magnetosphere

The Earth’s Magnetic Fields
The Dipole Magnetic Field
Motion of charged particles in a Dipole Magnetic Field
The Radiation Belts
The boundary and the tail of the Magnetosphere

Motion of Charged Particle in a Dipole Magnetic field

A charged particle moving with velocity *V* at an angle *ψ*, called **patch angle**, to a magnetic field will experience the **Lorentz Force** *F*.



Thank You !