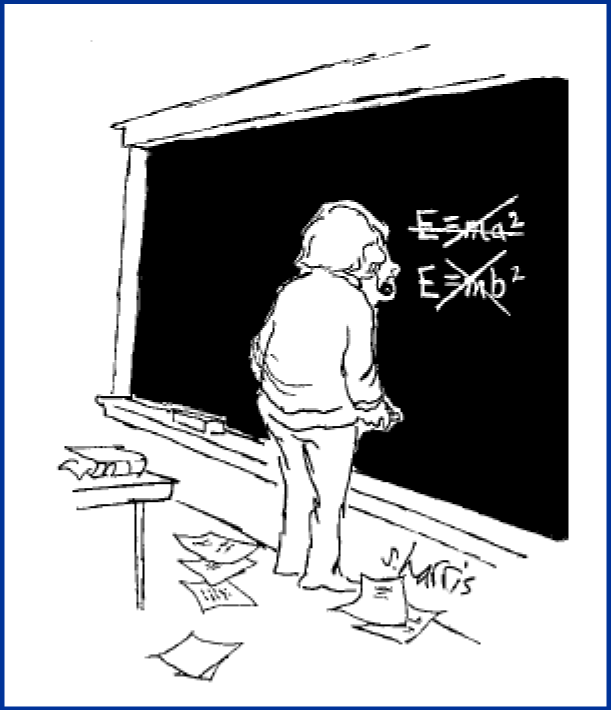
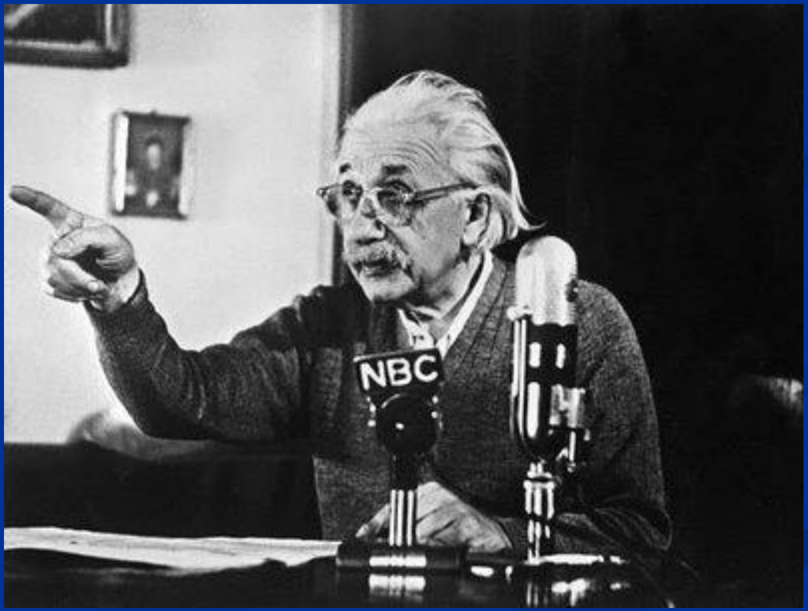
Special Theory of Relativity



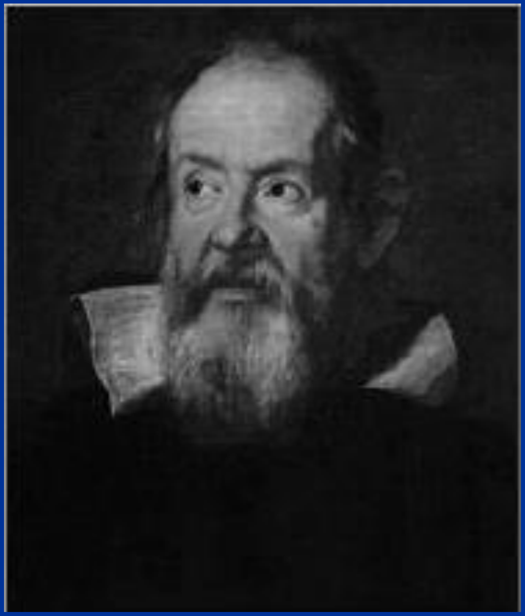


9th Lecture

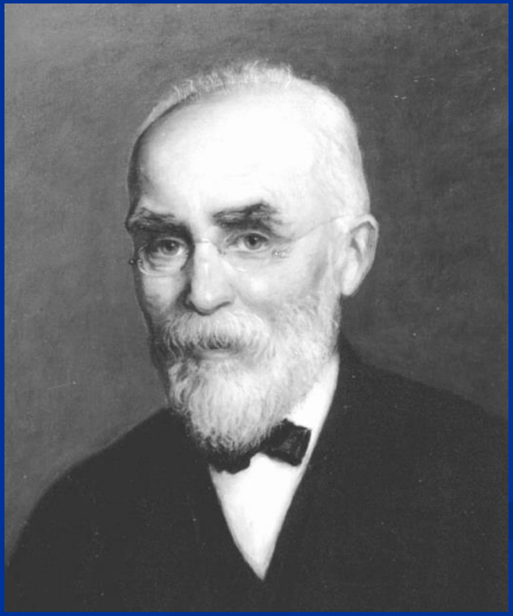
Transformations Equation

Transformation equations are used to transform between the coordinates of two reference frames. There are two types of transformation equations.

1. Galilean Transformation Equation (without relativistic effect!)
2. Lorentz Transformation Equation (with relativistic effect!)



Galileo Galilei



Hendric Lorentz

Galilean Transformation Equations

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Galilean Transformation Equations

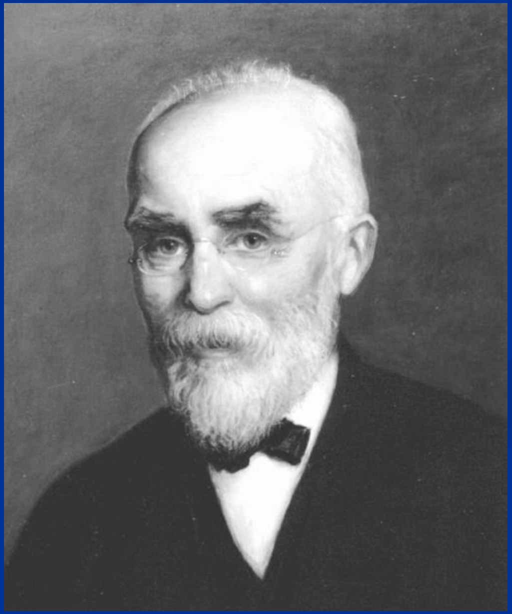
$$x = x' + vt'$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

Inverse Galilean Transformation Equations

They enable us to relate a measurement in one inertial reference frame to another. For example, suppose we measure the velocity of a vehicle moving in the x-direction in system S, and we want to know what would be the velocity of the vehicle in S'.

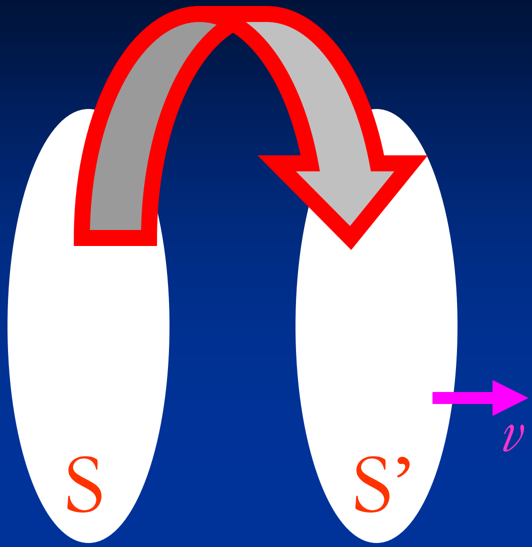
Transformations Equation

Lorentz Transformation Equation (with relativistic effect!)



Hendric Lorentz

Lorentz Transformation Equations



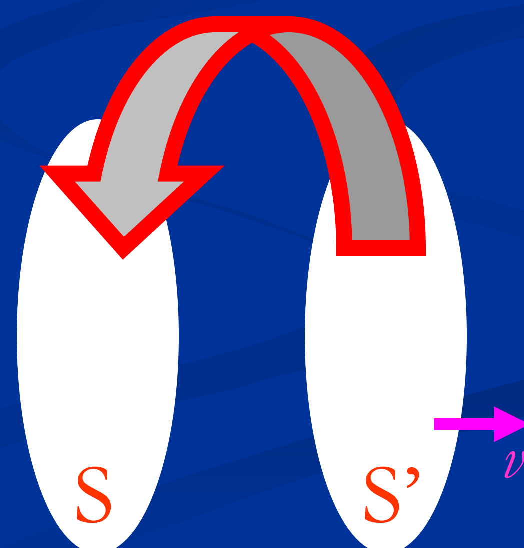
$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Transformation Equations



$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

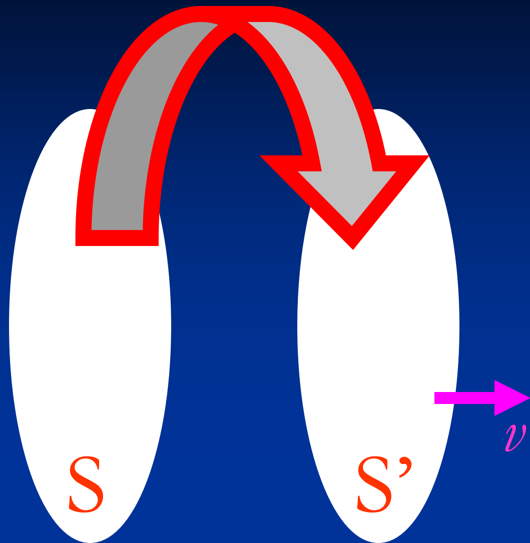
$$y = y'$$

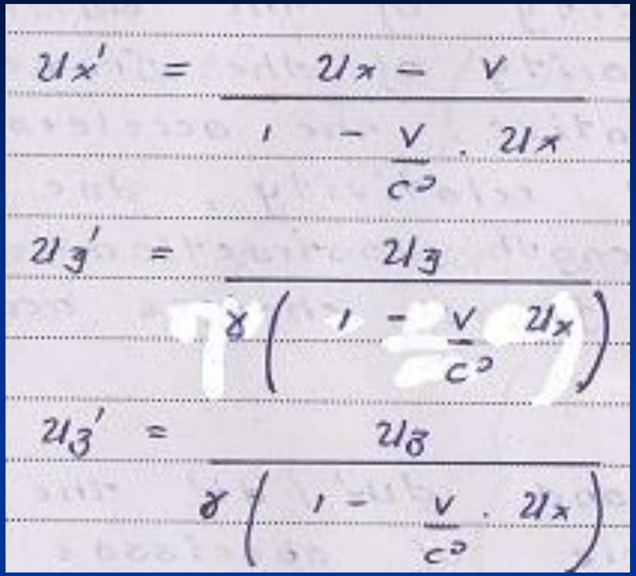
$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Inverse Transformation Equations

Lorentz Velocity Transformation Equations



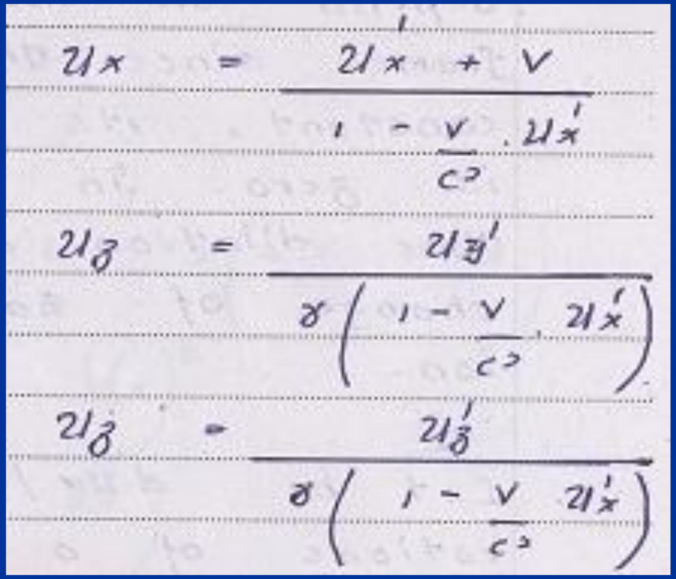


$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2} \cdot u_x}$$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2} u_x\right)}$$

$$u_z' = \frac{u_z}{\gamma\left(1 - \frac{v}{c^2} \cdot u_x\right)}$$

Lorentz Velocity Transformation Equations

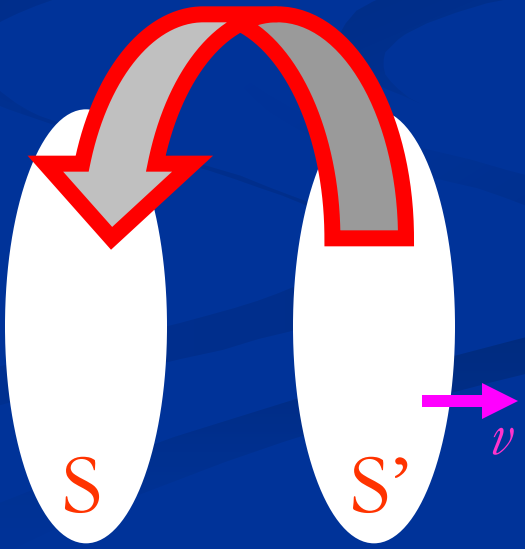


$$u_x = \frac{u_x' + v}{1 - \frac{v}{c^2} \cdot u_x'}$$

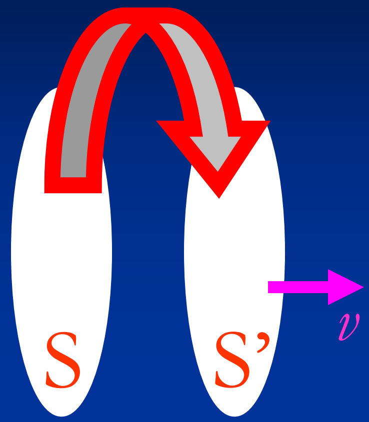
$$u_y = \frac{u_y'}{\gamma\left(1 - \frac{v}{c^2} u_x'\right)}$$

$$u_z = \frac{u_z'}{\gamma\left(1 - \frac{v}{c^2} u_x'\right)}$$

Lorentz Inverse Velocity Transformation Equations



Lorentz Acceleration Transformation Equations



$$a'_x = \frac{a_x}{\gamma^3\left(1 - \frac{v}{c^2}u_x\right)^3}$$

$$a'_y = \frac{a_y\left(1 - \frac{v u_x}{c^2}\right) + \frac{v}{c^2}u_y a_x}{\gamma^2\left(1 - \frac{v}{c^2}u_x\right)^3}$$

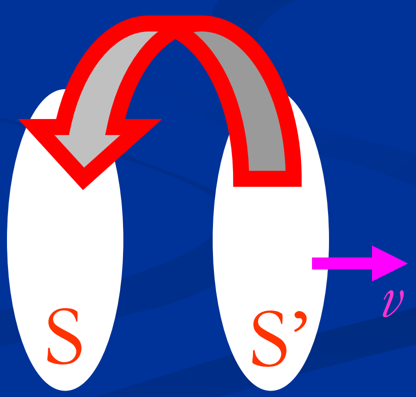
$$a'_z = \frac{a_z\left(1 - \frac{v u_x}{c^2}\right) + \frac{v}{c^2}u_z a_x}{\gamma^2\left(1 - \frac{v}{c^2}u_x\right)^3}$$

Lorentz Acceleration Transformation Equations

$$a_x = \frac{a'_x}{\gamma^3\left(1 + \frac{v}{c^2}u'_x\right)^3}$$

$$a_y = \frac{a'_y\left(1 + \frac{v u'_x}{c^2}\right) - \frac{v}{c^2}u'_y a'_x}{\gamma^2\left(1 + \frac{v}{c^2}u'_x\right)^3}$$

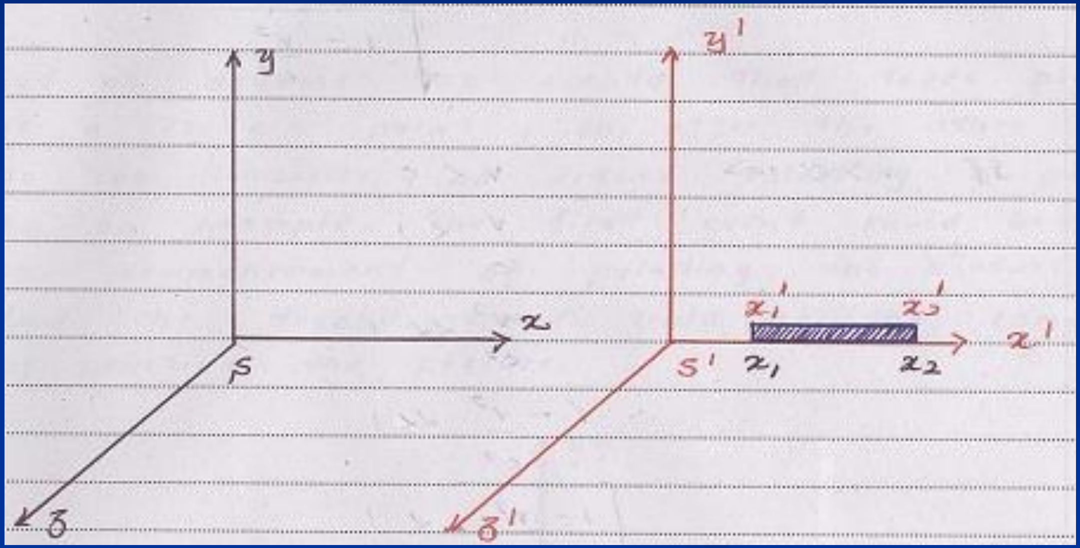
$$a_z = \frac{a'_z\left(1 + \frac{v u'_x}{c^2}\right) - \frac{v}{c^2}u'_z a'_x}{\gamma^2\left(1 + \frac{v}{c^2}u'_x\right)^3}$$



Lorentz Inverse Acceleration Transformation Equations

Length Contraction (using Lorentz Transformation)

Length contraction is the observation that a moving object appears shorter than a stationary object.



Let us assume there is a rod of length L with respect to the stationary frame S', is moving with constant velocity v.

Length with respect to the S' frame = L_0 = $x_2^1 - x_1^1$

Length with respect to the S frame = L = $x_2 - x_1$

Length Contraction (using Lorentz Transformation)

Using Lorentz transformation equations;

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x^1 = \gamma(x - vt)$$

Therefore, $x_1^1 = \gamma(x_1 - vt_1)$ and $x_2^1 = \gamma(x_2 - vt_2)$

$$x_2^1 - x_1^1 = \gamma(x_2 - vt_2) - \gamma(x_1 - vt_1)$$

$$x_2^1 - x_1^1 = \gamma(x_2 - x_1) - \gamma v(t_2 - t_1)$$

Where, $t_1 = t_2$

Therefore, $x_2^1 - x_1^1 = \gamma(x_2 - x_1)$

$$L_o = \gamma L$$

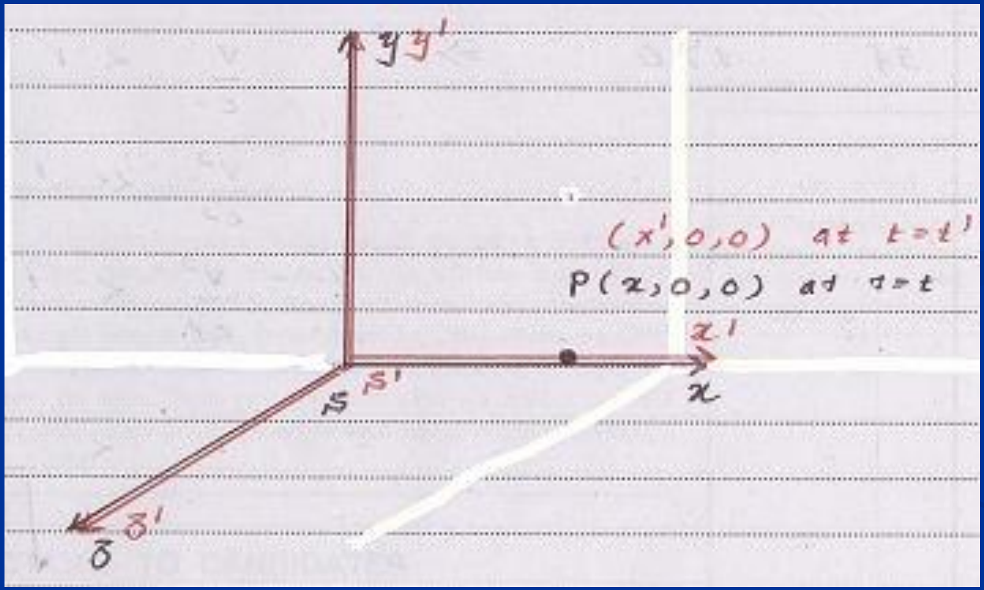
$$L_0 > L$$

L_0: Length measured by an observer in the Frame S'

L: Length measured by an observer in the Frame S

Time Dilation (using Lorentz Transformation)

Let us assume two events that takes place at a certain point, one after the other. We can consider, an artist painting a picture as an example. The first event could be the commencement of painting the picture and the second event could be the completion of painting the picture.



Time interval with respect to the S' frame $= t =$ $t_2^1 - t_1^1$

Time interval with respect to the S frame $= to =$ $t_2 - t_1$

Time Dilation (using Lorentz Transformation)

Using Lorentz transformation equations;

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t^1 = \gamma\left(t - \frac{v}{c^2}x\right)$$

Therefore, $t_1^1 = \gamma\left(t_1 - \frac{v}{c^2}x_1\right); \quad x_1 = x$ and $t_2^1 = \gamma\left(t_2 - \frac{v}{c^2}x_2\right); \quad x_2 = x$

$$t_2^1 - t_1^1 = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) - \gamma\left(t_1 - \frac{v}{c^2}x_1\right)$$

$$t_2^1 - t_1^1 = \gamma(t_2 - t_1) - \gamma\frac{v}{c^2}(x_2 - x_1)$$

Where,

$$x_1 = x_2$$

Therefore, $t_2^1 - t_1^1 = \gamma(t_2 - t_1)$

$$t = \gamma t_0$$

$$t > t_o$$

Time interval measured by an observer in the Frame S' (t)

Time interval measured by an observer in the Frame S (t_o)

Relative Motion for the two bodies in Relativity

Let us assume two objects are moving in an opposite direction to each other,

Using Lorentz transformation equations;

$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2} \cdot u_x}$$

For this example; $U_x^1 = V_A$, $U_x = -V_B$ and $v = V_{(B,A)}$

Direction of B is opposite to the A

$$U_x^1 = \frac{U_x - v}{1 - \frac{U_x v}{c^2}} \Rightarrow V_A = \frac{(-V_B) - v}{1 - \frac{(-V_B)v}{c^2}} \Rightarrow -v = \frac{V_A + V_B}{1 + \frac{V_A V_B}{c^2}}$$

$$-v = \frac{V_A + V_B}{1 + \frac{V_A V_B}{c^2}}$$

This v denotes $V(B, A)$. $V(B, A)$ has a negative value. :. The direction of V(B, A) should be the opposite direction. :. V(A, B) is +ve;

$$v = V_{(A,B)} = \frac{V_A + V_B}{1 + \frac{V_A V_B}{c^2}}$$

Or $v = V_{(B,A)}$

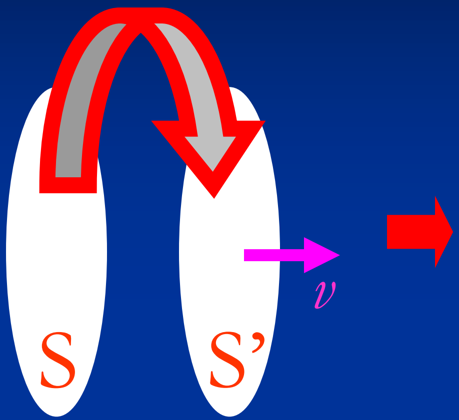
Let us assume two objects are moving in a same direction,

For this example; $U_x^1 = V_A$,

$U_x = V_B$ and $v = V_{(B,A)}$

$$v = V_{(B,A)} = \frac{V_A - V_B}{1 - \frac{V_A V_B}{c^2}}$$

Find the Energy & Momentum in S' frame w. r. t. S frame



$$E^1 = \gamma(v)\left[E - P_x v\right]$$

$$P_x^1 = \gamma(v)\left[P_x - \frac{Ev}{c^2}\right],$$

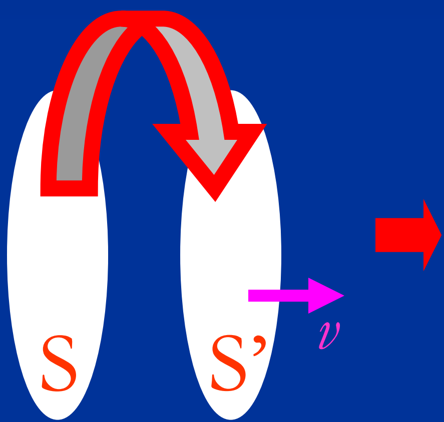
$$P_y^1 = P_y$$ and $$P_z^1 = P_z$$

Where, $\gamma(v) = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$, $E = mc^2 = \dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}c^2 = \gamma(v)m_o c^2$

and $P_x = mU_x = \dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}U_x = \gamma(v)m_o U_x$

Lorentz Invariant

A quantity that remains unchanged by a Lorentz transformation is said to be Lorentz invariant. Such quantities play on especially important role in special theory of relativity. The norm of any four vector is Lorentz Invariant.



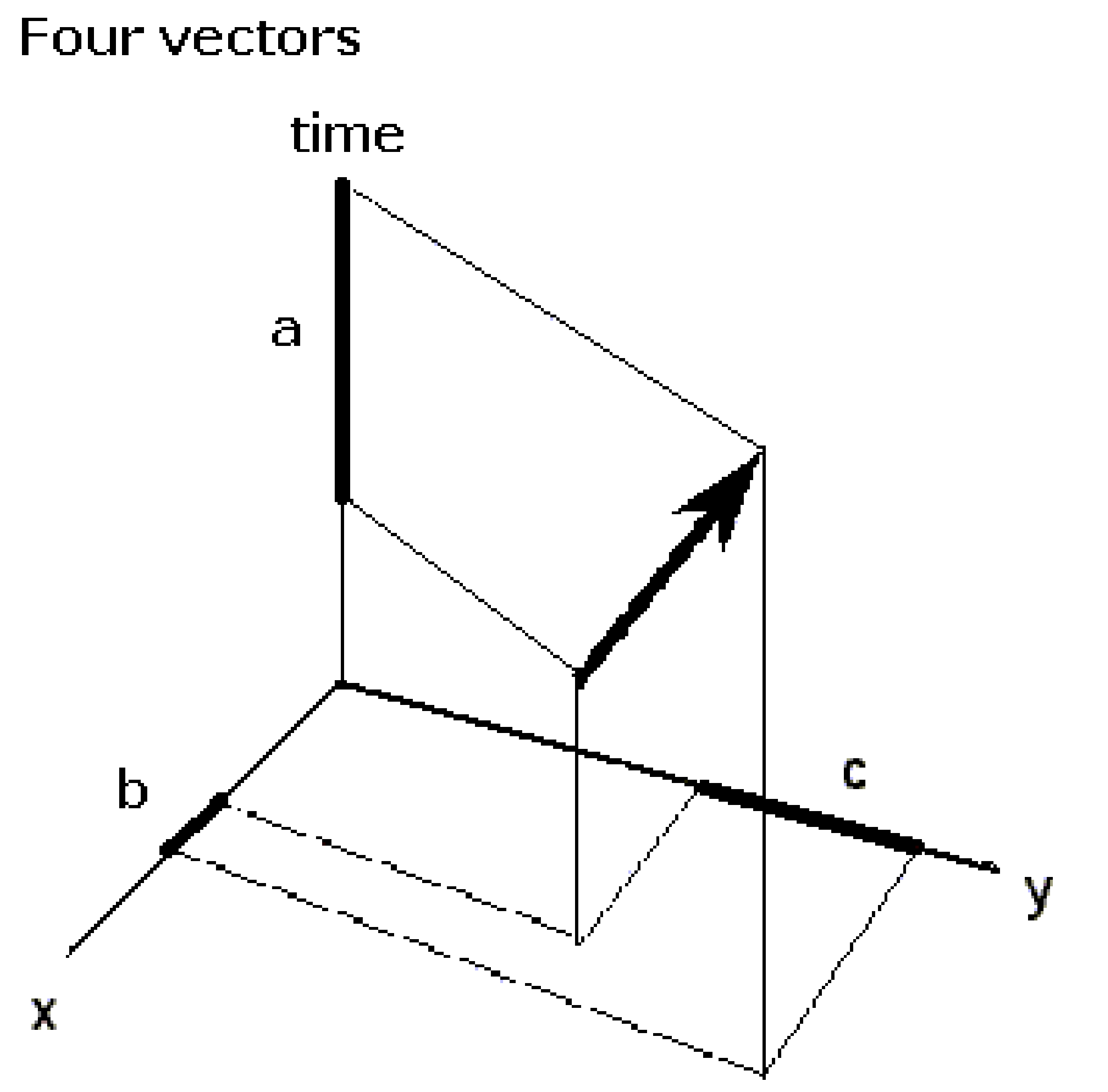
$$E'^2 - p'^2 c^2 = E^2 - p^2 c^2$$

and

$$c^2 t'^2 - r'^2 = c^2 t^2 - r^2$$

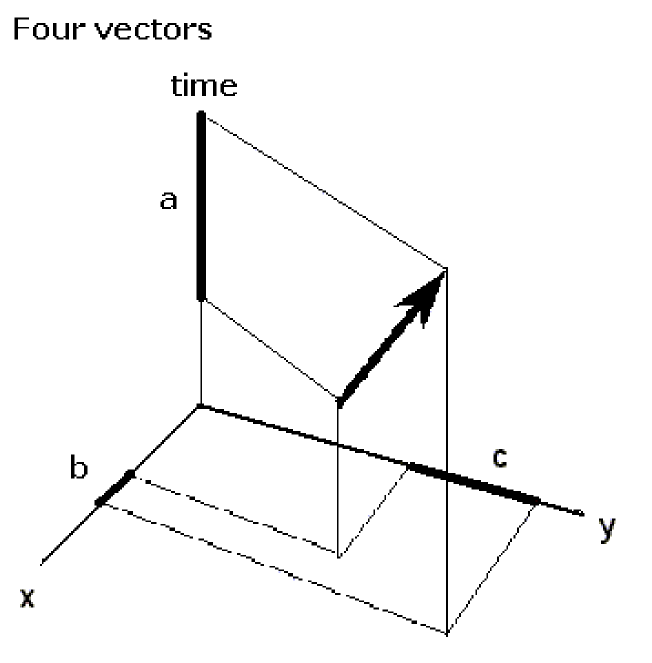
Four Vectors / Four Cdts Systems

Four Vectors / Four Cdts Systems



A four vector is a displacement in both space and time.

Four Vectors / Four Cdts Systems



A four vector is a displacement in both space and time.

In the theory of relativity, a four-vector is a vector in a four-dimensional real vector space, called Minkowski Space. It differs from a vector in that it can be transformed by Lorentz Transformation. The usage of the four-vector name tacitly assumes that its components refer to a standard basis. The components transform between these bases as the space and time coordinates differences, $(\Delta x, \Delta y, \Delta z, \Delta t)$ under spatial translations, rotations and boosts [a change by a constant velocity to another inertial reference frame]

Minkowski Four-Dimensional Space ("World")

We can characterize the Lorentz Transformation still more simply if we introduce the imaginary *Sqrt[-1] = i* *ct* in place of t, as time variable. If in accordance with this, we insert,

$$
\begin{aligned}
x_1 &= x \\
x_2 &= y \\
x_3 &= z \\
x_4 &= ict
\end{aligned}
$$

Where, $i = \sqrt{-1}$

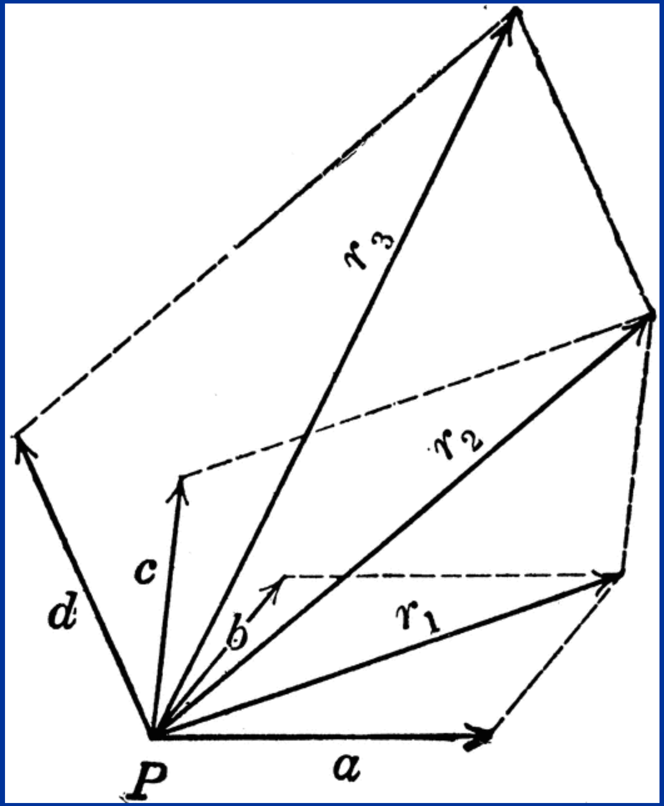
and similarly for the accented system K', then the condition which is identically satisfied by the transformation can be expressed, thus;

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = x_1^{1^2} + x_2^{1^2} + x_3^{1^2} + x_4^{1^2}$$

That is, by the afore-mentioned choice of "co-ordinates" is transformation into this equation.

Minkowski Four-Dimensional Space ("World")

We see from the above equation that the imaginary time co-ordinate X_4 enters into the condition of transformation in exactly the same way as the space co-ordinates X_1, X_2 and X_3. It is due to this fact that, according to the theory of relativity, the " time " X_4 enters into natural laws in the same form as the space co-ordinates X_1, X_2 and X_3.



A four-dimensional continuum described by the co-ordinates X_1, X_2 and X_3 was called "**World**" by Minkowski, who also termed a point-event a "World-point". From a "happening" in three-dimensional space, Physics becomes, as it were, an "existence" in the four-dimensional "World".

Causality

Thank You !