Space & Atmospheric Physics

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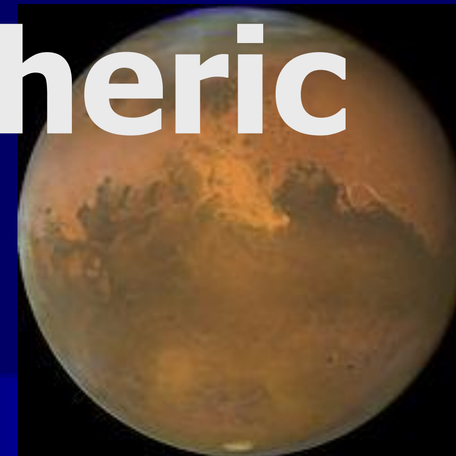
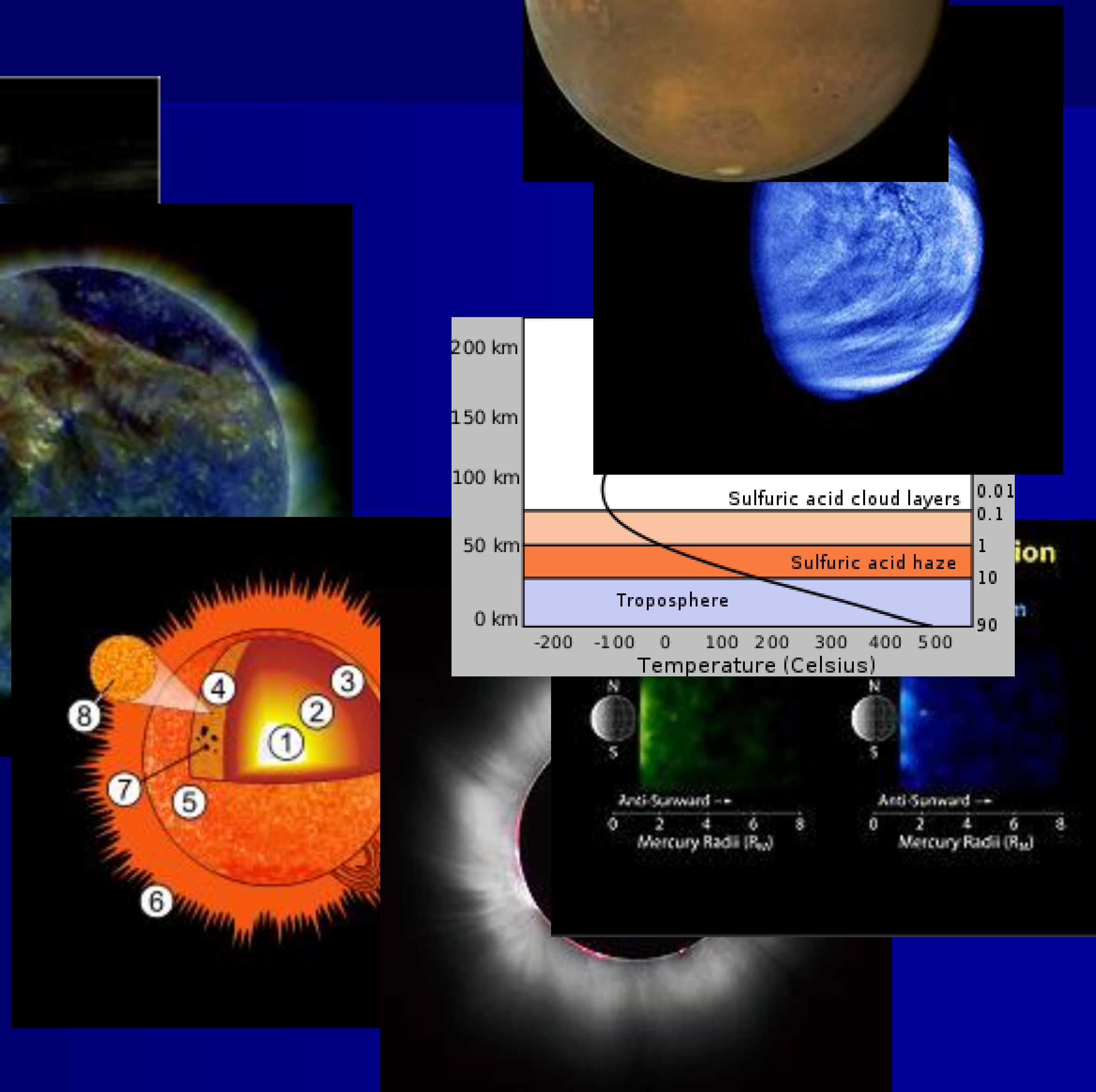
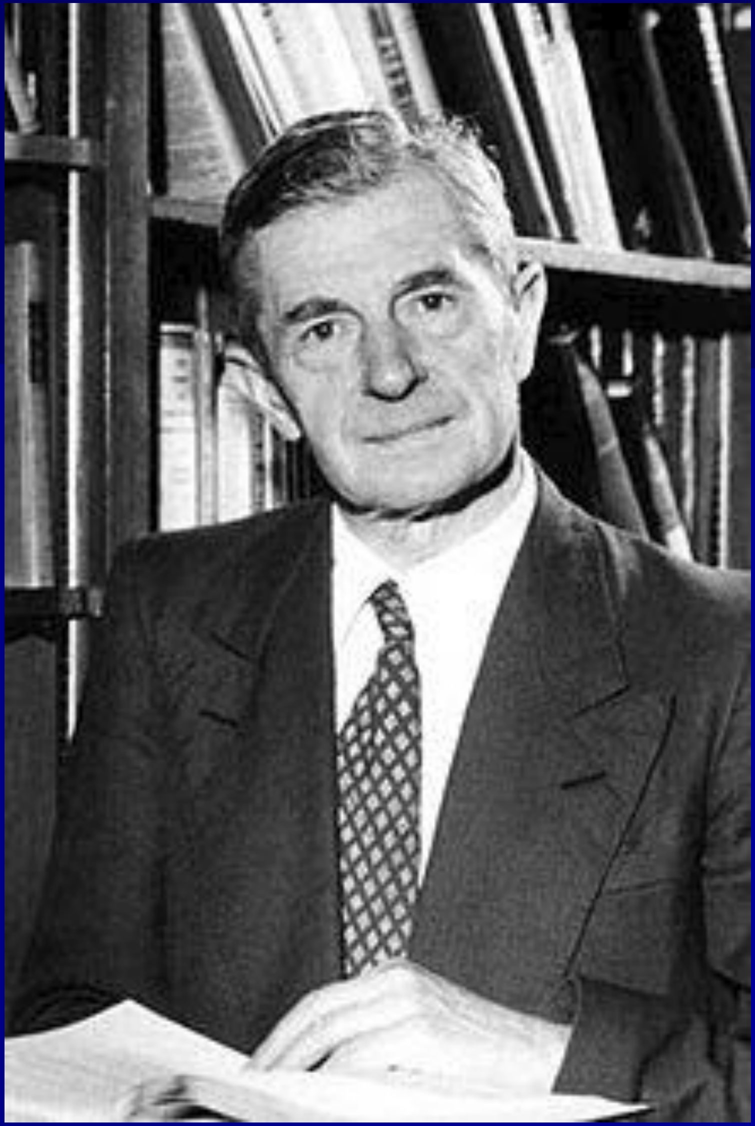




Lecture – 06 B

Chapman layer Theory

Sydney Chapman FRS (29 January 1888 – 16 June 1970) was a British mathematician and geophysicist. His work on the kinetic theory of gases, solar-terrestrial physics, and the Earth's ozone layer has inspired a broad range of research over many decades. He was Chief Professor of Mathematics at Imperial College London between 1924 and 1946.

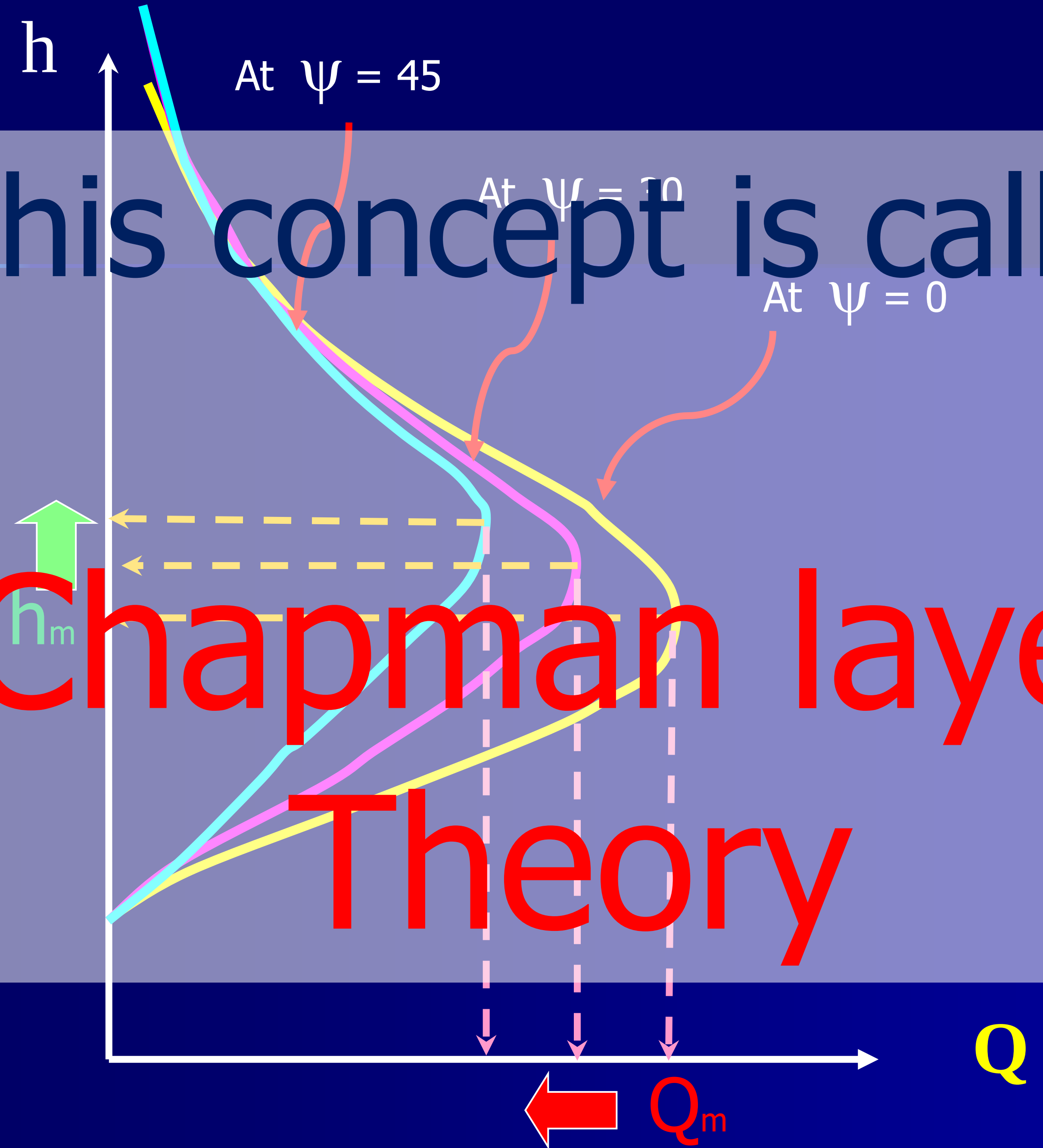


If ψ is increasing, the maximum value of the Electron Production Rate is decreasing. For that Molecular Number Density of the ionosphere should be decreasing.

.: Region of the Q_{max} is going to far away from the Earth surface. Because N should be decrees. Because h is low, N is high and h is high, N is low.

This concept is called

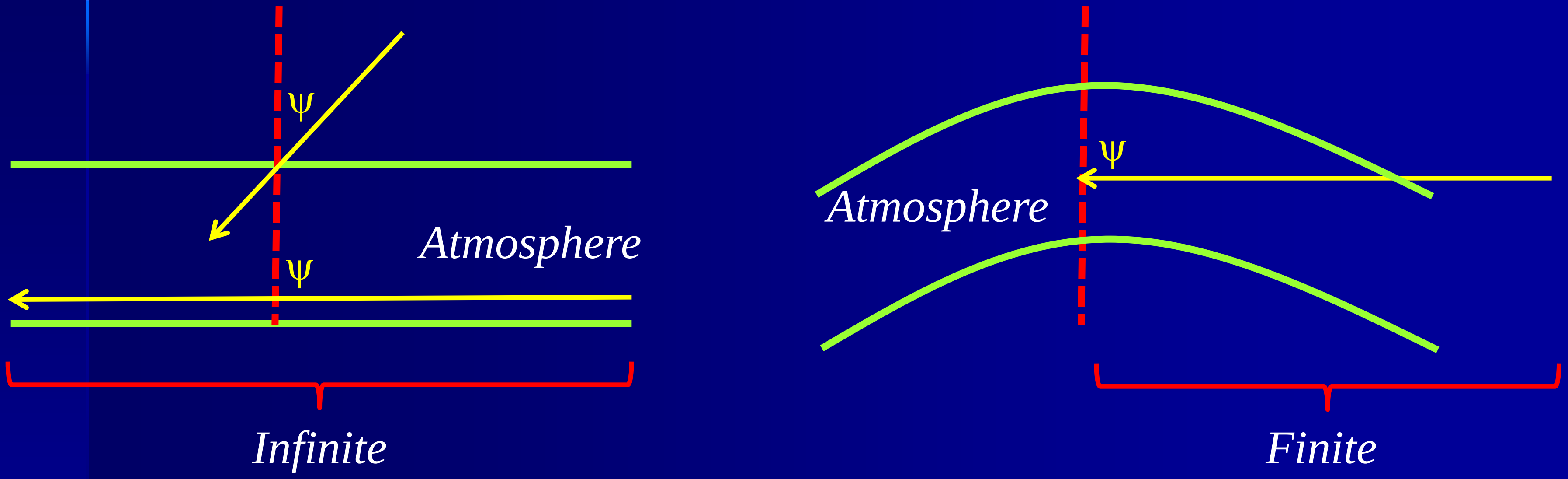
Chapman layer Theory



Chapman layer Theory

Electron Production Rate (Q)

If ψ (**angle of elevation** OR **Zenith Angle**) is high values (~90°), our plate assumption is not corrected.



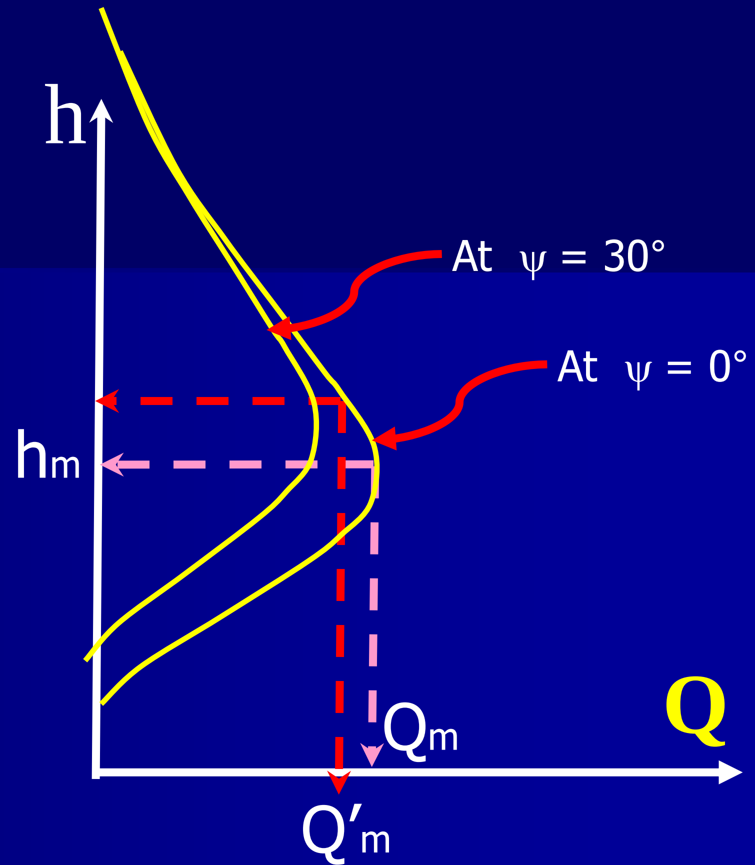
If ψ = 90°, according to our formula and logics, N → 0 !

That means Q_{max} is going to infinity. This is theoretical. But practically this should be large value; but not infinity.

Chapman layer Theory

Electron Production Rate (Q)

If we want to find the value of Q'_m (using the graph), ψ should be zero. Because ψ>0, there is no point on the graph when $Q=Q'_m$ according to the graph.



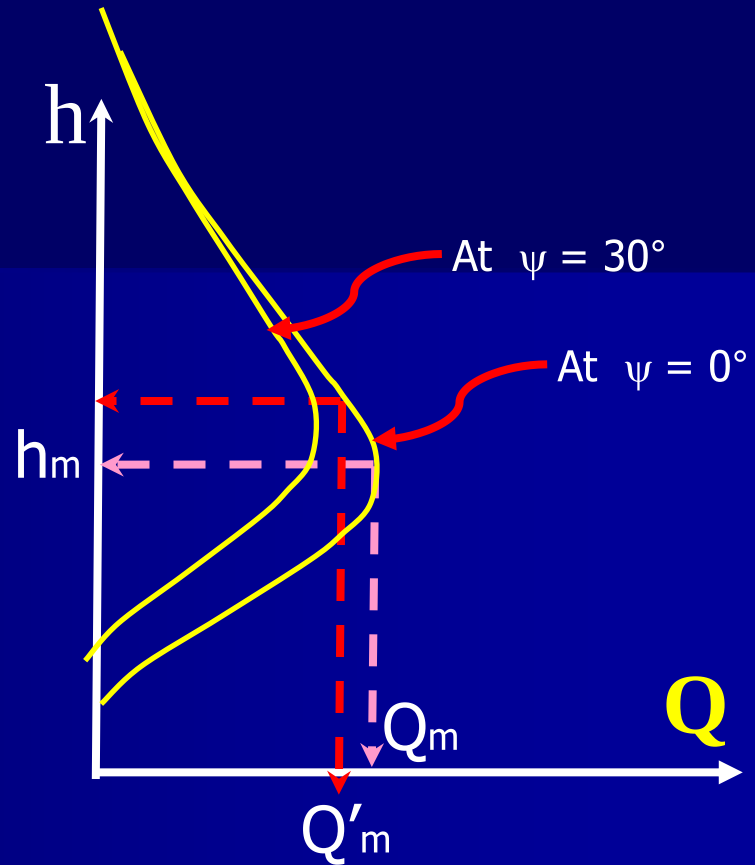
That means, if we want to find the value of h corresponding Q'_m :
It is depend on the "Time" of the day,
Eg: at 12:00 pm
at 1:00 pm
at 2:00 pm
. . .

- **At the night** there is **no height to corresponding to our value Q'_m** ! Because our graph [according to Chapman Layer Theory] does not exist at night.

Chapman layer Theory

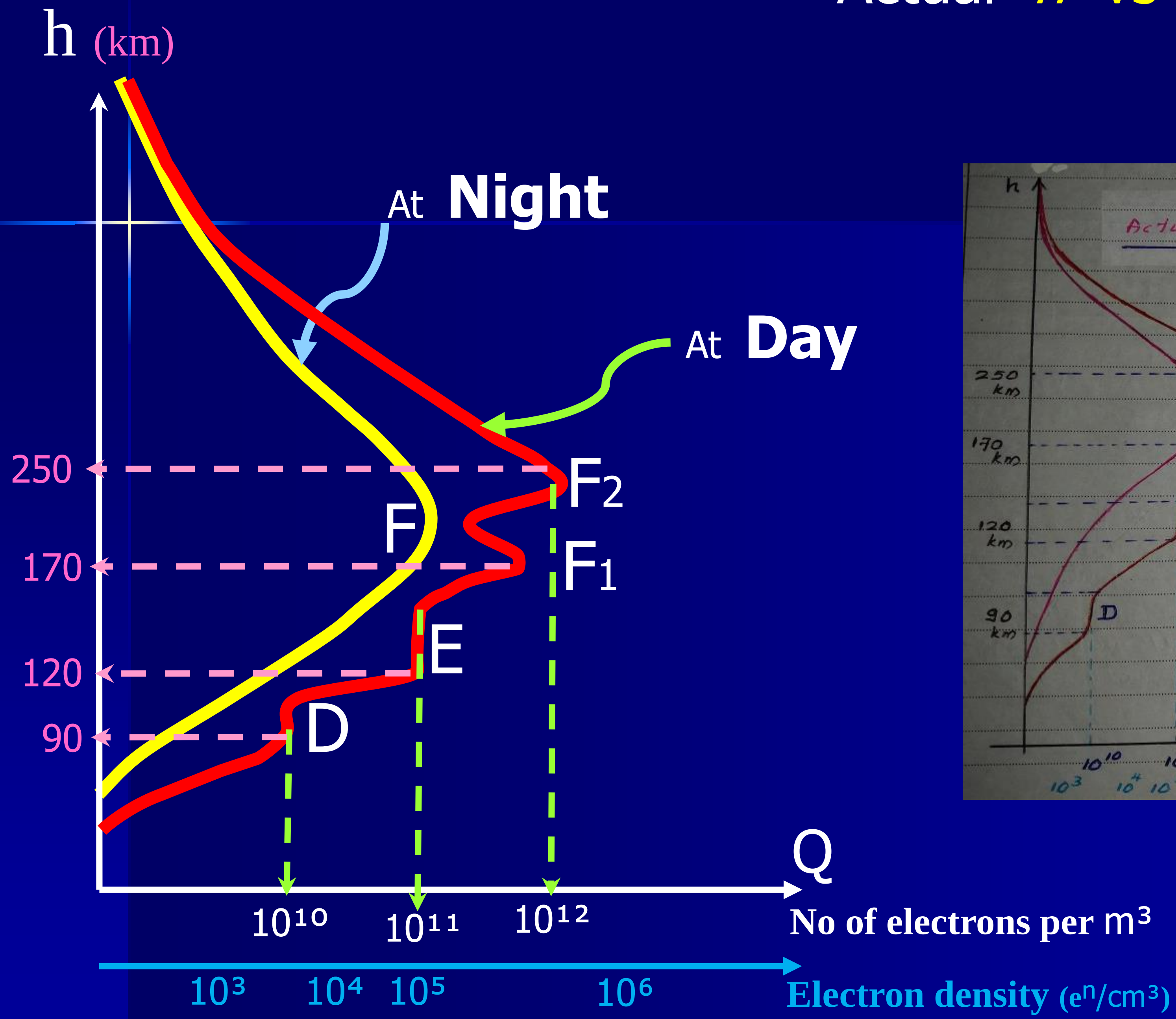
Electron Production Rate (Q)

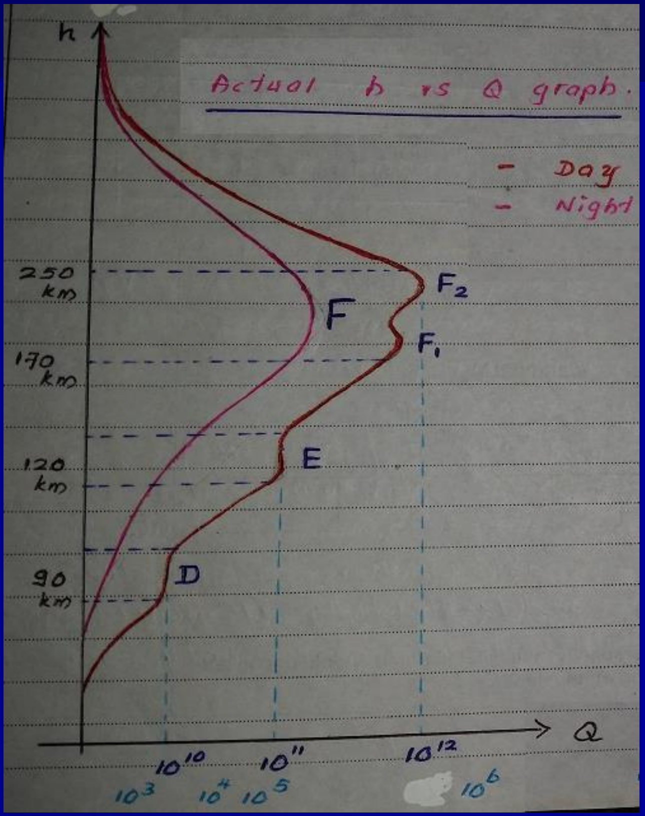
- There are **so many types of gasses in the atmosphere** of the Earth. As a result, the graph of h vs Q **should be contained several peaks**.
- Also if we assume there is a **monochromatic wavelength** comes from the Sun. This is wrong. There should be several peaks of the graph of h vs Q, because of there are **several wavelengths comes from the Sun to ionized the gasses**.

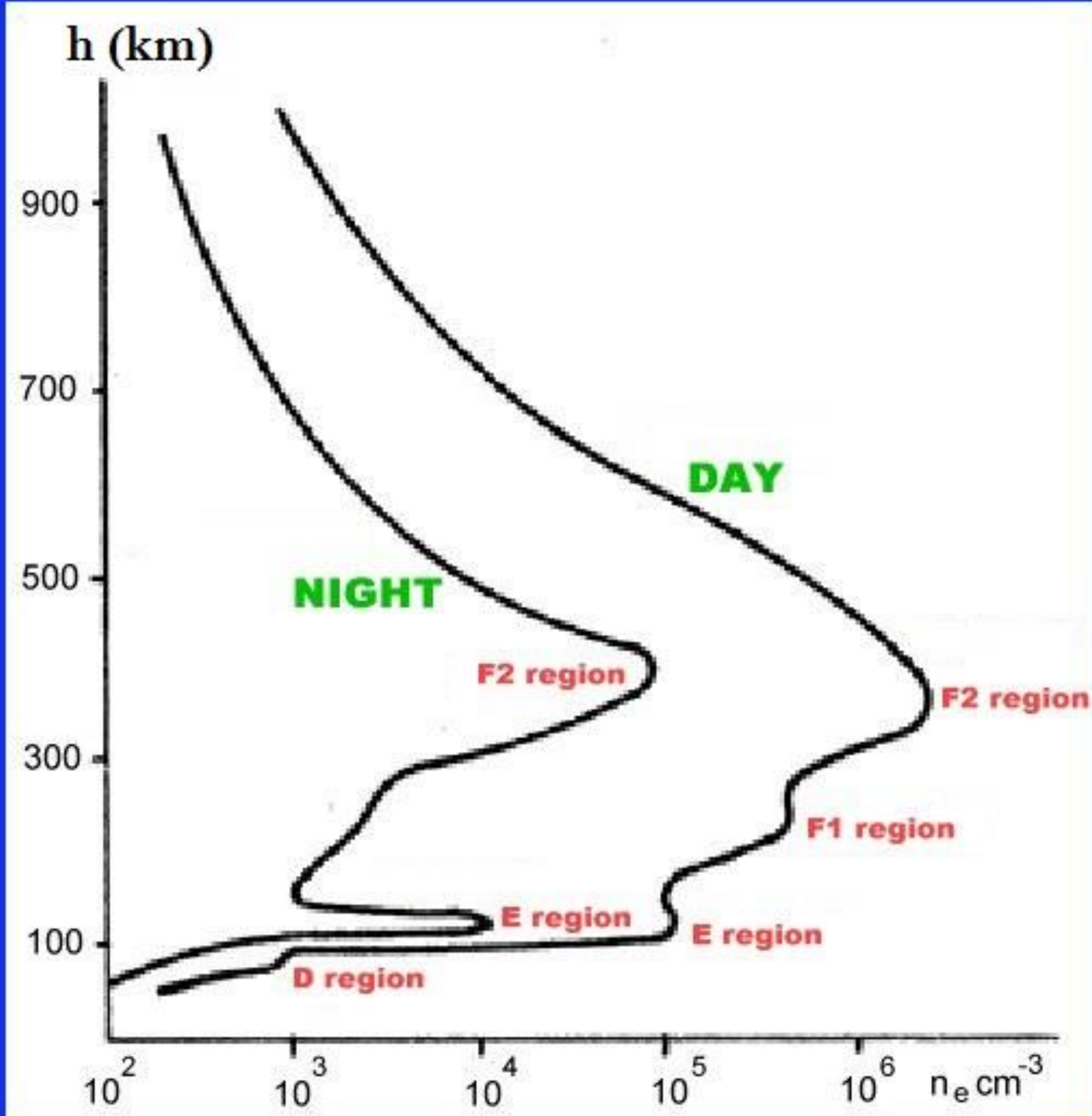


:. We should consider all the effects that we discussed, before plotting the graph of h vs Q.

Actual *h* vs *Q* graph



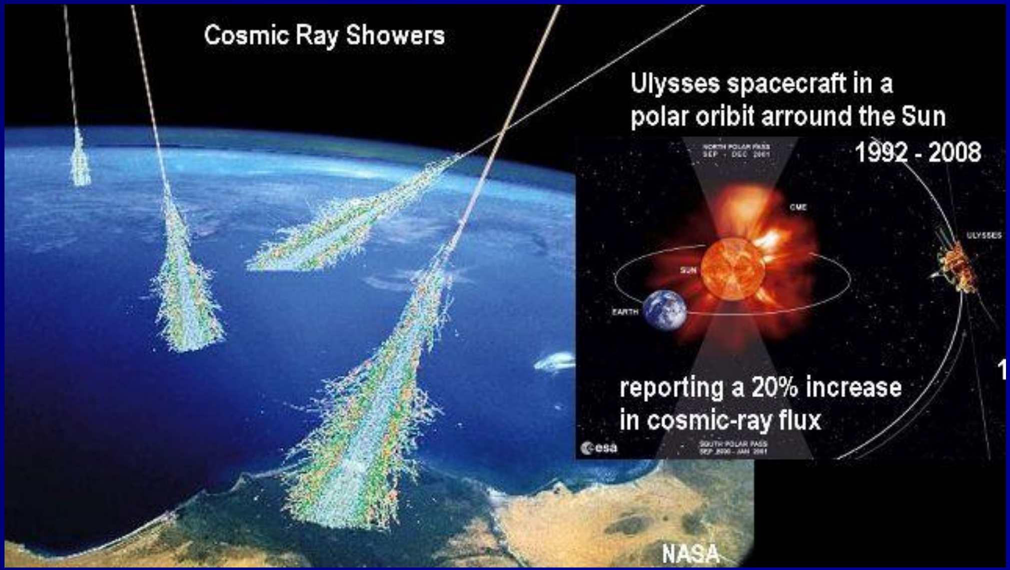



h (km)
900
700
500
300
100
DAY
NIGHT
F2 region
F2 region
F1 region
E region
E region
D region
10^2
10^3
10^4
10^5
10^6
$n_e\ cm^{-3}$

Chapman layer Theory

- We can not find the value of Q (Electron Production Rate) at night using our derived formula. Because, if $\psi > 90°$ our is formula failed !

$$Q = \frac{\eta \cdot I_{\infty}}{e\,H} e^{\left(1 - Z - \sec\psi \cdot e^{-Z}\right)}$$



- Galactic Cosmic Rays :

Galactic Cosmic Rays comes from the Sun and this radiation is spread all over the Universe. As a result this Galactic Cosmic Radiations comes to the Earth. At night there is no rays comes from the Sun, but Galactic Cosmic Radiations comes to the Earth at night. Therefore, there are several number of ionized electrons may exist at the night !

Plasma Frequency

Practically all of our knowledge about the ionosphere has come through radio sounding. Only in the late fifties and early sixties some measurements of local electron densities were made in the **upper ionosphere** using rockets and satellites, but even these methods have now been abandoned in favor of the more efficient **top-side sounder** satellites which again use radio waves to probe the top side of the ionosphere.

Let us consider an ionized layer with an uniform electron density N and radio waves of frequency f incident normally (at right angles) upon the layer. If the frequency is above a limiting frequency f_p the waves will pass through the layer, whereas if $\mathbf{f < f_p}$, the waves will be reflected back. This critical frequency is called the **Plasma Frequency, f_p** and is proportional to the square root of the **electron density, N** of the Layer

$$f_p \;\alpha\; N^{1/2}$$

Plasma Frequency

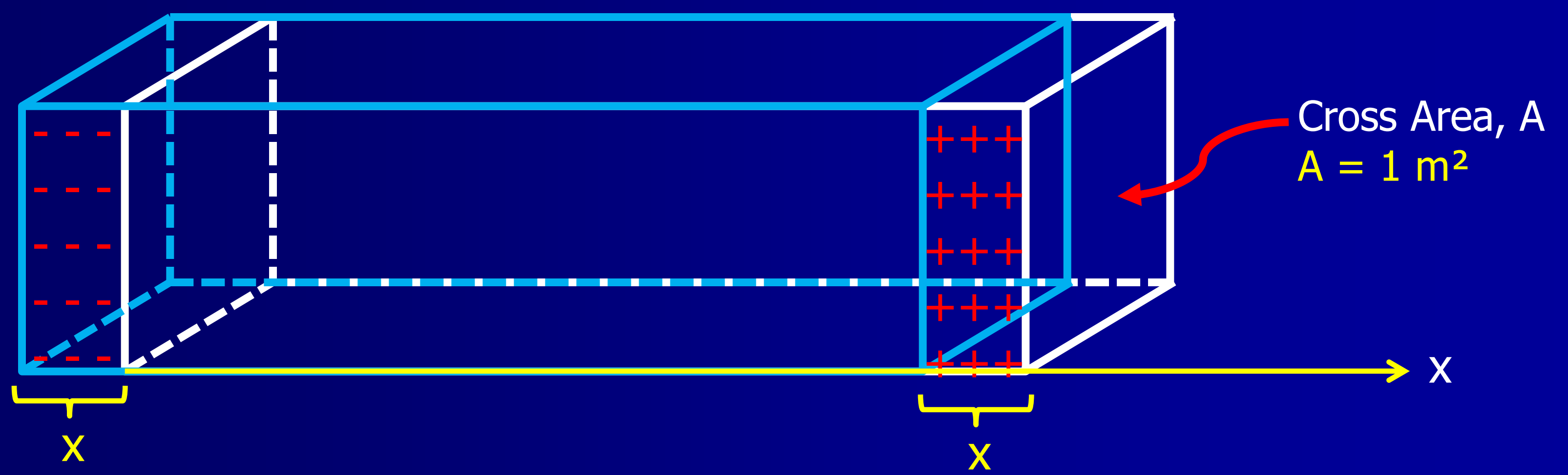
Plasma is the name given to a **mixture of** electrons, ions and neutral particles. When an electromagnetic wave such as the radio wave enters into a plasma, its electric field tends to set the charge particles in motion. The ions, which are about **10^4** times heavier than the electrons, respond very little to the weak field of the wave and can be considered as stationary. The light electrons, on the other hand, react readily to the **-eE** force acting on them. (Where -e is the negative charge of an electron)

Let N_i and N_e be the initial number of densities of the ions and electrons. Since the ionosphere is neutral we can set,

$$N_i = N_e = N$$

Plasma Frequency

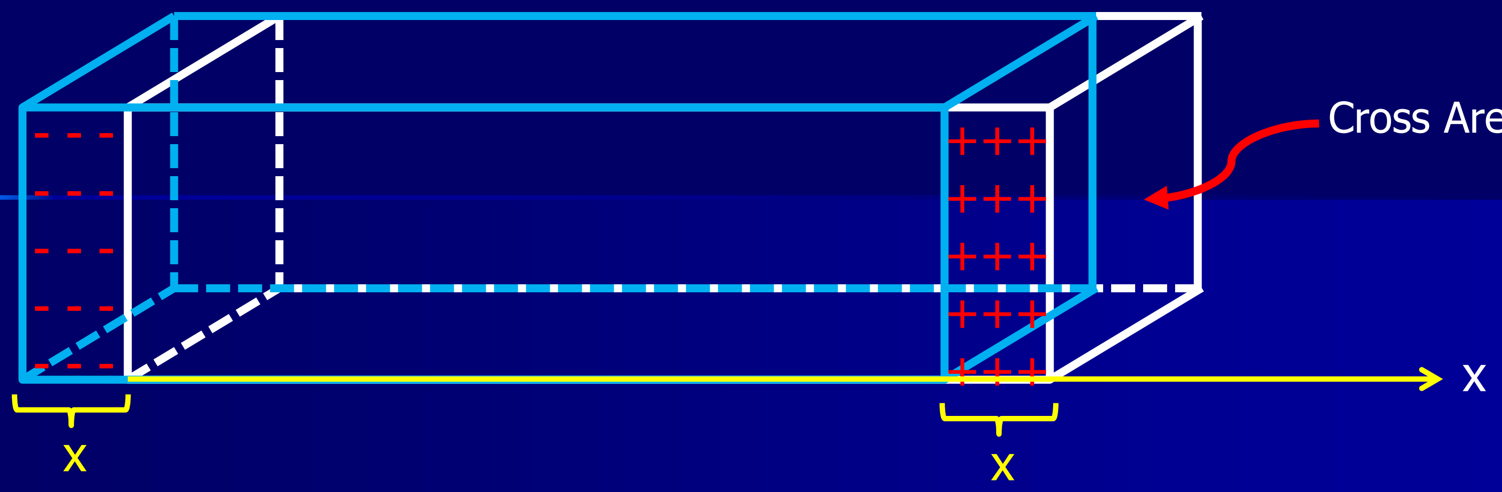
Assuming charge distribution of +ve ions and –ve electrons are separated like the following figure,



For instance, consider a one-dimensional situation in which a slab consisting entirely of one charge species is displaced from its quasi-neutral position by an infinitesimal distance x,

Volume of the charge distribution = $x \times 1\ m^2$

Plasma Frequency



Number of charges in the Volume = $x \times N$

(Where *N* is electron (charge) density)

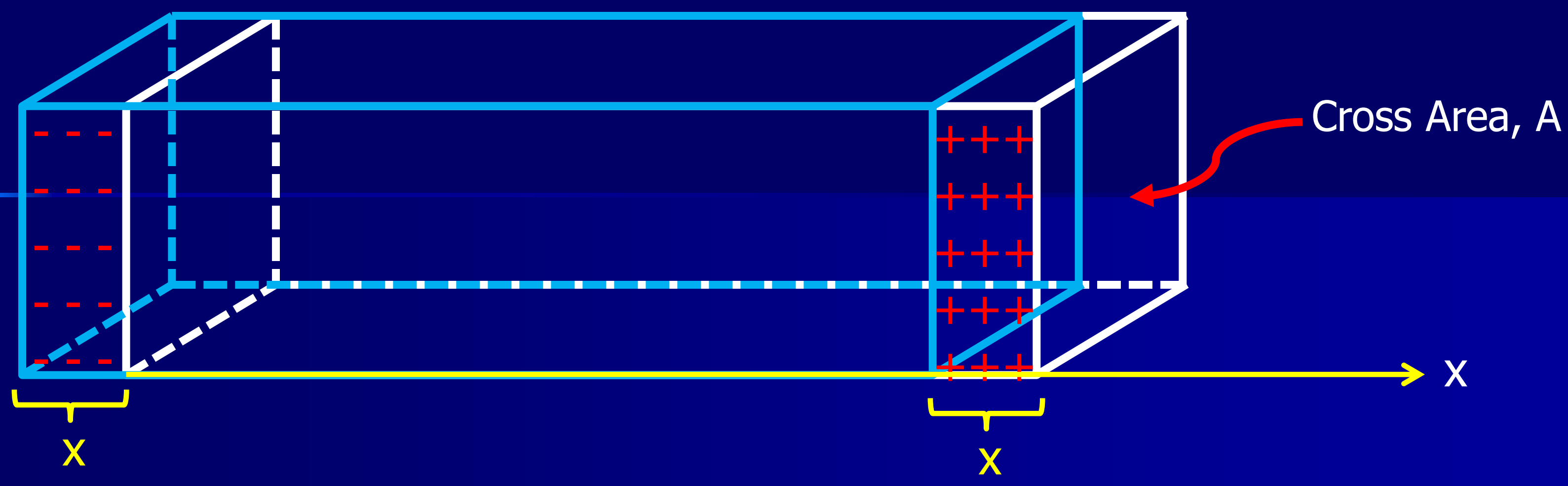
.: Surface charge density = $e \times xN$

(Where *e* is charge of an electron)

$$\sigma = eNx$$

(The resulting charge density which develops on the leading face of the slab)

Plasma Frequency



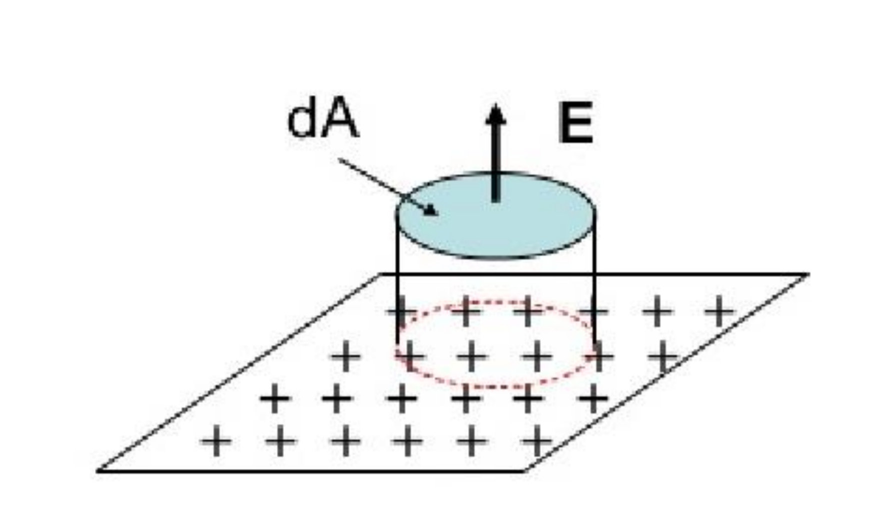
(As equal and opposite charge density develops on the opposite face. The x-direction electric field, generally inside the slab : [Using Gauss law, if we consider this is like a parallel plate situation])

$$E_x = -\frac{\sigma}{\varepsilon_o}$$

(This negative sign for the direction)

Proof -> P. T. O

Plasma Frequency



Using Gauss law,

$$\int_S \mathbf{E}.d\mathbf{S} = \frac{Q_{encl}}{\varepsilon_o}$$

We consider this is like a **parallel plate** situation :

$$E \cdot dA = \frac{\sigma.dA}{\varepsilon_o}$$

$$\mathbf{E} = \frac{\sigma}{\varepsilon_o}$$

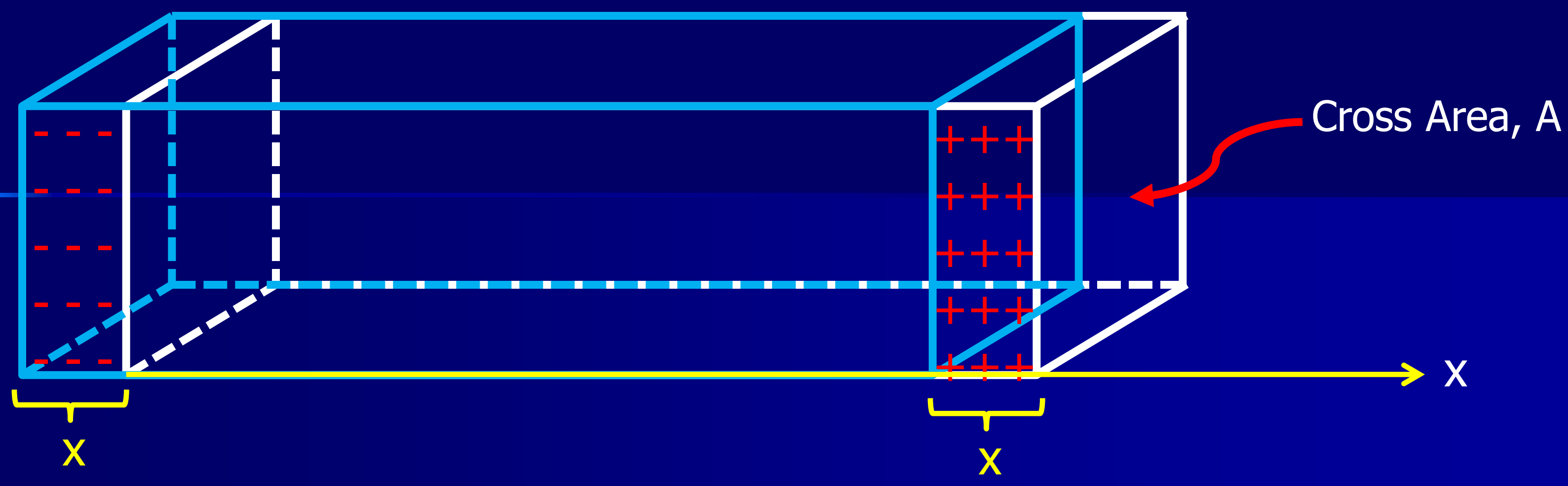
For our case :

$$E_x = -\frac{\sigma}{\varepsilon_o}$$

(This negative sign for the direction)

Force on an electron $F = e \times E$

Plasma Frequency



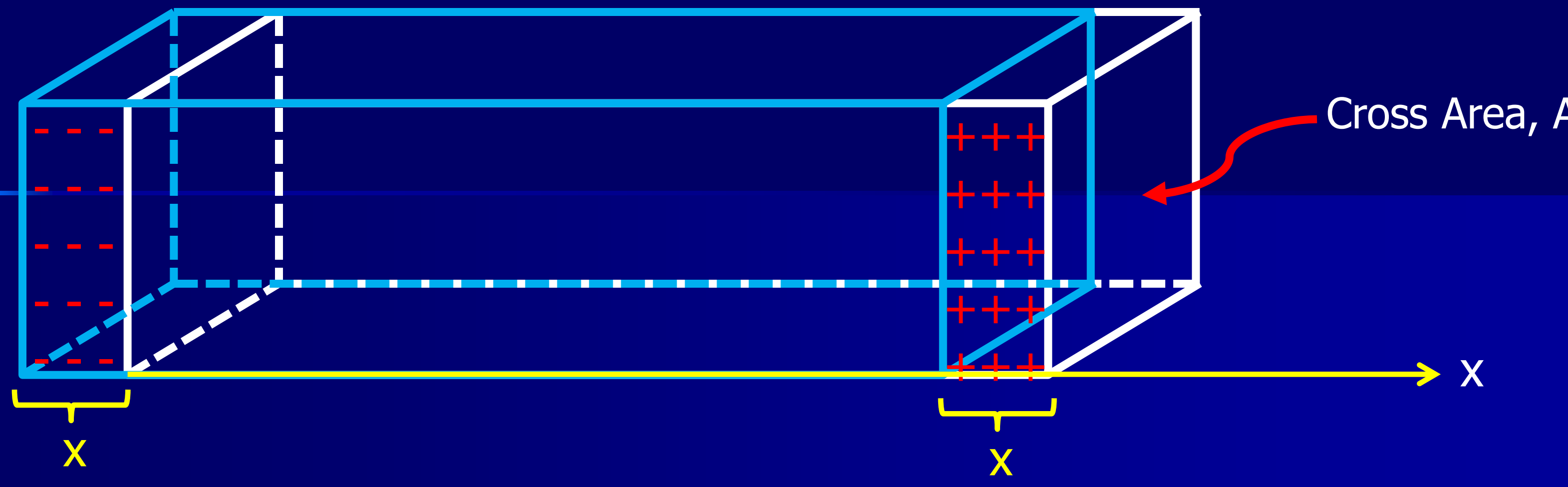
$$F = e \times E$$ ➡ $$F = e \times \left(-\frac{\sigma}{\varepsilon_o} \right)$$ ➡ $$F = e \times \left(-\frac{eNx}{\varepsilon_o} \right)$$

➡ $$F = -\frac{e^2 Nx}{\varepsilon_o}$$

Thus, Newton’s 2nd law applied to an individual particle inside the slab yields,

$$\vec{F} = m\vec{a}$$

Plasma Frequency



$$\vec{F} = m\vec{a}$$

$$\Rightarrow \quad -\frac{e^2 N x}{\varepsilon_o} = m\frac{d^2 x}{dt^2}$$

$$\Rightarrow \quad \ddot{x} = -\frac{e^2 N}{\varepsilon_o m} x$$

This is the equation of the Simple Harmonic Oscillation;

$$\ddot{x} = -\omega^2 x$$

Plasma Frequency

Then, the Angular Plasma Frequency;

$$\omega_p{}^2 = \frac{e^2 N}{\varepsilon_o m}$$

and the Plasma Frequency;

$$f_p = \frac{\omega_p}{2\pi}$$

$$\Rightarrow \quad f_p = \frac{e}{2\pi(\varepsilon_o m)^{1/2}} N^{1/2}$$

The *plasma frequency*, is the most fundamental time-scale in plasma physics. Clearly, there is a different plasma frequency for each species. However, the relatively fast electron frequency is, by far, the most important, and references to "the plasma frequency" in text-books invariably mean the *electron* plasma frequency.

Plasma Frequency

$$f_p = \frac{e}{2\pi(\varepsilon_o m)^{1/2}} N^{1/2}$$

A constant !

Where,

$$e = 1.6\times10^{-19}\,C$$

$$m = 9.1\times10^{-31}\,kg$$

$$\varepsilon_o = 8.85\times10^{-12}\,F\,m^{-1}$$

Then,

$$\frac{e}{2\pi(\varepsilon_o m)^{1/2}} = 8.97 \cong 9$$

$$f_p = 9\,N^{1/2}$$

Where, f_p is the Plasma Frequency of the medium (is measured in Hz)

N is the Molecular Number Density of the medium (is measured in e^n / m^3)

Eg : If electron density at some height is 10^{12} e^n/m^3, Find the plasma frequency of the medium at that height.

$$f_p = 9 N^{1/2} \Rightarrow f_p = 9\times\left(10^{12}\right)^{1/2} \Rightarrow f_p = 9\times10^6$$

$$\Rightarrow f_p = 9\ MHz$$

That means, if we send a Radio Wave of frequency 9 MHz , it is reflected from the region of the atmosphere when the electron density is 10^{12} e^n/m^3.
That height is situated at **F** (actually **F_2** region)

But if we send **UHF (300 MHz)** or **VHF (30 MHz)** signal (Radio Wave); the wave goes through the ionosphere without any reflection !


Thank You !
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