Space & Atmospheric Physics

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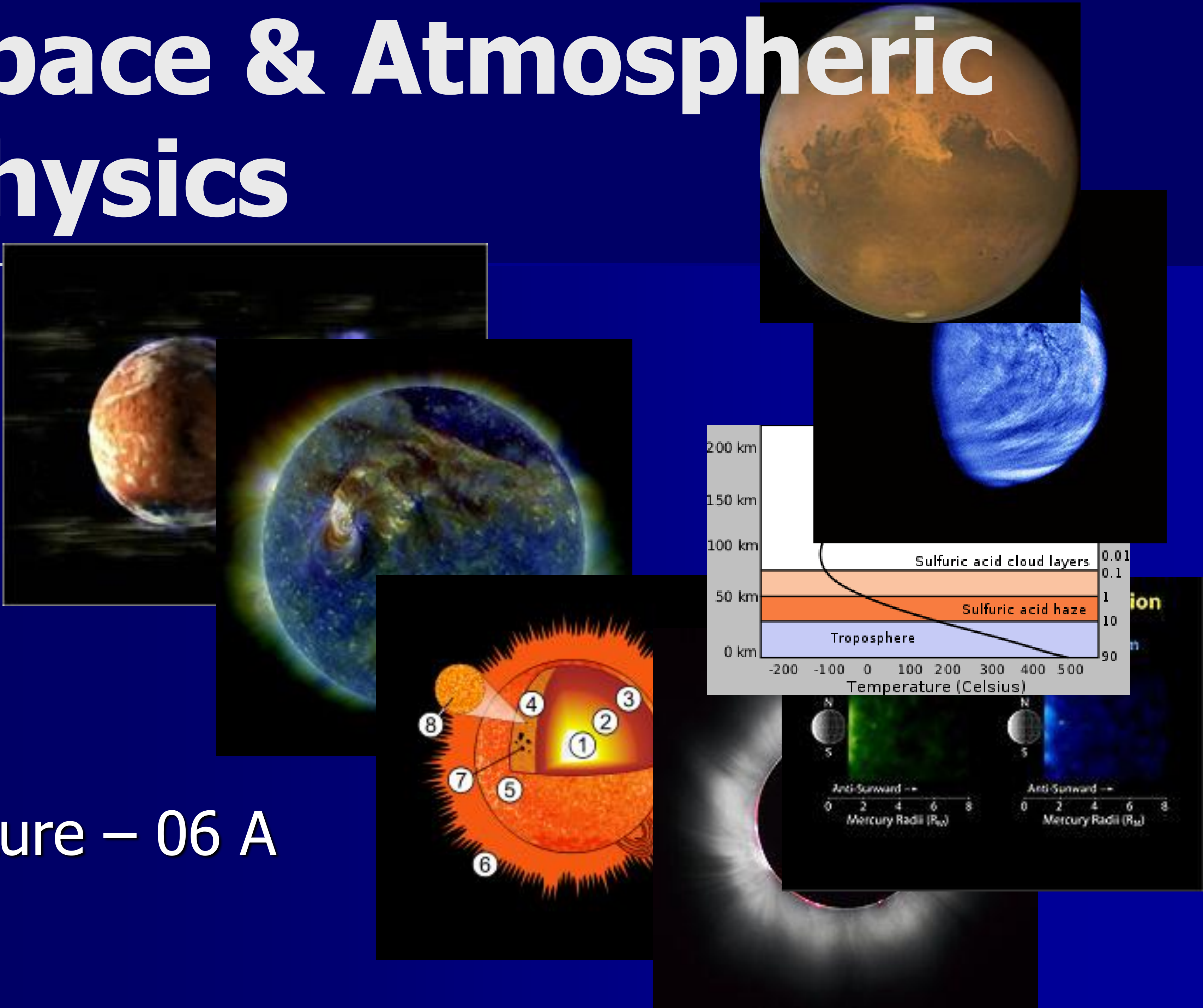


Lecture – 06 A

PHY 497 2.0 – Space & Atmospheric Physics

Continuous Assignment – 06

Show that the variation of molecular number density $N(h)$ with the altitude h of an isothermal atmosphere which is consisting of only one type of gas molecules of mass m can be expressed as,

$$N(h) = N_0 \; e^{-\frac{h}{H}} \qquad \text{Where,} \quad H = \frac{kT}{\bar{m}g}$$

The temperature T and the acceleration of gravity g are constants.

In the Earth's atmosphere, the major constituents are nitrogen and oxygen having average molecular mass of 4.8×10^{-26} kg. The total number density $N_0 = 2.54\times10^{19}$ cm^{-3} at the ground.

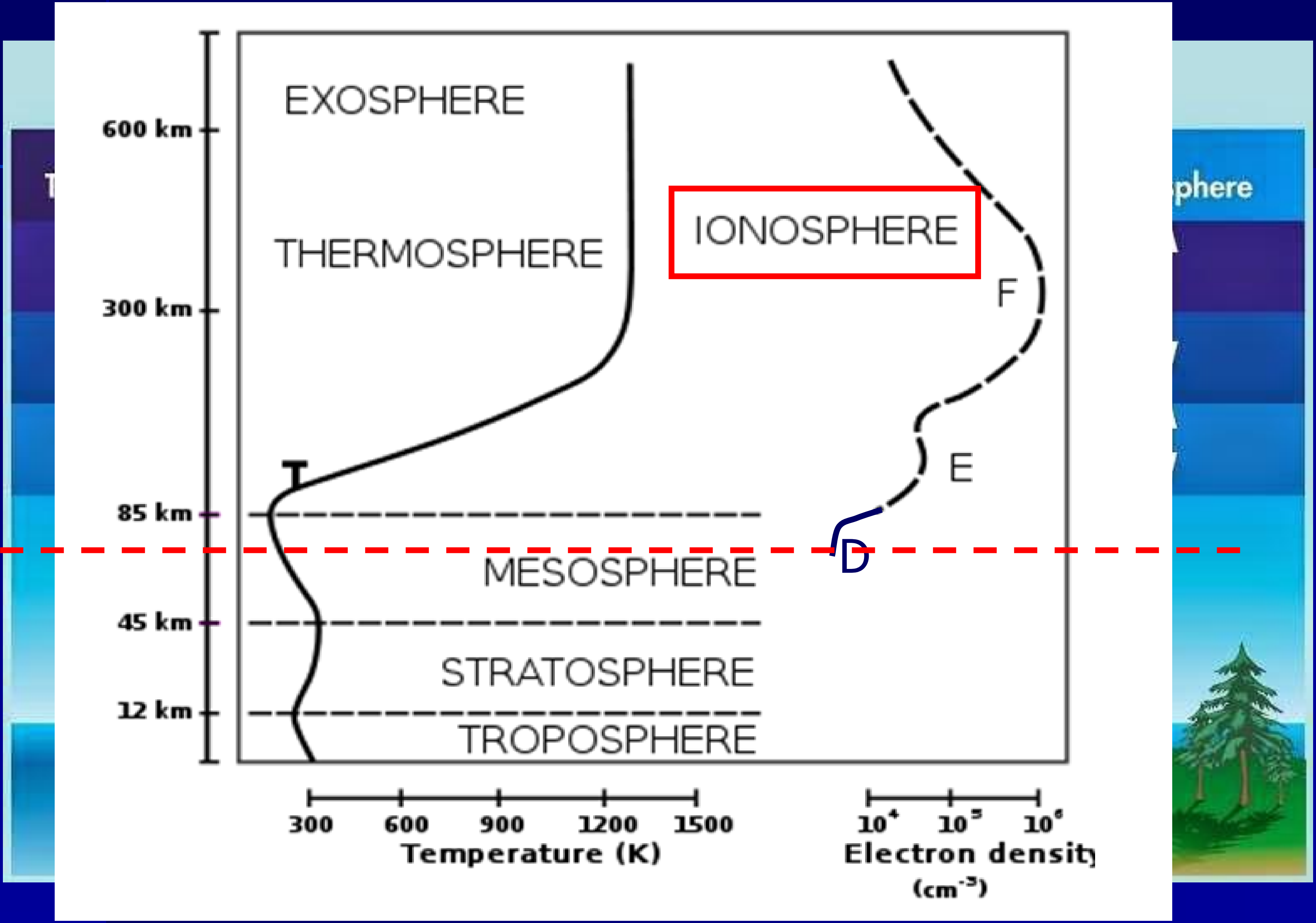
Estimate the molecular number density at an altitude of 8.4 km.

$g = 9.8$ ms^{-2}, $k = 1.38\times10^{-23}$ JK^{-1} and $T = 288$ K

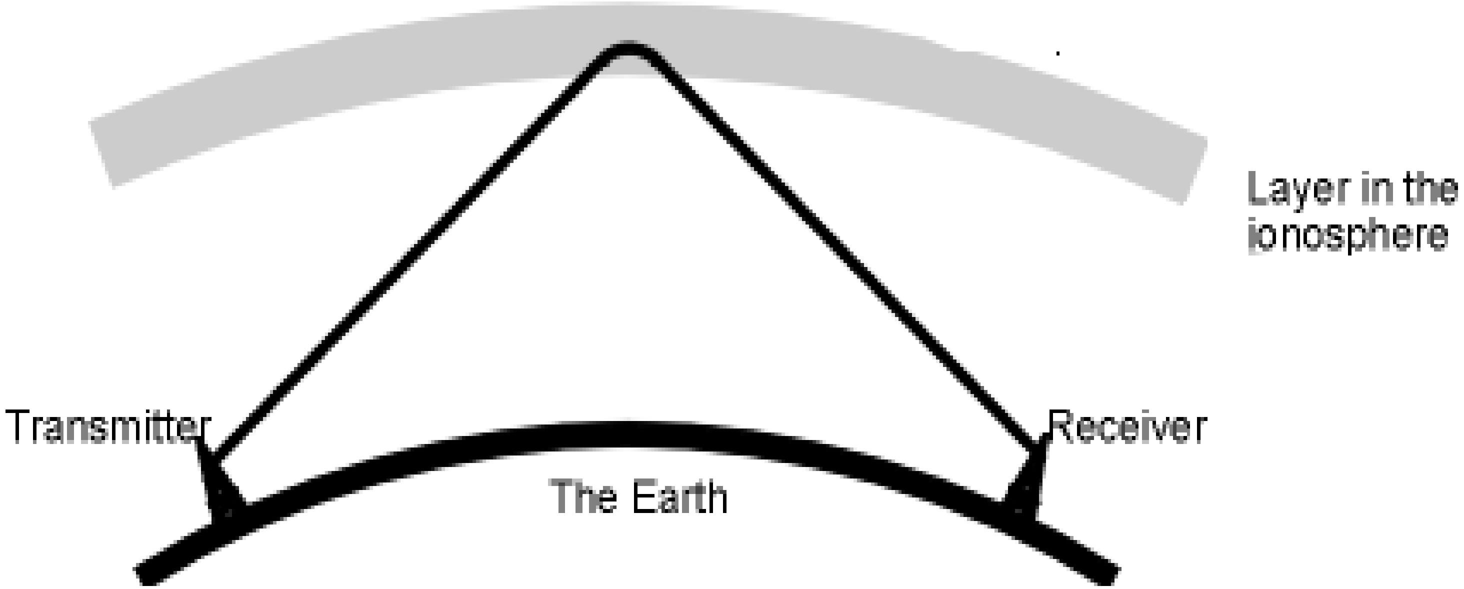
The Ionosphere

Introduction
The Chapman Layer Theory
Plasma Frequency
Collision Frequency and Absorption
The Structure of the Ionosphere and the Plasmasphere
Regular and Irregular Variations of the Ionosphere

Relationship of the atmosphere and ionosphere

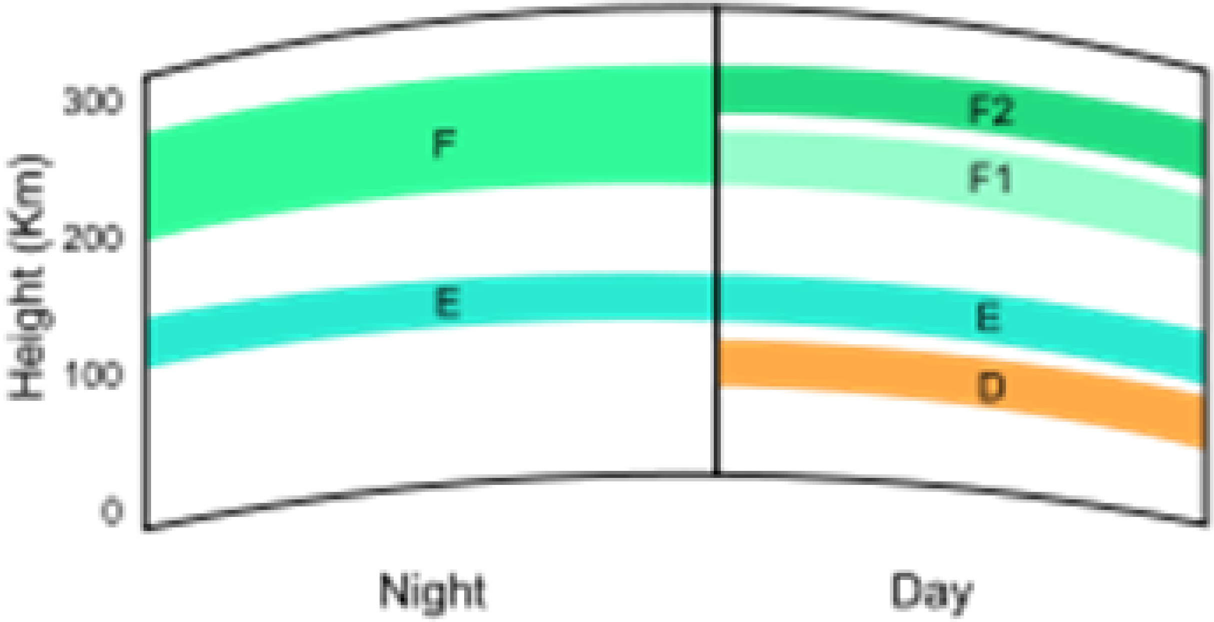


The Ionosphere acting as a reflector of radio waves making possible radio telecommunication over the horizon.



Refraction of a radio signal as it enters an ionised region

The ionospheric layers



Ionospheric layers.

The Ionosphere

Introduction
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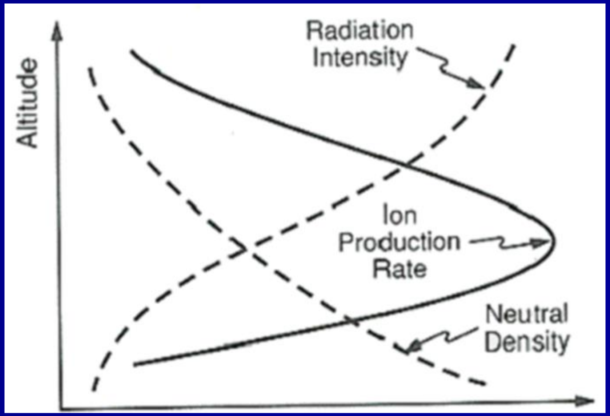
Chapman layer Theory

The Ionization of the atmosphere

The ionization of the atmosphere is produced primarily by the Sun's Ultraviolet and X-ray radiation. **The rate q at which ion-electron pairs are produced per unit volume is proportional to the intensity of the ionizing radiation I and the number density N_n of the neutral atmosphere**, i.e.:

$$\boxed{q \,\alpha\, I \cdot N_n}$$

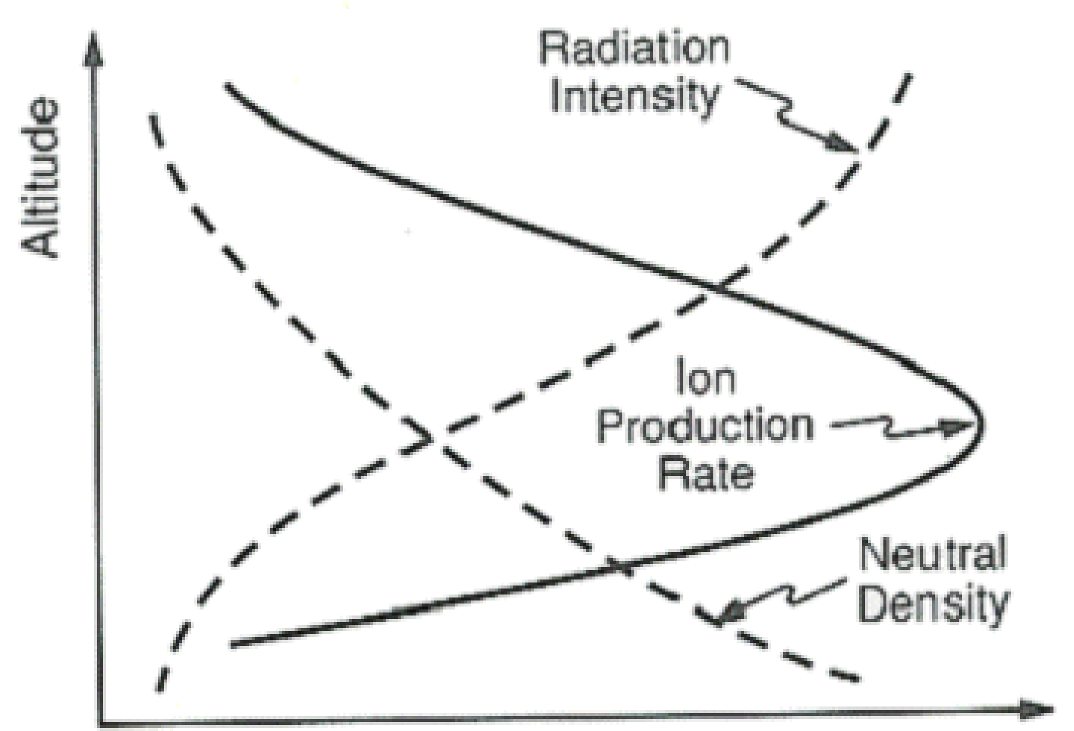
As seen from the following diagram, at high altitudes q **is very small because N_n is very small.** As the ionizing radiation penetrates deeper into the more dense layers of the atmosphere, q reaches a maximum q_m at a height h_m where I and N_n reach the best possible combination.



Chapman layer Theory

The Ionization of the atmosphere

Below this altitude, the intensity of the ionizing radiation drops rapidly because the energy is spent for the ionization of the atmosphere. As I decreases, q also decreases and finally vanishes near **70 km**.



Chapman layer Theory

The Ionization of the atmosphere

Chapman in 1931 produced a very neat theoretical treatment of the problem. In his simplified model, Chapman assumed,

- an isothermal,
- horizontally stratified atmosphere,
- composed of a single gas, which is been ionized by
- monochromatic radiation from the Sun.

It is obvious that this model is an **over simplification** of the actual conditions.

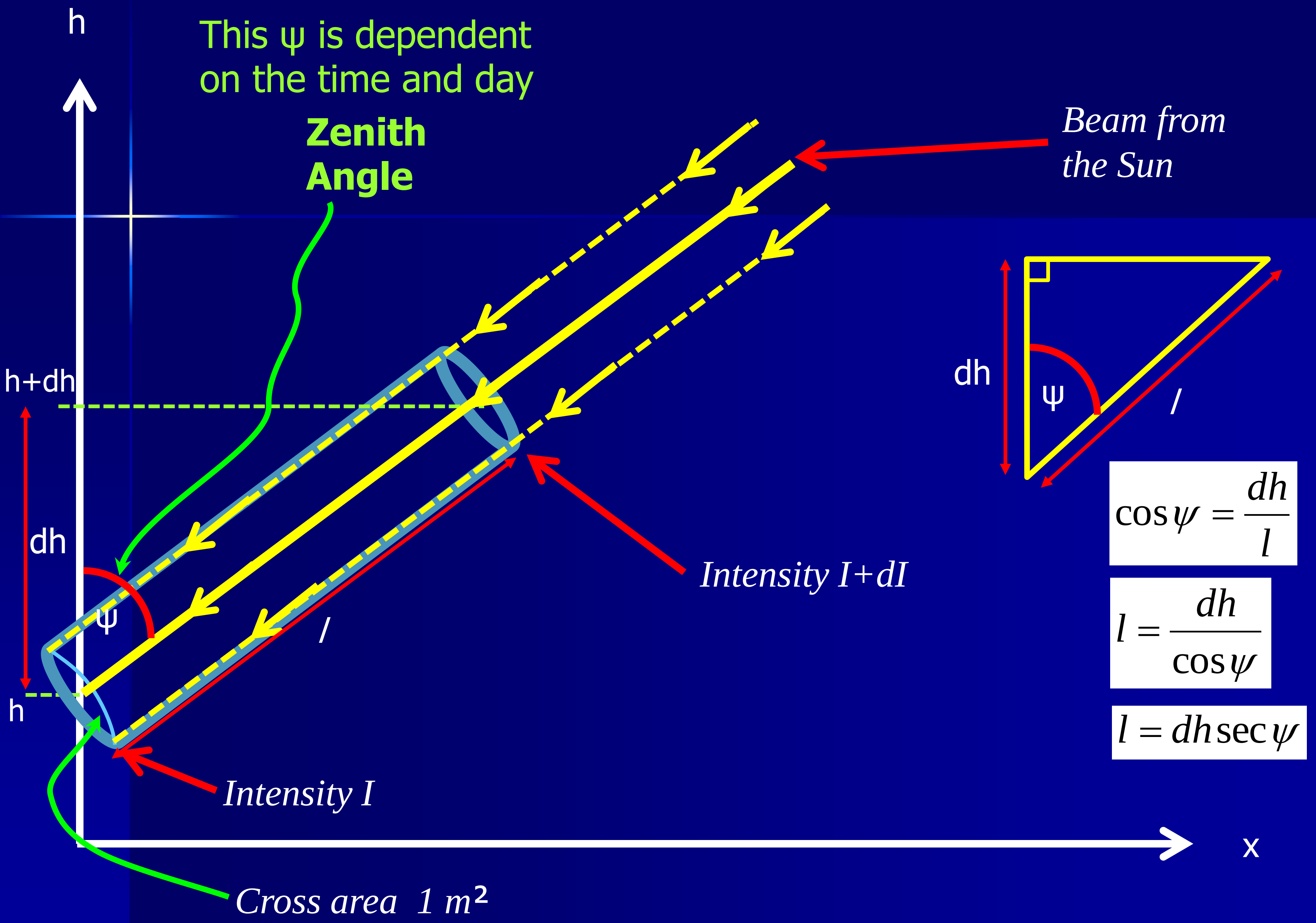
Chapman layer Theory

The Chapman Layer Theory in 1931 is a very good example of an **ingenious mathematical formulation** of a very complicated physical problem.

Intensity of Ionizing Radiation :

Let us first compute the absorption sustained by a beam of ionizing radiation at a height *h*. Let the beam have **unit cross-section** and ψ be the angle the beam makes with the vertical (called **Zenith Angle**). The energy of the beam expanded to ionized neutral particles between h and h+dh will be proportional to the intensity of the beam at this height I(h).

P.T.O



This ψ is dependent on the time and day

Zenith Angle

Beam from the Sun

Intensity I+dI

Intensity I

Cross area 1 m²

$$\cos\psi = \frac{dh}{l}$$

$$l = \frac{dh}{\cos\psi}$$

$$l = dh\sec\psi$$

If the molecular number density is N,
Number of molecules in the selected area :

$$= N \times V$$

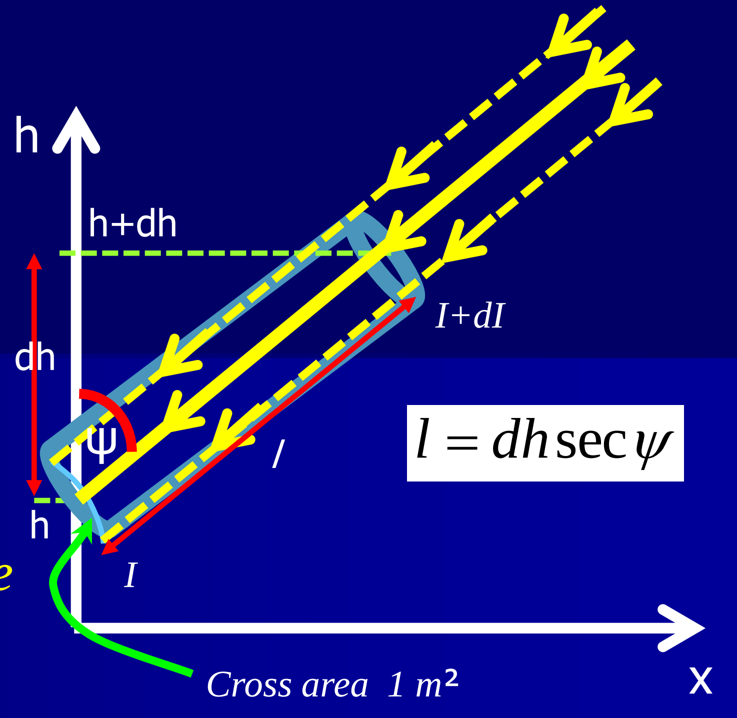
$$= N \times l \times 1$$

$$= N dh \sec\psi$$

Total cross section : $= \sigma_a \times N dh \sec\psi$

Where σ_a *is Absorption Cross-section*

(Absorption Cross-section is the ability of a molecule to absorb a photon of a particular wavelength and polarization)



$$l = dh \sec\psi$$

The amount of radiation absorbed in this selected layer will be,

$$dI = I \times \sigma_a N dh \sec\psi$$

For the total region : Integrating from the height h to ∞,

$$\int_{I=I}^{I=I_\infty} \frac{dI}{I} = \int_{h=h}^{h=\infty} \sigma_a N \sec\psi \; dh$$

(Assume the intensity of ionizing radiation at infinity is I_∞ *and intensity of ionizing radiation at h is I)*

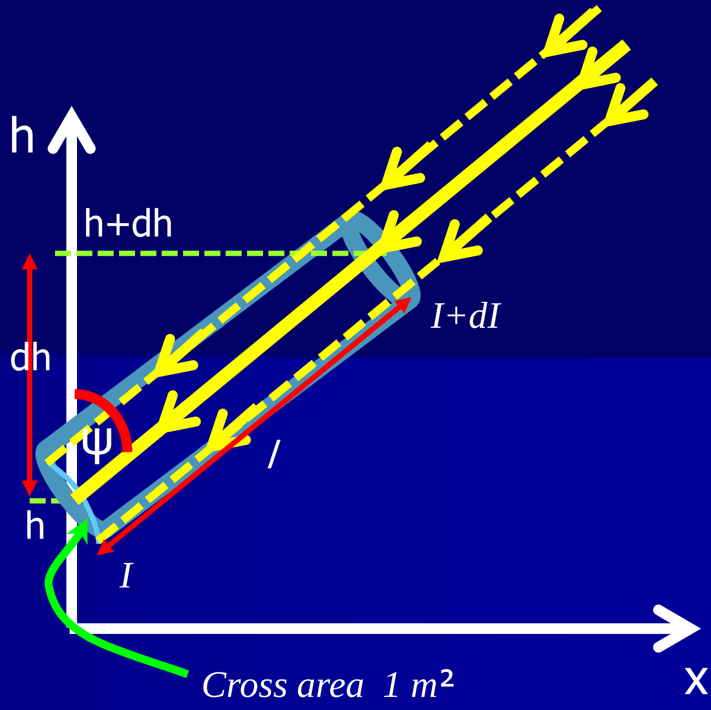
$$\int_{I=I}^{I=I_\infty} \frac{dI}{I} = \int_{h=h}^{h=\infty} \sigma_a N \sec\psi \; dh$$

$$\left[\ln I\right]_{I=I}^{I=I_\infty} = \sigma_a \sec\psi \int_{h=h}^{h=\infty} N dh$$

$$\ln\left(\frac{I_\infty}{I}\right) = \sigma_a \sec\psi \int_{h=h}^{h=\infty} N dh$$

$$\ln\left(\frac{I}{I_\infty}\right) = -\sigma_a \sec\psi \int_{h=h}^{h=\infty} N dh$$

$$I = I_\infty e^{\left(-\sigma_a \sec\psi \int_{h=h}^{h=\infty} N dh\right)}$$



Chapman layer Theory

Intensity of Ionizing Radiation

Intensity of Ionizing Radiation at infinity

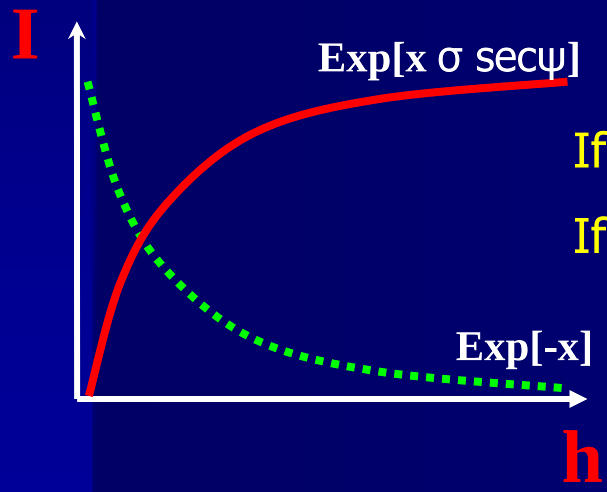
Absorption Cross-section

Zenith Angle

Molecular Number Density

Intensity of Ionizing Radiation at height h

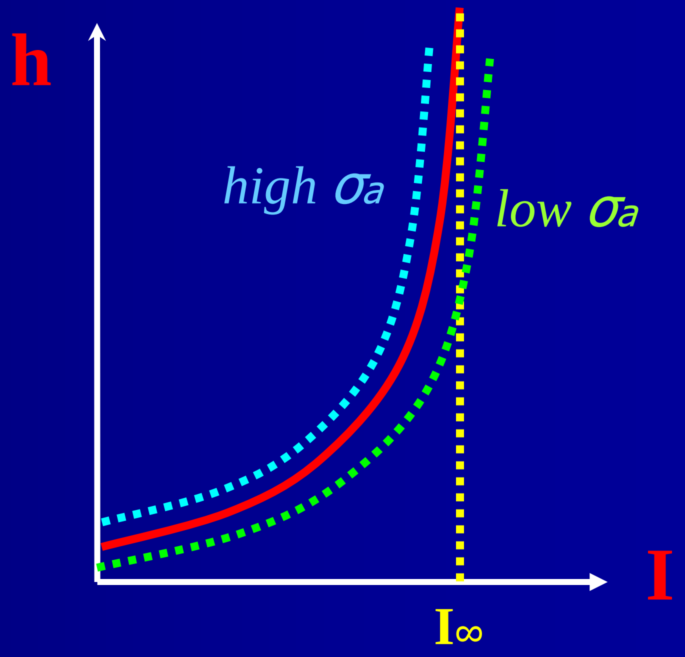
$$I = I_{\infty} \cdot e^{-\sigma_a \sec\psi \int_{h=0}^{\infty} N \cdot dh}$$



If h ↑ , σ ↓ and

If h ↑ , ψ ↑

Ionization Radiation (I)



Ionization Wavelength (λ) :

Ionization of O, O_2 , NO and N_2 in the Earth atmosphere due to radiation at a particular wavelength from the Sun. This wavelength is called “Ionization Wavelength”.

For Ionization

Given wavelength to ionized

$$\lambda < \lambda_i$$

Wave length that is corresponding maximum energy of ionized of a specific material

Material	Required wavelength for ionized
N_2	796 Å
O	911 Å
O_2	1118 Å
NO	1340 Å

N_2 *is the more difficult material is to be ionized !*

Ionization Efficiency (η) :

The ratio of the number of ions formed to the number of electrons or protons used in an ionization process OR no of ion-paires per unit absorbed energy.

$$\eta = \frac{No\ of\ ion - pairs\left(e^{n}s\right)}{Absorbed\ energy}$$

If $\lambda > \lambda_i$ → $\eta = 0$ (Because there are no ionized irons)

If $\lambda < \lambda_i$ → $\eta > 0$ (Because there are ionized irons in this case)

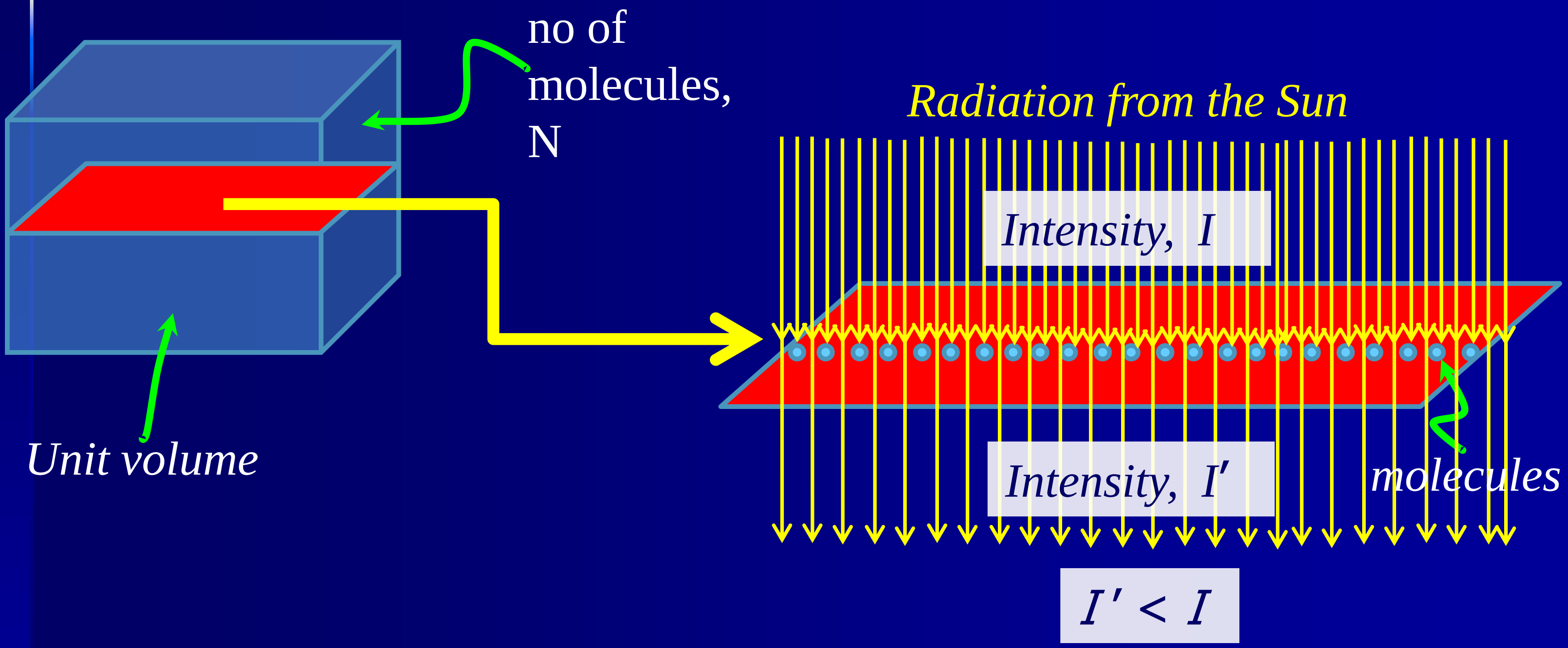
$$\eta = \frac{No\ of\ ion - pairs\left(e^{n}s\right)}{Absorbed\ energy}$$

→ $No\ of\ ion - pairs\left(e^{n}s\right) \quad \alpha \quad Absorbed\ energy$

Electron Production Rate (Q)

Absorbed Intensity (dI)

If we assume there are N no of molecules in an unit volume!



no of molecules, N

Unit volume

Radiation from the Sun

Intensity, I

Intensity, I'

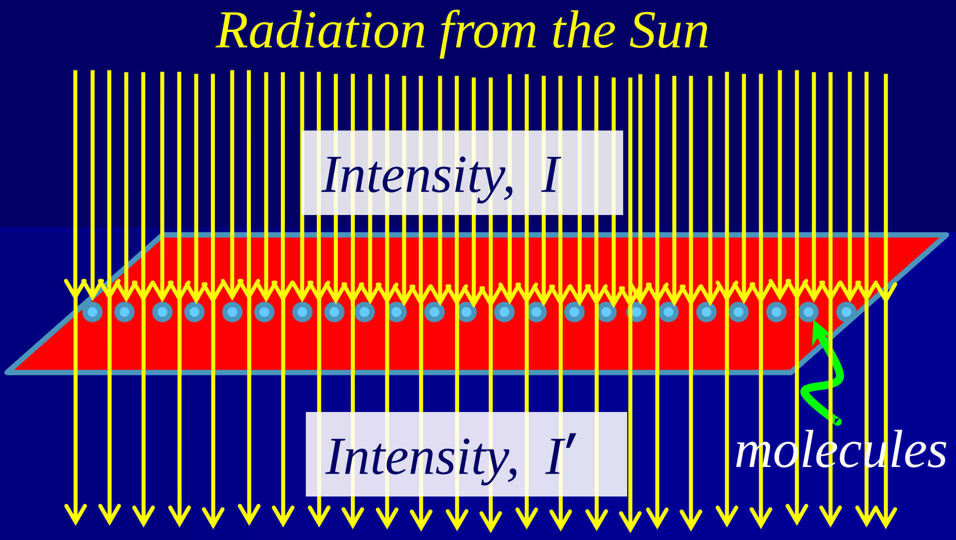
molecules

$I' < I$

Intensity of the Radiation from the Sun (I) comes from the upside to the selected molecules layer. The intensity I' goes through that layer to the downside. $I > I'$ because the amount of $I - I'$ (=dI) radiation intensity stopped by the molecular layer.

Electron Production Rate (Q)

Absorbed Intensity (*dI*)



$I' < I$

Assume σ_a is the Absorption Cross-section area corresponding to the molecules.

Block intensity from the area $N\sigma_a$

Cross Area of the molecules in the Unit Area

$$\frac{dI}{I} = \frac{N\sigma_a}{1}$$

Intensity from the Sun

Intensity I comes to the cross area $1m^2$

Where, $dI = I - I'$.

Absorbed Intensity

$$dI = \overbrace{N\sigma_a}^{\text{Cross Area of the molecules in the Unit Area}} I$$

Intensity of the Radiation from the Sun

Electron Production Rate (Q)

No of ion pairs (electrons or positive ions) produce in an unit volume per second is called **Electron Production Rate** (Q)

$$Q = \eta \times dI$$

$$Q = \eta \times N \sigma_a I$$

$\eta \times \sigma_a =$ Ionization Cross Section (σ_i)

If the gas is not Ionized; Ionization Cross Section, $\sigma_i = 0$ because $\eta = 0$.

Electron Production Rate (Q)

Electron Production rate

$$Q = \eta \times N \sigma_a I$$

Intensity of Ionizing Radiation at height h

$$I = I_\infty \cdot e^{-\sigma_a \sec\psi \int_{h=0}^{\infty} N \cdot dh}$$

$$\therefore \quad Q = \eta \times N \sigma_a \left(I_\infty \cdot e^{-\sigma_a \sec\psi \int_{h=0}^{\infty} N \cdot dh} \right)$$

But we know;

$$\int_{h=0}^{\infty} N \cdot dh = NH$$

Where H is the "Scale Height"

Electron Production Rate (Q)

$$\int_{h=0}^{\infty} N \cdot dh = NH$$

Total number of molecules from surface of the Earth to infinity !

$$\therefore \quad Q = \eta \sigma_a N \, I_\infty \cdot e^{-\sec\psi . \sigma_a NH}$$

$$\therefore \quad Q = \frac{\eta \sigma_a N}{e^1} I_\infty \cdot e^{1-\sec\psi . \sigma_a NH}$$

$$\therefore \quad Q = \frac{\eta \boxed{\sigma_a NH}}{eH} I_\infty \cdot e^{1-\sec\psi . \boxed{\sigma_a NH}}$$

Substitute ; $\boxed{\sigma_a NH} = e^{-Z}$ *Where Z is (some) height*

Electron Production Rate (Q)

$$\therefore \quad Q = \frac{\eta e^{-Z}}{eH} I_{\infty} \cdot e^{1-\sec\psi . e^{-Z}}$$

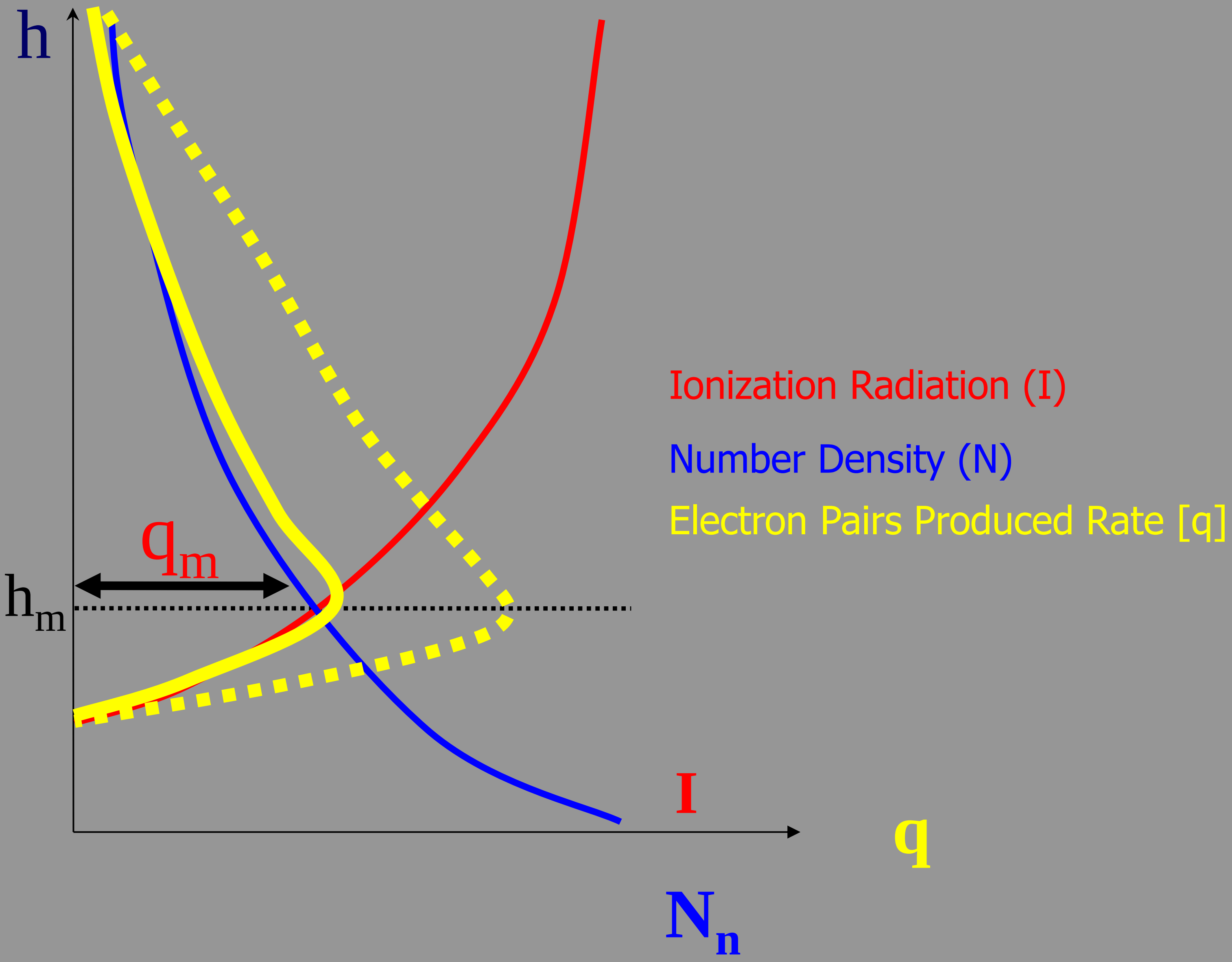
Where N and Z are dependent variables, because

$$e^{-Z} = \sigma_a NH$$

Production rate at any point

$$Q = \frac{\eta \cdot I_{\infty}}{eH} e^{\left(1-Z-\sec\psi \cdot e^{-Z}\right)}$$

Ionization Radiation (I), Number Density (N) and Electron Pairs Produced Rate [q]



Find the value of Q_m

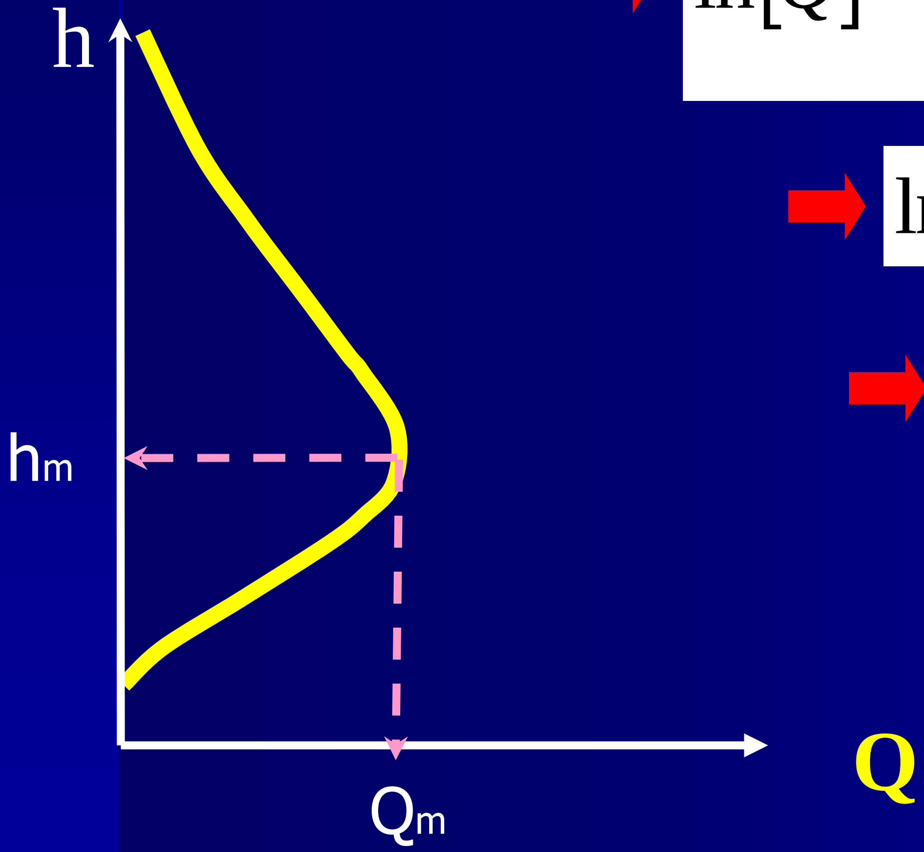
$$Q = \frac{\eta \cdot I_\infty}{eH} e^{\left(1 - Z - \sec\psi \cdot e^{-Z}\right)}$$

$$\ln[Q] = \ln\left[\frac{\eta \cdot I_\infty}{eH} e^{\left(1 - Z - \sec\psi \cdot e^{-Z}\right)}\right]$$

$$\ln[Q] = \ln\left[\underbrace{\frac{\eta \cdot I_\infty}{eH}}_{\text{constant, } c}\right] + \ln\left[e^{\left(1 - Z - \sec\psi \cdot e^{-Z}\right)}\right]$$

$$\ln[Q] = \underbrace{c + 1}_{\substack{\text{constant,} \\ C}} - Z - \sec\psi \cdot e^{-Z}$$

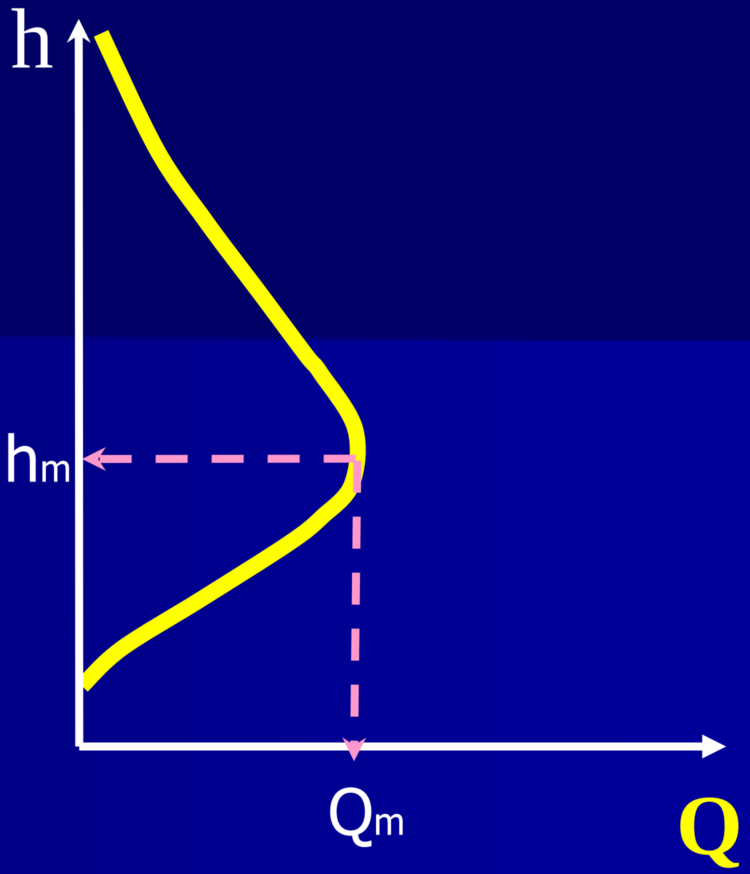
$$\ln[Q] = C - Z - \sec\psi \cdot e^{-Z}$$



For find the maximum ;

$$\frac{d\left(\ln[Q]\right)}{dz} = 0$$

Find the value of Q_m



$$\ln[Q] = C - Z - \sec\psi \cdot e^{-Z}$$

$$\frac{d(\ln[Q])}{dz} = \frac{d\left(C - Z - \sec\psi \cdot e^{-Z}\right)}{dz}$$

$$\frac{d(\ln[Q])}{dz} = -1 - \sec\psi \cdot e^{-Z}(-1)$$

For find the maximum ;

$$\frac{d(\ln[Q])}{dz} = 0$$

$$\Rightarrow \cos\psi = e^{-Z}$$

We know,

$$e^{-Z} = \sigma_a N H$$

Find the value of Q_m

$$Q = \frac{\eta \cdot I_\infty}{e H} e^{\left(1 - Z - \sec\psi \cdot e^{-Z}\right)}$$

01

For find the maximum ;

$$\frac{d\left(\ln[Q]\right)}{dz} = 0$$

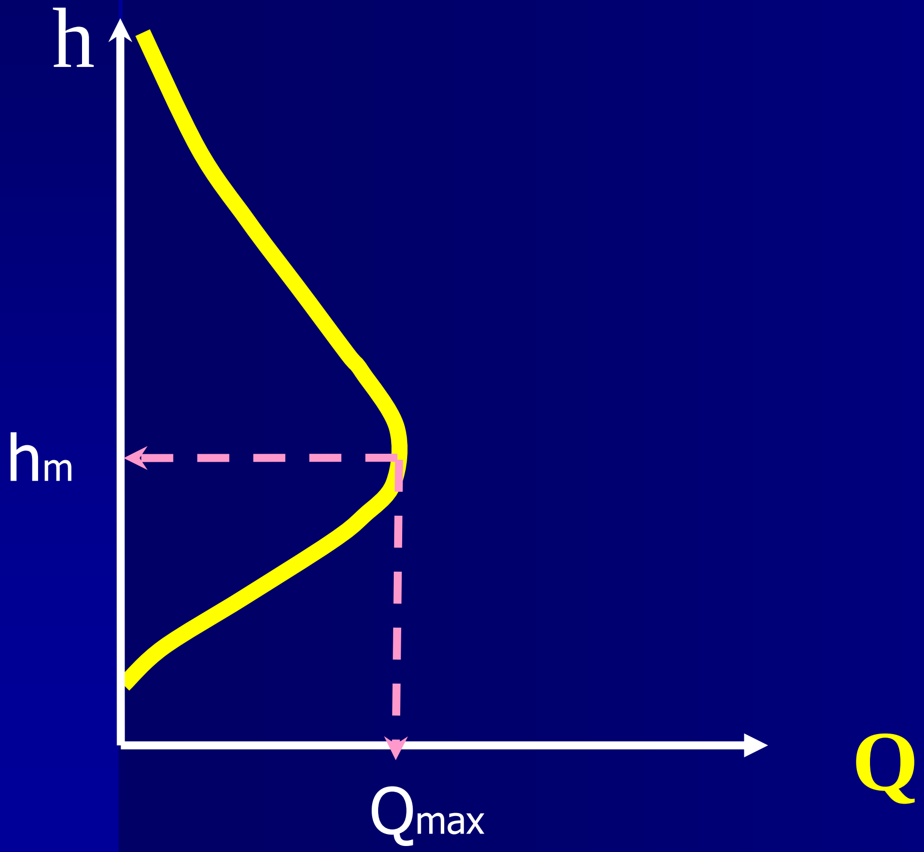
$$\Rightarrow \cos\psi = e^{-Z}$$

We know,

$$e^{-Z} = \sigma_a N H$$

Using equation – 01 :

$$\Rightarrow Q_{\max} = \frac{\eta \cdot I_\infty}{e H} \cos\psi$$



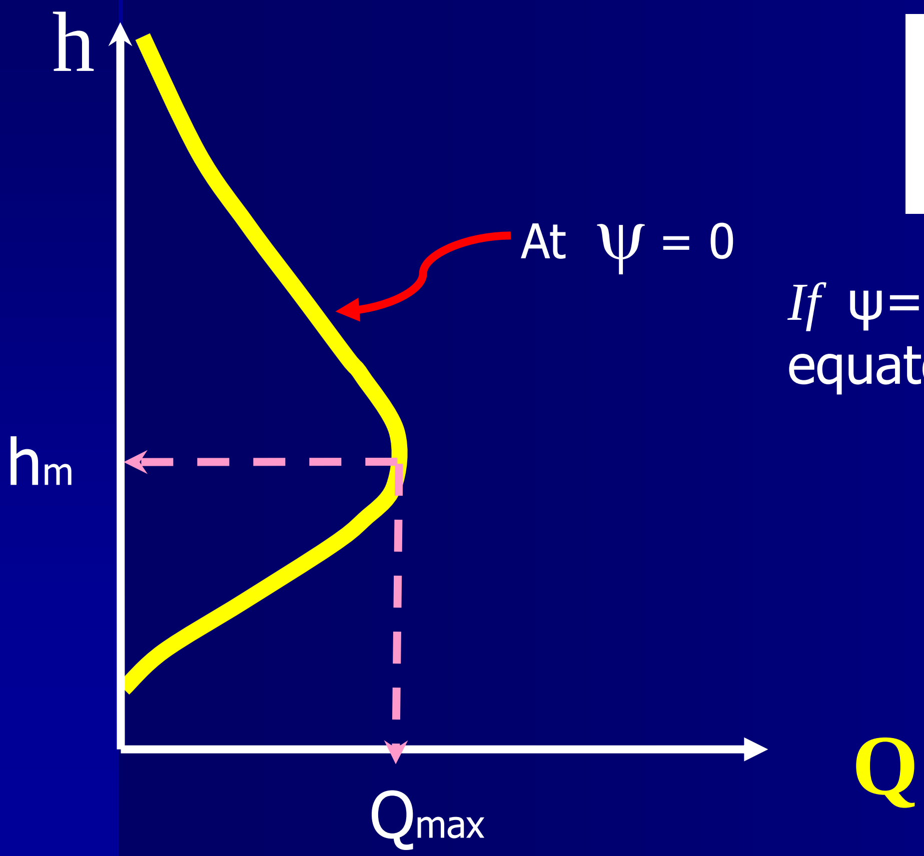
Find the value of Q_m

$$Q_{\max} = \frac{\eta \cdot I_{\infty}}{e\,H}\cos\psi$$

➡ $\cos\psi = \sigma_a N H$

Production Rate Q :

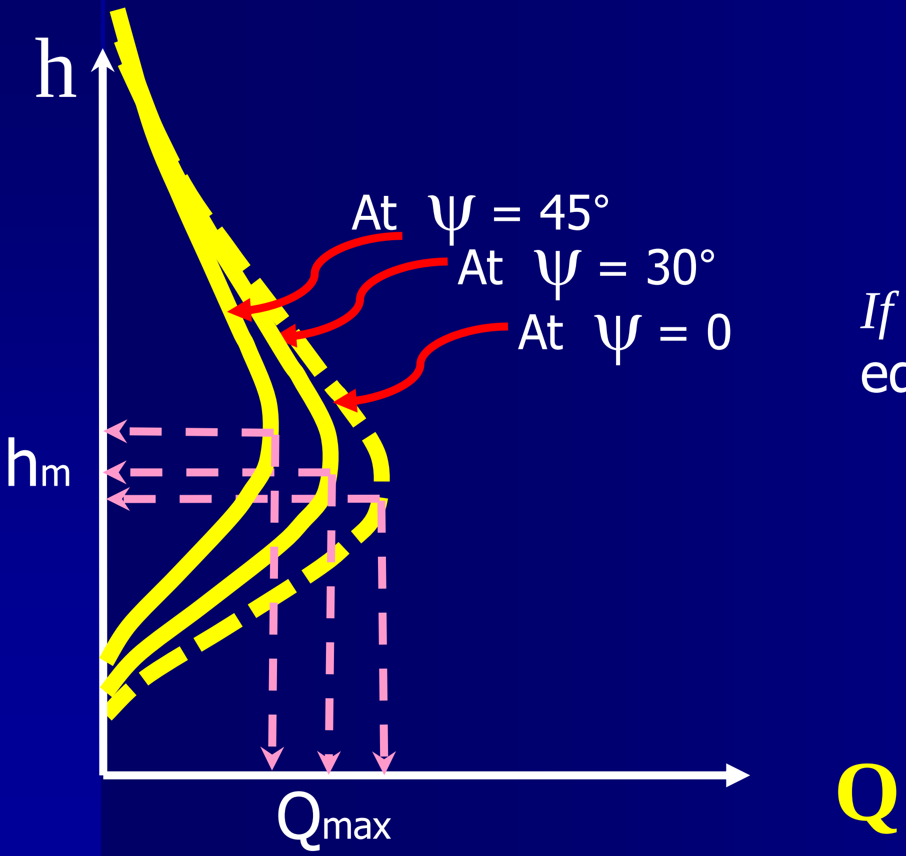
$$Q = Q_{\max}\, e^{\left(1-\sec\psi\cdot e^{-Z}\right)}$$



If ψ=0°, Then the Sun is directly up on the equator *:*

➡ $$Q_{\max} = \frac{\eta \cdot I_{\infty}}{e\,H}(1)$$

Find the value of Q_m



$$Q_{\max} = \frac{\eta \cdot I_\infty}{e H} \cos \psi$$

If ψ=30°, Then the Sun is 30° from the equator :

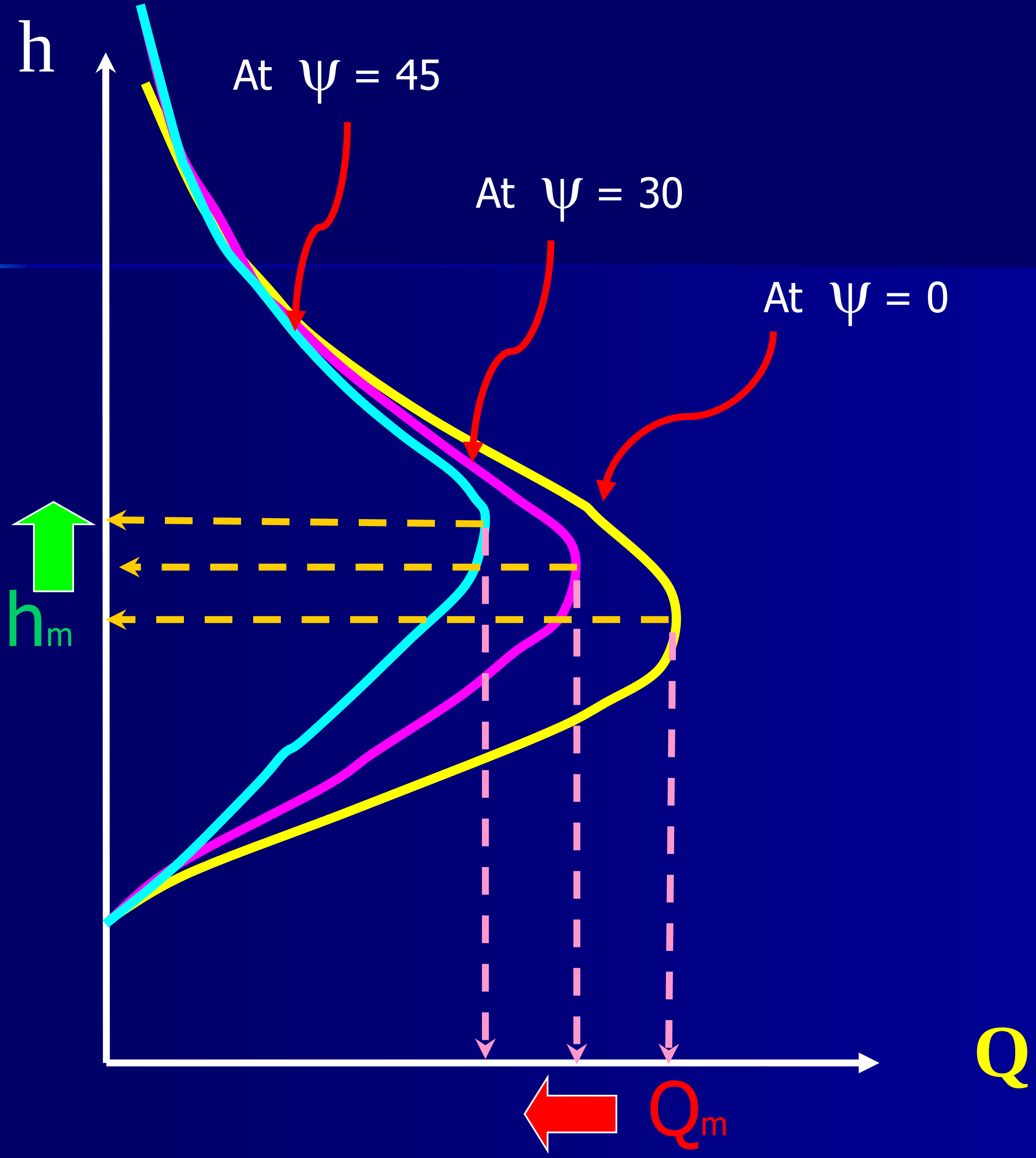
$$\Rightarrow Q_{\max} = \frac{\eta \cdot I_\infty}{e H} (Cos 30)$$

$$\Rightarrow Q_{\max} = \frac{\eta \cdot I_\infty}{e H} (0.8660)$$

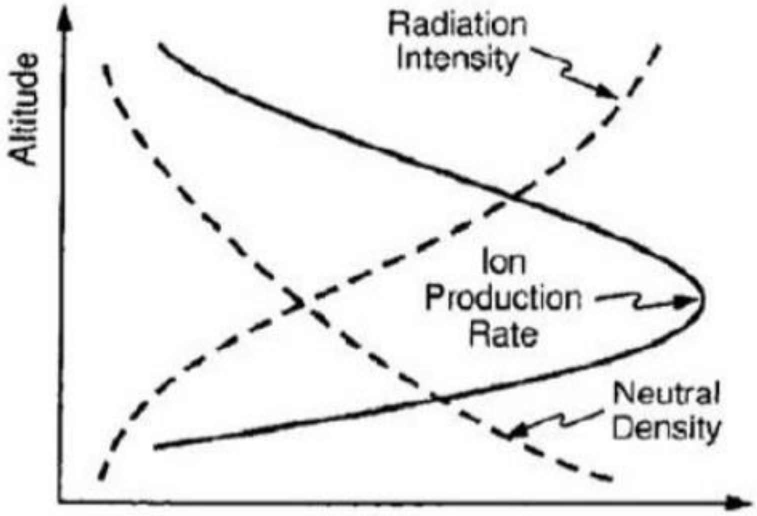
If ψ=45°, Then the Sun is 45° from the equator :

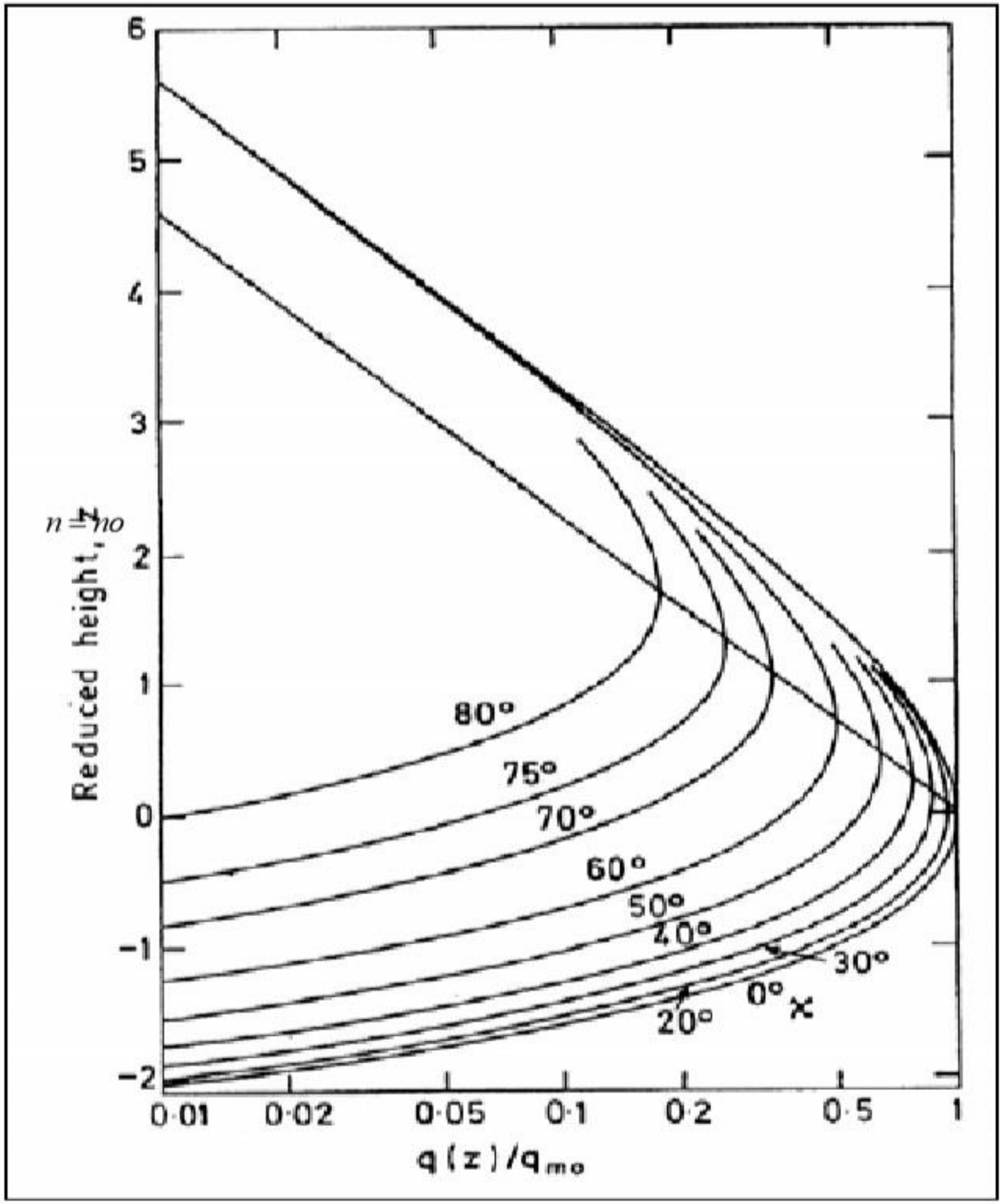
$$\Rightarrow Q_{\max} = \frac{\eta \cdot I_\infty}{e H} (Cos 45)$$

$$\Rightarrow Q_{\max} = \frac{\eta \cdot I_\infty}{e H} (0.7071)$$


h
At ψ = 45
At ψ = 30
At ψ = 0
h_m
Q
Q_m

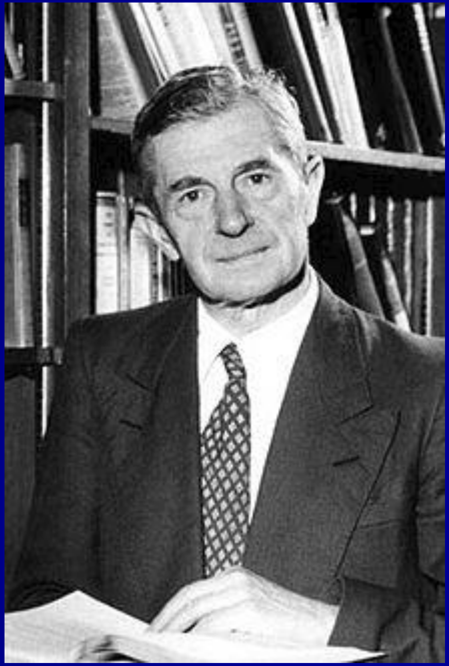
Chapman's Production Profile



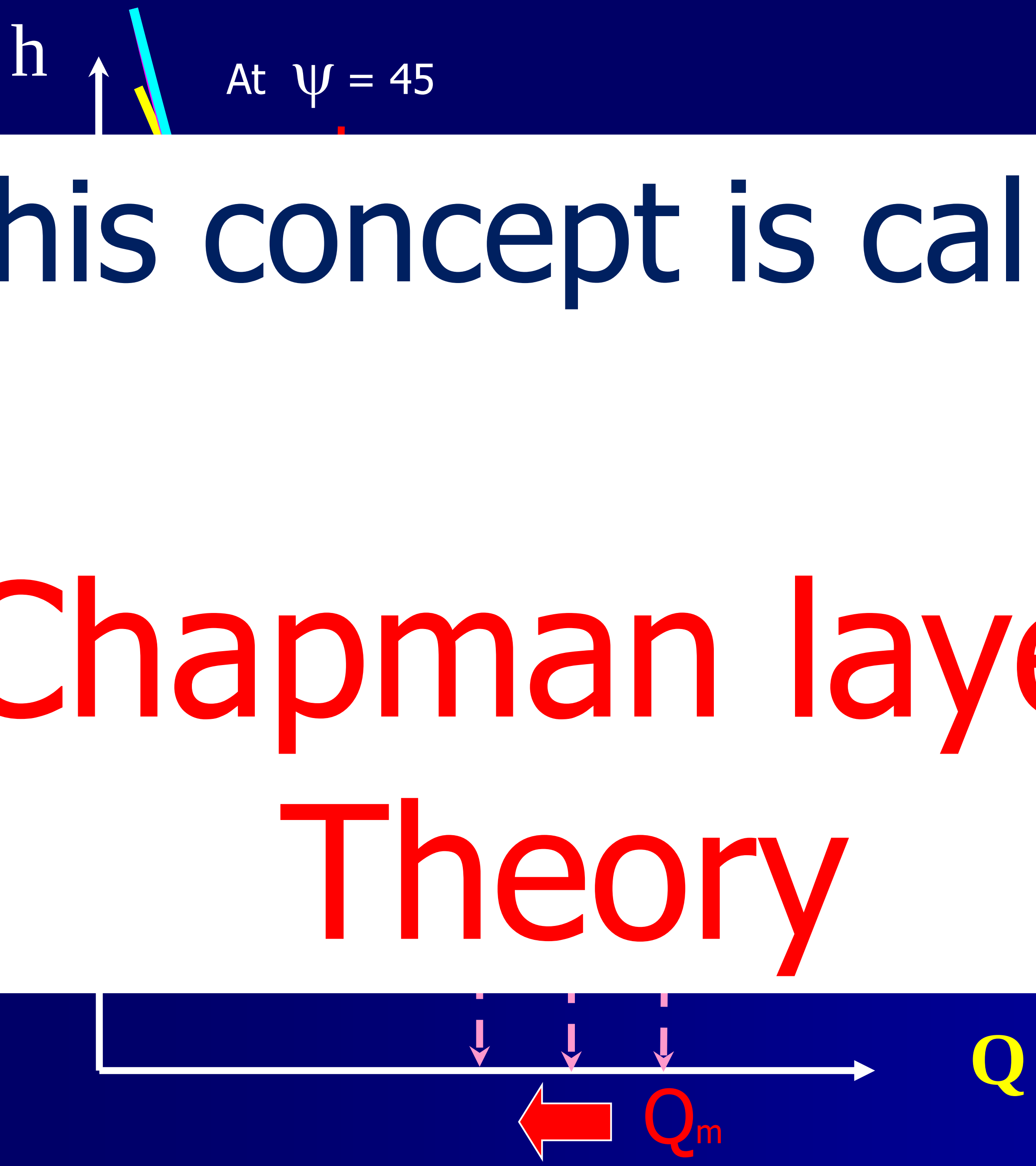


That means ψ is increasing, the maximum value of the Electron Production Rate is decreasing. For that Molecular Number Density of the ionosphere should be decreasing.

.: Region of the Q_{max} is going to far away from the Earth surface. Because N should be decrees. Because h is low, N is high and h is high, N is low.



Sydney Chapman FRS (29 January 1888 – 16 June 1970) was a British mathematician and geophysicist. His work on the kinetic theory of gases, solar-terrestrial physics, and the Earth's ozone layer has inspired a broad range of research over many decades. He was Chief Professor of Mathematics at Imperial College London between 1924 and 1946.



This concept is called

Chapman layer Theory

Thank You !

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