## Special Theory of Relativity


$6^{\text {th }}$ Lecture

## Special Theory of Relativity

## Einstein's Two Postulates in STR

Postulate 01 : The Principle of Relativity:
The laws of physics must be the same in all inertial reference frames.

The laws of Physics are the same for all observers in uniform motion relative to one another.

Postulate 02 : The constancy of the speed of light :

The speed of light in vacuum has the same value, $c=3 \times 10^{\wedge} 8$ $\mathrm{m} / \mathrm{s}$ in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

## Measurement of Time in STR


$\Delta t_{o}=\Delta t_{P} \frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$
Where, $v$ is the Relative
Speed of the Two Frames

This equation express the fact that for the moving observer the period of the clock is shorter than in the frame of the ground observer itself !

$$
\left.\Delta t_{O}\right\rangle \Delta t_{P}
$$

Time interval w. r. t the stationary frame

Time interval w. r. t the moving frame

## Time Dilation



## This is called Time Dilation!



$$
f_{o}=f_{s} \frac{1}{\gamma(1-\beta \cos \theta)}
$$

$$
\text { where, } \gamma=\frac{1}{\sqrt{1-\beta^{2}}}
$$

$$
\text { and } \quad \beta=\frac{v}{c}
$$

This is the general form of the Doppler's Effect in STR!

## Relativistic Mass

If we assume a body with mass $m$ is moving with a constant velocity, $v$.
Using the relationship between Relativistic Energy - Momentum:

$$
E^{2}=p^{2} c^{2}+m_{o}^{2} c^{4}
$$

Where, $E=m c^{2} \quad$ and, $\quad p=m v$

$$
\left(m c^{2}\right)^{2}=(m v)^{2} c^{2}+m_{o}^{2} c^{4}
$$



## Experimental Proofs :

The first verification of the increase in mass with velocity came from the experimental work of Kaufmann in 1902 and 1906 and particularly, that of Bucherer in 1909.

## Sommerfeld's theory of Atomic Orbits:

This verification of the mass increased predicted by the STR was proposed by Sommerfeld in 1916.

## Atomic Accelerators :

Early in 1952 the Brookhaven
National Laboratory announced its success in accelerating protons (nuclei of H atoms) up to 0.95 c . As a result the mass of the proton was increased to about three times its original mass.

And in June 1952 the California
Institute of Technology announced it had succeeded in accelerating electrons to 0.9999999 c. The corresponding mass increase was about 900 times its original mass.

## Twin Paradox

In Physics, the twin paradox is a thought experiment in special relativity, in which a twin makes a journey into space in a high speed rocket and returns home to find he has aged less than his identical twin who stayed on Earth.


This result appears puzzling because each twin sees the other twin as travelling and so, according to the theory of special relativity, paradoxically each should find the other to have aged more slowly!

## Transformations Equation



Galiteo Galilei


Hendric Lorentr.

## Transformations Equation

Transformation equations are used to transform between the coordinates of two reference frames. There are two types of transformation equations.

1. Galilean Transformation Equation (without relativistic effect!)
2. Lorentz Transformation Equation (with relativistic effect!)


Galileo Galilei


Hendric Lorents.

## Galilean Transformation



> The Galilean transformation is used to transform between the coordinates of two reference frames which differ only by constant relative motion within the constructs of Newtonian physics. This is the passive transformation point of view. The equations below, although apparently obvious, break down at speeds that approach the speed of light owing to physics described by relativity theory.

Galileo formulated these concepts in his description of uniform motion. The topic was motivated by Galileo's description of the motion of a ball rolling down a ramp, by which he measured the numerical value for the acceleration of gravity near the surface of the Earth.
P. T. O

## Galilean Transformation



The notation below describes the relationship under the Galilean transformation between the coordinates $(x, y, z, t)$ and ( $\left.x^{\prime}, y^{\prime}, z^{\prime}, t\right)$ of a single arbitrary event, as measured in two coordinate systems $S$ and $\mathrm{S}^{\prime}$, in uniform relative motion (velocity $v$ ) in their common $x$ and $x^{\prime}$ directions, with their spatial origins coinciding at time $\mathrm{t}=\mathrm{t}^{\prime}=0$ :

$$
\begin{aligned}
& x^{\prime}=x-v t \\
& y^{\prime}=y \\
& z^{\prime}=z \\
& t^{\prime}=t
\end{aligned}
$$

## Galilean Inverse Transformation



$$
\begin{aligned}
& x^{\prime}=x-0 t \\
& y^{\prime}=3 \\
& z^{\prime}=3 \\
& t^{\prime}=t
\end{aligned}
$$

$$
\begin{aligned}
& \text { Galilean Transformotion } \\
& \text { Equadions }
\end{aligned}
$$

$$
\begin{aligned}
& x=x^{\prime}+z z^{\prime} \\
& y=y^{\prime} \\
& z=z^{\prime} \\
& t=z^{\prime}
\end{aligned}
$$

Inverse

> Galileon

Transformotion
Equo \%ons


## Galilean Transformation Equations



They enable us to relate a measurement in one inertial reference frame to another. For example, suppose we measure the velocity of a vehicle moving in the $x$-direction in system $S$, and we want to know what would be the velocity of the vehicle in $S^{\prime}$.

## Galilean Velocity Transformation Equations

$$
\begin{aligned}
& U_{x}=\frac{d x}{d t} \\
& U_{y}=\frac{d y}{d t} \\
& U_{z}=\frac{d z}{d t}
\end{aligned}
$$

$$
\begin{aligned}
& U_{x}^{\prime}=\frac{d x^{\prime}}{d t^{\prime}} \\
& U_{y}^{\prime}=\frac{d y^{\prime}}{d t^{\prime}} \\
& U_{z}^{\prime}=\frac{d z^{\prime}}{d t^{\prime}}
\end{aligned}
$$

$$
2 I_{x^{\prime}}=u_{x}-v
$$

$$
u_{y^{\prime}}=u_{y}
$$

$$
213^{\prime}=213
$$

$$
\begin{aligned}
& u_{x}=u_{x}^{\prime}+v \\
& u_{3}=2 z_{y}^{\prime} \\
& u_{3}=2 u_{x}^{\prime}
\end{aligned}
$$

Galilean Velocity
Transformation Equations

Inverse Galilean Velocity
Transformation Equodions

## Galilean Velocity Transformation Equations



$$
\begin{aligned}
& 2 u_{x}^{\prime}=u_{x}-v \\
& u_{y}^{\prime}=u_{y} \\
& 2 u_{3}^{\prime}=u_{3}
\end{aligned}
$$

$$
\begin{aligned}
& \text { Galilean Velocity } \\
& \text { Transformation Equations }
\end{aligned}
$$



Galilean Acceleration Transformation Equations

$$
\begin{array}{ll}
a_{x}=\frac{d U_{x}}{d t} & a_{x}^{\prime}=\frac{d U_{x}^{\prime}}{d t^{\prime}} \\
a_{y}=\frac{d U_{y}}{d t} & a_{y}^{\prime}=\frac{d U_{y}^{\prime}}{d t^{\prime}} \\
a_{z}=\frac{d U_{z}}{d t} & a_{z}^{\prime}=\frac{d U_{z}^{\prime}}{d t^{\prime}}
\end{array}
$$

$$
\begin{aligned}
& a_{x}^{\prime}=a_{x} \\
& a_{3}^{\prime}=a_{y} \\
& a_{8}^{\prime}=a_{3}
\end{aligned}
$$

Galilean Accelerator Transformed tron Equation

$$
\begin{aligned}
& a_{x}=a_{x}^{\prime} \\
& a_{y}=a_{y}^{\prime} \\
& a_{3}=a_{3}^{\prime}
\end{aligned}
$$

Inverse Galzieon Accelerator Froosformation Equation

Galilean Acceleration Transformation Equations


$$
\begin{aligned}
& a_{x}^{\prime}=a_{x} \\
& a_{3}^{\prime}=a_{3} \\
& a_{8}^{\prime}=a_{3}
\end{aligned}
$$

Galilean Accelerator Transform alton
Eq*adion

$$
\begin{aligned}
& a_{x}=a_{x}^{\prime} \\
& a_{y}=a_{y}^{\prime} \\
& a_{3}=a_{3}^{\prime}
\end{aligned}
$$

Inverse Galilean Accelerator Trousformatloo Equation

## Lorentz Transformation



In physics (1904), the Lorentz transformation or Lorentz-Fitzgerald transformation describes how, according to the theory of special relativity, different measurements of space and time by two observers can be converted into the measurements observed in either frame of reference.

The Lorentz transformation was originally the result of attempts by Lorentz and others to explain how the speed of light was observed to be independent of the reference frame, and to understand the symmetries of the laws of electromagnetism. Albert Einstein later re-derived the transformation from his postulates of special relativity. The Lorentz transformation supersedes the Galilean transformation of Newtonian physics, which assumes an absolute space and time. According to special relativity, the Galilean transformation is a good approximation only at relative speeds much smaller than the speed of light.

## Lorentz Transformation Equations

Consider two observers $O$ and $O^{\prime}$, each using their own Cartesian coordinate system to measure space and time intervals. $O$ uses $(t, x, y, \gamma)$ and $O^{\prime}$ uses $\left(t^{\prime}, x^{\prime}, y^{\prime}\right.$ ,$\left.z^{\prime}\right)$. Assume further that the coordinate systems are oriented so that, in 3 dimensions, the $x$-axis and the $x^{\prime}$-axis are collinear, the $y$-axis is parallel to the $y^{\prime}$ axis, and the $\tau^{-a x i s}$ parallel to the $z^{\prime}$-axis. The relative velocity between the two observers is $v$ along the common $x$-axis. Also assume that the origins of both coordinate systems are the same, that is, coincident times and positions. If all these hold, then the coordinate systems are said to be in standard configuration. A between the forward Lorentz Transformation and the inverse Lorentz Transformation can be achieved if coordinate systems are in. The symmetric form highlights that all physical laws should remain unchanged under a Lorentz transformation.


## Lorentz Transformation Equations



These are the simplest forms. The Lorentz transformation for frames in standard configuration can be shown to be:
where:


Lorentz Transformation Equations



I orentz Transformation


Lorentz Velocity Transformation Equations



## Lorentz Acceleration Transformation Equations



Lorentr. Acceleration Transformation Equations


## Length Contraction (using Lorentz Transformation)

Length contraction is the observation that a moving object appears shorter than a stationary object.


Let us assume there is a rod of length $L$ with respect to the stationary frame $S^{\prime}$, is moving with constant velocity $\nu$.

Length with respect to the $S^{\prime}$ frame $=L_{0}=x_{2}^{1}-x_{1}^{1}$
Length with respect to the $S$ frame $=L=x_{2}-x_{1}$

## Length Contraction (using Lorentz Transformation)

Using Lorentz transformation equations;


$$
x^{1}=\gamma(x-v t)
$$

Therefore,

$$
x_{1}^{1}=\gamma\left(x_{1}-v t_{1}\right) \quad \text { and } \quad x_{2}^{1}=\gamma\left(x_{2}-v t_{2}\right)
$$

$$
x_{2}^{1}-x_{1}^{1}=\gamma\left(x_{2}-v t_{2}\right)-\gamma\left(x_{1}-v t_{1}\right)
$$

$$
x_{2}^{1}-x_{1}^{1}=\gamma\left(x_{2}-x_{1}\right)-\gamma v\left(t_{2}-t_{1}\right)
$$

Where, $t_{1}=t_{2}$

Therefore,

$$
\begin{aligned}
x_{2}^{1}-x_{1}^{1} & =\gamma\left(x_{2}-x_{1}\right) \\
L_{o} & =\gamma L
\end{aligned}
$$

Length measured by an $L_{o}>L \quad$ Length measured by an observer in the Frame S' observer in the Frame S

## Time Dilation (using Lorentz Transformation)

Let us assume two events that takes place at a certain point, one after the other. We can consider, an artist painting a picture as an example. The first event could be the commencement of painting the picture and the second event could be the completion of painting the picture.


Time interval with respect to the $S^{\prime}$ frame $=t=t_{2}^{1}-t_{1}^{1}$
Time interval with respect to the $S$ frame $=t_{0}=t_{2}-t_{1}$

## Time Dilation (using Lorentz Transformation)

Using Lorentz transformation equations;


$$
t^{1}=\gamma\left(t-\frac{v}{c^{2}} x\right)
$$

Therefore, $t_{1}^{1}=\gamma\left(t_{1}-\frac{v}{c^{2}} x_{1}\right) ; \quad x_{1}=x \quad$ and $\quad t_{2}^{1}=\gamma\left(t_{2}-\frac{v}{c^{2}} x_{2}\right) ; \quad x_{2}=x$

$$
t_{2}^{1}-t_{1}^{1}=\gamma\left(t_{2}-\frac{v}{c^{2}} x_{2}\right)-\gamma\left(t_{1}-\frac{v}{c^{2}} x_{1}\right)
$$

$$
t_{2}^{1}-t_{1}^{1}=\gamma\left(t_{2}-t_{1}\right)-\gamma \frac{v}{c^{2}}\left(x_{2}-x_{1}\right)
$$

Where,

$$
x_{1}=x_{2}
$$

Therefore, $\quad t_{2}^{1}-t_{1}^{1}=\gamma\left(t_{2}-t_{1}\right)$

$$
t=\gamma t_{0}
$$

Time interval measured by an $t>t_{0}$ observer in the Frame S'

Time interval measured by an observer in the Frame S

## Relative Motion for the two bodies in Relativity

Let us assume two objects are moving in an opposite direction to each other,

$$
\stackrel{A}{\ominus} V_{A} \quad V_{B} \stackrel{B}{\square}
$$

Using Lorentz transformation equations;


For this example;

$$
U_{x}^{1}=V_{A}, \quad U_{x}=-V_{B} \quad \text { and } \quad v=V_{(B, A)}
$$

Direction of $B$ is opposite to the $A$



This $v$ denotes $V(B, A) . V(B, A)$ has a negative value. :. The direction of $\mathrm{V}(B, A)$ should be the opposite direction. :. $\mathrm{V}(A, B)$ is + ve;

$$
\longrightarrow v=V_{(A, B)}=\frac{V_{A}+V_{B}}{1+\frac{V_{A} V_{B}}{c^{2}}}
$$

$$
\text { Or } v=V_{(B, A)}
$$

Let us assume two objects are moving in a same direction,
$\stackrel{\mathrm{A}}{\stackrel{ }{\square}} \mathrm{VA}_{\mathrm{B}}^{\stackrel{\mathrm{B}}{\longrightarrow}} \mathrm{VB}_{\mathrm{B}}$
For this example;

$$
U_{x}^{1}=V_{A}
$$

$$
U_{x}=V_{B} \quad \text { and } \quad v=V_{(B, A)}
$$

$$
v=V_{(B, A)}=\frac{V_{A}-V_{B}}{1-\frac{V_{A} V_{B}}{c^{2}}}
$$

Find the Energy \& Momentum in S' frame w. r. t. S frame


$$
E^{1}=\gamma(v)\left[E-P_{x} v\right]
$$

$$
P_{x}^{1}=\gamma(v)\left[P_{x}-\frac{E v}{c^{2}}\right],
$$

$$
P_{y}^{1}=P_{y} \text { and } P_{z}^{1}=P_{z}
$$

Where, $\gamma(v)=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}$,


$$
\text { and } P_{x}=m U_{x}=\frac{m_{o}}{\sqrt{1-\frac{v^{2}}{c^{2}}}} U_{x}=\gamma(v) m_{o} U_{x}
$$

## Lorentz Invariant

A quantity that remains unchanged by a Lorentz transformation is said to be Lorentz invariant. Such quantities play on especially important role in special theory of relativity. The norm of any four vector is Lorentz Invariant.


$$
\begin{aligned}
& E^{1^{2}}-p^{1^{2}} c^{2}=E^{2}-p^{2} c^{2} \\
& c^{2} t^{1^{2}}-r^{1^{2}}=c^{2} t^{2}-r^{2}
\end{aligned}
$$

## Four vectors

## Four Vectors / Four Cdts

 Systemstime


A four vector is a displacement in both space and time.


