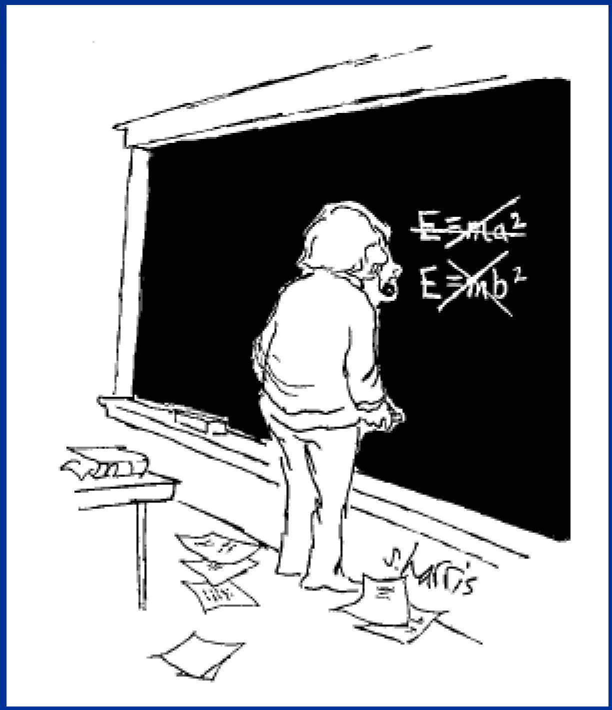
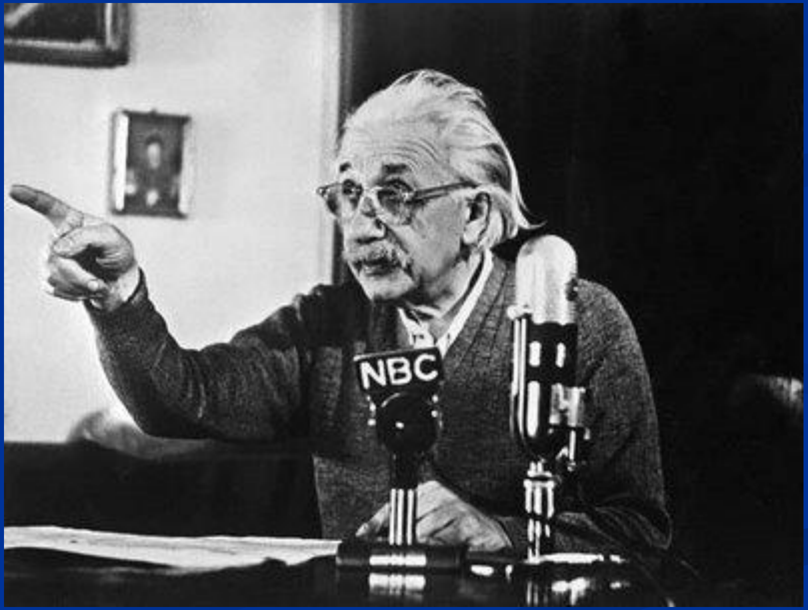
# Special Theory of Relativity



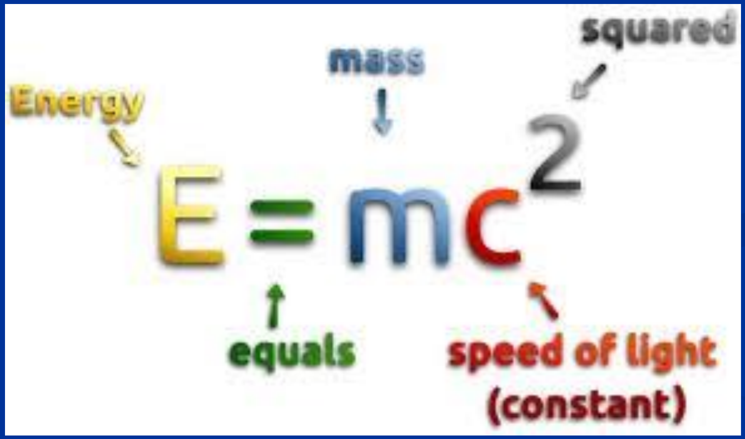




## 6th Lecture

# The mass – energy equivalence

In Physics, mass – energy equivalence is the concept that the mass of a body is a measure of its energy content.

Albert Einstein proposed mass – energy equivalence in 1905. The equivalence is described by the famous equation,




The equation **$E = m c^2$** indicates that energy always exhibits mass in whatever from the energy takes. It does not imply that mass may be "**converted**" to energy, for modern theory holds that neither mass or energy may be destroyed, but only moved from one location to another.

# The mass – energy equivalence


4-meter-tall sculpture of Einstein's 1905 $E = mc^2$ formula at the 2006 Walk of Ideas, Berlin, Germany.

*Find the mass-equivalence energy of a 1kg.*

*Using,* $E = mc^2$

$$E = (1)\,(3\times10^8)^2$$

$$E = 9\times10^{16}\ J$$

This is a very large energy. Using this energy, we can vaporize **~10^10 kg** of water at the room temperature (**30°C**)! $\because E = ms\theta + mL$

# Relativistic Energy - Momentum Equation

**What is relativistic energy ???**

The energy of a moving body as measured by an observer in the same frame of reference as the body.

**What is relativistic momentum ???**

The momentum of a moving body as measured by an observer in the same frame of reference as the body.

**Derivation of the Relativistic Energy - Momentum equation :**

*Momentum = Mass x Velocity*

$$p = mv$$

$$m = \frac{p}{v}$$

P.T.O

*Using Einstein's equivalence Mass-Energy equation;*

$$E = mc^2 \rightarrow E = \frac{p}{v} c^2 \rightarrow E = p c^2 \frac{1}{v}$$

01

*Work done = Force x Distance* $\rightarrow$ $dE = F\,dx$

*Using Newton's second law,* $F = ma$

$$\rightarrow F = m\frac{dv}{dt}$$

$$\rightarrow F = \frac{d(mv)}{dt} \rightarrow F = \frac{dp}{dt}$$

*Then work done ,* $dE = F\,dx \rightarrow dE = \frac{dp}{dt} dx$

$$\rightarrow dE = dp\frac{dx}{dt} \rightarrow dE = dp\,.\,v \rightarrow \frac{1}{v} = \frac{dp}{dE}$$

P.T.O

*Substitute this 1/v expression to equation 01;*

$$E = pc^2 \frac{dp}{dE}$$ *because,* $$\frac{1}{v} = \frac{dp}{dE}$$ ➡ $$E = pc^2 \frac{1}{v}$$

➡ $$E\,.dE = pc^2.dp$$

*Integrating the above equation;*

$$\int E\,.dE = \int pc^2.dp \quad + \quad k$$

➡ $$\frac{E^2}{2} = c^2 \frac{p^2}{2} + k$$

To find the value *k*, apply a boundary condition:

If $t = 0$, $v$ should be $0$. Therefore, $p = mv = 0$ and $E = E_o = m_o c^2$,
Where, $m_o$ is the *Rest Mass* of the body.

➡ $$\frac{E_o^2}{2} = k$$

P.T.O

$$\frac{E^2}{2} = c^2 \frac{p^2}{2} + k \quad \Rightarrow \quad \frac{E_o^2}{2} = k$$

$$\Downarrow$$

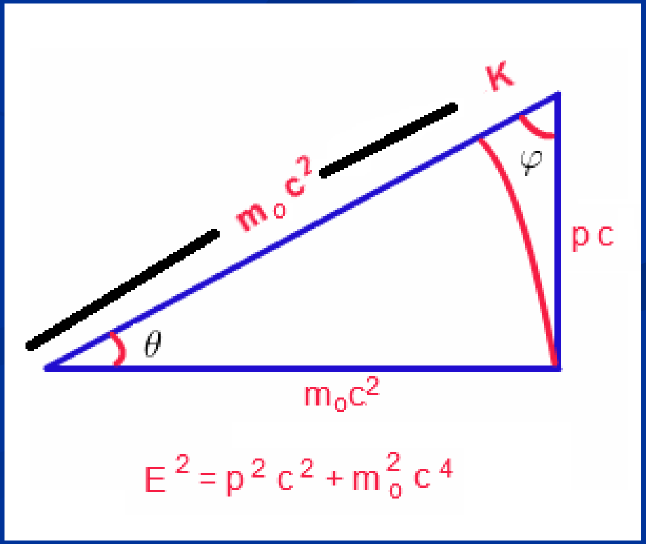
$$\frac{E^2}{2} = c^2 \frac{p^2}{2} + \frac{E_o^2}{2}$$

$$\Rightarrow \quad E^2 = p^2c^2 + E_o^2$$

$$\Rightarrow \quad E^2 = p^2c^2 + m_o^2c^4$$

Finally we got the above relationship. This is called,

“**Relativistic Energy-Momentum equation**”



## "Relativistic Energy-Momentum equation"

$$E^2 = p^2c^2 + m_o^2c^4$$

*Where,*

**$E$ – Relativistic Energy**

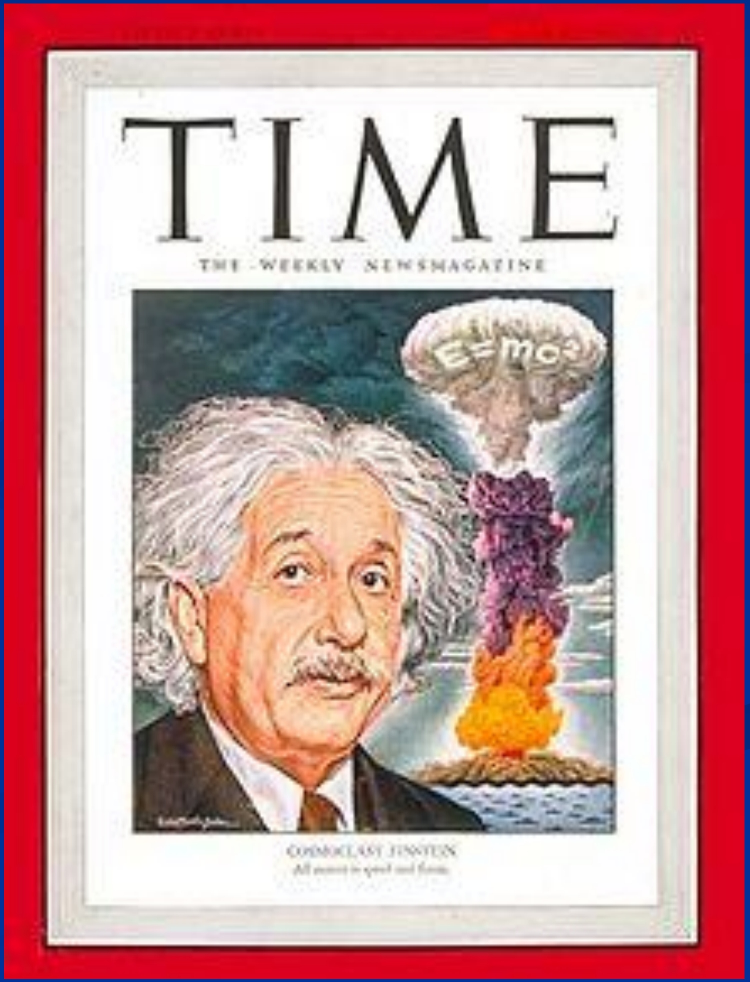
The energy of the body which is moving with velocity $v$.

**$p$ – Relativistic Momentum**

The momentum of the body which is moving with velocity $v$.

**$m_o$ – Rest Mass of the body**

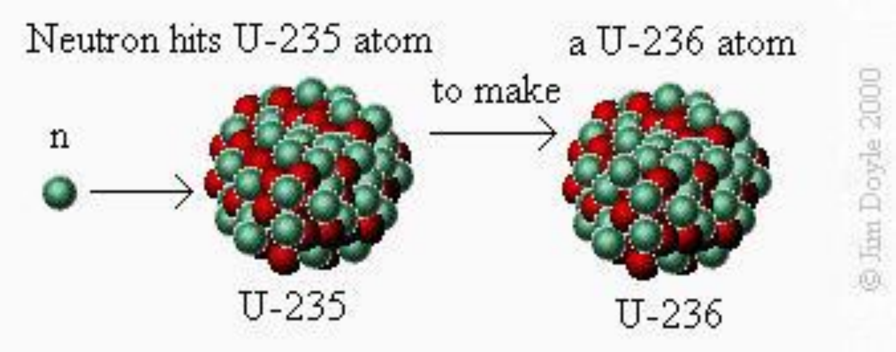
The mass of the body which is at rest.



# Practical Application of Equivalence of mass and energy

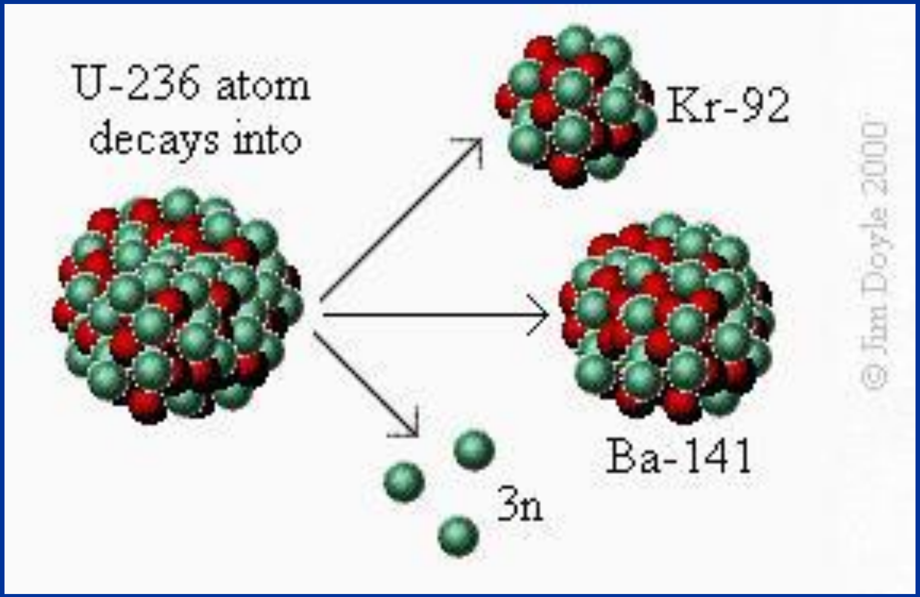
Nuclear fission takes place when a heavy atomic nucleus, such as uranium, breaks into two or more smaller pieces with the release of some energy. During this process some of the mass of the original atom is converted into energy in accordance with the equation **$E = mc^2$**.

First we fire a neutron (n) at the uranium-235 (U-235) atom so that it sticks to it. After a short while the uranium-235 splits into an atom of barium-141 (Ba-141), an atom of krypton-92 (Kr-92) and three neutrons. We can show this schematically. Firstly, we see a neutron striking a uranium-235 atom to make a uranium-236 atom:



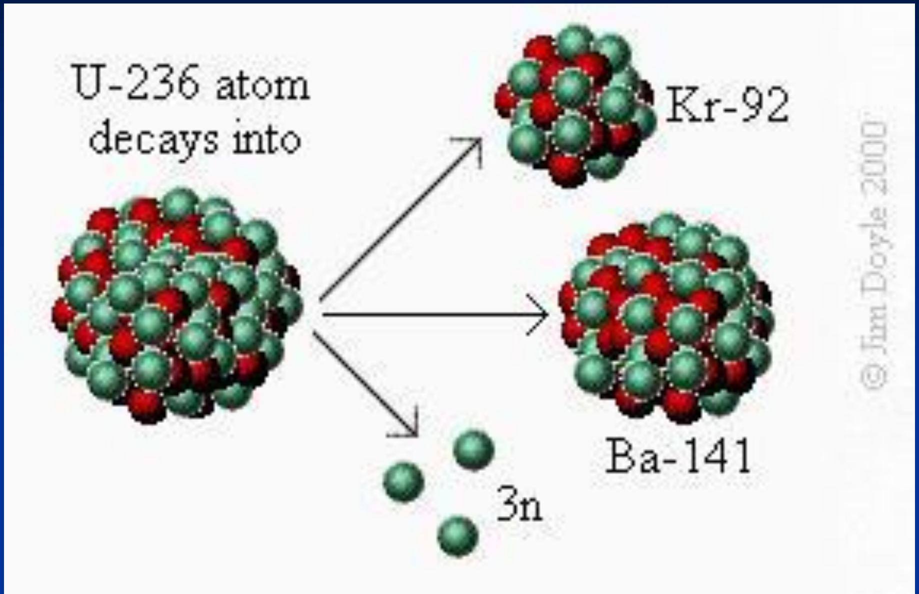
# Practical Application of Equivalence of mass and energy

Secondly, the new uranium-236 atom rapidly decays into an atom of Ba-141 (barium), an atom of Kr-92 (krypton) and three neutrons:



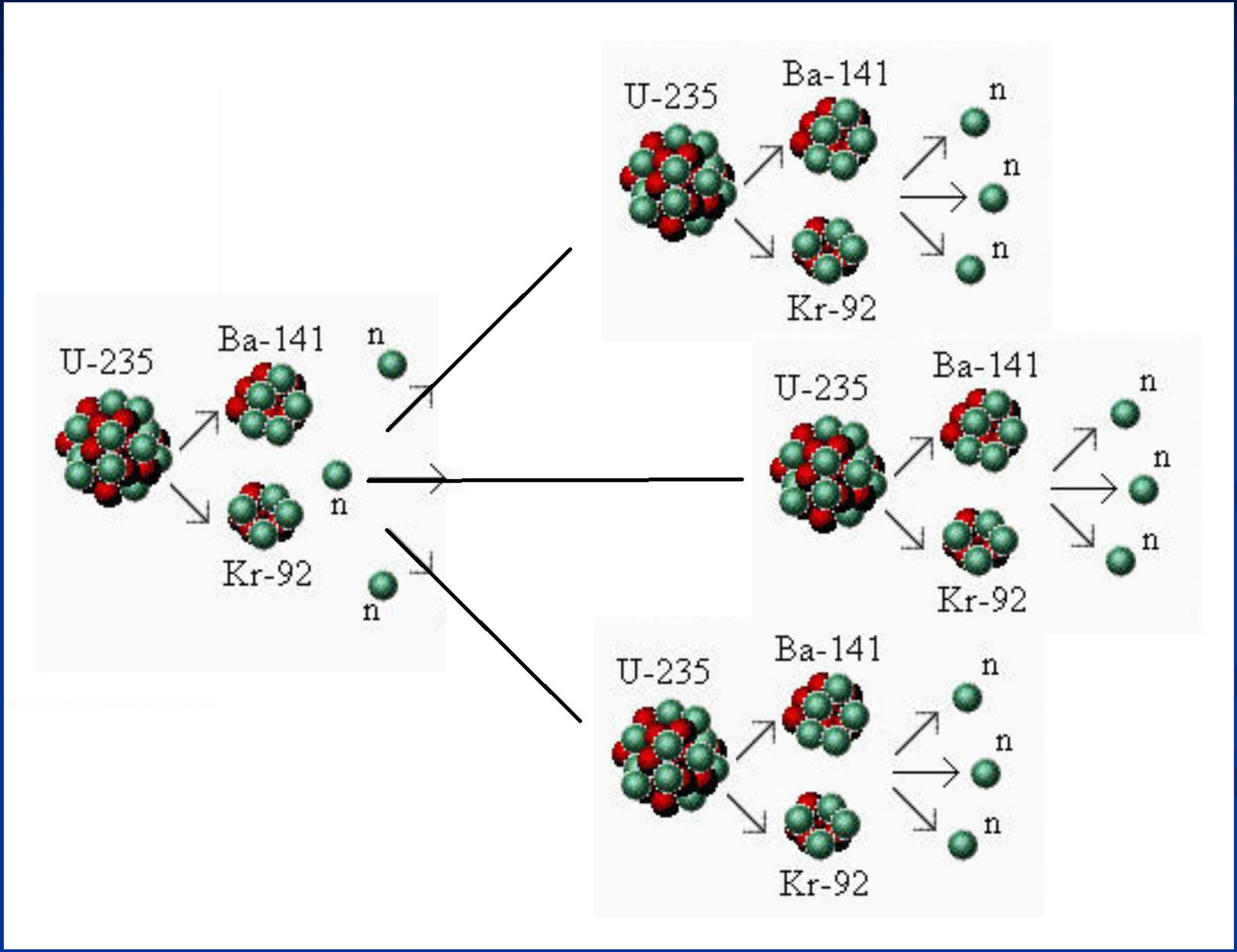

The resulting particles all have kinetic energy. This energy comes from converting a little of the mass of the original atom into energy and can be measured using **$E = mc^2$**. When this is done, the amount of energy typically released in the case of U-235 is around 200 MeV (0.0000000003204 Joules).

# Practical Application of Equivalence of mass and energy




That, it seems, is a very tiny amount of energy. However, it is about a *million times* more energy than is released by the burning of one molecule of petrol (gas) in a car's engine. Put another way, if you currently use a tank of petrol each week but could use the energy provided by one tank of uranium-235 fission instead, you wouldn't need to re-fill your car for over 19,000 years.

# Practical Application of Equivalence of mass and energy



## Nuclear Power Stations :

Nuclear power stations use another element that can undergo nuclear fission, that of plutonium (Pu), which is also used in most modern atomic bombs. This element is slightly heavier than uranium, but doesn't occur naturally in anything like sufficient quantities to be useful and so is made, or synthesized, from uranium together with other particles. The method is surprisingly simple in that U-238 is bombarded with either neutrons until some stick and transmute (i.e. change) and so form Pu-239, or the nuclei of heavy hydrogen (1 neutron and 1 proton) are used to bombard U-238 to form Pu-238. There are intermediate stages of transmutation in each case, but what remains at the end is isotopes of plutonium, either Pu-239 or Pu-238, and with each nucleus containing 94 protons.

# Nuclear Power Stations :

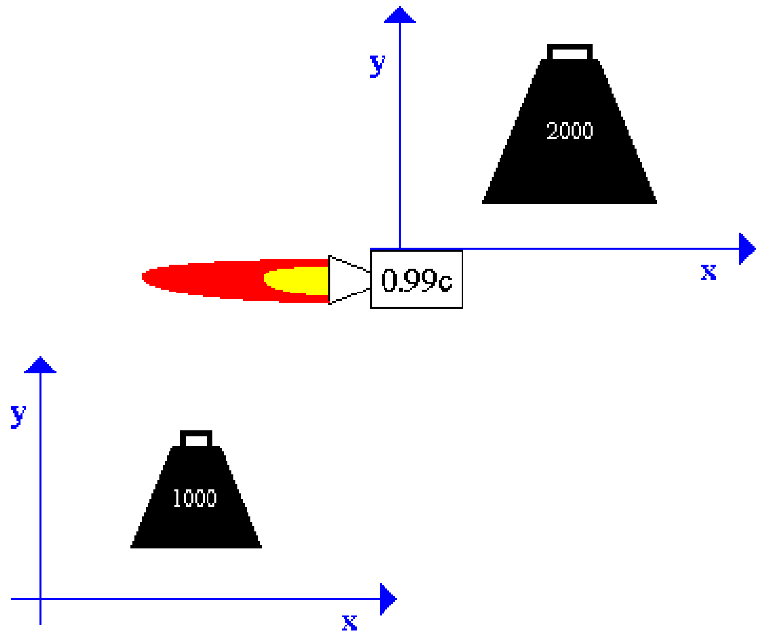
In this way the vast bulk (99.3%) of mined uranium, i.e. U-238, can still be used to provide energy. The process of turning uranium into plutonium is sometimes referred to as "breeding", and "breeder reactors" are used for this purpose.

A nuclear power station works in pretty much the same way as any other power station, only the energy source is different. Generally, rods of fissionable material (plutonium) are pushed towards each other until a *controlled* amount of heat is produced. This heat is used to produce steam, which is forced at high pressure through a set of turbine wheels. The turbine wheels are connected to a generator and electricity is produced.

# Relativistic Mass

If any body is moving with some velocity, that body has a mass (greater than its mass at rest) is called Relativistic Mass of the body!

In special theory of relativity, an object that has a mass cannot travel at the speed of light. As the object approaches the speed of light, the object's energy and momentum increase without bound.



an objects mass increases as it approaches the speed of light

# Relativistic Mass

If we assume a body with mass $m$ is moving with a constant velocity, $v$.

Using the relationship between Relativistic Energy - Momentum:

$$E^2 = p^2c^2 + m_o^2c^4$$

Where, $E = mc^2$ and, $p = mv$

$$(mc^2)^2 = (mv)^2c^2 + m_o^2c^4$$

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# Relativistic Mass

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

*Where,*

***m* – Relativistic Mass of the body**

The mass of the body which is moving with velocity *v*.


**$m_o$ – Rest Mass of the body**

***v* – Relativistic Speed of the body**

# Relativistic Mass

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = m_o\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

*Using Taylor series expansion,*

$$m = m_o\left(1-\frac{v^2}{c^2}\right)^{-1/2} = m_o\left(1-\left(-\frac{1}{2}\right)\frac{v^2}{c^2}+\left(\frac{3}{2}\right)\left(-\frac{1}{2}\right)^2\left(\frac{v^2}{c^2}\right)^2-\left(\frac{5}{2}\right)\left(-\frac{1}{2}\right)^3\left(\frac{v^2}{c^2}\right)^3+\dots\right)$$

$$m = m_o\left(1-\frac{v^2}{c^2}\right)^{-1/2} \approx m_o\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$$

$$m \approx m_o\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$$

*Because,*

$$\dots\left(\frac{v^2}{c^2}\right)^2 << \left(\frac{v^2}{c^2}\right) << 1$$

# Relativistic Mass

Rest mass of an electron is *9.01x10⁻³¹ kg*. An electron is accelerated to a speed of *0.8 c* by using a particle accelerator. Calculate the mass of the moving electron.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = 9.01 \times 10^{-31} \times \frac{1}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}}$$

$$m = 9.01 \times 10^{-31} \times 1.66$$

$$m = 1.5 \times 10^{-30}\ kg$$

**The Mass is increased!**

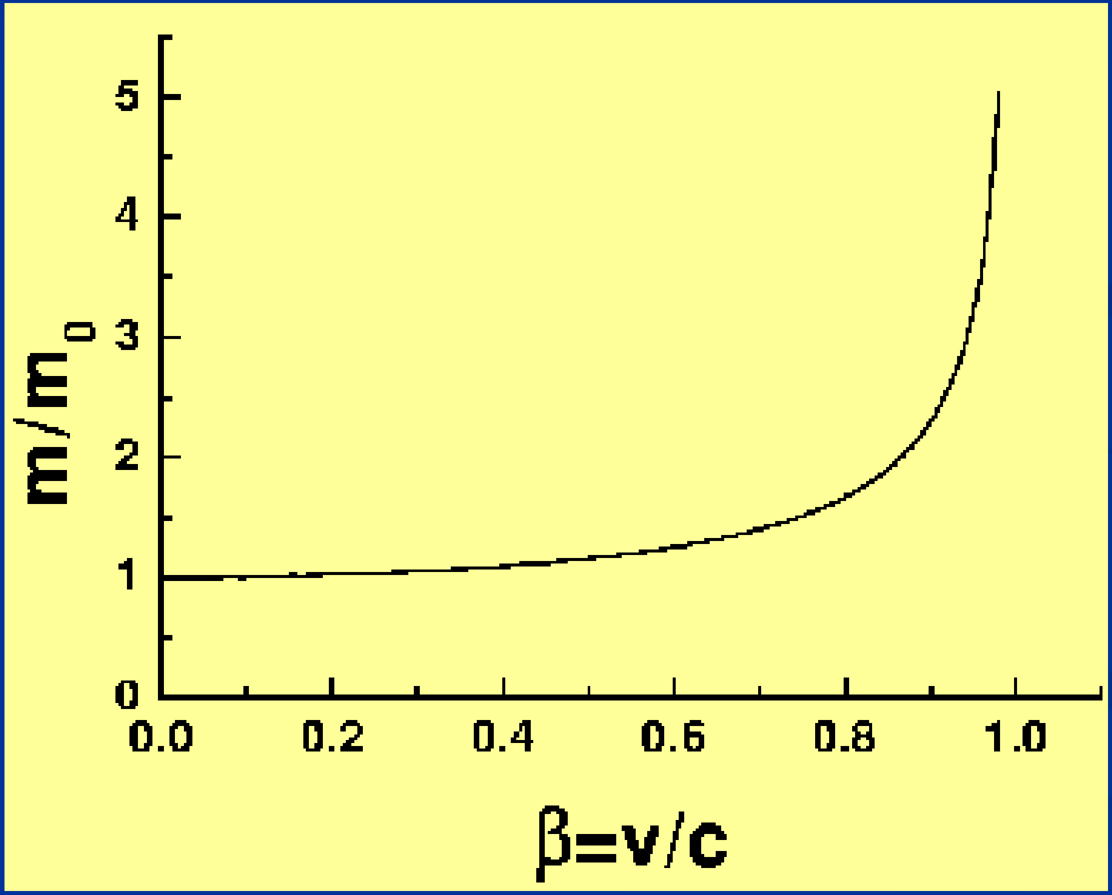
# Relativistic Mass

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = m_o\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$\frac{m}{m_o} = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$\frac{m}{m_o} = \left(1-\beta^2\right)^{-\frac{1}{2}}$$

*Where,* $\beta = \frac{v}{c}$



# Experimental Proofs :

The first verification of the increase in mass with velocity came from the experimental work of Kaufmann in 1902 and 1906 and particularly, that of Bucherer in 1909. They were working on something entirely unrelated to relativity – or so they thought. It had been known for some time that certain substances, radium for one, were constantly shooting off three different types of small particles, or rays. Such substances are called **radioactive**. They were investigating the particular type of radiation known as beta rays and were attempting to determine just what these were. In doing so, they found the velocities with which individual particles making up the radiation were ejected from radioactive substances, the amount of electric charge on each and the mass of each.

The velocities were found to be comparable to the velocity of light; they also found that the higher the velocity, the greater the mass of the particle. Hence, they obtained many different beta particles, each with a different mass. They found that rest mass was the same for each particle.

# Experimental Proofs :

This result constituted the first experimental proof of ,

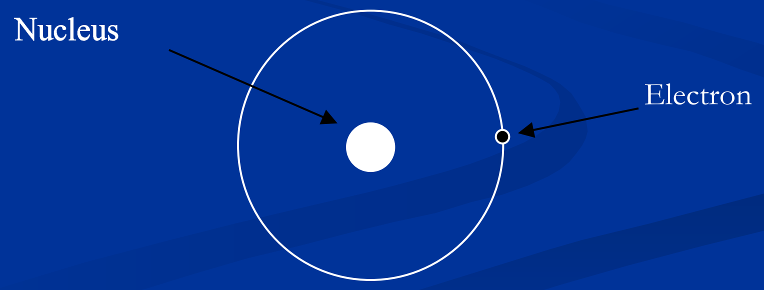
$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Equation and the first verification of the special theory of relativity.

**Sommerfeld's theory of Atomic Orbits**:

This verification of the mass increased predicted by the STR was proposed by Sommerfeld in 1916.

Previous to this time, the Bohr Theory (1913) had pictured the atom as consisting of a **nucleus at the centre** with the **electrons moving in circles about the nucleus**.



# Experimental Proofs :

## Sommerfeld's theory of Atomic Orbits…

However, Sommerfeld showed that it was more correct to assume that, in general, the electron paths were not circles but **ellipses**, and that the electron **revolved about the nucleus**, which was situated at the one of the foci of the ellipse, in the same way that planets revolve around the Sun as in the following figure.

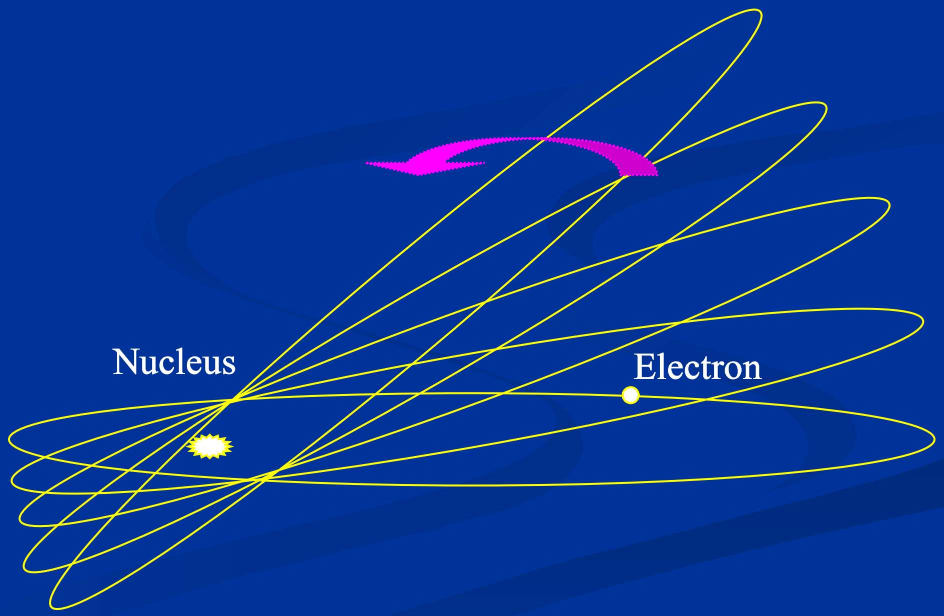

It had been shown by Kepler in 1609 that when a planet revolves around the Sun, the velocity of the planet changes from a minimum to maximum, that amount of the variation depending on the flatness, or ellipticity, of the orbit. Now, since the velocity changes, the mass increase formula says that the mass of the electron should change and the greater the variation in velocity, the greater will be this change in mass.

# Experimental Proofs :

## Sommerfeld's theory of Atomic Orbits…

Average velocity of the electron in its orbit about the nucleus is about one one-hundredth the velocity of light, so that for a fairly flat orbit the change in velocity, and consequent change in mass, is small but detectable.

Sommerfeld showed mathematically that the net effect of this change in mass is that the electron will not keep revolving around the nucleus in the same elliptical path over and over again like the Earth does around the Sun, but the ellipse will slowly rotate and the electron will describe a **Rosette Patten** as shown.



# Experimental Proofs :

**Sommerfeld's theory of Atomic Orbits**…

It might seem at first glance as if it is impossible to determine the path of a single electron about a nucleus, since not only do we have no way of chopping off a single atom from a substance, but it would be impossible to see such an atom – even with the most powerful microscope we have.

In experimental work a prism by itself is insufficient for creating the best possible spectrum, since much greater precision is needed. An instrument called a spectroscope is used which contains a prism plus other necessary devices to help gain this high precision. The spectrum produced by a spectroscope shows a number of lines called Spectral Lines, which are scattered throughout the various colours of the spectrum.

*Thank You !*