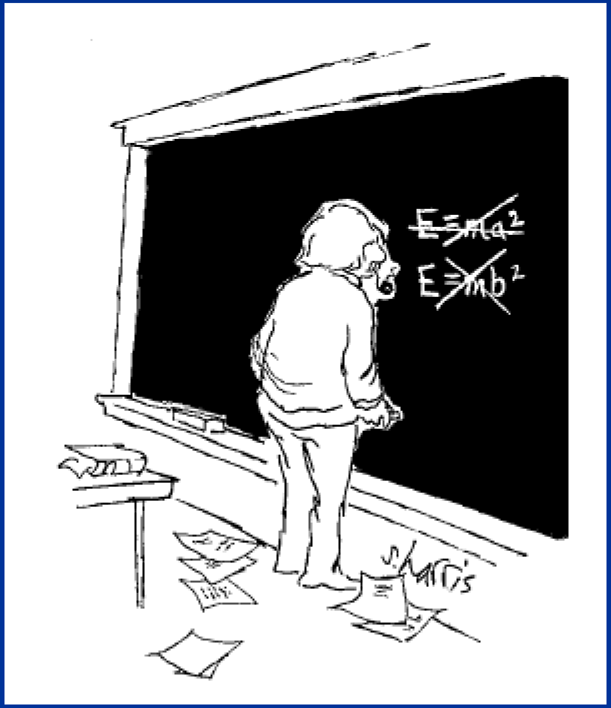
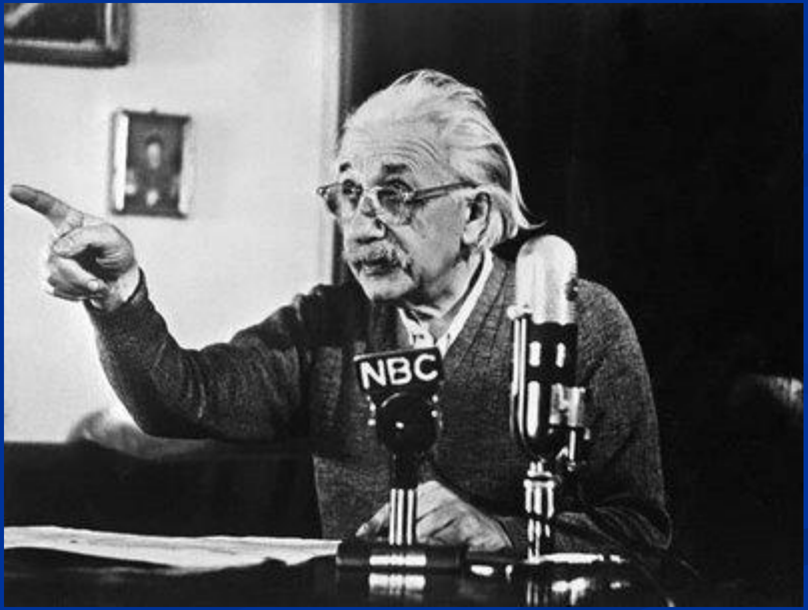
Special Theory of Relativity





4th Lecture

Special Theory of Relativity

Einstein's Two Postulates in STR

Postulate 01 : The Principle of Relativity:

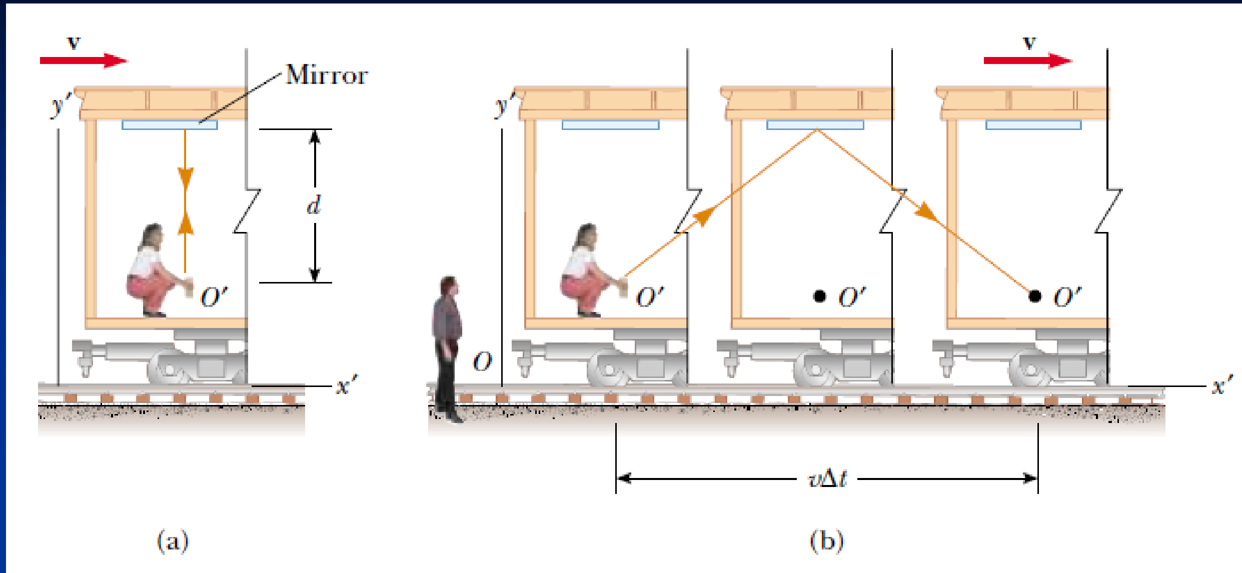
The laws of physics must be the same in all inertial reference frames.

The laws of Physics are the same for all observers in uniform motion relative to one another.

Postulate 02 : The constancy of the speed of light :

The speed of light in vacuum has the same value, c = 3 x 10^8 m/s in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Measurement of Time in STR



$$\Delta t_O = \Delta t_P \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where, v is the Relative Speed of the Two Frames

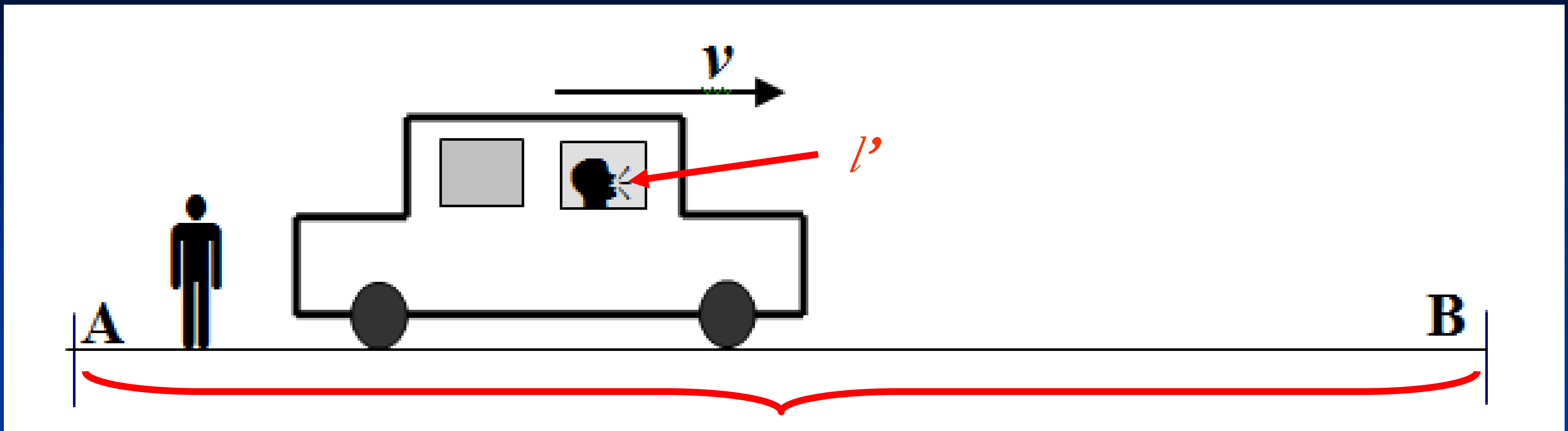
$$\Delta t_O \rangle \Delta t_P$$

Time interval w. r. t the stationary frame

Time interval w. r. t the moving frame

This is called Time Dilation !

Measurement of Length in STR



$$t_{im} = \frac{l}{v}$$

$$t_{pro} = \frac{l^1}{v}$$

$$t_{im} = t_{pro} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \quad l^1 = l \sqrt{1 - \frac{v^2}{c^2}}$$

If $v > 0$ $\Rightarrow$ $l^1 < l$

Length measured by an observer in the car

Length measured by observer in the Earth

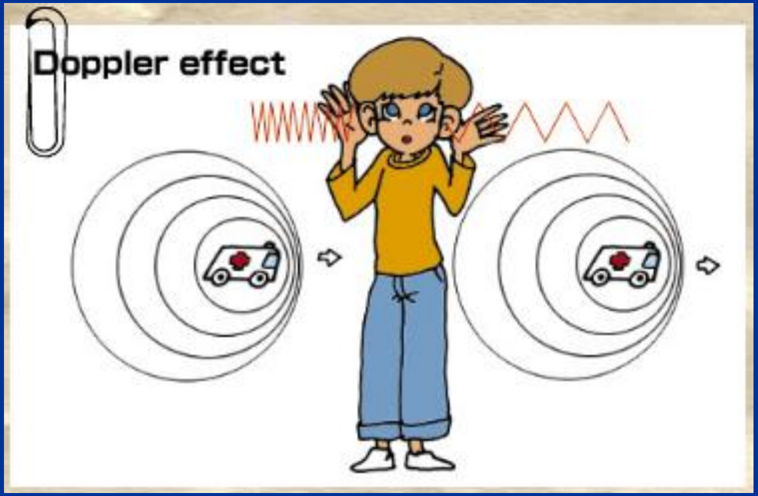
This is called Length Contraction !

Doppler's Effect in STR

Doppler's Effect for Sound Waves :

The Doppler's effect (or Doppler's shift), named after Austrian physicist Christian Doppler who proposed it in 1842, is the change in frequency of a wave for an observer moving relative to the source of the wave.

It is commonly heard when a vehicle sounding a siren or horn approaches, passes and recedes from the observer.



In classical physics (wave in a medium), where the source and the receiver velocities are not supersonic, the relationship between observed frequency f_0 and emitted frequency (or source frequency) f_s is given by,

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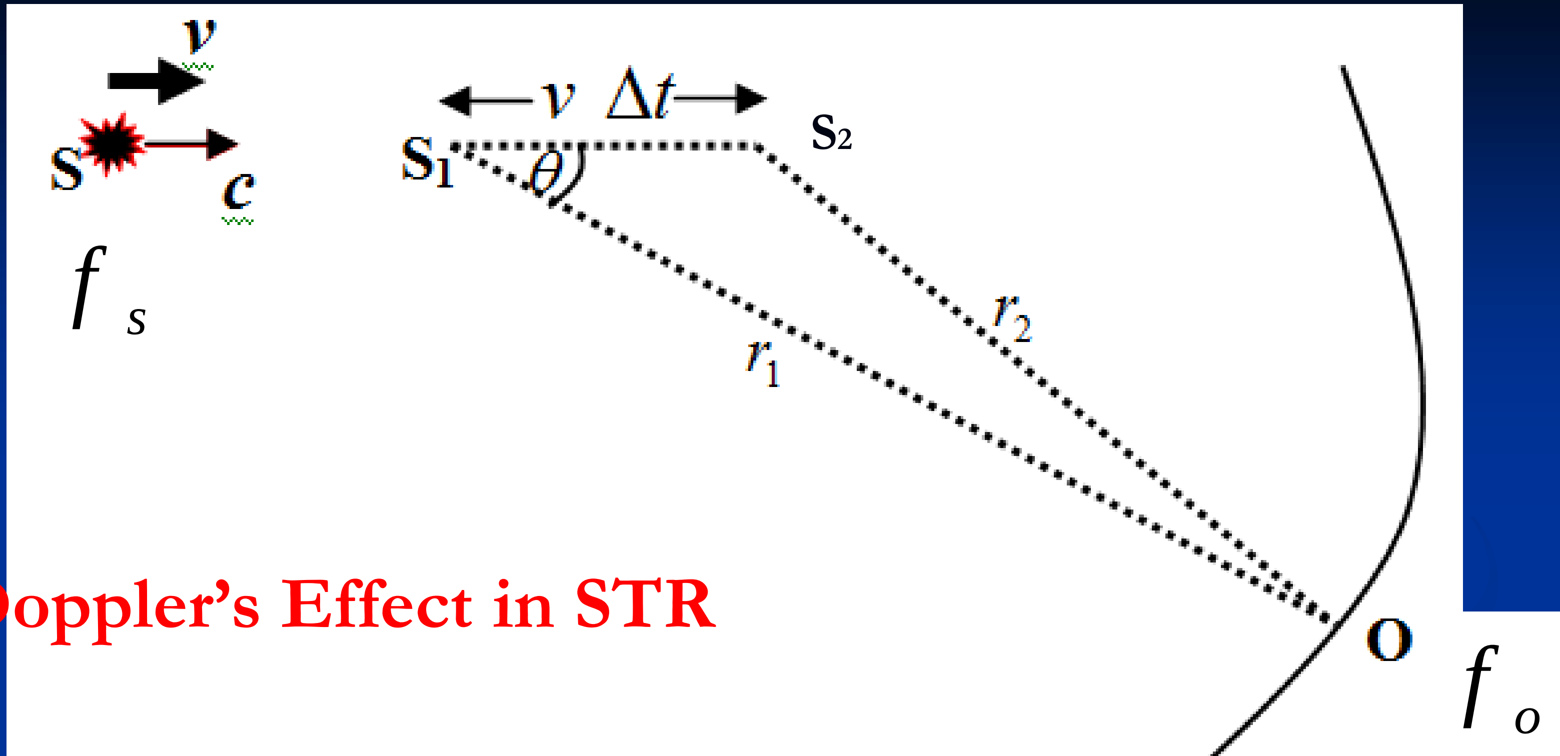
Doppler's Effect in STR



$$f_o = f_s \frac{1}{\gamma(1-\beta\cos\theta)}$$

where, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

and $\beta = \dfrac{v}{c}$

This is the general form of the Doppler's Effect in STR!

Special Cases:

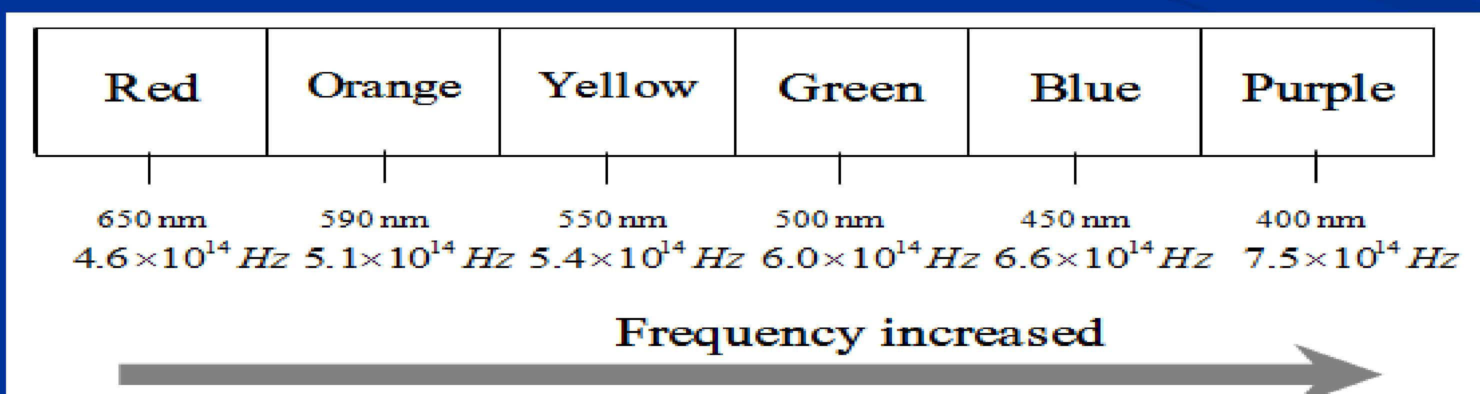
(a) If the source is directly approaching the observer:

Then, $\theta = 0$ and $\cos\theta = 1$ ➡ $f_o = f_s \dfrac{1}{\gamma(1-\beta)}$ where, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

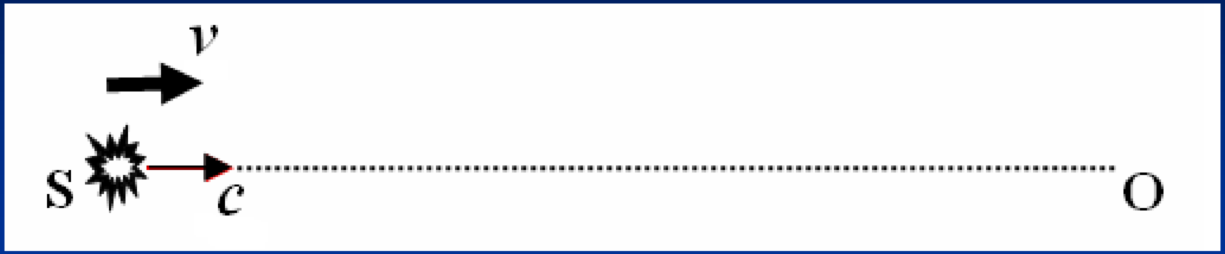
$\beta = \dfrac{v}{c}$

➡ $$f_o = f_s \sqrt{\frac{1+\beta}{1-\beta}}$$ ➡ $f_o > f_s$

➡ **Thus, the frequency appears to increase !**

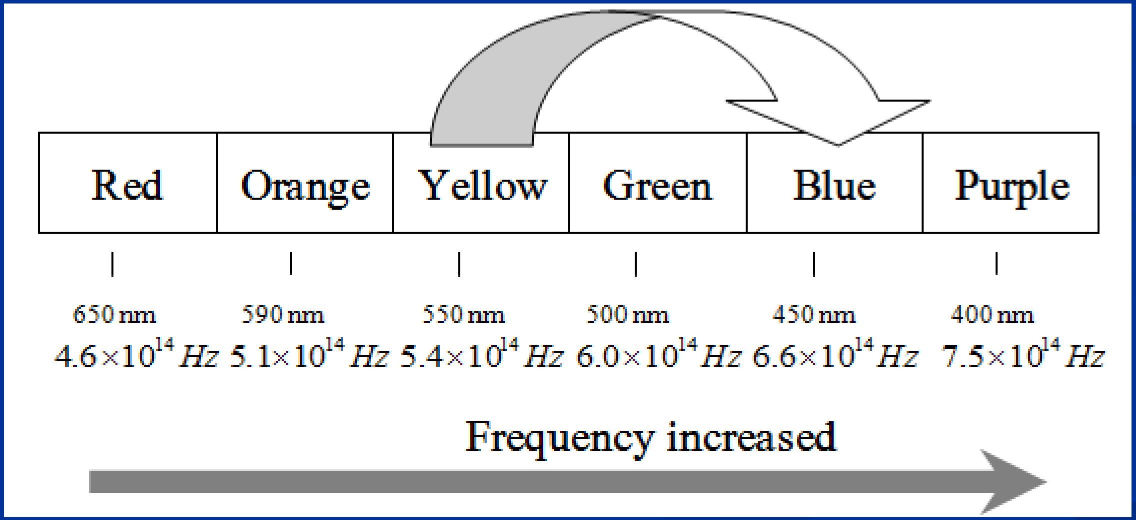


(a) If the source is directly approaching the observer:



$$f_o > f_s$$

Thus, the frequency appears to increase !



This is called frequency shift !

Special Cases:

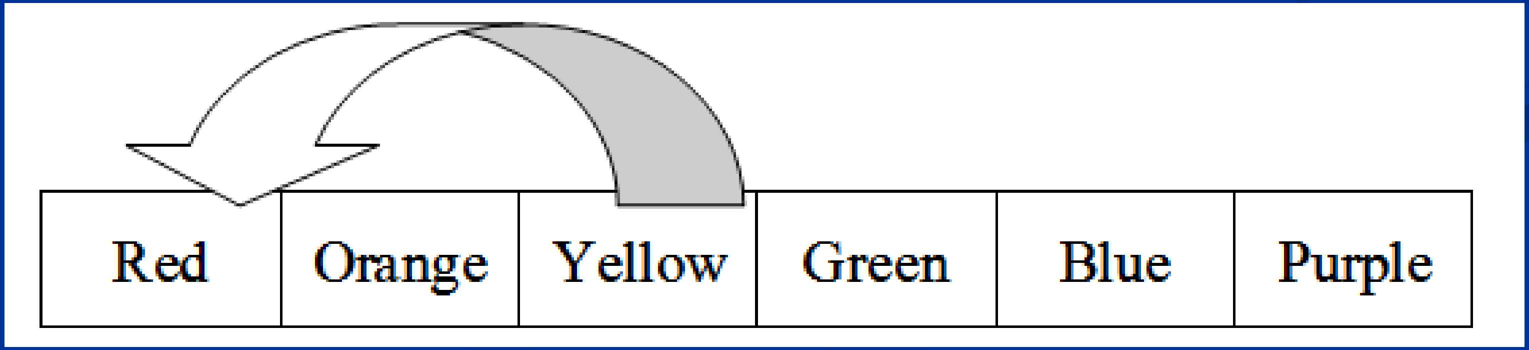
(b) If the source is receding directly from the observer:

Then, $\theta = \pi$ and $\cos\theta = -1$ ➡ $$f_o = f_s \frac{1}{\gamma(1+\beta)}$$ where, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$\beta = \frac{v}{c}$

➡ $$f_o = f_s \sqrt{\frac{1-\beta}{1+\beta}}$$

➡ **Thus, the frequency appears to decrease !**



Example :

A yellow coloured vehicle appears as a green coloured vehicle to a stationary observer due to its speed. Find the velocity of the vehicle. (Wavelengths of yellow and green light are 550nm and 500nm respectively)

Is the above incident practically possible??? Briefly explain your answer.

Example :

A yellow coloured vehicle appears as a green coloured vehicle to a stationary observer due to its speed. Find the velocity of the vehicle. (Wavelengths of yellow and green light are 550nm and 500nm respectively)

Is the above incident practically possible??? Briefly explain your answer.

Source frequency = true colour of the vehicle

f_s = *frequency of the yellow colour*

For E-M Waves : $v = f\lambda$ ➡ $f = \dfrac{c}{\lambda}$

➡ $$f_s = \frac{3 \times 10^{8}\, ms^{-1}}{550 \times 10^{-9}\, m}$$

➡ $$f_s = 5.45 \times 10^{14}\, Hz$$

Example :

Observed frequency = appeared colour of the vehicle

f_o = *frequency of the green colour*

For E-M Waves : $v = f\lambda$ ➡ $f = \dfrac{c}{\lambda}$

➡ $f_o = \dfrac{3\times 10^{8}\ ms^{-1}}{500\times 10^{-9}\ m}$

➡ $f_o = 6.00\times 10^{14}\ Hz$

$f_o = 6.00\times 10^{14}\ Hz$ > $f_s = 5.45\times 10^{14}\ Hz$

That means, $f_o > f_s$

Then, the frequency appears to increase! That means, the car is directly approaching to the observer! ∴ Using the Doppler's equation,

➡ $f_o = f_s\sqrt{\dfrac{1+\beta}{1-\beta}}$

Example :

$$f_o = f_s \sqrt{\frac{1+\beta}{1-\beta}}$$

$$6.00\times10^{14} = 5.45\times10^{14}\sqrt{\frac{1+\beta}{1-\beta}}$$

$$1.1 = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$1.21 = \frac{1+\beta}{1-\beta}$$

$$\beta = \frac{0.21}{2.21}$$

$$\beta = \frac{0.21}{2.21}$$

$$\frac{v}{c} = \frac{0.21}{2.21}$$

$$\frac{v}{c} = 0.095$$

$$v = 0.095\,c$$

$$v = 2.8\times10^{7}\ ms^{-1}$$

This is a practically impossible velocity !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Hansi Creation
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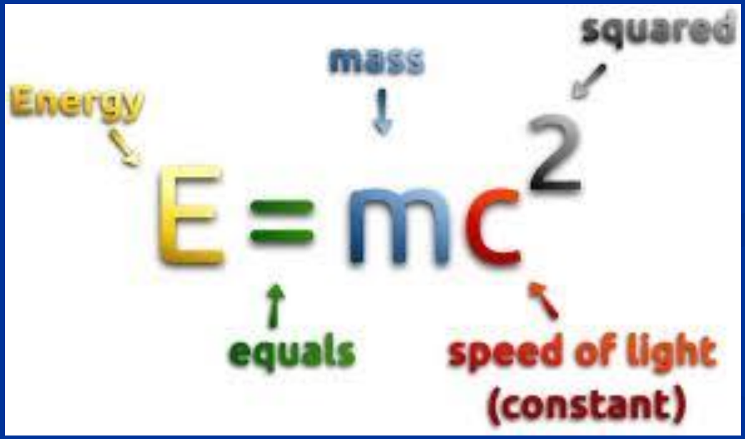
Artist : Sunil Edirisinghe
Hansi Creation

The mass – energy equivalence

In Physics, mass – energy equivalence is the concept that the mass of a body is a measure of its energy content.

Albert Einstein proposed mass – energy equivalence in 1905. The equivalence is described by the famous equation,

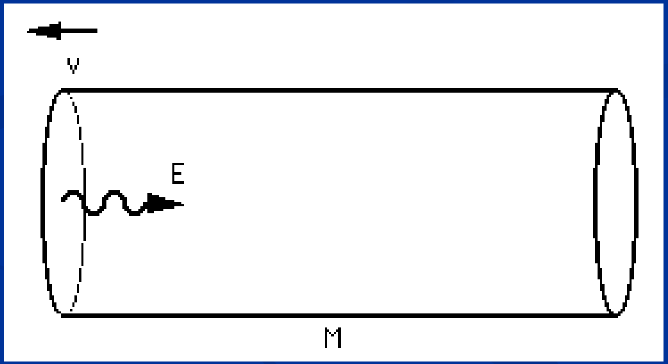


The equation **$E = m c^2$** indicates that energy always exhibits mass in whatever from the energy takes. It does not imply that mass may be "**converted**" to energy, for modern theory holds that neither mass or energy may be destroyed, but only moved from one location to another.

Proof of $E = mc^2$ [Einstein's Box]

This is a Hypothetical Experiment.

Before Einstein, it was known that a beam of light pushes against matter; that is known as Radiation Pressure. This means the light has momentum, E/c. Einstein used this fact to show that radiation (light) energy has an equivalent mass.



Consider a cylinder of mass M. A pulse of light with energy E is emitted from the left side. The cylinder recoils to the left with velocity V. If the mass of cylinder is large, it doesn't move far before the light reaches the other side. So, the light must travel a distance L, requiring time $t = L/c$. In this time, the cylinder travels a distance x.

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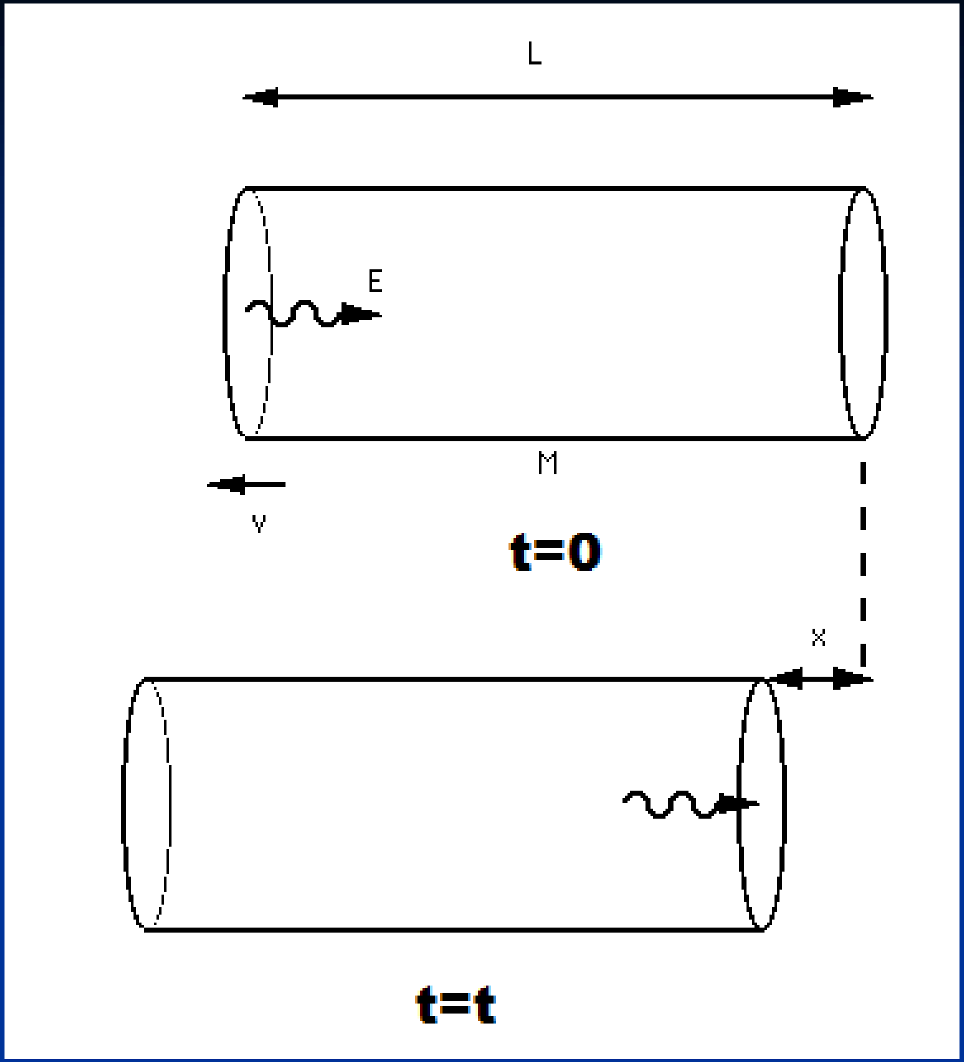


Momentum of the photon give the momentum to the cylinder.

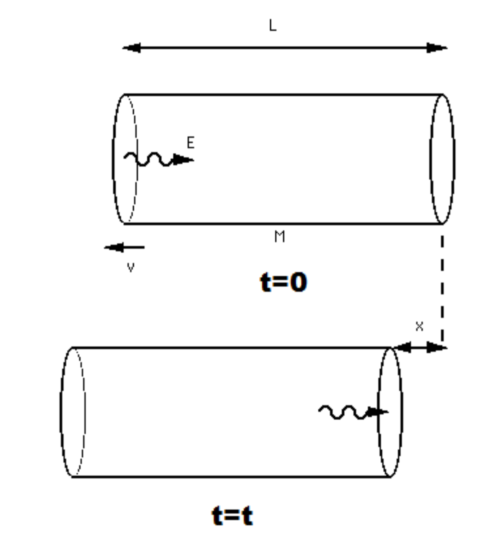
Momentum of the photon = Momentum of the Box

$$\frac{E}{c} = M V$$ 01

Time for light beam to cross the cylinder = $$t = \frac{L}{c}$$ 02

Distance traveled by the cylinder = $$x = V t$$ 03

P.T.O



Lets assume the photon (light pulse) has a mass *m*.

Using the law of conservation of momentum;

$$mc + M(-V) = 0$$

Where, $V = \frac{x}{t}$ *and* $c = \frac{L}{t}$

$$Mx = mL$$

Using the equation 01 & 03, $M = \frac{E}{Vc}$ *and* $x = Vt$

$$E = mc^2$$

Einstein was not the first to propose a mass-energy relationship. However, Einstein was the first scientist to propose the **$E = mc^2$** formula and the first to interpret mass-energy equivalence as a fundamental principle that follows from the relativistic symmetries of Space & Time!

The mass – energy equivalence

4-meter-tall sculpture of Einstein's 1905 $E = mc^2$ formula at the 2006 Walk of Ideas, Berlin, Germany.

Find the mass-equivalence energy of a 1kg.

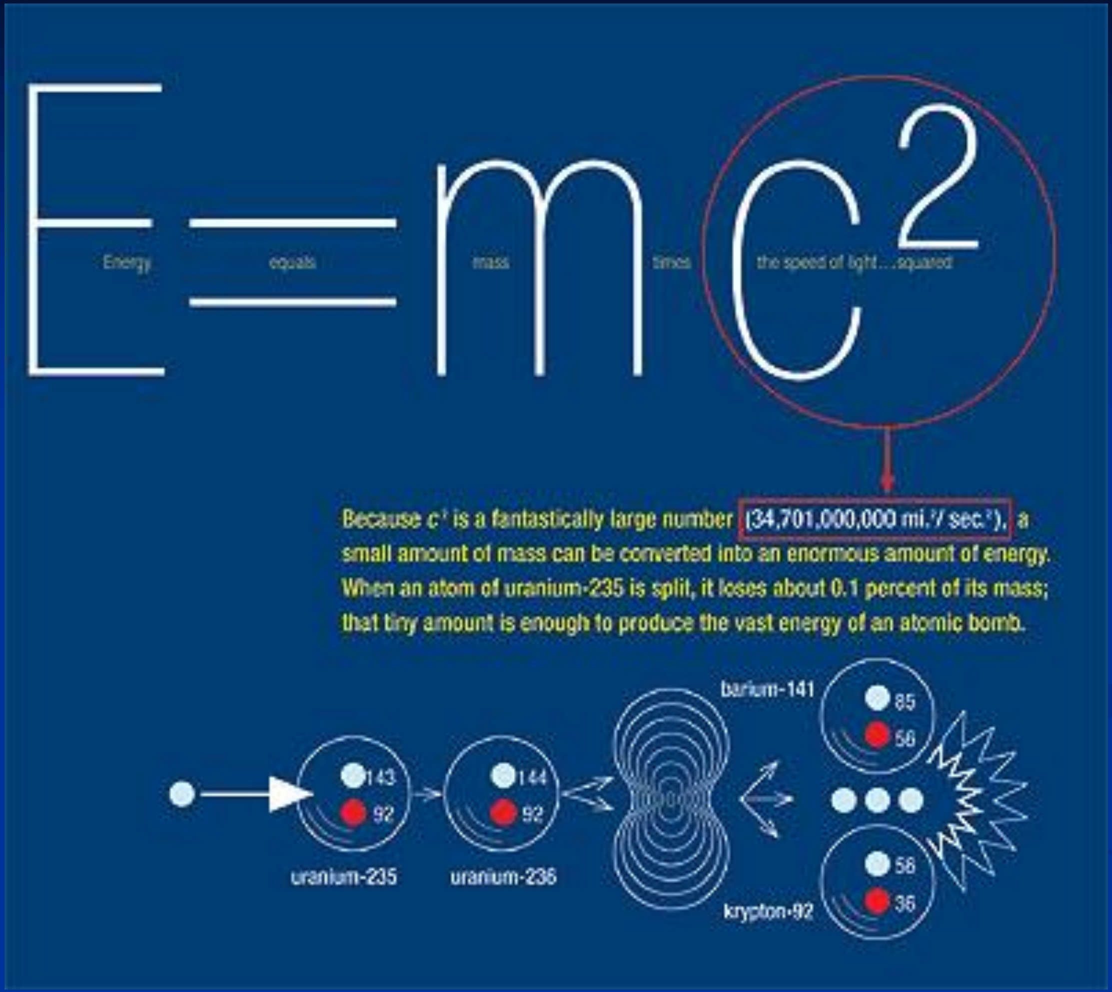
Using, $E = mc^2$

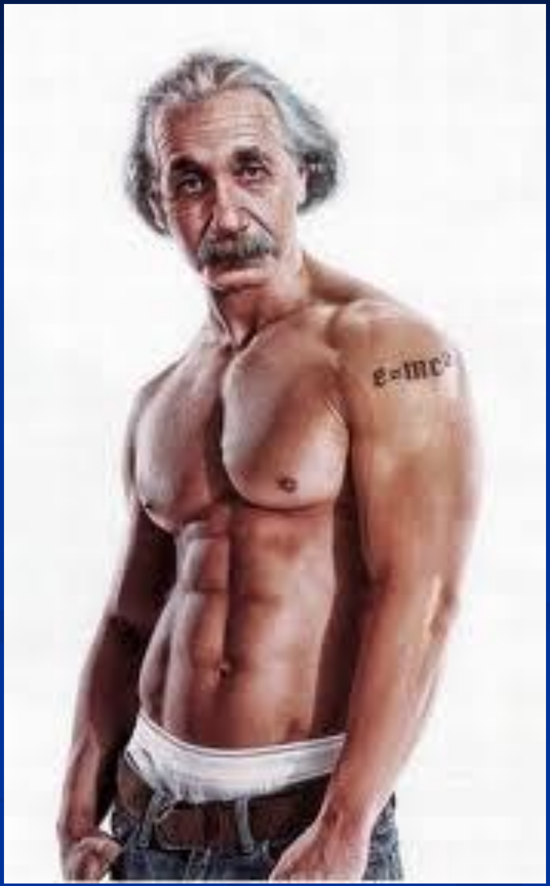
$$E = (1)(3\times10^8)^2$$

$$E = 9\times10^{16}\ J$$

This is a very large energy. Using this energy, we can vaporize **~10^10 kg** of water at the room temperature (**30°C**)! $\because E = ms\theta + mL$

Equivalence of mass and energy





Einstein put forward new ideas regarding the relationship between space, time, mass and energy which have come to be known as the theory of relativity. It had long been accepted that matter could not be destroyed. This assumption was expressed in the *law of conservation of matter*, which states that the total quantity of matter in the universe is fixed and cannot be increased or decreased by human agency.

Thank You !