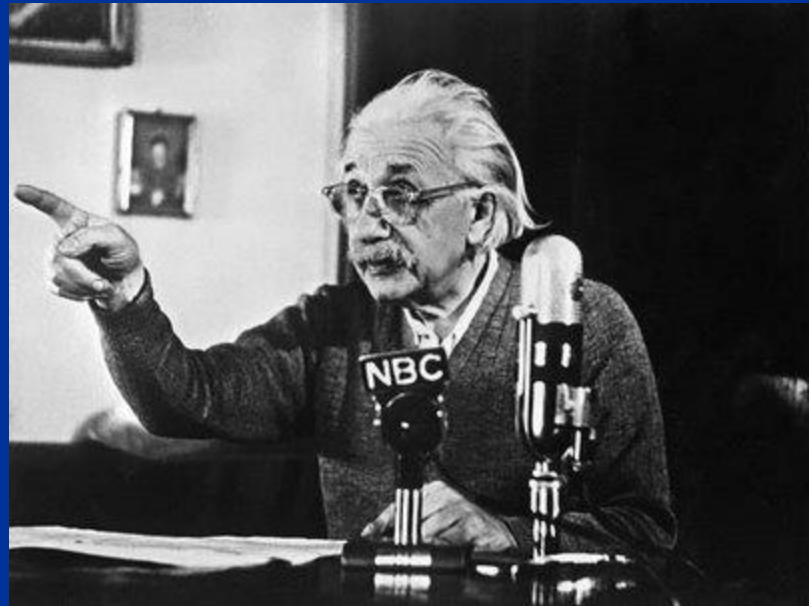


Special Theory of **Relativity**



4th Lecture

Special Theory of Relativity

Einstein's Two Postulates in STR

Postulate 01 : The Principle of Relativity:

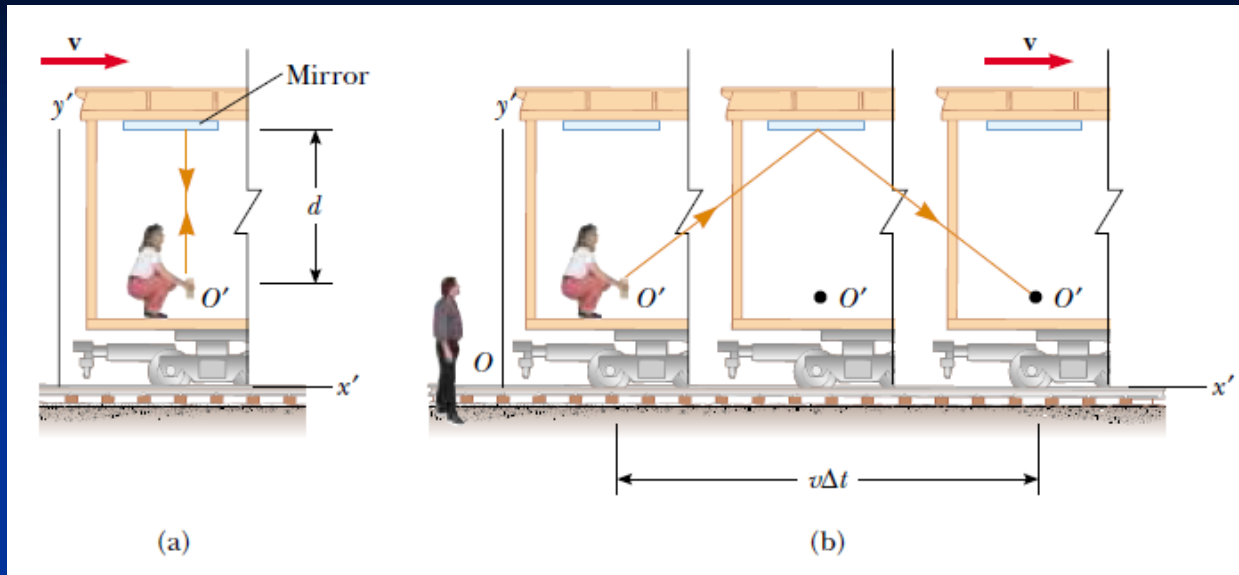
The laws of physics must be the same in all inertial reference frames.

The laws of Physics are the same for all observers in uniform motion relative to one another.

Postulate 02 : The constancy of the speed of light :

The speed of light in vacuum has the same value, $c = 3 \times 10^8$ m/s in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Measurement of Time in STR



$$\Delta t_O = \Delta t_P \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where, v is the Relative Speed of the Two Frames

$$\Delta t_O > \Delta t_P$$

Time interval
w. r. t the
stationary
frame

Time interval
w. r. t the
moving frame

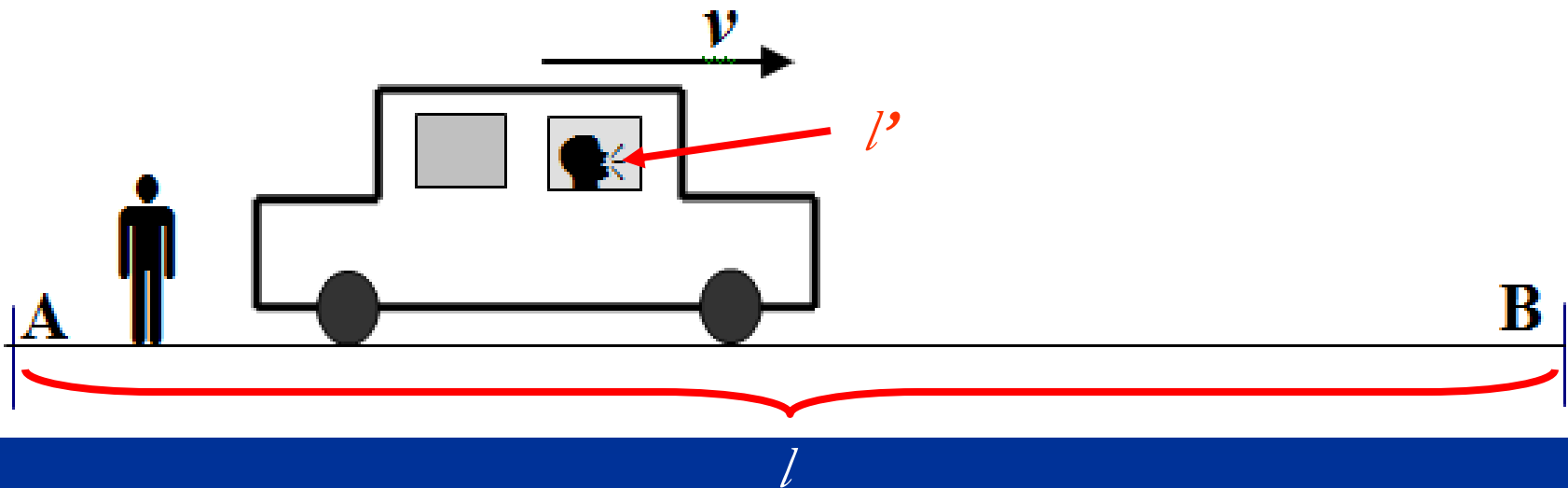
This is called Time Dilation !

Measurement of Length in STR

So, time is relative ! What about distance ???

In order to think about this note that when we say that the distance between two objects is l we imagine measuring the position of these objects simultaneously... but simultaneously is relative, so **we can expect distance to be a relative concept too!**

Length contraction is the observation that a moving object appears shorter than a stationary object! Like time dilation, length contraction is a consequence of the postulates of relativity. Length contraction and time dilation are related, and introduce a relation between length and time (Or space and time, if you prefer!)

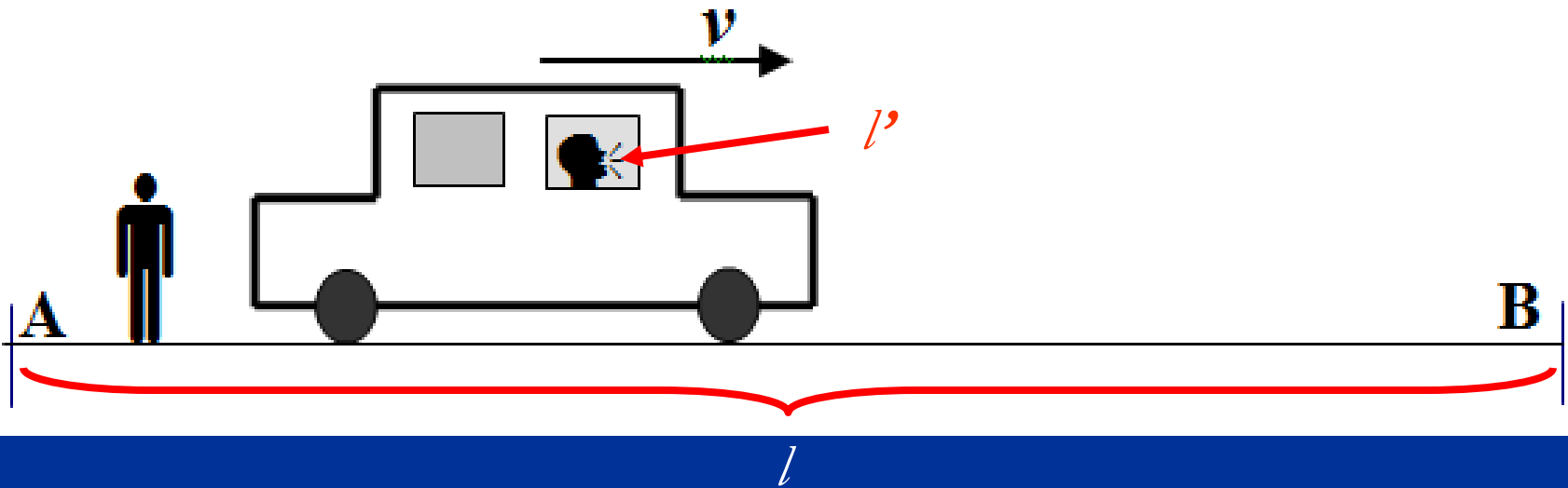


Suppose a car is moving with a uniform speed v relative to an observer in a stationary frame. This observer notes that it covers a distance l between two check posts (A & B) in time t . As two different clocks (one at each post) are involved in the measurement of this time, this time is improper time.

$$t_{im} = \frac{l}{v}$$

The Observer in the car covers the distance between the check-posts in proper time, such that:

$$t_{im} = t_{pro} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Thus, the distance l' that the observer in the car measures is given by,

$$t_{pro} = \frac{l'}{v}$$

(l' is measured by using **speedometer in the car** !)

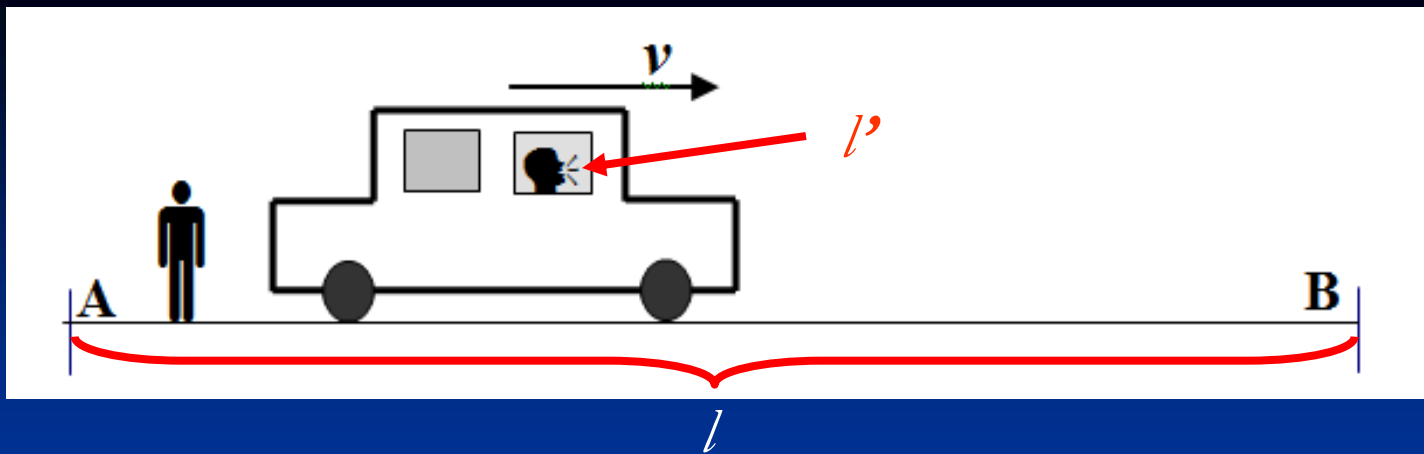
$$t_{im} = t_{pro} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$$\frac{l}{v} = \frac{l'}{v} \times \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$$l' = l \sqrt{1 - \frac{v^2}{c^2}}$$



$$l^1 = l \sqrt{1 - \frac{v^2}{c^2}}$$

This equation is called
relativistic length equation!

If $v > 0$



$$\frac{v}{c} < 1$$



$$\frac{v^2}{c^2} < 1$$



$$1 - \frac{v^2}{c^2} < 1$$



$$\sqrt{1 - \frac{v^2}{c^2}} < 1$$



$$l^1 = l (< 1)$$



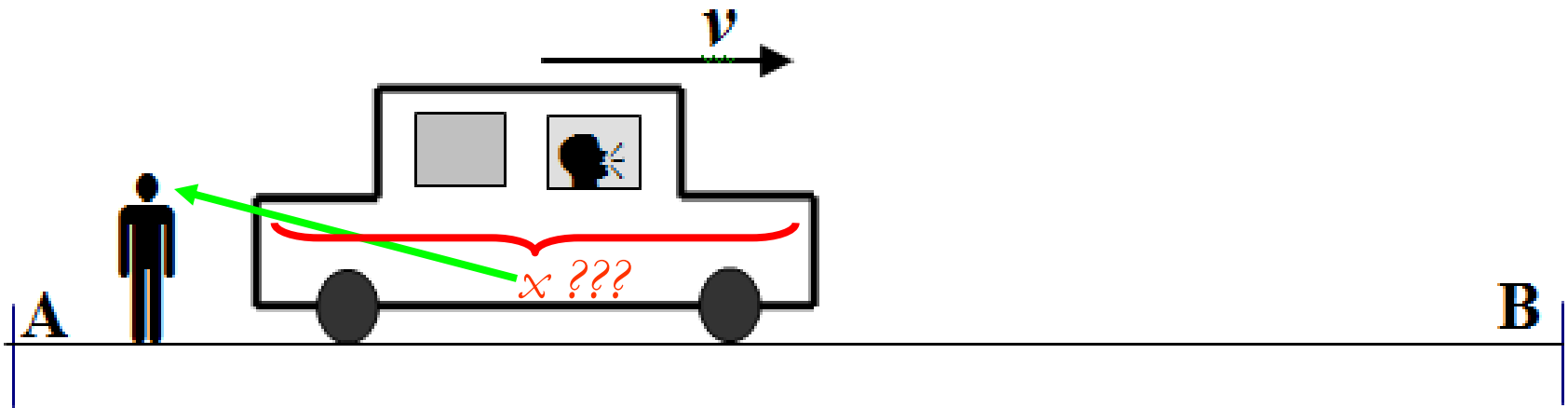
$$l^1 < l$$

**This is called
Length Contraction !**

Length measured
by an observer in
the car

Length measured
by observer on
the Earth

What is the length of the car as seen by an observer on the Earth ???



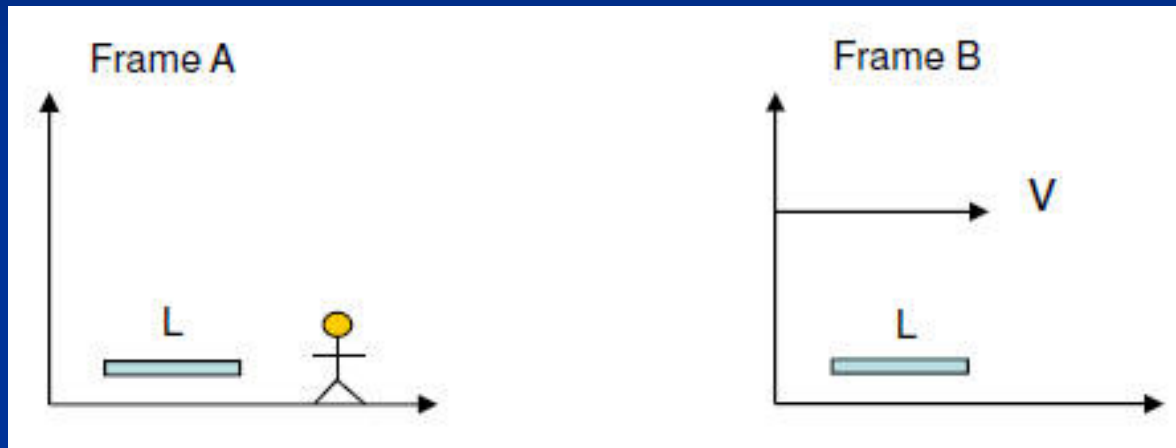
Is it **less than** the original length ???

Is it **greater than** the original length ???

Is it **equal** to the original length ???

Length Contraction

Length contraction is the observation that a moving object appears shorter than a stationary object.



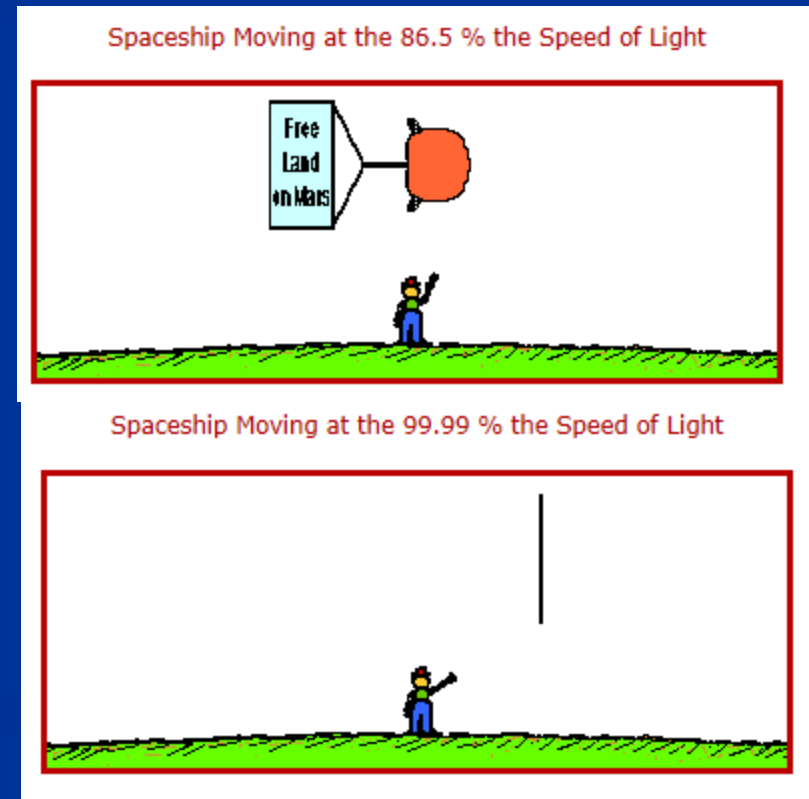
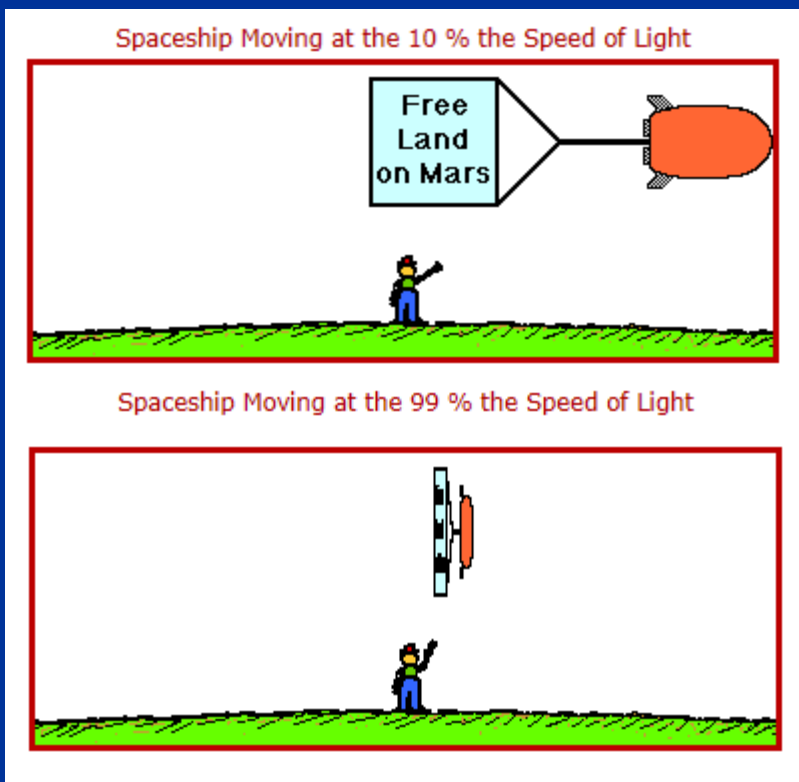
Length contraction – an observer in frame A sees the stick in frame B as shorter than the stick in frame A.

$$l^1 < l$$

Length measured by an observer
in the Frame A

Length measured by an observer in
the Frame B

One of the peculiar aspects of Einstein's theory of special relativity is that the length of objects moving at relativistic speeds undergoes a contraction along the dimension of motion. An observer at rest (relative to the moving object) would observe the moving object to be shorter in length. That is to say, that an object at rest might be measured to be 200 feet long; yet the same object when moving at relativistic speeds relative to the observer/measurer would have a measured length which is less than 200 ft. This phenomenon is not due to actual errors in measurement or faulty observations. The object is actually contracted in length as seen from the *stationary reference frame*. The amount of contraction of the object is dependent upon the object's speed relative to the observer.



Example:

A 200 m long train passes through a tunnel 100 m long with the constant speed $c/4$.

- (a) What is the length of the tunnel as seen by an observer in the train ???
- (b) What is the length of the train as seen by an observer on the Earth ???

(a) $Length\ (\text{Tunnel, E}) = 100\text{ m}$
 $Length\ (\text{Tunnel, Train}) = l\ ?$

Using relativistic length equation :

$$l^1 = l \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = 100 \sqrt{1 - \frac{\left(\frac{c}{4}\right)^2}{c^2}}$$



$$l = 100 \times 0.9682$$



$$l = 96.82\text{ m}$$

Example:

A 200 m long train passes through a tunnel 100 m long with the constant speed $c/4$.

- (a) What is the length of the tunnel as seen by an observer in the train ???
- (b) What is the length of the train as seen by an observer on the Earth ???

(b)

Length (Train, E) = 200 m at rest

Length (Train, E) = l ? is moving

Using relativistic length equation :

$$l^1 = l \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = 200 \sqrt{1 - \frac{\left(\frac{c}{4}\right)^2}{c^2}}$$



$$l = 200 \times 0.9682$$



$$l = 193.64\text{ m}$$

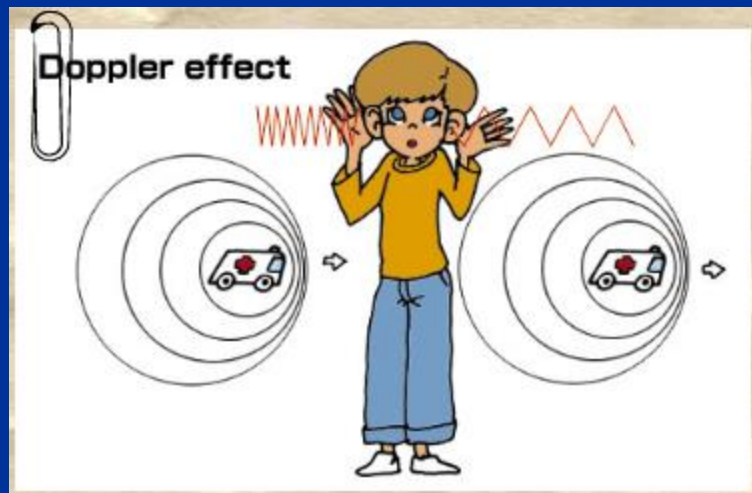
Doppler's Effect in STR



Doppler's Effect for Sound Waves :

The Doppler's effect (or Doppler's shift), named after Austrian physicist **Christian Doppler** who proposed it in 1842, is the change in frequency of a wave for an observer moving relative to the source of the wave.

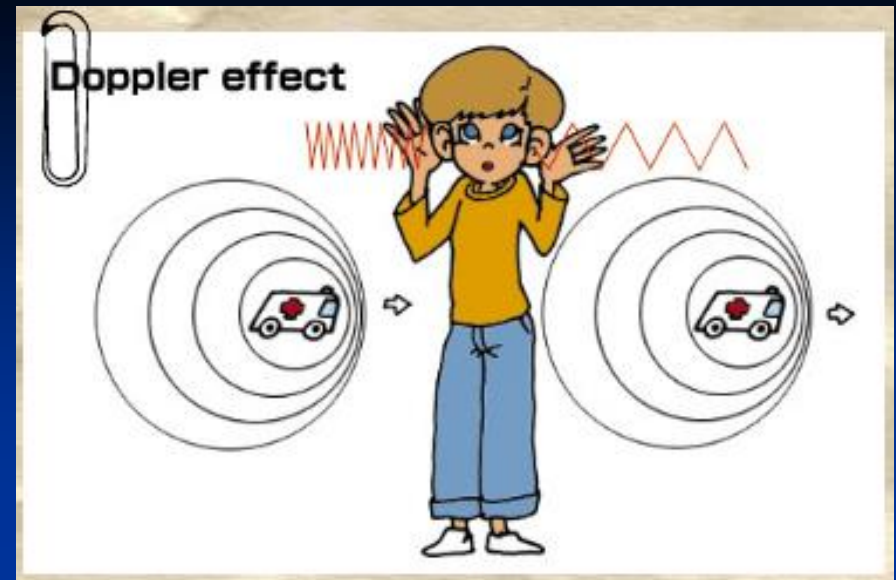
It is commonly heard when a vehicle sounding a siren or horn approaches, passes and recedes from the observer.



In classical physics (wave in a medium), where the source and the receiver velocities are not supersonic, the relationship between observed frequency f_o and emitted frequency (or source frequency) f_s is given by,

Doppler's Effect for Sound Waves :

$$f_o = f_s \left(\frac{v \pm v_1}{v \mp v_2} \right)$$



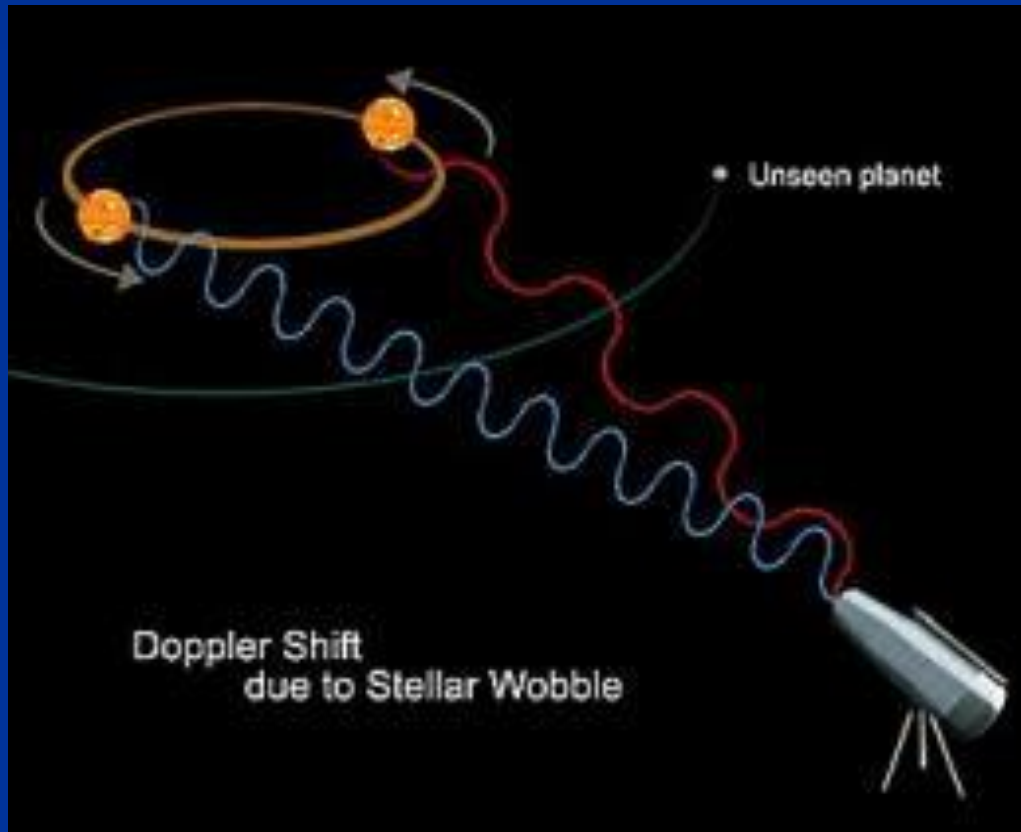
Where,

- v is the velocity of waves in the medium.
- v_1 is the velocity of the receiver relative to the medium; **positive if the receiver is moving towards the source.**
- v_2 is the velocity of the source relative to the medium; **positive if the source is moving away from the receiver.**

The frequency is decreased if either is moving away from the other!

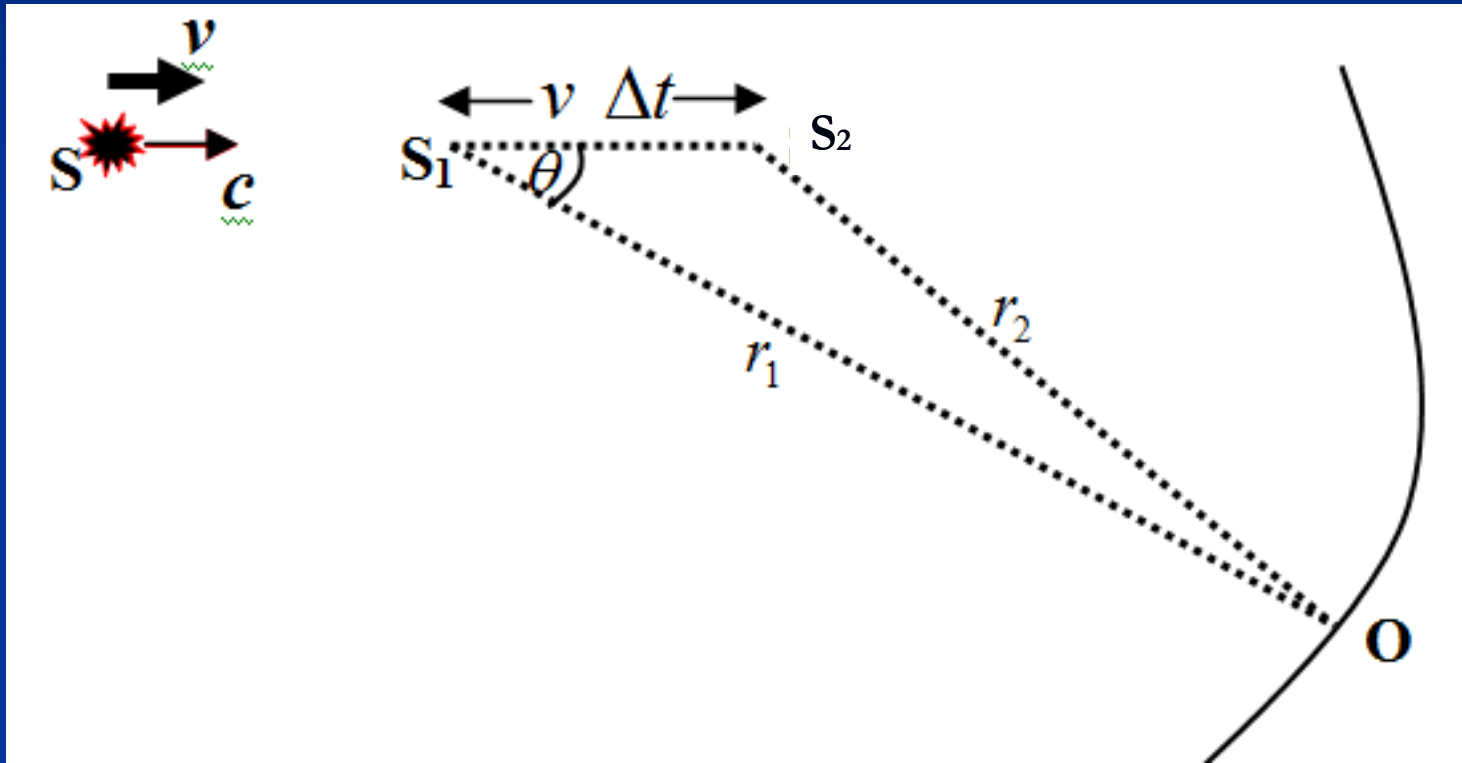
Doppler's Effect in STR

The relativistic Doppler's effect is the change in frequency (and the wavelength) of light, caused by the relative motion of the source and the observer (as in the classical Doppler's effect), when taking into account effects of the Special Theory of Relativity!

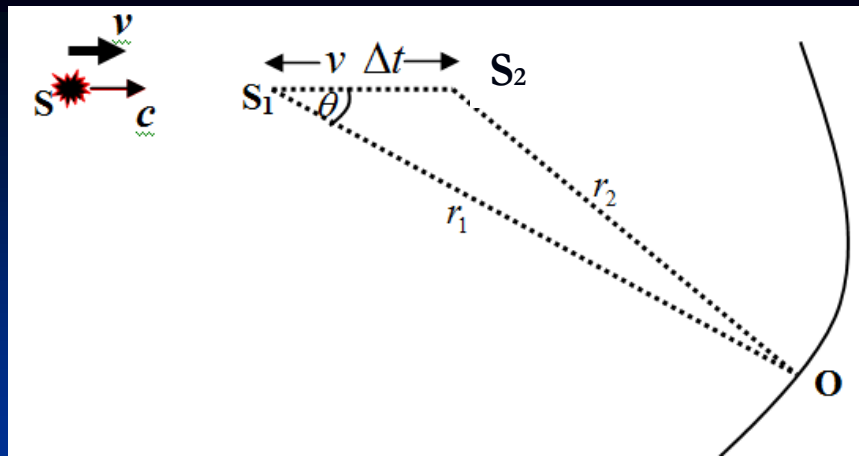


If a light source in uniform motion approaches or recedes a stationary observer then the frequency of light is observed to change. This is known as **Doppler effect** in STR.

We assume a frame in which the source is moving and observer is stationary. Consider a source S of frequency f_s moving with velocity v . The source emits periodic waves. Suppose that when the source is at S_1 it emits a signal. This reaches the observer O after a time r_1/c .



When the signal reaches O the source has moved S_2 . Suppose the time taken for the source to move from S_1 to S_2 is Δt . Then $S_1S_2 = v \Delta t$.



$$S_1 S_2 = v \Delta t$$

This is the distance covered by the source before giving out the next signal. When the source is at S_2 it gives out a signal which is received by the observer at O .

Thus, the time for the second signal to be received at O is,

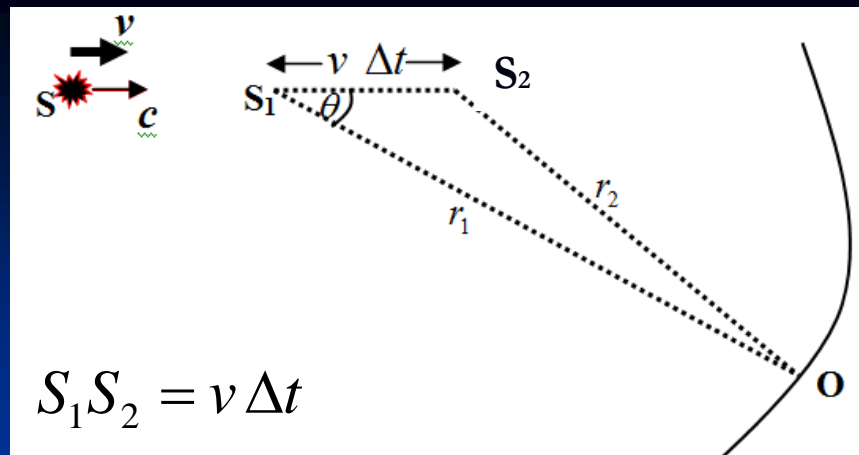
$$\Delta t + \frac{r_2}{c}$$

The difference in time for the two signals to reach O is then,

$$\Delta t_o = \Delta t + \frac{r_2}{c} - \frac{r_1}{c}$$



$$\Delta t_o = \Delta t + \frac{(r_2 - r_1)}{c}$$



From the figure we note that, (Using Cosine Theorem)

$$r_2^2 = r_1^2 + (v\Delta t)^2 - 2r_1(v\Delta t)\cos\theta$$

As $r_1 \gg v\Delta t$, we get, $v\Delta t \approx (v\Delta t)\cos\theta$

$$\Rightarrow r_2^2 = r_1^2 - 2r_1(v\Delta t)\cos\theta + ((v\Delta t)\cos\theta)^2$$

$$\Rightarrow r_2^2 = (r_1 - (v\Delta t)\cos\theta)^2$$

$$\Rightarrow r_2 = r_1 - (v\Delta t)\cos\theta$$

$$\Rightarrow r_2 - r_1 = -(v\Delta t)\cos\theta$$

Connect equation 01 and 02 :

$$\Delta t_o = \Delta t + \frac{(r_2 - r_1)}{c} \quad \rightarrow \quad \Delta t_o = \Delta t + \frac{-(v\Delta t)\cos\theta}{c}$$
$$\quad \rightarrow \quad \Delta t_o = \Delta t \left(1 - \frac{v\cos\theta}{c} \right) \quad \longrightarrow \quad 03$$

This is the time difference between two successive signals received at O, i.e.: $1 / \Delta t_o$ is the observed frequency of emission.

$$f_o = \frac{1}{\Delta t_o}$$

We now need to connect t_s with the actual frequency of emission by the source, f_s . We realize that $1/f_s$ is the time difference between two successive signals emitted by the source and is therefore a proper time in the frame of the source. On the other hand, Δt is the time interval between the emission of two consecutive signals as seen from the stationary frame.

Using the Relativistic Time equation :

$$\Delta t = \Delta t_s \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$$\Delta t_s = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$



$$\frac{1}{f_s} = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$

Using equation 03 ;

$$\Delta t_o = \Delta t \left(1 - \frac{v \cos \theta}{c} \right)$$

We get,

$$\Delta t = \Delta t_o \frac{1}{\left(1 - \frac{v}{c} \cos \theta \right)}$$

Then we have,

$$\frac{1}{f_s} = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$



$$\frac{1}{f_s} = \Delta t_o \frac{1}{\left(1 - \frac{v}{c} \cos \theta \right)} \sqrt{1 - \frac{v^2}{c^2}}$$



$$\frac{1}{f_s} = \Delta t_o \frac{1}{\left(1 - \frac{v}{c} \cos \theta\right)} \sqrt{1 - \frac{v^2}{c^2}}$$



$$\frac{1}{f_s} = \Delta t_o \frac{1}{\gamma(1 - \beta \cos \theta)}$$

where,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and,

$$\beta = \frac{v}{c}$$

As the frequency f_o observed by the observer is $1/\Delta t_o$, we obtain,

$$f_o = \frac{1}{\Delta t_o}$$

$$\frac{1}{f_s} = \Delta t_o \frac{1}{\gamma(1 - \beta \cos \theta)}$$



$$\frac{1}{f_s} = \frac{1}{f_o} \frac{1}{\gamma(1 - \beta \cos \theta)}$$



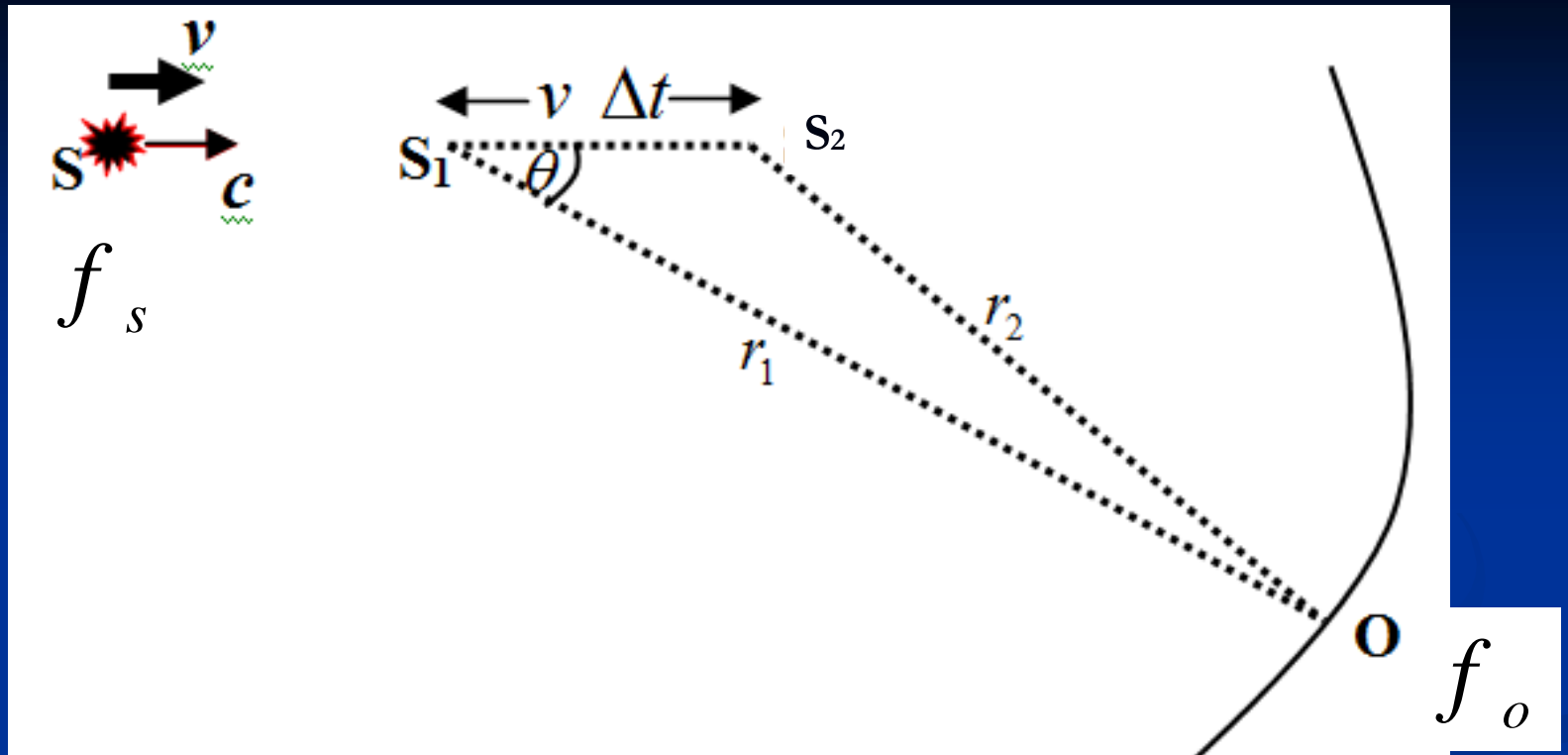
$$f_o = f_s \frac{1}{\gamma(1 - \beta \cos \theta)}$$

where,

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\beta = \frac{v}{c}$$

This is the general form of the Doppler's Effect in STR!



$$f_o = f_s \frac{1}{\gamma(1 - \beta \cos \theta)}$$
 where, $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$
 and $\beta = \frac{v}{c}$

This is the general form of the Doppler's Effect in STR!

Special Cases:

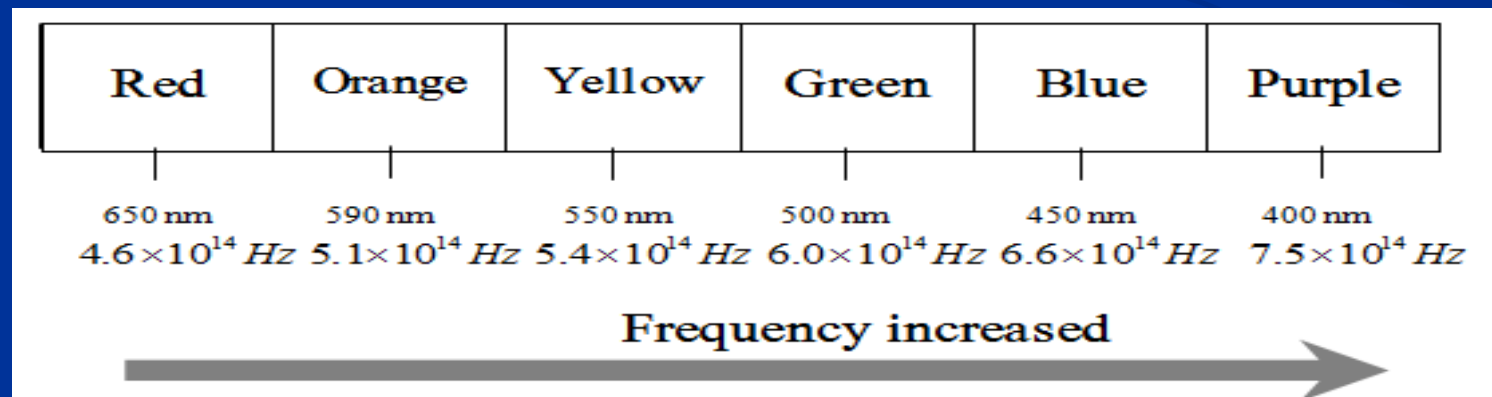
(a) If the source is directly approaching the observer:



Then, $\theta = 0$ and $\cos \theta = 1$ $\Rightarrow f_o = f_s \frac{1}{\gamma(1-\beta)}$ where, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$$\Rightarrow f_o = f_s \sqrt{\frac{1+\beta}{1-\beta}} \Rightarrow f_o > f_s \quad \beta = \frac{v}{c}$$

\Rightarrow Thus, the frequency appears to increase !

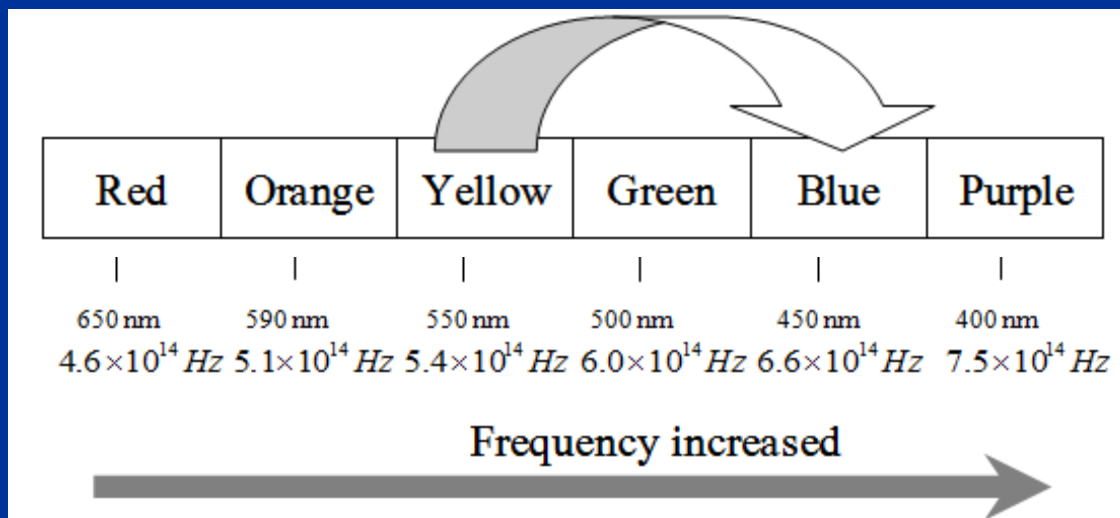


(a) If the source is directly approaching the observer:



$$f_o > f_s$$

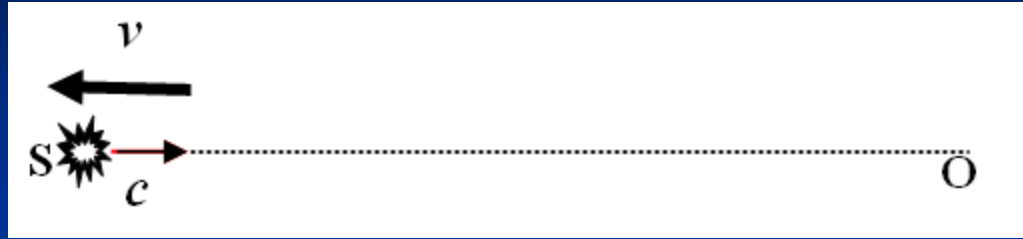
➔ Thus, the frequency appears to increase !



This is called frequency shift !

Special Cases:

(b) If the source is receding directly from the observer:

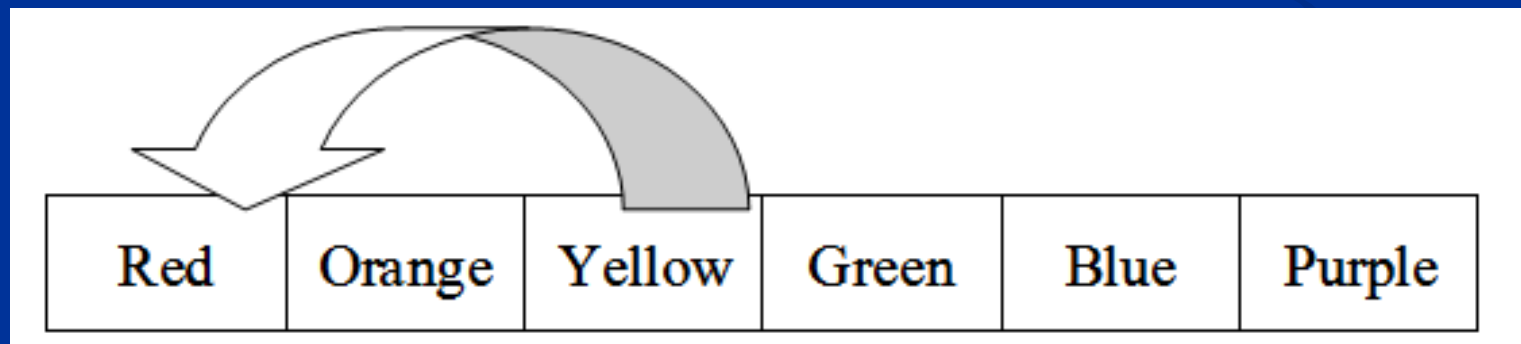


Then, $\theta = \pi$ and $\cos \theta = -1$ \Rightarrow $f_o = f_s \frac{1}{\gamma(1 + \beta)}$ where, $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$

$$\Rightarrow f_o = f_s \sqrt{\frac{1 - \beta}{1 + \beta}}$$

$$\beta = \frac{v}{c}$$

\Rightarrow Thus, the frequency appears to decrease !



Example :

A yellow coloured vehicle appears as a green coloured vehicle to a stationary observer due to its speed. Find the velocity of the vehicle. (Wavelengths of yellow and green light are 550nm and 500nm respectively)

Is the above incident practically possible??? Briefly explain your answer.



Thank You !