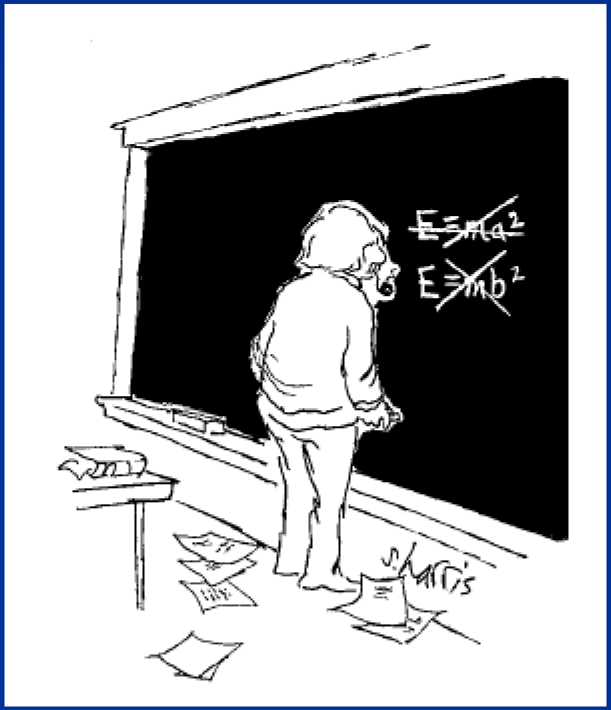
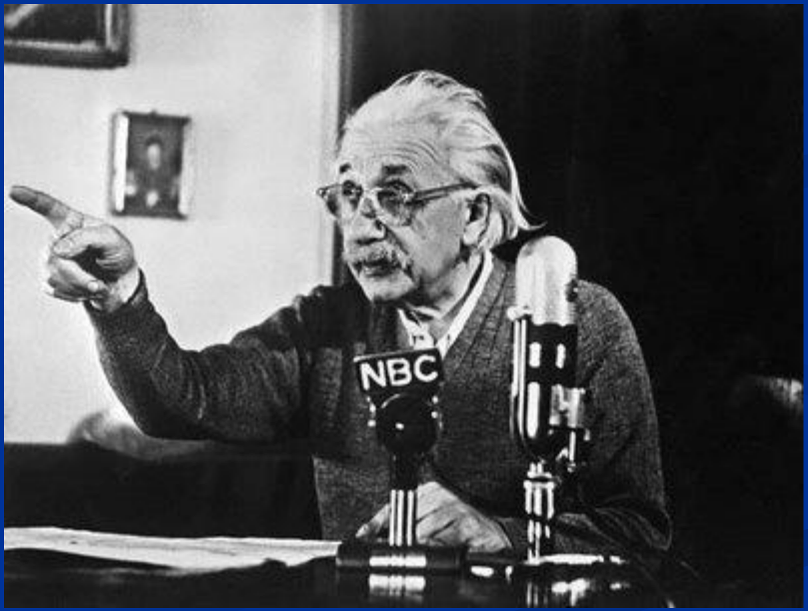
Special Theory of Relativity





3rd Lecture

Special Theory of Relativity

Einstein's Two Postulates in STR

Postulate 01 : The Principle of Relativity:

The laws of physics must be the same in all inertial reference frames.

The laws of Physics are the same for all observers in uniform motion relative to one another.

Postulate 02 : The constancy of the speed of light :

The speed of light in vacuum has the same value, $c = 3 \times 10^8$ m/s in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Measurement of Time in STR

What is Time ???

Time is the indefinite continued progress of existence and events that occur in apparently **irreversible** succession from the **past** through the **present** to the **future**!

Time is a component quantity of many measurements used to sequence events, to compare the durations of events and the intervals between them, and to quantify rates of change of quantities in material reality or in the conscious experience.

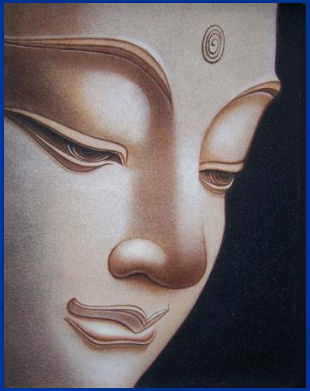
The fundamental unit of time in the International System of Units (SI) is the second (symbolized s or sec).

One second elapses during the occurrence of exactly 9,192,631,770 (9.192631770×10^9) cycles of the radiation produced by the transition between two levels of the cesium 133 atom.

Measurement of Time in STR

What is Time ???

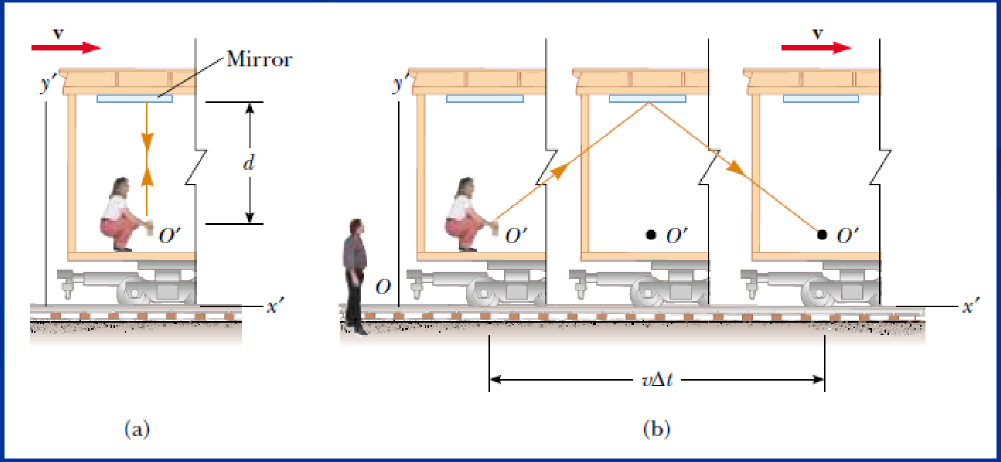
The Gap between two events is called Time (difference) ! – Lord Buddha!



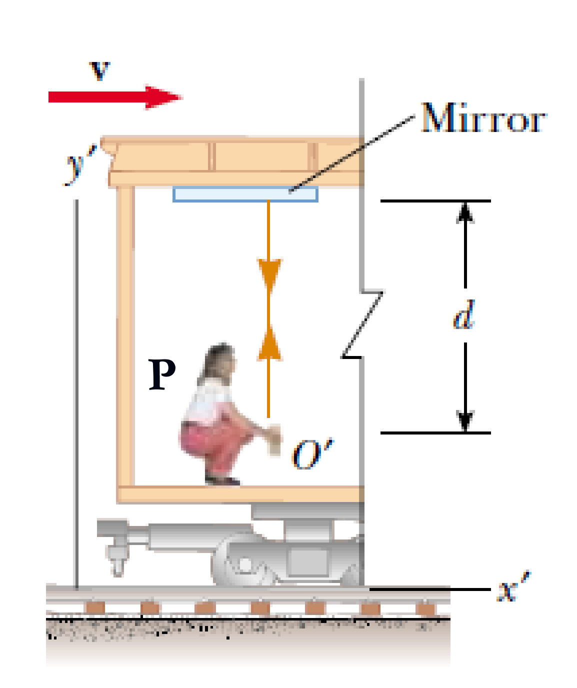
There is no belief in a personal god. Buddhists believe that nothing is fixed or permanent and that change is always possible. The path to Enlightenment is through the practice and development of morality, meditation and wisdom. Buddhists believe that life is both endless and subject to impermanence, suffering and uncertainty. These states are called the *tilakhana*, or the three signs of existence. Existence is endless because individuals are reincarnated over and over again, experiencing suffering throughout many lives.

Measurement of Time in STR

Now consider the same pair of events as viewed by observer *O* in a second frame, as shown in Figure b. According to this observer, the mirror and laser are moving to the right with a speed v, and as a result the sequence of events appears entirely different. By the time the light from the laser reaches the mirror, the mirror has moved to the right a distance $v(\Delta t_o/2)$, where Δt_o is the time it takes the light to travel from *O'* to the mirror and back to *O'* as measured by *O*. In other words, *O* concludes that, because of the motion of the vehicle, if the light is to hit the mirror, it must leave the laser at an angle with respect to the vertical direction. Comparing Figure a and Figure b, we see that the light must travel farther in (b) than in (a).



For the Observer, P on the vehicle :

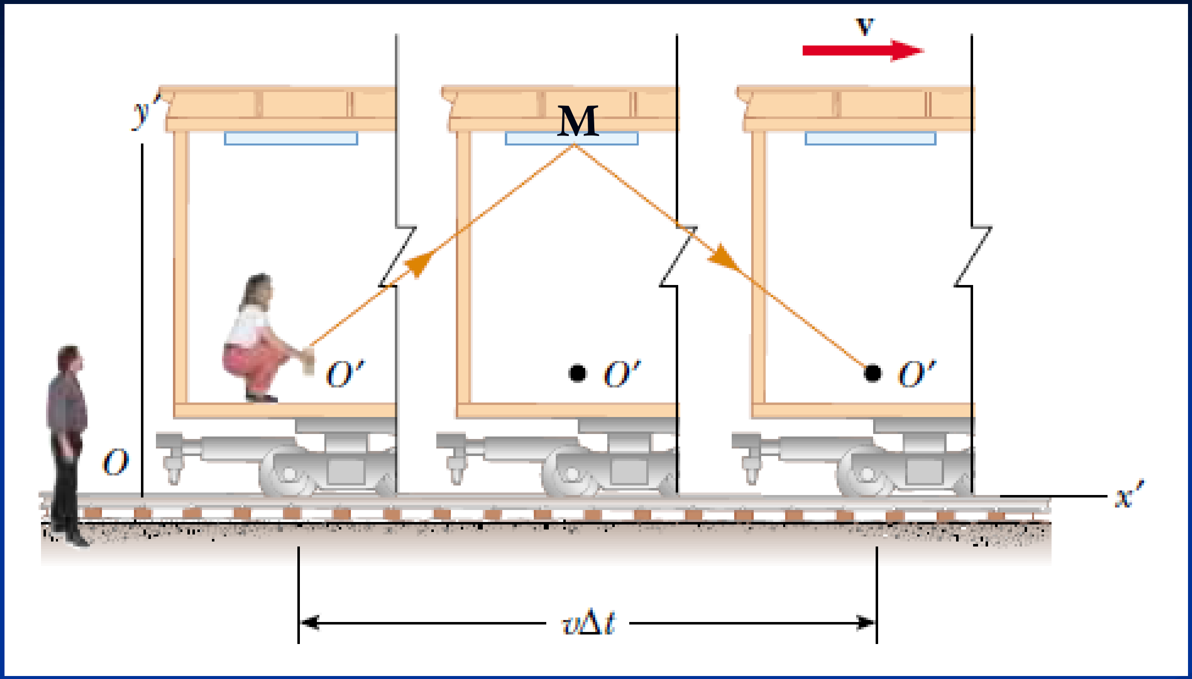


In the frame where the clock is moving, the light pulse traces out a path of length $2d$ and the period of the clock is $2d$ divided by the speed of light, c .

$$\Delta t_P = \frac{2d}{c}$$

where Δt_P is the time it takes the light to travel from O' to the mirror and back to O' as measured by P.

For the Observer, O on the ground :



Total time for the light pulse to trace its path with respect to O.

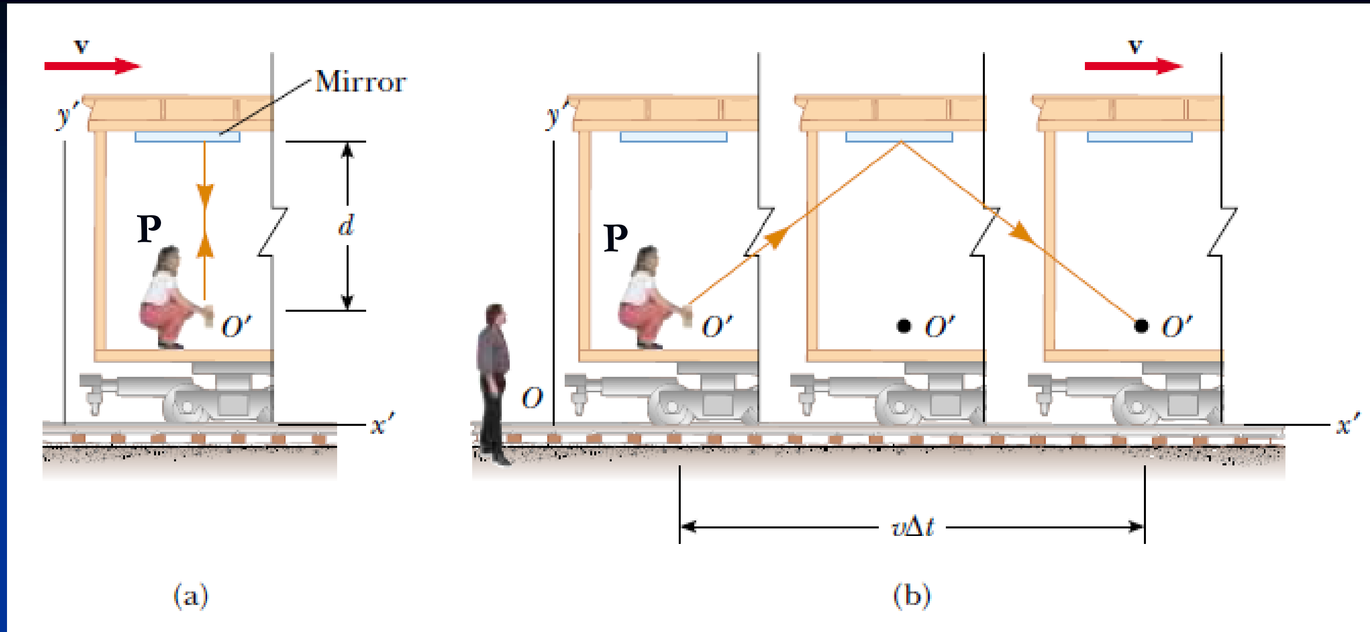
$$\Delta t_O = \frac{O^1MO^1}{velocity}$$

(Using postulate 01)

$$\Delta t_O = \frac{2\left(d^2 + \left[v\dfrac{\Delta t_O}{2}\right]^2\right)^{1/2}}{c}$$

(Using postulate 02)

$$\Delta t_O = \frac{2d}{c}\frac{1}{\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}$$



$$\Delta t_O = \Delta t_P \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

This equation express the fact that for the moving observer the period of the clock is shorter than in the frame of the ground observer itself !

Where, v is the Relative Speed of the Two Frames

$$\Delta t_O > \Delta t_P$$

Time interval w. r. t the stationary frame

Time interval w. r. t the moving frame

Time Dilation

Suppose,

$$t_2 = t_1 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

This equation is called relativistic time equation!

If v > 0 ➡ $\frac{v}{c} < 1$ ➡ $\frac{v^2}{c^2} < 1$ ➡ $1-\frac{v^2}{c^2} < 1$

➡ $\sqrt{1-\frac{v^2}{c^2}} < 1$ ➡ $t_2 = t_1 \frac{1}{(<1)}$

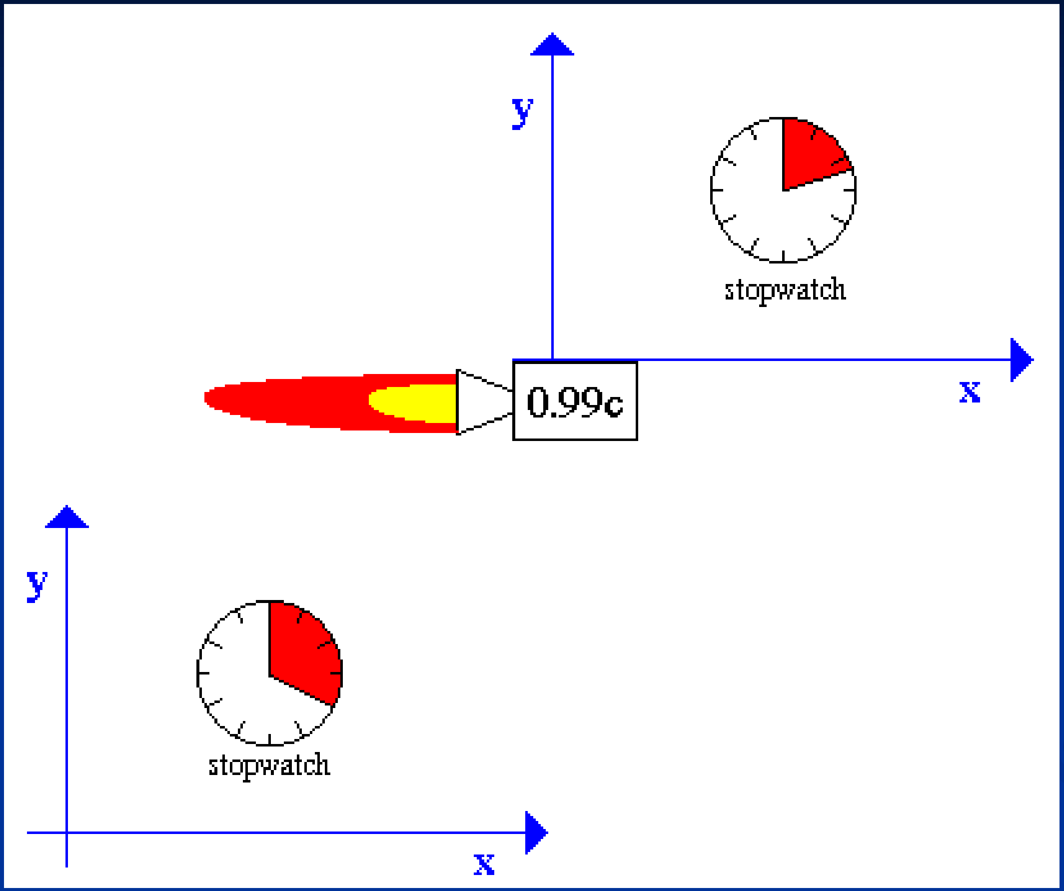
$$t_2 > t_1$$

t_2: Time interval w. r. t the stationary frame

t_1: Time interval w. r. t the moving frame

This is called Time Dilation !

Time Dilation



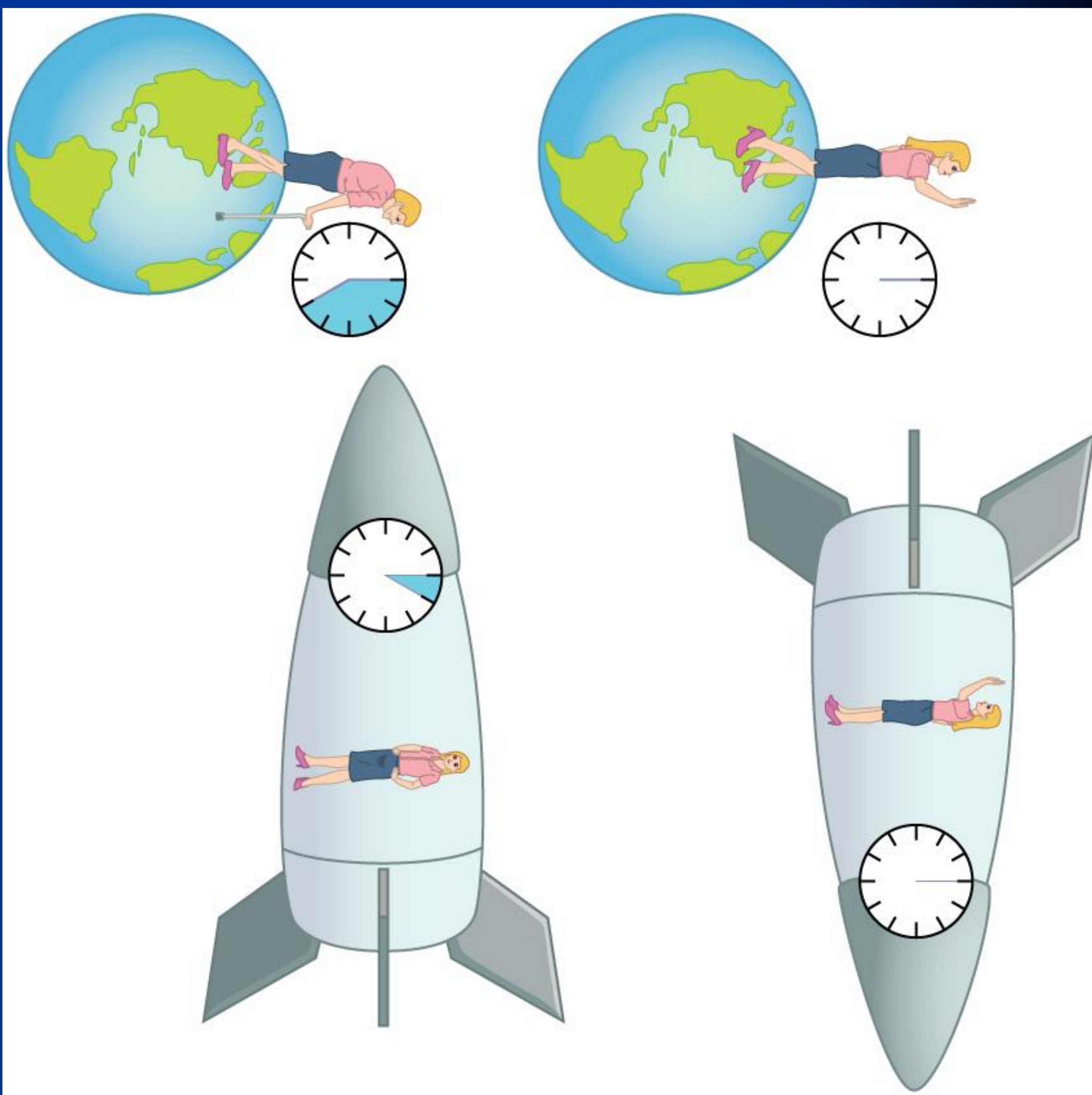
$$t_2 > t_1$$

Time interval w. r. t the stationary frame

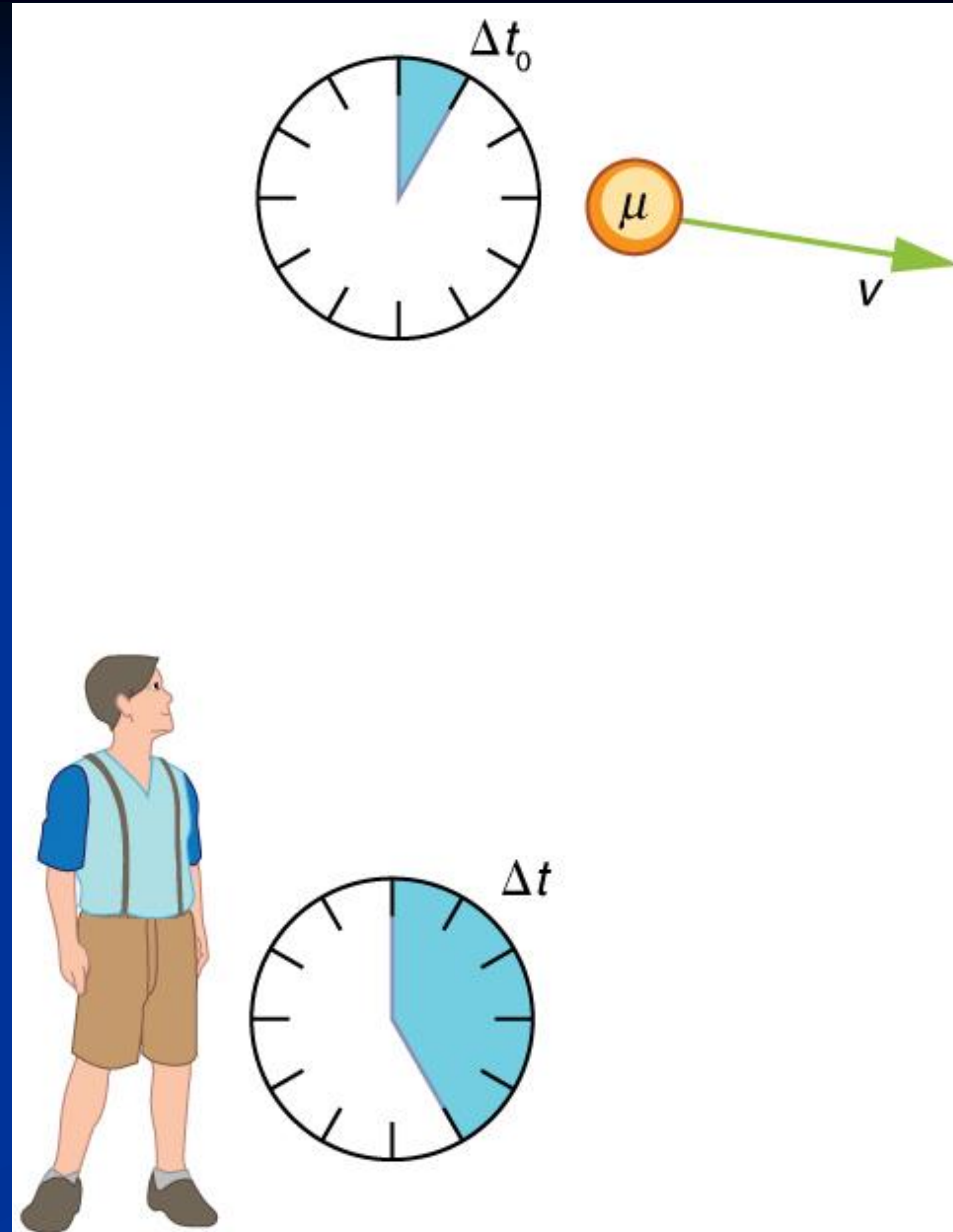
Time interval w. r. t the moving frame

Time dilation is an actual difference of elapsed time between two events as measured by observers either moving relative to each other.

Time Dilation



Time Dilation



Proper Time & Improper Time

Proper Time : In relativity, proper time is time measured by a single clock between events that occur at the same place as the clock. It depends not only on the events but also on the motion of the clock between the events!

Improper Time : Time measured with two clocks or a single moving clock!

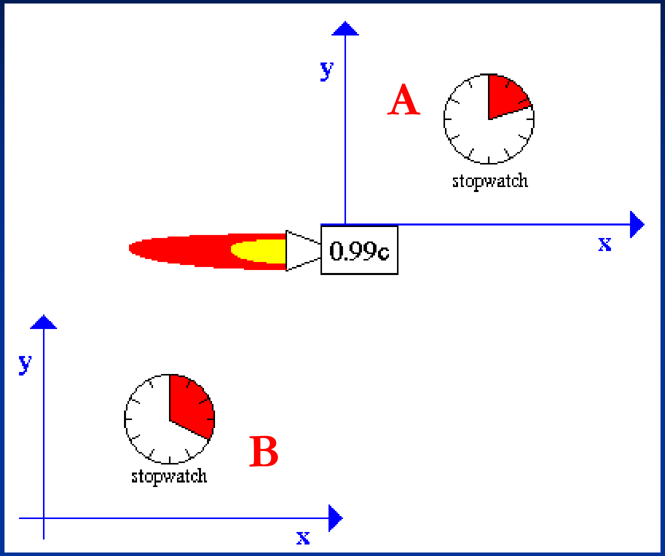
$$\text{Improper Time} = \text{Proper Time} \times \sqrt{1-\frac{v^2}{c^2}}$$

This means, that time is dilated (extended) in motion. This gives rise to two questions,

(01) Do moving clocks really run slow ???

(02) The clock mechanism affected by motion ???

Proper Time & Improper Time



The above results tells us that " *moving clocks run slow* ". When the clock is moving one is measuring improper time. A stationary clock measures proper time. It seems as through there is a Clock Paradox in question **01** started above. For instance, suppose an observer A is moving with a uniform velocity and another observer B is "stationary". Then according to A his watch runs slower than that of B, as a consequence of measurement of proper and improper time !

Question **02** started above, regarding whether the clock mechanism is affected during motion is simply non-sense. As our frames are inertial all physical laws, including mechanics remain unaltered.

Example : A particle X, which is created in a particle accelerator, travels a total distance of $100.0\ m$ between two detectors in $410\ ns$ as measured in the laboratory frame before decaying into other particles. What is the lifetime of the particle X as measured in its own frame ???

Velocity of the particle X w.r.t lab frame :

$$v = \frac{distance}{time}$$

$$v = \frac{100\,m}{410\,ns}$$

$$v = 2.44 \times 10^8\ ms^{-1}$$

Using relativistic time equation :

$$t_2 = t_1 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t_1 = ?$

$t_2 = 410 \times 10^{-9}\ s$

$v = 2.44 \times 10^8\ ms^{-1}$

$$t_1 = t_2 \sqrt{1 - \frac{v^2}{c^2}}$$

$$t_1 = 410 \sqrt{1 - \frac{(2.44 \times 10^8)^2}{(3.0 \times 10^8)^2}}$$

$$t_1 = 238\,ns$$

Experiment on Time Dilation :

The experimental evidence for time dilation was provided by Ives and Stilwel (1938) who measured the change in frequency of spectral lines emitted by fast moving atoms. The effect observed was small as the velocities of atoms was only about c/2; but it was convincing.

The real, conclusive evidence came from the experiment of Rossi & Hall (1941) involving **muons**, and we discuss this below :

The muon is a charged particle with mass about 200 times that of an electron. It decays to an electron plus a neutrino – antineutrino pair.

$$\mu \rightarrow e + \gamma_{\mu} + \overline{\gamma_{\mu}}$$

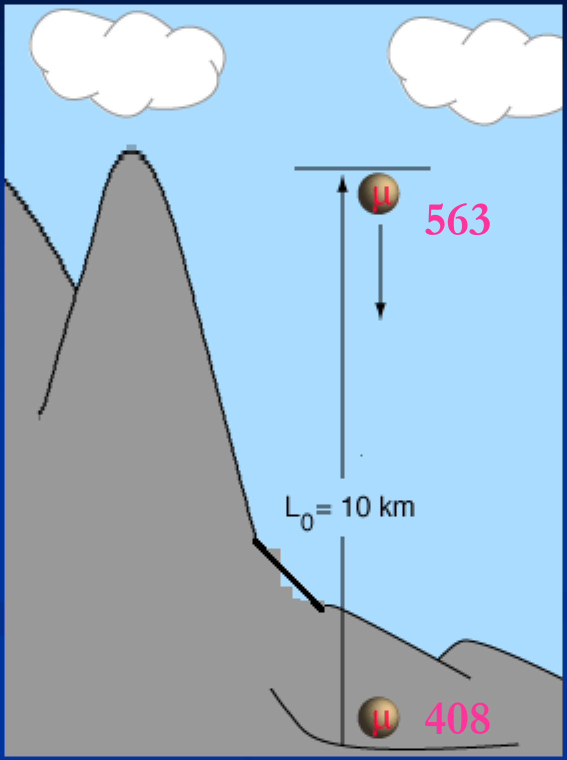
Antineutrino

Neutrino

and the half life for the muon decay is *1.53 x 10^(-6) s*. (This means that if there are N_o number of muons at time t = 0, then after *1.53 x 10^(-6) s*, there will be $N_o/2$ muons left. The rest $N_o/2$ would have decayed.)

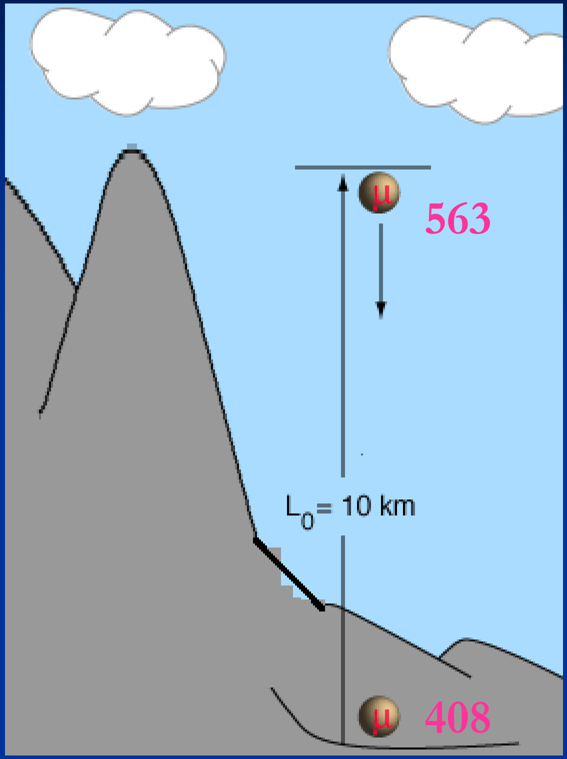
Experiment on Time Dilation…

Cosmic rays contain large number of muons at high altitudes and they mostly travel vertically downward at speeds comparable with that of light. The experiment of Rossi & Hall consisted in measuring the number of muons and their time of flight at the top of **Mountain Washington** (height 6265 ft) in New Hampshire, and at 10 ft above sea level. Only those muons with speed between 0.9950 c and 0.9954 c were counted. This number at the mountain top was found to be 563±10 per hour.



However, at sea level the number of muons was found to be 408±9 per hour. **It must be said that in the experiment Rossi & Hall did not detect the 408±9 muons remaining out of the 563±10 per hour**. The ground experiment was done at some outer place which was close to the mountain.

Experiment on Time Dilation…



Thus, the measured time of the muon flight, which is the improper time is,

$$t_{im} = \frac{(6265 - 10)\,ft}{0.9952 \times 3 \times 10^{8} \times 3.28\ (ft/m)}$$

$$t_{im} = 6.4 \times 10^{-6}\, s$$

This experiment verifies that,

$$t_{improper} \rangle\ t_{proper}$$

This experiment clearly shows that time dilation can be a very significant effect for clocks that are in high speed relative motion!

$$t_A = t_B \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

```
In[1]:= tb = 10; (* Time in Seconds *)
        c = 3*10^(8); (* Velocity of Light *)
        v = 60; (* Speed of the vehicle *)
        ta = N[tb*(1/Sqrt[1 - v^2/c^2]), 50] (* Time in Seconds *)

Out[3]= 10.000000000000200000000000006000000000000200000000
```

13^{th} Decimal Point

```
In[4]:= tb = 10; (* Time in Seconds *)
        c = 3*10^(8); (* Velocity of Light *)
        v = c/10; (* Speed of the particle *)
        ta = N[tb*(1/Sqrt[1 - v^2/c^2]), 50] (* Time in Seconds *)

Out[5]= 10.050378152592120754893735565668747527051783471483
```

2^{nd} Decimal Point

```
In[6]:= tb = 10; (* Time in Seconds *)
        c = 3*10^(8); (* Velocity of Light *)
        v = c/2; (* Speed of the particle *)
        ta = N[tb*(1/Sqrt[1 - v^2/c^2]), 50] (* Time in Seconds *)

Out[7]= 11.547005383792515290182975610039149112952035025403
```

Measurement of Length in STR

So, time is relative ! What about distance ???

Thank You !