Space Physics

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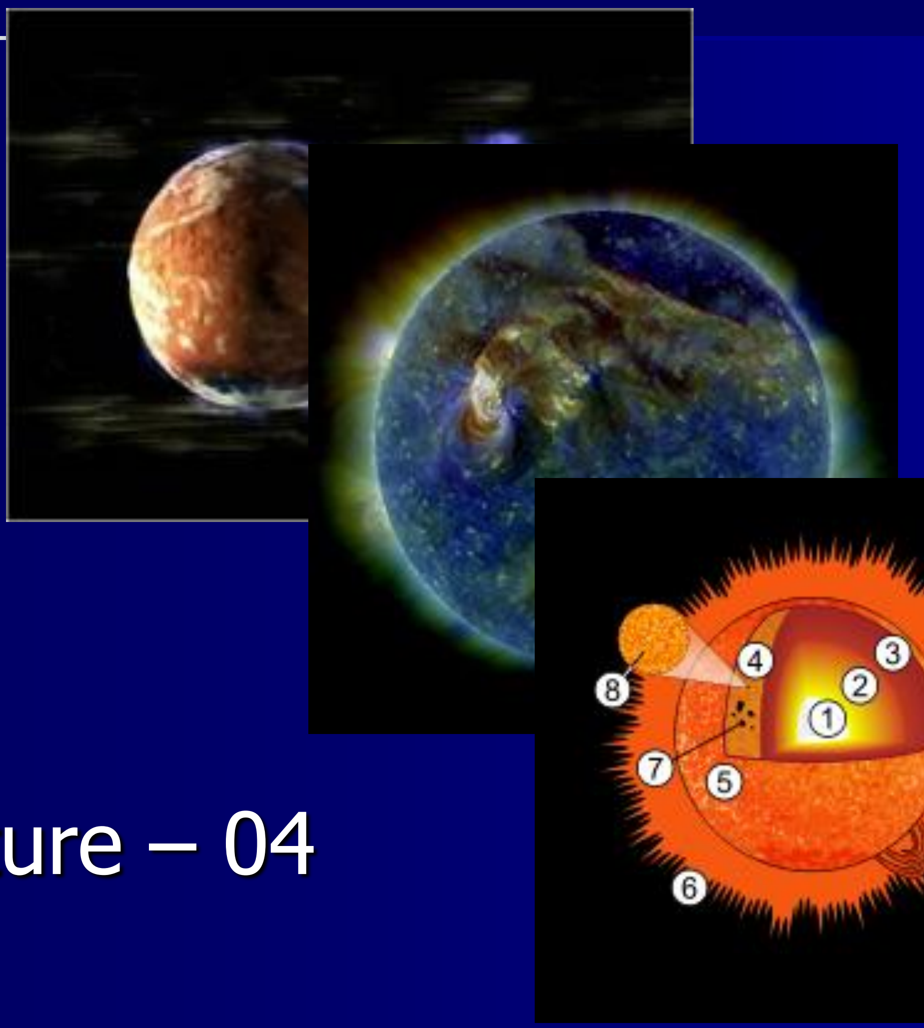
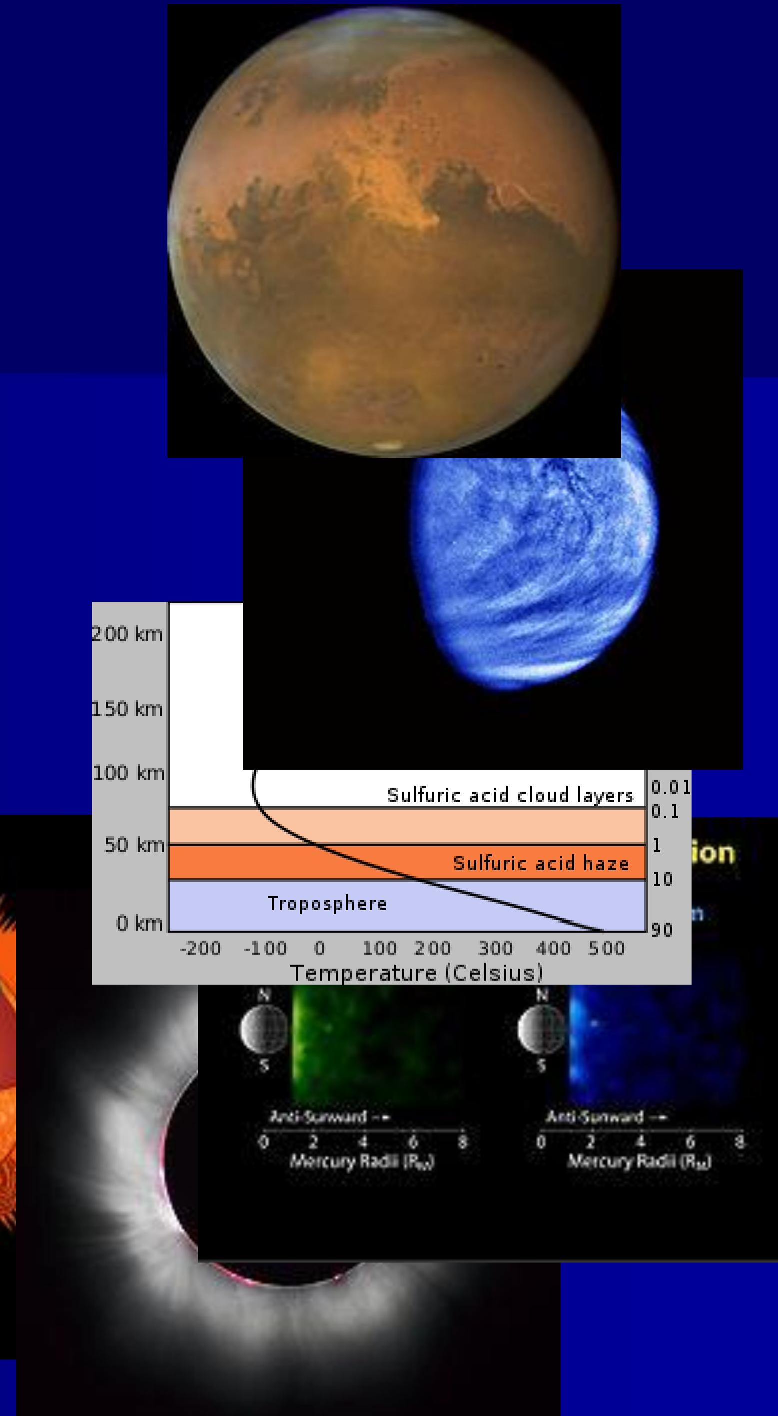


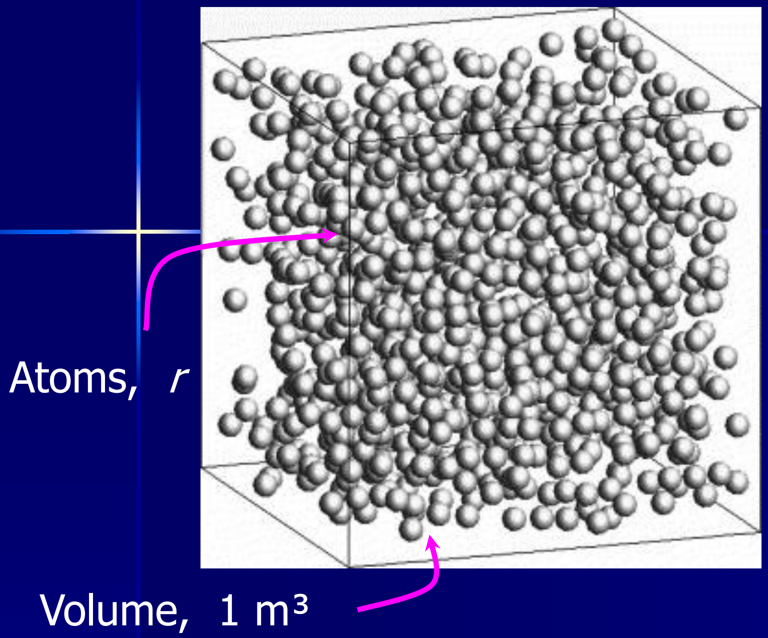


Lecture – 04

Earth Atmosphere

Retaining of Gases in the Earth
Major / Minor constituents
Barometric Equation
Scale Height
Number Density Profiles
Atmospheric Regions
Temperature Profiles
Retaining of Gases

Density of the Atoms



Assume there are *r* atoms in this volume

Masses of the atoms are:

$$m_1, m_2, m_3, \ldots, m_r$$

Number densities of those atoms are:

$$N_1, N_2, N_3, \ldots, N_r$$

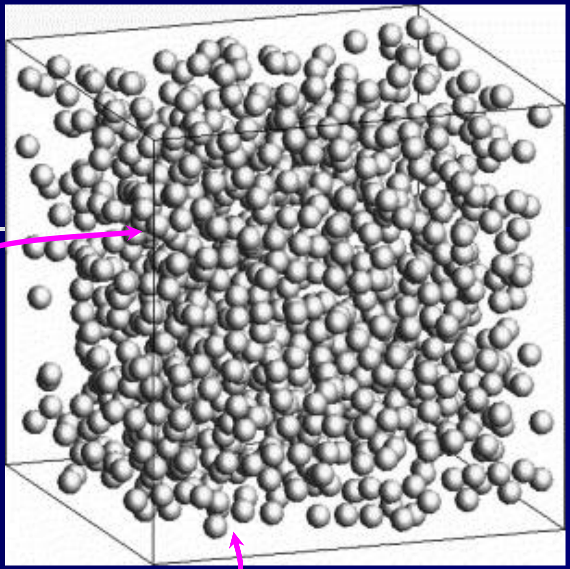
Total Mass of the atoms in the above volume:

$$m_1.N_1 + m_2.N_2, + m_3.N_3, \ldots, + m_r.N_r$$

(This is called the **density** because we consider the unit volume)

Total Molecular Number density: $N = N_1 + N_2, + N_3, \ldots, + N_r$

Density of the Atoms



Atoms, r

Volume, 1 m³

Total Mass per unit volume

Mean Molecular mass : $\overline{m}$

$$\overline{m} = \frac{Total\ \ Mass}{Total\ \ Molecular\ \ Number\ \ Density}$$

$$\overline{m} = \frac{m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r}{N_1 + N_2 + N_3 + ... + N_r}$$

$$\overline{m} = \frac{m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r}{N}$$

$$N.\overline{m} = m_1.N_1 + m_2.N_2 + m_3.N_3 + ... + m_r.N_r$$

Density

Density of the Atoms

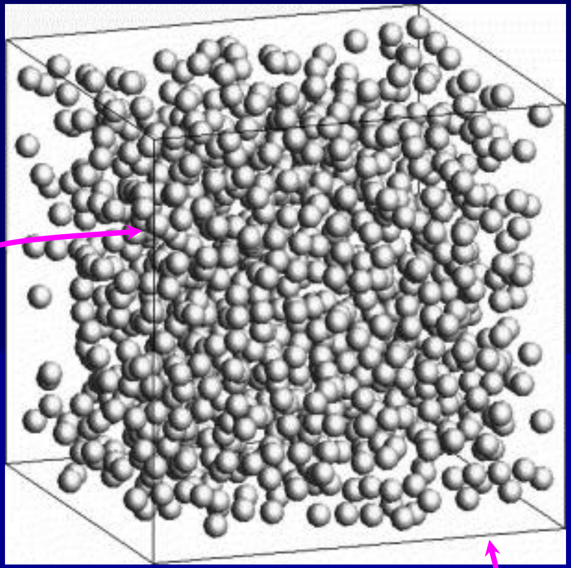
Mean Molecular
Number Density

Density

$$\rho = N \times \overline{m}$$

Total Molecular Number Density

Atoms, *r*



Volume, 1 m³

For the Ideal Gas

$$PV = nRT$$

Number of molecules
per volume, V

$$PV = \frac{NV}{N_o} RT$$

Avogadro Number (Number of
molecules in a molecular weight)

Boltzmann Constant

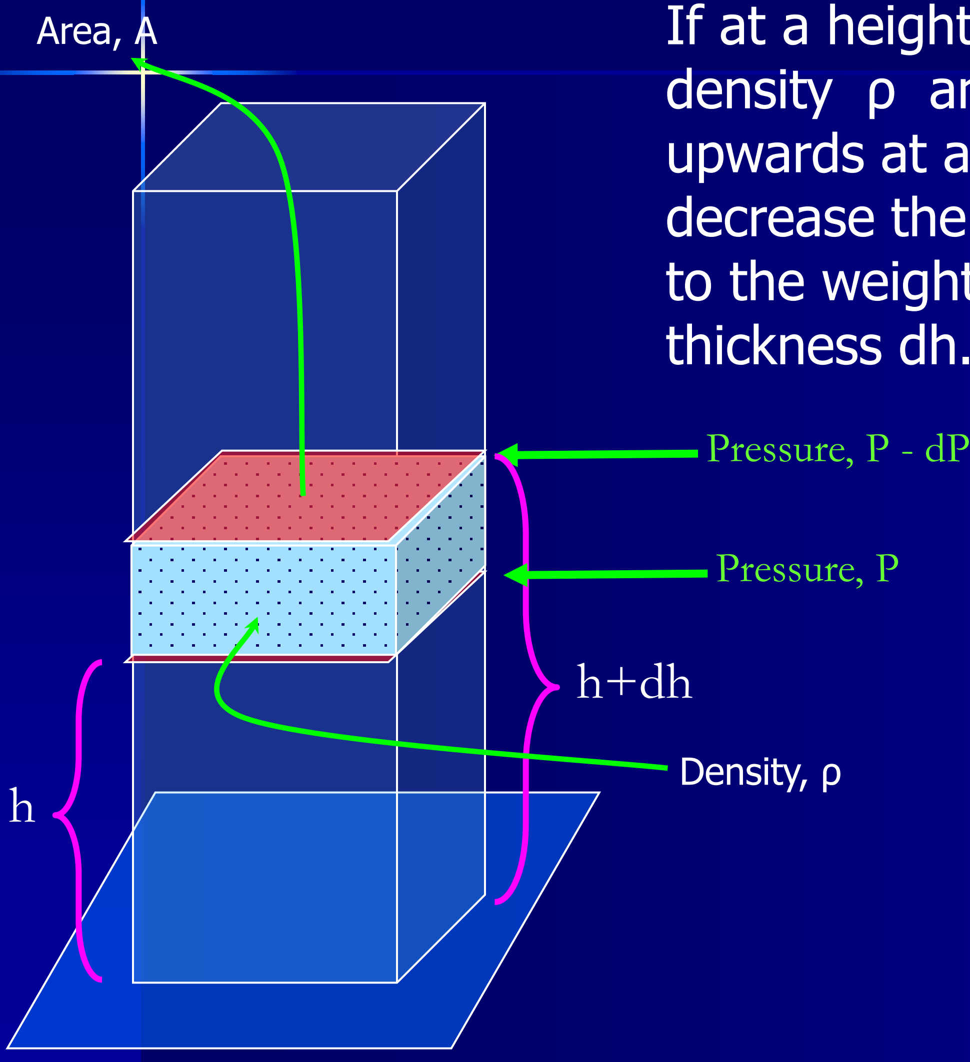
$$P = NkT$$

Where,

$$k = \frac{R}{N_o}$$

Pressure Profile

The pressure at the Earth's surface (or at higher levels) is a result of the weight of the overlying atmosphere [force per unit area]. If at a height of h the atmosphere has density ρ and pressure P then moving upwards at an infinitesimally small dh will decrease the pressure by amount dP equal to the weight of the layer of atmosphere of thickness dh.



3-D View

$$\frac{\text{Pressure of the Lower Layer} = \text{Pressure of the Higher Layer} + \text{Weight of the air molecules in the selected part}}{\text{Cross area of the selected part}}$$

$$\boxed{\cancel{P}} = \boxed{\cancel{P} - dP} + \boxed{\frac{\cancel{A}.dh.\rho.g}{\cancel{A}}}$$

Pressure Profile

$$dP = -\rho g . dh$$

This minus (-) sign indicates that as the height h is increases, the pressure P is decreases.

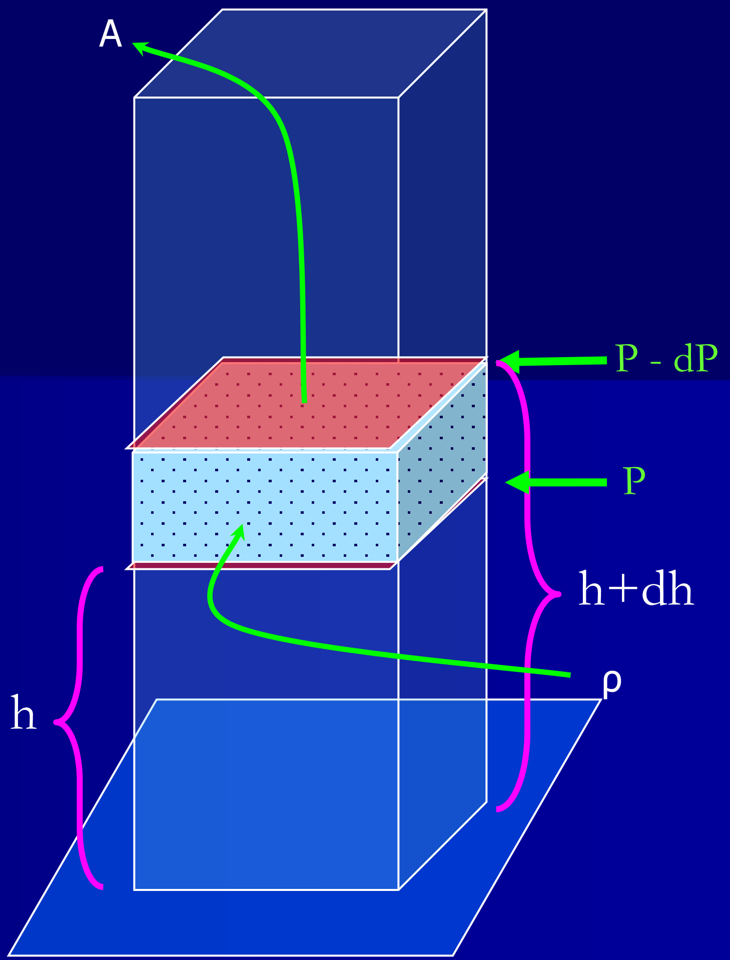
Where g is used to denote the acceleration due to gravity. For small dh it is possible to assume g to be constant. Also, $\rho = N \times \overline{m}$

$$dP = -N . \overline{m} . g . dh \qquad 1$$

Also, we know

$$P = NkT \qquad 2$$

Using 1 & 2 ; $\dfrac{dP}{P} = -\dfrac{\overline{m} g}{kT} dh$



3-D View

Pressure Profile

$$\frac{dP}{P} = -\frac{\overline{m}g}{kT}\,dh$$

The Pressure at height h can be written as:

$$P(h) = P_o\, e^{\frac{-\overline{m}g}{kT}h}$$

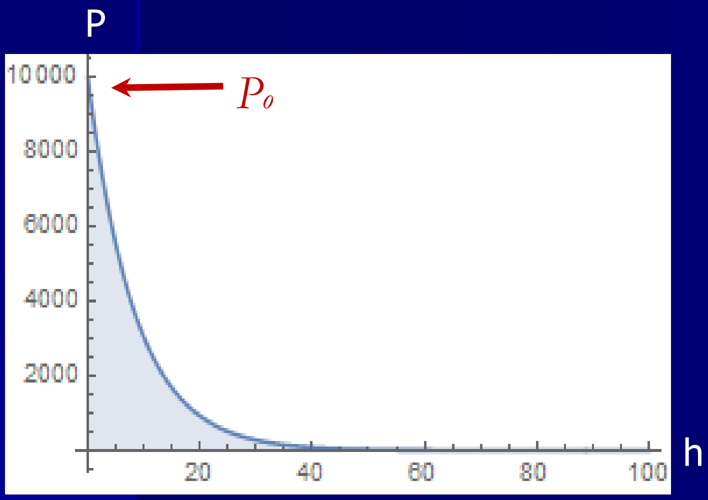
This is the general formula as the Pressure at height; This translate as the pressure decreasing exponentially with height !

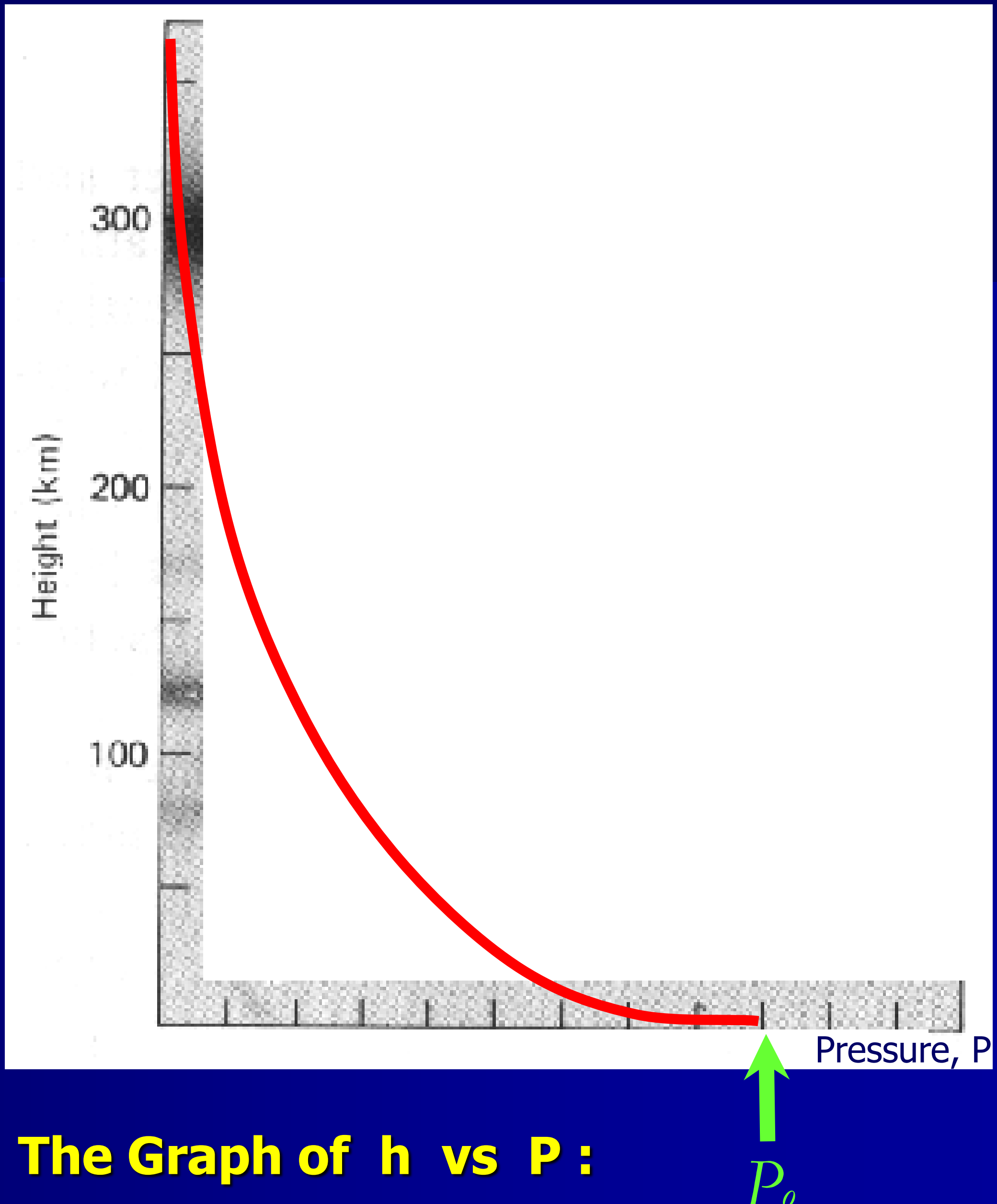
If h=0 then P=Po (1); That means Po is the pressure at h=0 level or The Ground Level.

Also $\frac{-\overline{m}g}{kT}h$ is independent of the units. That means $\frac{kT}{\overline{m}g}$ is also a some height !

The Graph of P vs h :

$$P(h) = P_o\, e^{\frac{-\overline{m}g}{kT}h}$$



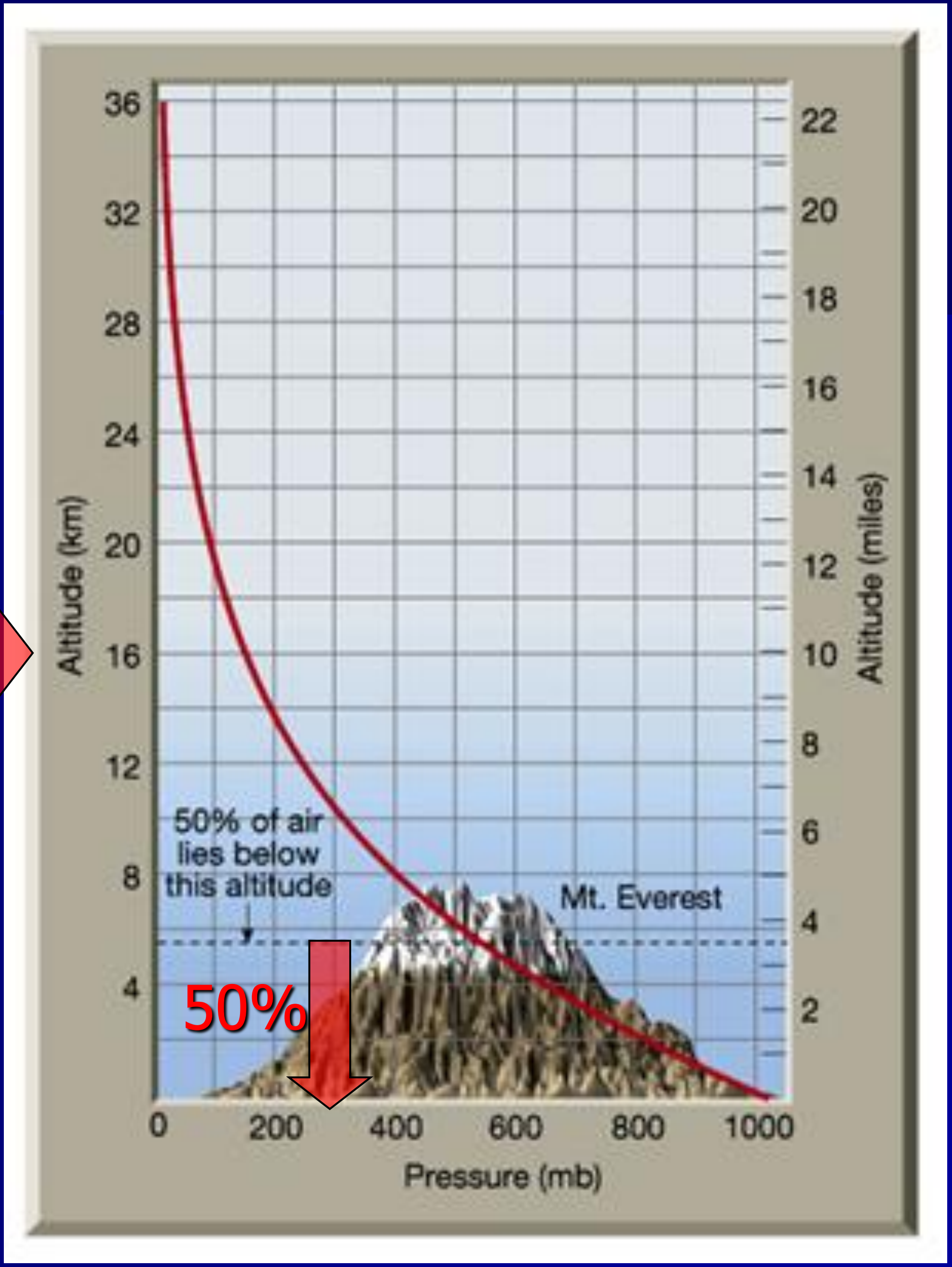


The Graph of h vs P :

The Graph of h vs P :

Percent sea level pressure	Altitude (km)
100	0
50	5.6
10	16.2
1	31.2
0.1	48.1
0.01	65.1
0.001	79.2
0.00003	100

Practical Values



Scale Height (H)

A scale height is a term often used in scientific context for a distance over which a quantity decreses by a factor of *e* (the base of natural logarithms). It is usually denoted by the capital letter H.

$$P(h) = P_o\, e^{\frac{-\overline{m}g}{kT}h}$$

For planetary atmosphere, it is the vertical distance upwards, over the which the pressure of the atmosphere decreases by a factor of e. The scale height remains constant for a particular temperature. It can be calculated by,

If P = Po/e then h = H,

$$\frac{P_o}{e} = P_o\, e^{\frac{-\overline{m}g}{kT}h} \Rightarrow H = \frac{kT}{\overline{m}g}$$

where:

- k = Boltzmann constant = 1.38×10^{-23} J·K^{-1}
- T = mean planetary surface temperature in kelvins
- $\overline{m}$ = mean molecular mass of dry air (units kg)
- g = acceleration due to gravity on planetary surface (m/s²)

Scale Height (H)

The Graph of Scale Heights vs P :

$$P(h) = P_o\, e^{\frac{-h}{H}}$$

If h = H, $P(H) = P_o\, e^{\frac{-H}{H}}$ ⇒ $P(H) = \frac{P_o}{e}$ ⇒ $0.36 P_o$

If h = 2H, $P(H) = P_o\, e^{\frac{-2H}{H}}$ ⇒ $P(H) = \frac{P_o}{e^2}$ ⇒ $0.13 P_o$

If h = 3H, $P(H) = P_o\, e^{\frac{-3H}{H}}$ ⇒ $P(H) = \frac{P_o}{e^3}$ ⇒ $0.04 P_o$

If h = 4H, $P(H) = P_o\, e^{\frac{-4H}{H}}$ ⇒ $P(H) = \frac{P_o}{e^4}$ ⇒ $0.01 P_o$

. . . .

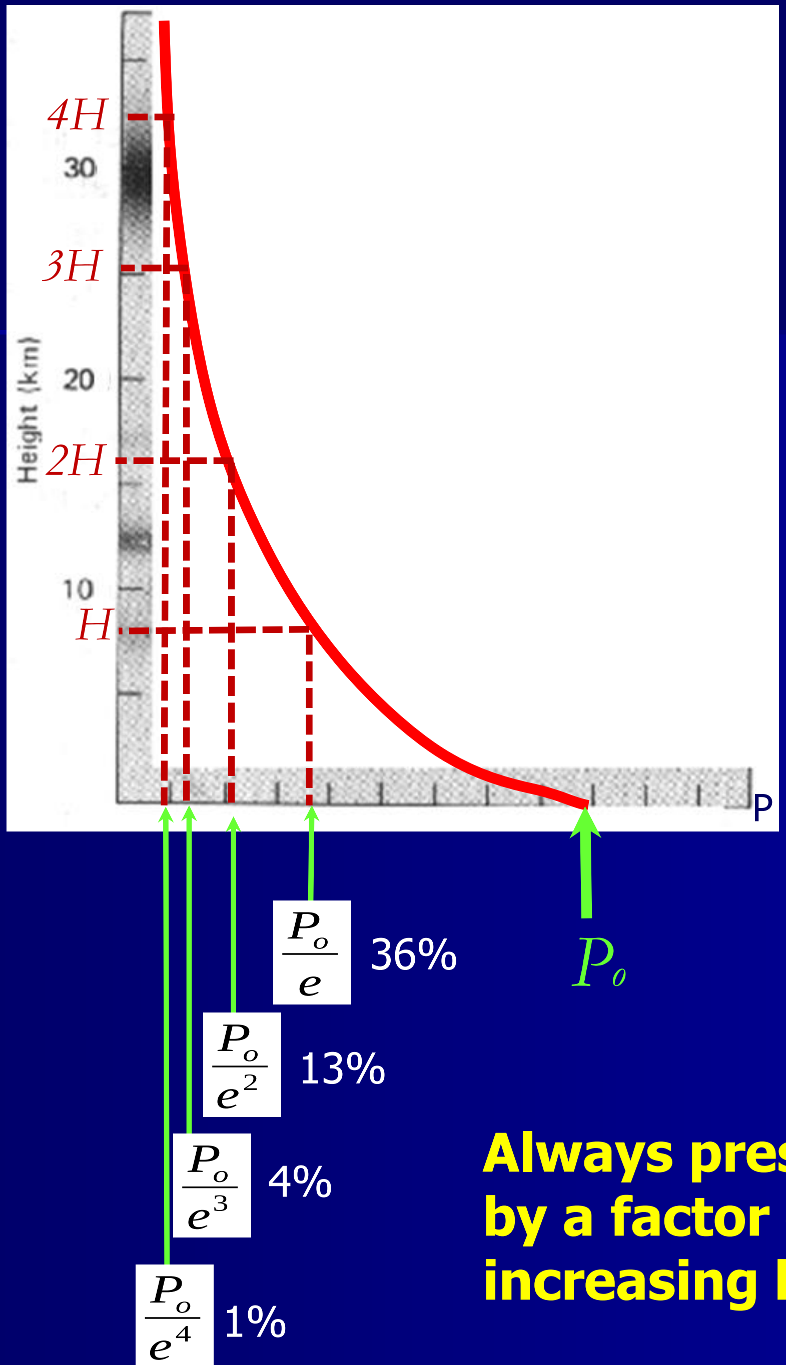
. . . .

. . . .

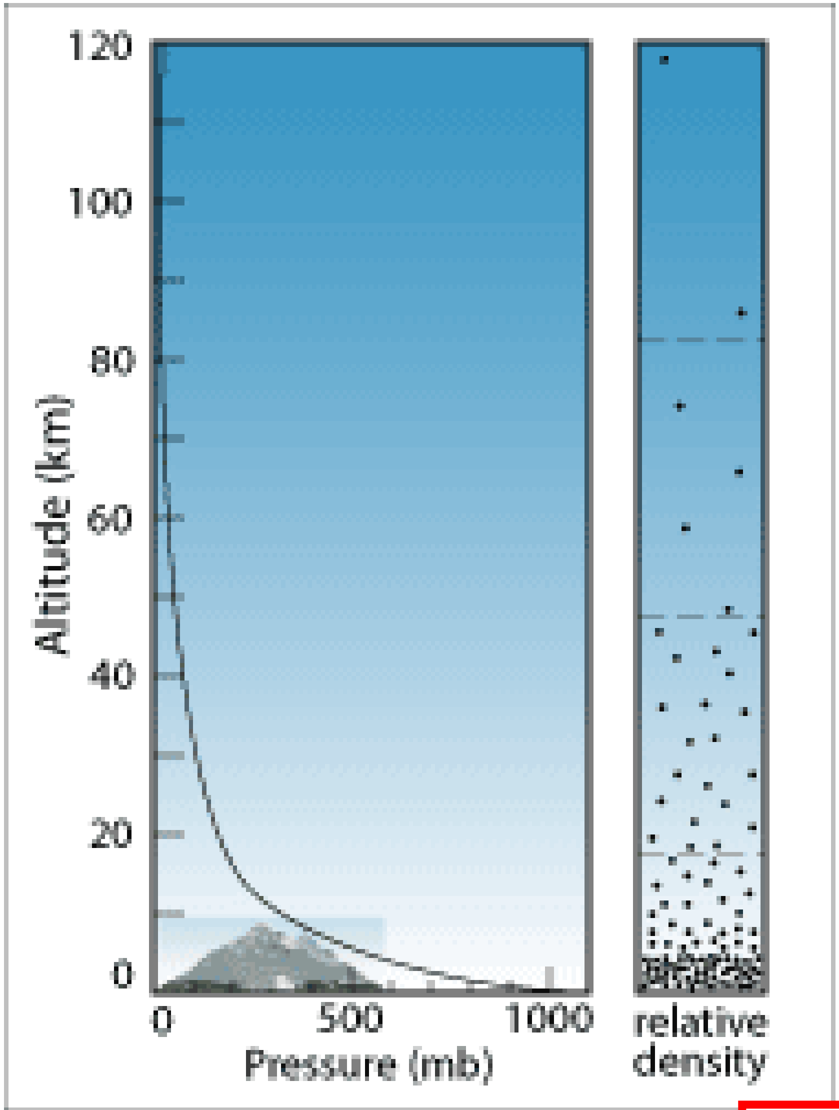
The Graph of Scale Heights vs P :

Height	Pressure	
H	Po / e	0.36 Po
2 H	Po / e^2	0.13 Po
3 H	Po / e^3	0.04 Po
4 H	Po / e^4	0.01 Po
5 H	Po / e^5	0.006 Po
………	…….	
n H	Po / e^n	

The Graph of H vs P :



Always pressure is decreasing by a factor of e when height is increasing by a multiplies of H



Pressure and density decrease rapidly with altitude.

Scale Height of the Earth, H

Temp, *T* vs Sca Hght, *H*

T (K)	H (m)
290	8500
273	8000
260	7610
210	6000

bars		millibars		atmospheres		millimeters of mercury
1.013 bar	=	1013 mb	=	1 atm	=	760 mm Hg

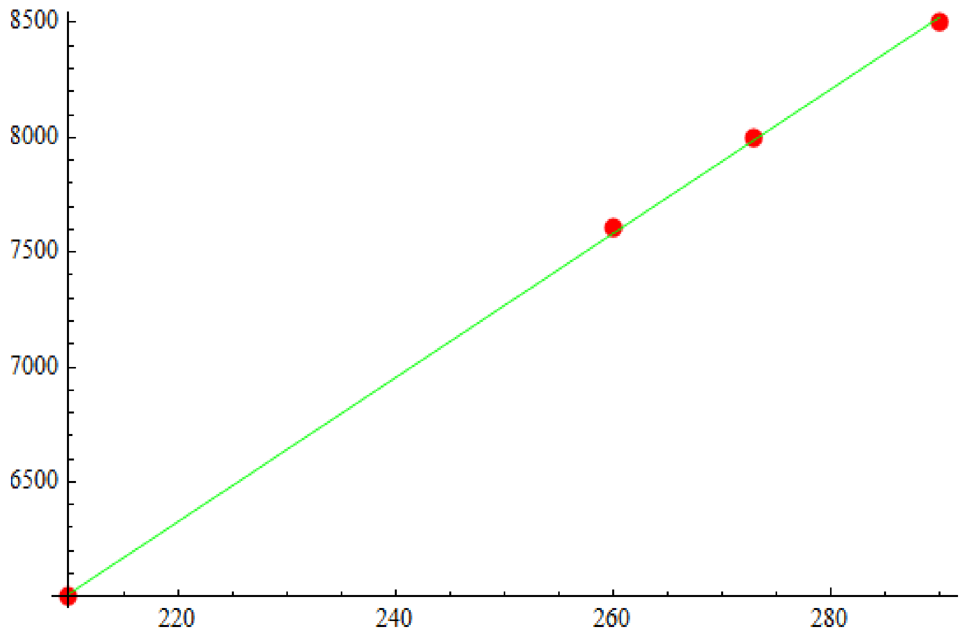
Correspondence of atmospheric measurement units.

Space Physics Lec 05 - Scale Height vs height.nb

In[8]:=
```
data = {{290, 8500}, {273, 8000}, {260, 7610}, {210, 6000}};
g1 = ListPlot[data, PlotStyle → {RGBColor[1, 0, 0], PointSize[0.02]}];
f = Fit[data, {t, 1}, t]
g2 = Plot[f, {t, data[[1, 1]], Last[data][[1]]}, PlotStyle → RGBColor[0, 1, 0]];
Show[g1, g2]
Print["The Scale Height is the function of Temperature in Kelvin (t)"]
Print[" H(t) = ", f]
```

Out[10]= $-582.316 + 31.403\,t$

Out[12]=



The Scale Height is the function of Temperature in Kelvin (t)

H(t) = $-582.316 + 31.403\,t$

Scale Height (H)

$$H = \frac{kT}{\overline{m}g}$$

where:

- k = Boltzmann constant = 1.38 x 10^{-23} J·K^{-1}
- T = mean planetary surface temperature in kelvins
- $\overline{m}$ = mean molecular mass of dry air (units kg)
- g = acceleration due to gravity on planetary surface (m/s²)

k = 1.4*10^(-23) J K^{-1}

T = 300 K

$\overline{m} = 5\times10^{-26}$ kg mol^{-1}

g = 10 m s^{-2}

$$H = \frac{\left(1.4\times10^{-23}\right)\times\left(300\right)}{\left(5.0\times10^{-26}\right)\times\left(10\right)}$$

$$H = 8.4km$$

Theoretically this *H* is a constant. But practically this *H* is not a constant. Because, the values of "**mean molecular mass**", "**acceleration due to gravity**" and "**mean planetary surface temperature**" are changing with respect to height from the Earth surface.

Eg: At which height from the surface of the Earth, which you can expect the atmosphere pressure which is half of that of the initial atmosphere pressure ?

Using the Pressure Equation : $$P(h) = P_o\, e^{\frac{-h}{H}}$$

Where, $H = 8.4km$

If P(h) = Po/2 when h=h, $$\frac{P_o}{2} = P_o\, e^{\frac{-h}{H}}$$ $\Rightarrow$ $$\frac{h}{H} = \ln(2)$$

$\Rightarrow$ $$h = 8.4 \times 0.6931$$

$\Rightarrow$ $$h = 5.822\,km$$

$\Rightarrow$ $$h = \sim 6km$$

	Height (km)	Pressure	
6 x 1	6	Po / 2	Po / 2^1
6 x 2	12	Po / 4	Po / 2^2
6 x 3	18	Po / 8	Po / 2^3
6 x 4	24	Po / 16	Po / 2^4
6 x 5	30	Po / 32	Po / 2^5
	………	…….	
	6 n	Po / 2^n	

Molecular Number Density

Using the Pressure Equation : $$P(h) = P_o\, e^{\frac{-h}{H}}$$

Where, $H = 8.4km$

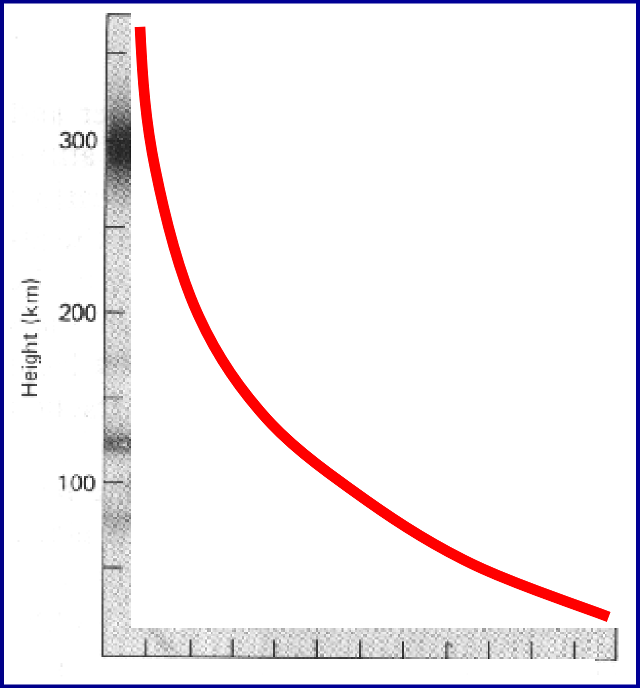
For the Ideal Gas

$$PV = nRT$$

$$P = NkT \quad \Rightarrow \quad N = \frac{P}{kT}$$

$$N(h) = \frac{P(h)}{kT} \quad \& \quad No = \frac{Po}{kT}$$

$$N(h) = N_o\, e^{-\frac{h}{H}}$$



Molecular Number Density

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$

If h = H,

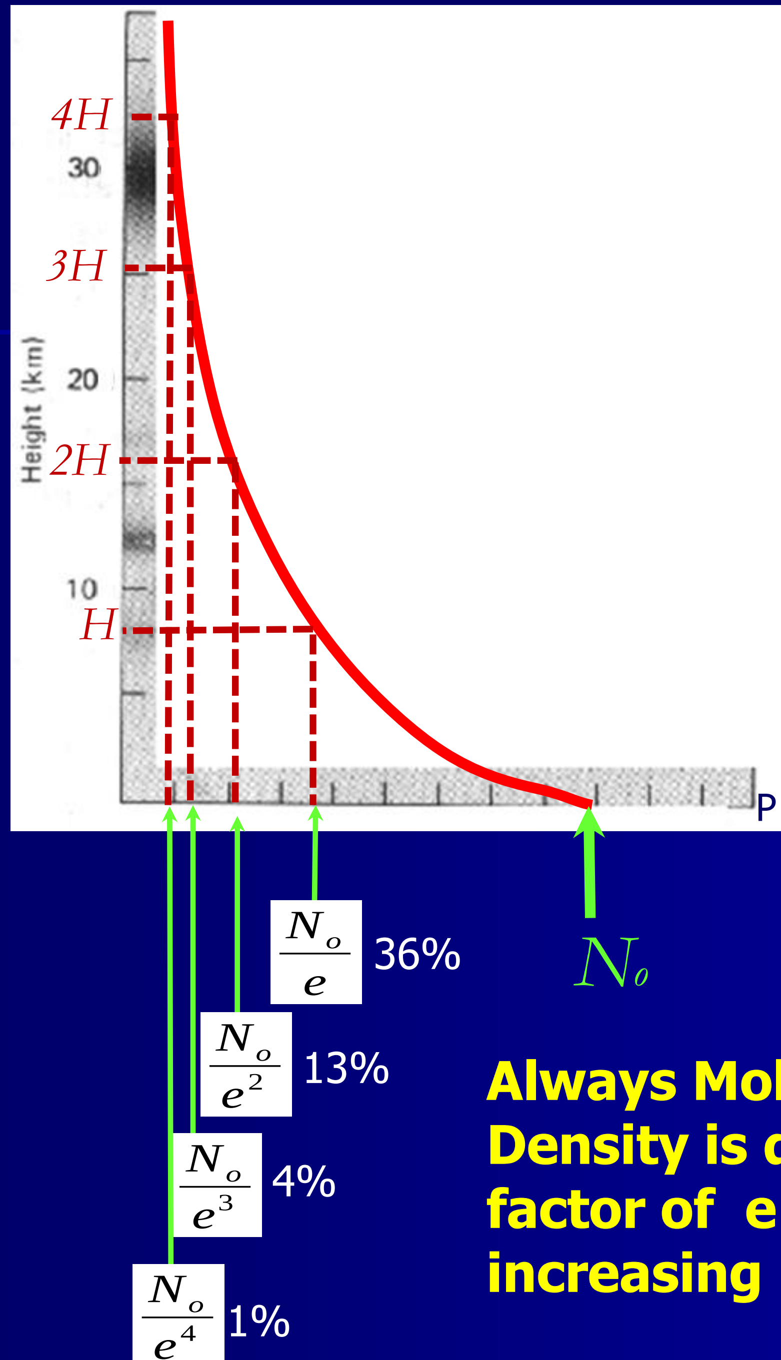
$$N(H) = N_o e^{\frac{-H}{H}}$$

$$N(H) = \frac{N_o}{e}$$

$$0.36 N_o$$

Height	Mol Num Den	
H	No / e	0.36 No
2 H	No / e^2	0.13 No
3 H	No / e^3	0.04 No
4 H	No / e^4	0.01 No
5 H	No / e^5	0.006 No
....	....	
n H	No / e^n	

The Graph of H vs N :

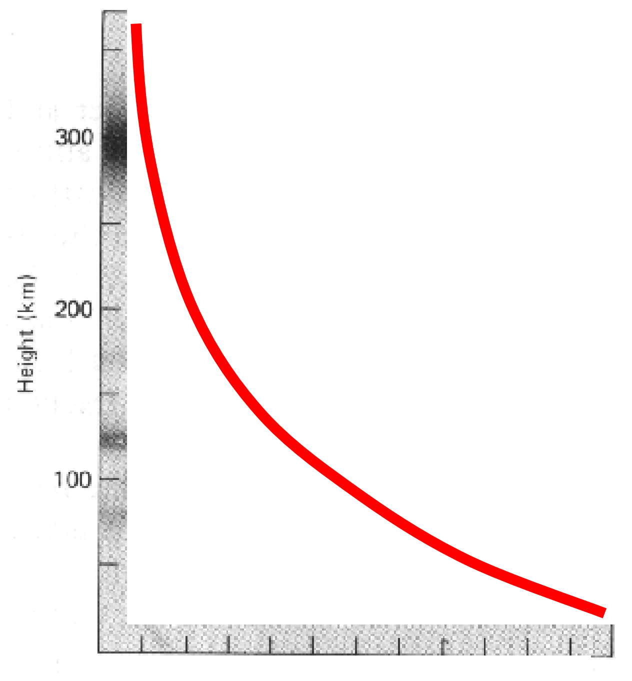


Always Molecular Number Density is decreasing by a factor of e when height is increasing by a multiplies of H

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$

Eg:
At which height from the surface of the Earth, which you can expect the Molecular Number Density which is half of that of the initial value of the Molecular Number Density ?



Molecular Number Density

If N(h) = No/2 when h=h,

$$\frac{N_o}{2} = N_o e^{\frac{-h}{H}}$$

$$h = \sim 6km$$

	Height (km)	Pressure	
6 x 1	6	No / 2	No / 2^1
6 x 2	12	No / 4	No / 2^2
6 x 3	18	No / 8	No / 2^3
6 x 4	24	No / 16	No / 2^4
6 x 5	30	No / 32	No / 2^5
	………	…….	
	6 n	No / 2^n	

Molecular Number Density

$$N(h) = N_o e^{-\frac{h}{H}}$$

If h=6 km Then N(h) = ?, $\Rightarrow$ $N = \dfrac{N_o}{2}$

If h=60 km Then N(h) = ?, $\Rightarrow$ $N = \dfrac{N_o}{2^{10}} =\sim \dfrac{N_o}{1000}$

If h=600 km Then N(h) = ?, $\Rightarrow$ $N = \dfrac{N_o}{2^{100}} =\sim \dfrac{N_o}{10^{30}}$

That means at 600 km height, the Molecular Number Density is (1/(10^30)) from its initial value.

Consider Linear Distance ;
At 600 km height, the Molecular Linear Distance is (1/(10^30))^(1/3) = (1/(10^10)) from its initial value.

$$= \left(\frac{1}{10^{30}}\right)^{\frac{1}{3}}$$

Molecular Number Density

$$= \left(\frac{1}{10^{30}} \right)^{\frac{1}{3}}$$

Linear Distance of the molecules = **Mean Free Path** ;

This is "Separation between two atoms"

Mean Free Path on the ground level $= 6.0 \times 10^{-8}$ m

Mean Free Path at altitude 600 km height from the ground level :

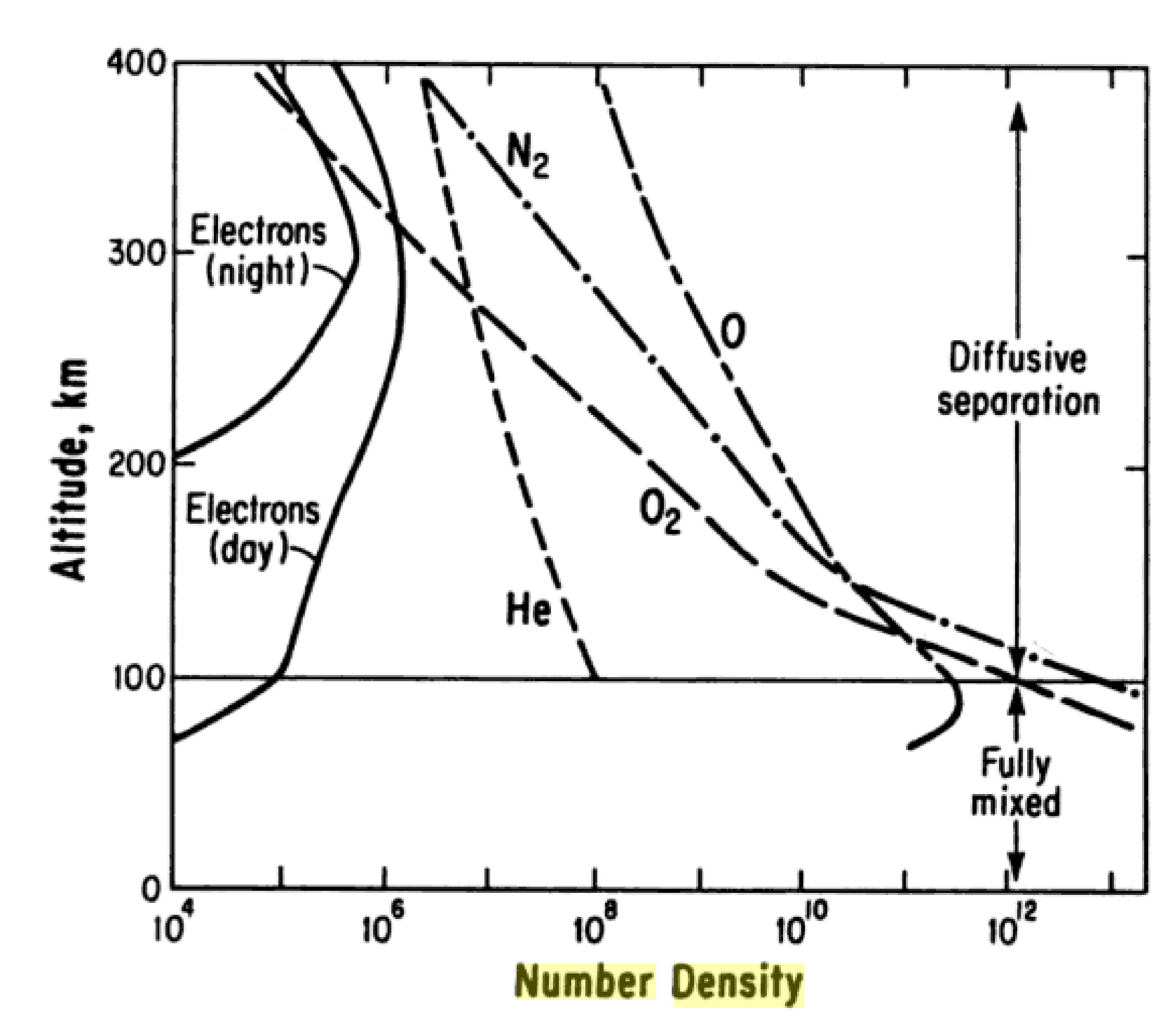
$$= 6 \times 10^{-8} \times \left(10^{30}\right)^{\frac{1}{3}}$$

$$= 6 \times 10^{-8} \times 10^{10}$$

$$= 600\,m$$

That means the **gap between two atoms** on that 600 km height (altitude) from the ground level is very high ! At that level there is no mean "**The gas**", because the **mean free path is very high** (600 m)

Molecular Number Density



Density

Using the Molecular Number Density Equation :

$$N(h) = N_o\, e^{-\frac{h}{H}}$$

Where, $H = 8.4km$

Mean Molecular Number Density

Density

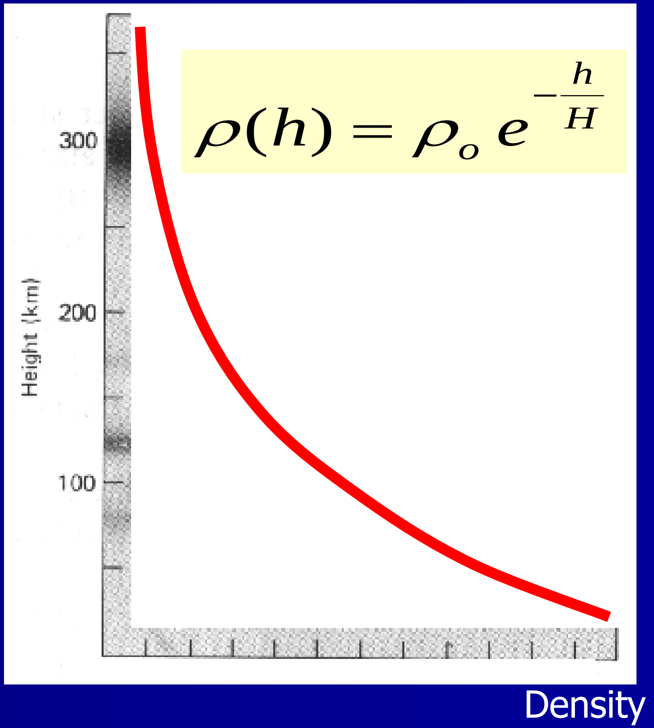
$$\rho = N \times \overline{m}$$

Total Molecular Number Density

$$\rho(h) = N(h) \times \overline{m}$$

&

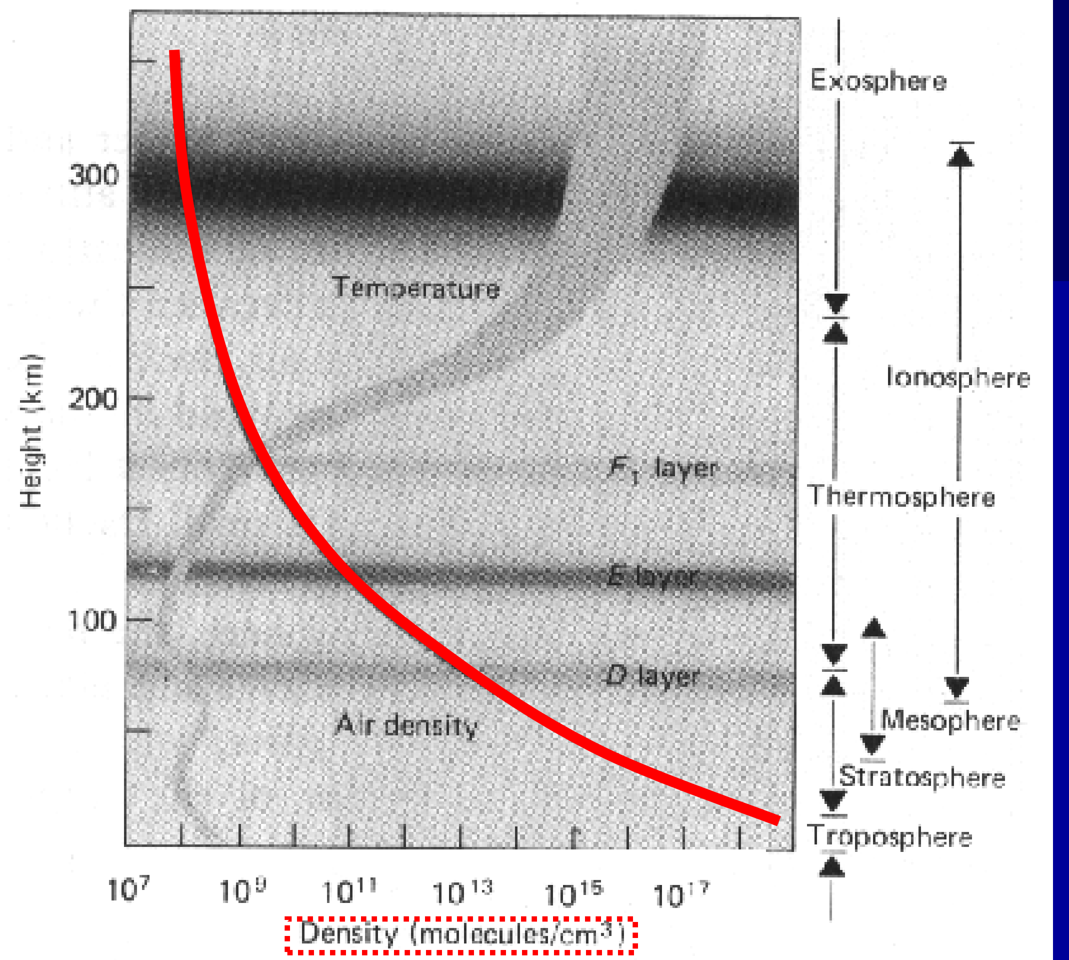
$$\rho_o = N_o \times \overline{m}$$



Density

$$\rho(h) = \rho_o \, e^{-\frac{h}{H}}$$

Where, $H = 8.4km$



Variations in the density of
Earth's neutral atmosphere with altitude

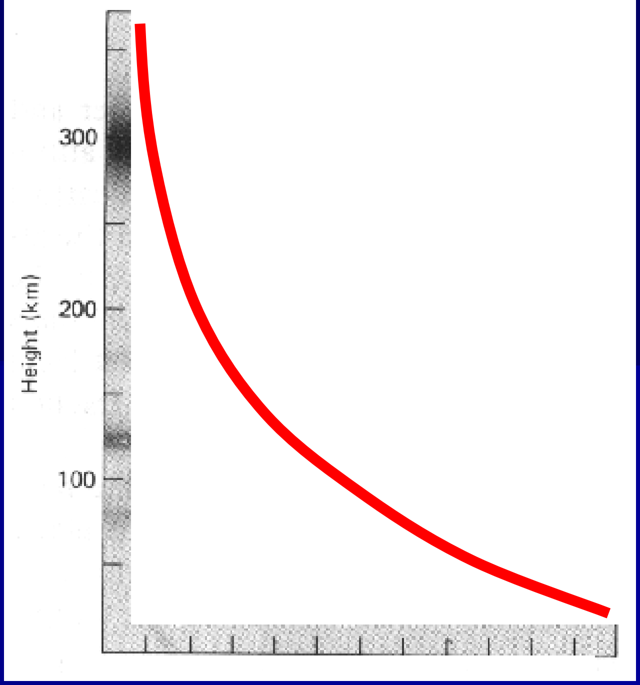
If h = H,

$$\rho(H) = \rho_o \, e^{\frac{-H}{H}} \Rightarrow \rho(H) = \frac{\rho_o}{e} \Rightarrow 0.36\rho_o$$

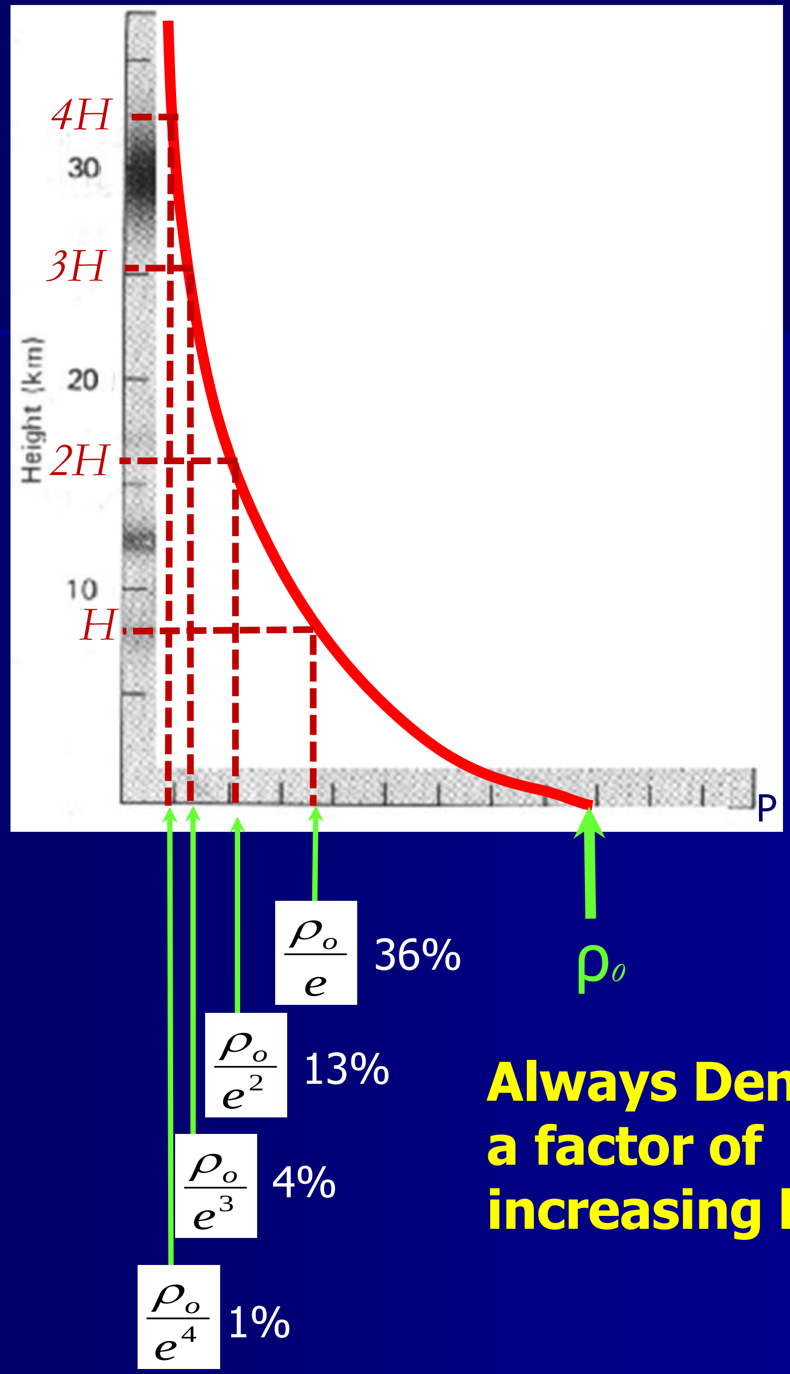
Density

$$\rho(h) = \rho_o \, e^{-\frac{h}{H}}$$

Height	Air Density	
H	ρo / e	0.36 ρo
2 H	ρo / e^2	0.13 ρo
3 H	ρo / e^3	0.04 ρo
4 H	ρo / e^4	0.01 ρo
5 H	ρo / e^5	0.006 ρo
………	…….	
n H	ρo / e^n	

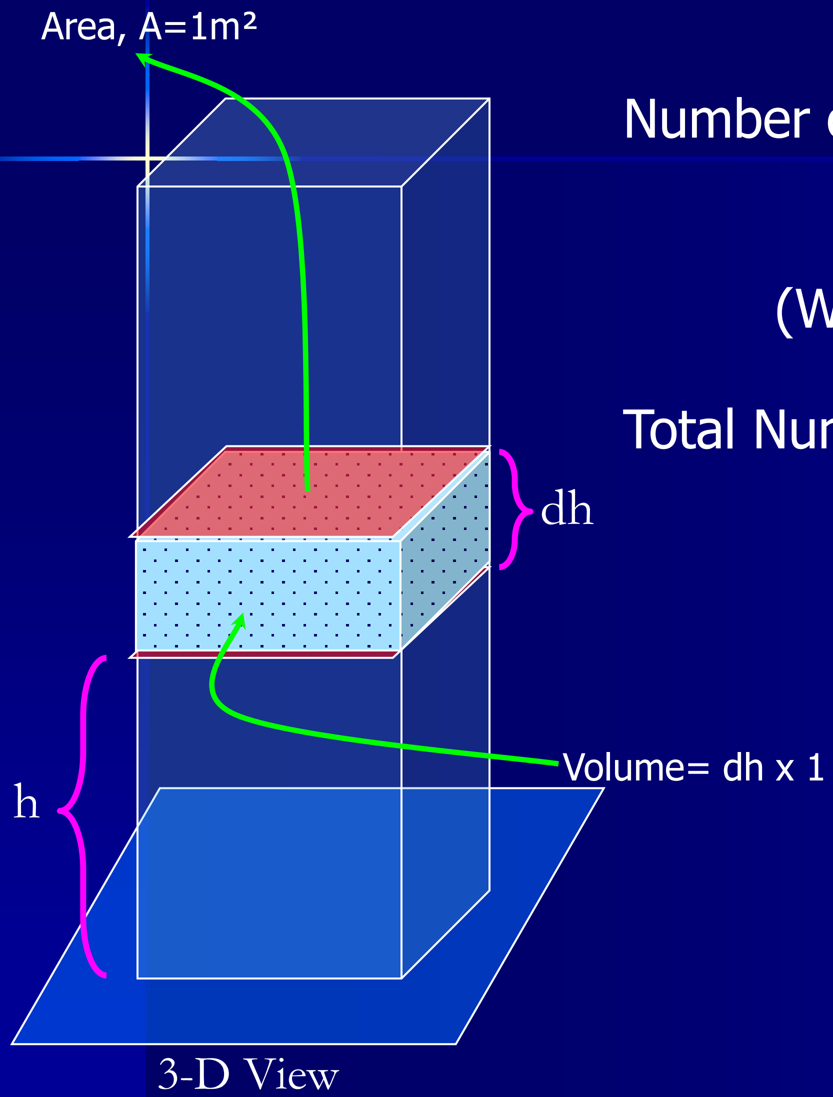


The Graph of H vs ρ :



Always Density is decreasing by a factor of e when height is increasing by a multiplies of H

Total Number of Molecules from Earth Surface to altitude h :



Number of molecules in a selected part =

$$N \times dh \times 1$$

(Where N is the molecular number density)

Total Number of molecules from h=h to h=∞

$$\int_{h=h}^{\infty} N.dh$$

Where,

$$N(h) = N_o e^{\frac{-h}{H}}$$

$$\int_{h=h}^{\infty} N_o e^{\frac{-h}{H}} .dh$$

Total Number of Molecules from Earth Surface to altitude h :

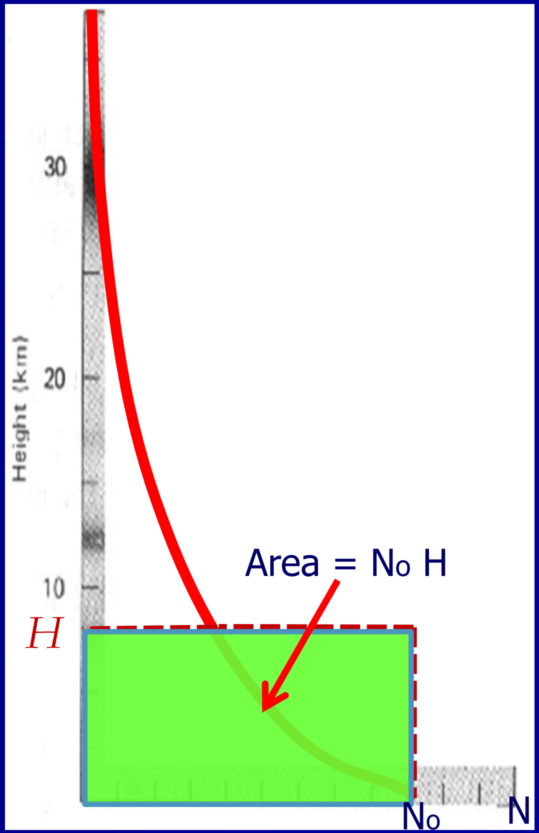
$$N_{\substack{Total \\ h\to\infty}} = N_o H \; e^{-\frac{h}{H}}$$

Case I :

$$N_{\substack{Total \\ 0\to\infty}} = N_o H$$

That means, if the molecular number density of the atmosphere of the Earth varies **linearly** without varying **exponentially**, the atmosphere of the Earth will diminish after ~8.4 km (a scale height).

This gives to us another definition for the Scale Height !



Total Number of Molecules from Earth Surface to altitude h :

$$\underset{h\to\infty}{N_{Total}} = N_o H \; e^{-\frac{h}{H}}$$

Case II :

$$\frac{\underset{h\to\infty}{N_{Total}}}{\underset{0\to\infty}{N_{Total}}} = \frac{N_o H \; e^{-h/H}}{N_o H} = e^{-h/H}$$

Fraction of the Number of Molecules from the specific height h.

If h=H km Then RATIO = ?,

$$\left(e^{-h/H} \right)_{h\to H} = e^{-\,H/H} = e^{-1}$$

~ 40 %

60 % of the total molecules exist bellow H (8.4 km) !

Total Number of Molecules from Earth Surface to altitude h :

If h=2H km Then RATIO = ?, $\left(e^{-h/H} \right)_{h \to 2H} = e^{-2H/H} = e^{-2}$

~ 15 %

85 % of the total molecules exist bellow 2H (16.8 km) !

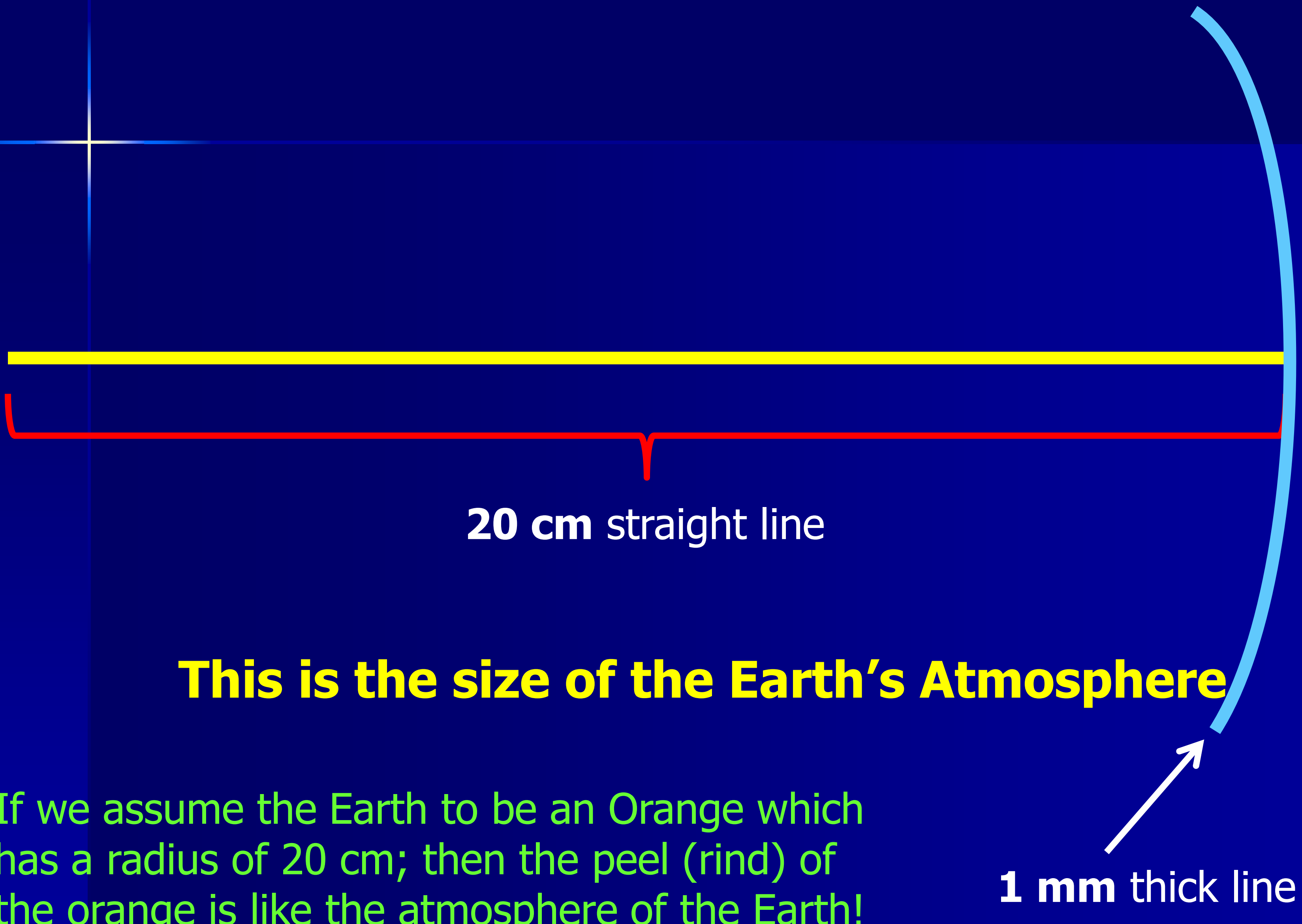
If h=3H km Then RATIO = ?, $\left(e^{-h/H} \right)_{h \to 3H} = e^{-3H/H} = e^{-3}$

~ 5 %

95 % of the total molecules exist bellow 3H (16.8 km) !

h (km)		N (h →∞) / N (0 →∞)	% below h
H	08.4	36.78	63.21
2 H	16.8	13.53	86.46
3 H	25.2	4.97	95.02
4 H	36.6	1.83	98.16
5 H	42.0	0.67	99.32
6 H	50.4	0.24	99.75
7 H	58.8	0.09	99.90
8 H	67.2	0.03	99.96
9 H	75.6	0.01	99.98
10 H	84.0	0.004	99.995

Sketch the size of the Earth’s Atmosphere



This is the size of the Earth’s Atmosphere

If we assume the Earth to be an Orange which has a radius of 20 cm; then the peel (rind) of the orange is like the atmosphere of the Earth!

Temperature Profile of the Earth


Thank You !
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