Space & Atmospheric Physics

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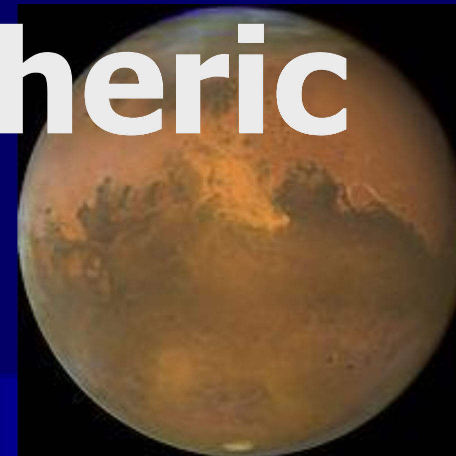
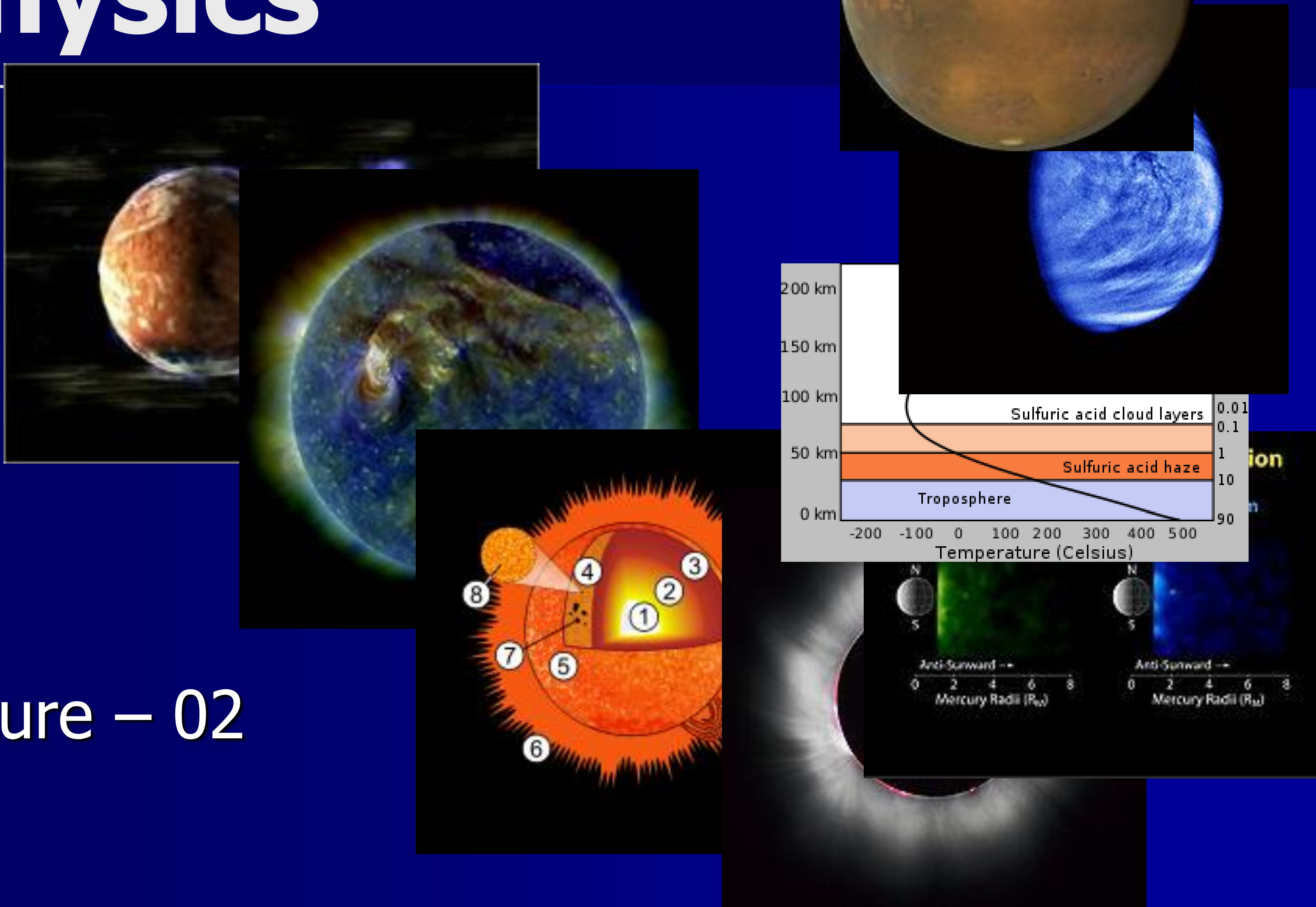




Lecture – 02

Planetary Atmospheres

Planetary Atmospheres

- Formation and Evolution of Planetary Atmospheres
- **The Structure of the Terrestrial Atmosphere**
- The Temperature of the Neutral Atmosphere
- The Escape of the Atmospheric Gases
- The Atmospheres of the Planets

The physical parameters of an average atmosphere.

TABLE 1.2-I

Altitude in km	Temperature in °K	Density in gr/cm^{-3}	Mean Mol. Weight	Pressure in dyn/cm^2	Mean Free Path in m	Accel. Grav. in cm/s^2
0	288	1.23×10^{-3}	28.96	1.01×10^{6}	6.63×10^{-8}	981
2	275	1.01×10^{-3}	28.96	7.95×10^{5}	8.07×10^{-8}	980
4	262	8.19×10^{-4}	28.96	6.17×10^{5}	9.92×10^{-8}	979
6	249	6.60×10^{-4}	28.96	4.72×10^{5}	1.23×10^{-7}	979
8	236	5.26×10^{-4}	28.96	3.57×10^{5}	1.55×10^{-7}	978
10	223	4.14×10^{-4}	28.96	2.65×10^{5}	1.96×10^{-7}	978
20	217	8.89×10^{-5}	28.96	5.53×10^{4}	9.14×10^{-7}	975
40	250	4.00×10^{-6}	28.96	2.87×10^{3}	2.03×10^{-5}	968
60	256	3.06×10^{-7}	28.96	2.25×10^{2}	2.66×10^{-4}	962
80	181	2.00×10^{-8}	28.96	1.04×10	4.07×10^{-3}	956
100	210	4.97×10^{-10}	28.88	3.01×10^{-1}	1.63×10^{-1}	951
140	714	3.39×10^{-12}	27.20	7.41×10^{-3}	2.25×10	939
180	1156	5.86×10^{-13}	26.15	2.15×10^{-3}	1.25×10^{2}	927
220	1294	1.99×10^{-13}	24.98	8.58×10^{-4}	3.52×10^{2}	916
260	1374	8.04×10^{-14}	23.82	3.86×10^{-4}	8.31×10^{2}	905
300	1432	3.59×10^{-14}	22.66	1.88×10^{-4}	1.77×10^{3}	894
400	1487	6.50×10^{-15}	19.94	4.03×10^{-5}	8.61×10^{3}	868
500	1499	1.58×10^{-15}	17.94	1.10×10^{-5}	3.19×10^{4}	843
600	1506	4.64×10^{-16}	16.84	3.45×10^{-6}	1.02×10^{5}	819
700	1508	1.54×10^{-16}	16.17	1.19×10^{-6}	2.95×10^{5}	796

Altitude in km	Density in gr/cm^{-3}
0	1.23×10^{-3}
2	1.01×10^{-3}
4	8.19×10^{-4}
6	6.60×10^{-4}
8	5.26×10^{-4}
10	4.14×10^{-4}

The graph of h (in km) vs Density

(in gr cm^(-3))

Enter the data as two 1D arrays….

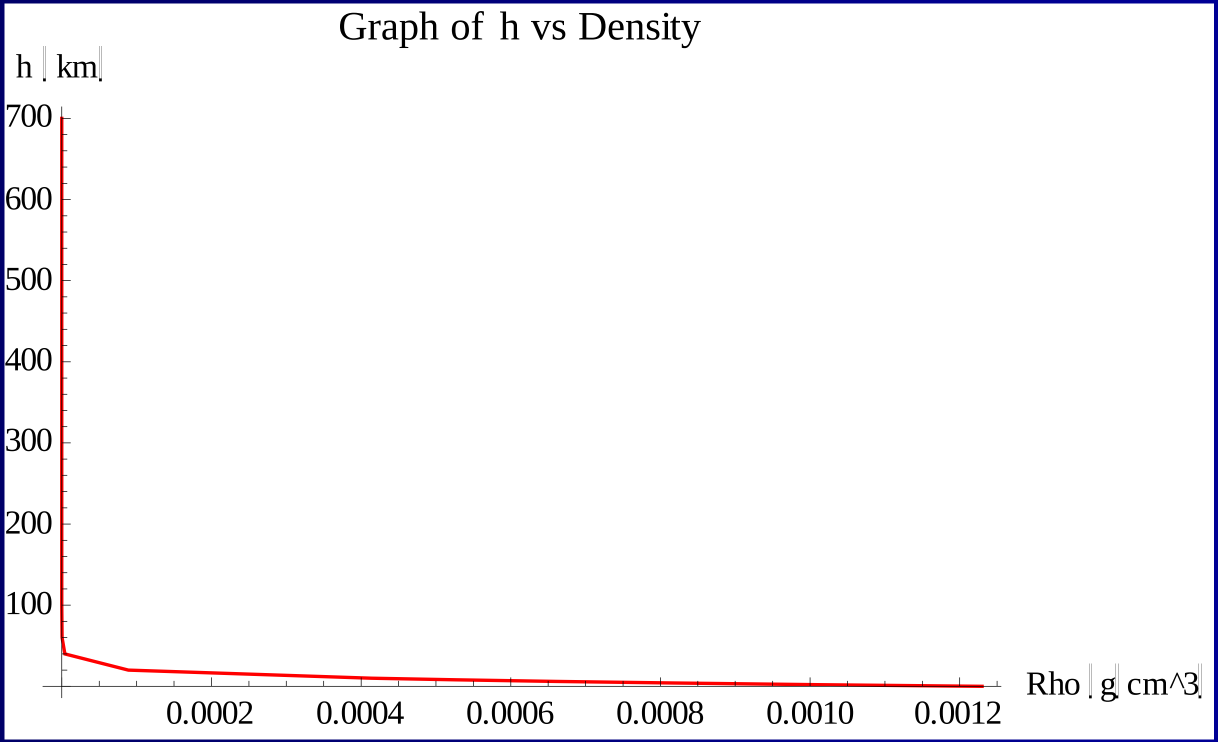
```
(* This program is to plot the Earth's parameters w.r.t
 altitude from the Earth Surface *)

hgh = {0, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100, 140, 180, 220,
   260, 300, 400, 500, 600, 700}; (* Altitude *)
rho = {1.23*10^(-3), 1.01*10^(-3), 8.19*10^(-4),
  6.60*10^(-4), 5.26*10^(-4), 4.14*10^(-4), 8.89*10^(-5),
  4.00*10^(-6), 3.06*10^(-7), 2.00*10^(-8), 4.97*10^(-10),
  3.39*10^(-12), 5.86*10^(-13), 1.99*10^(-13),
  8.04*10^(-14), 3.59*10^(-14), 6.50*10^(-15),
  1.58*10^(-15), 4.64*10^(-16), 1.54*10^(-16)};
(* Density *)
```

The graph of h (in km) vs Density (in gr cm^(-3))

```
d1 = Transpose[{rho, hgh}]; (* to get the h vs Rho data set *)
ListPlot[d1, PlotJoined → True,
 PlotStyle → {RGBColor[1, 0, 0], PointSize[0.02]},
 PlotLabel → "Graph of h vs Density",
 AxesLabel → {"Rho (g/cm^3)", "h (km)"}]
(* To Plot the h vs Rho graph *)
```

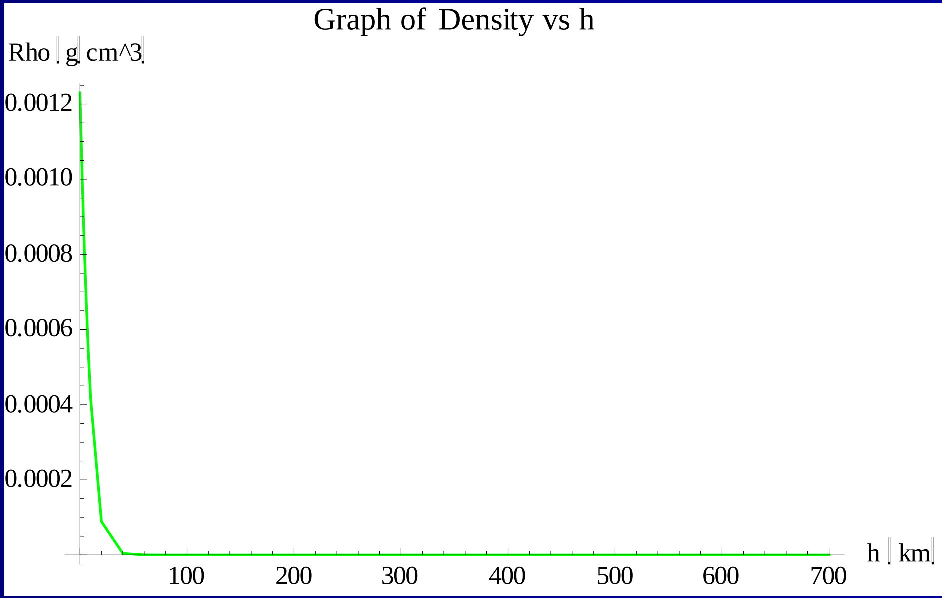
Plot the graph.....



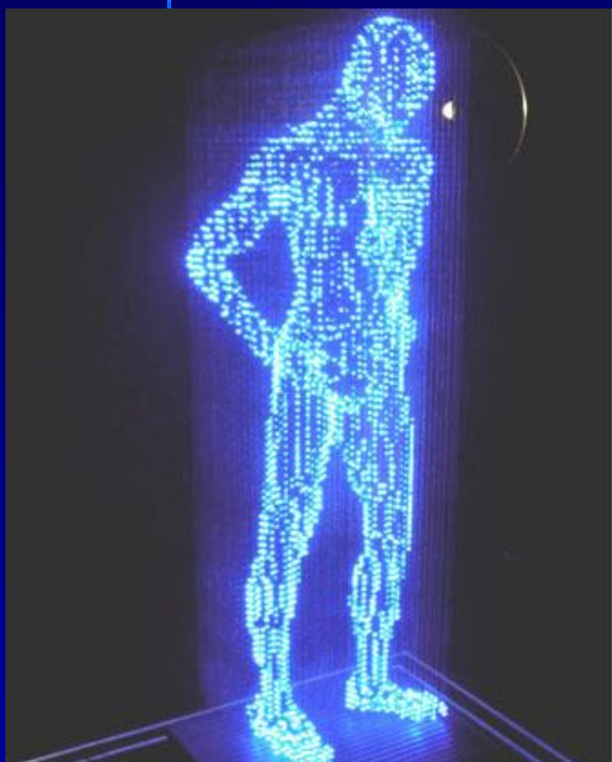
To model the data set …

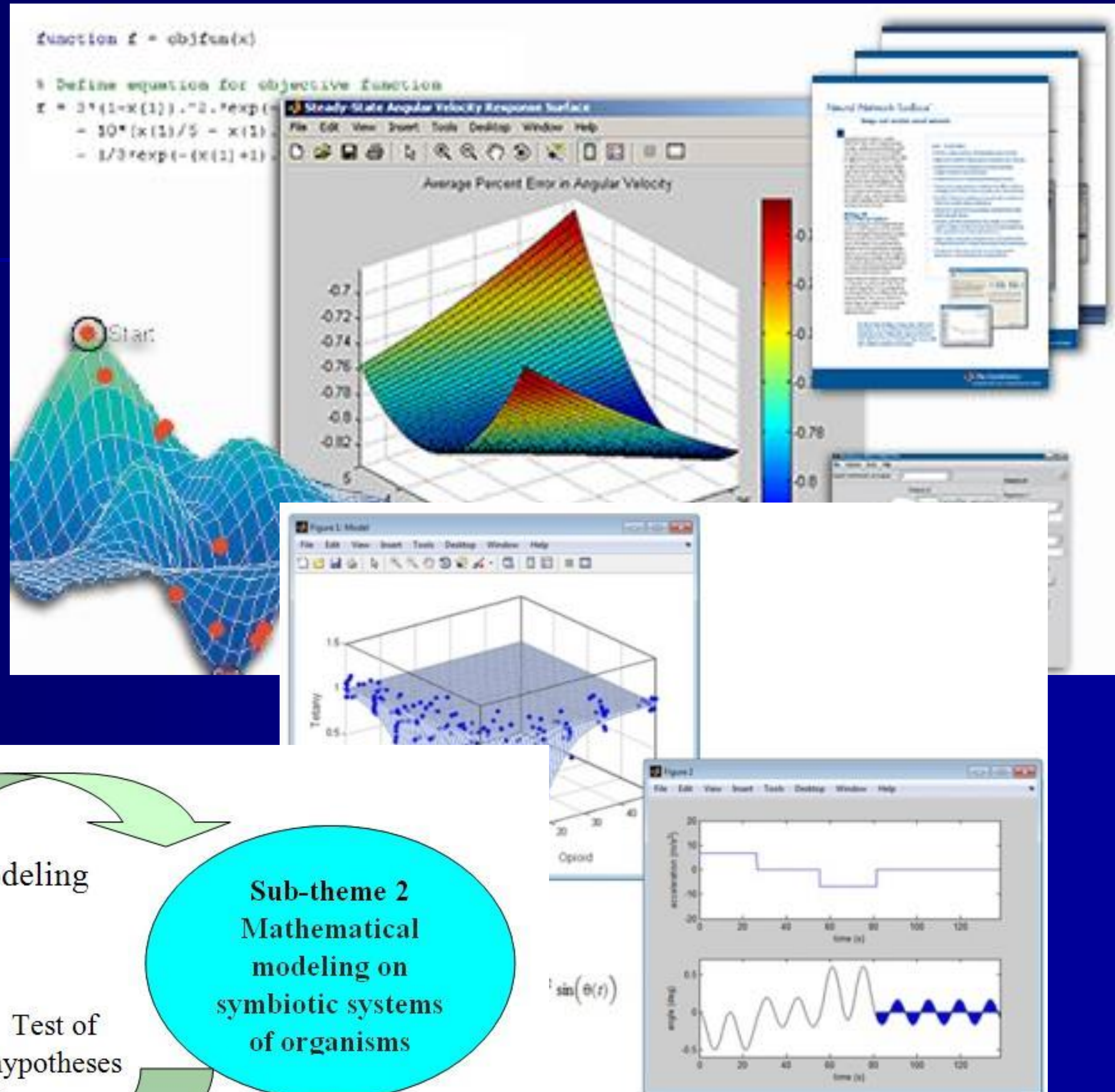
```
(* To model the data set in Rho vs h format *)
data = Transpose[{hgh, rho}];
g1 = ListPlot[data, PlotJoined → True,
  PlotStyle → {RGBColor[0, 1, 0], PointSize[0.02]},
  PlotLabel → "Graph of Density vs h",
  AxesLabel → {"h (km)", "Rho (g/cm^3)"}]
(* To Plot the Rho vs h graph *)
```

To Model …..

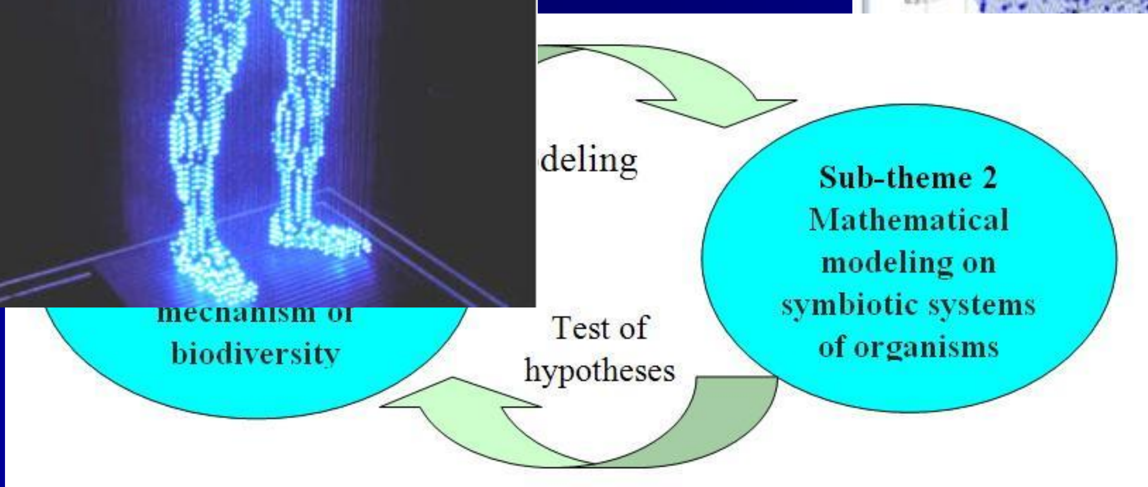


Modeling…





function f = objfun(x)
% Define equation for objective function
Steady-State Angular Velocity Response Surface
Average Percent Error in Angular Velocity
Start
Opioid

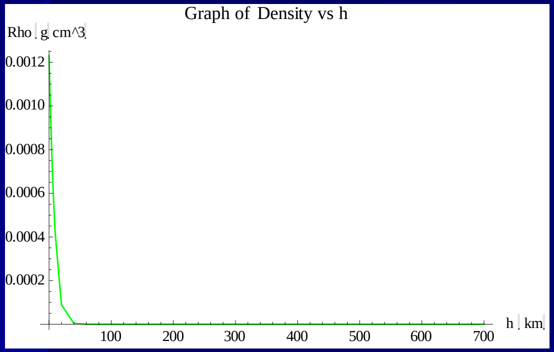


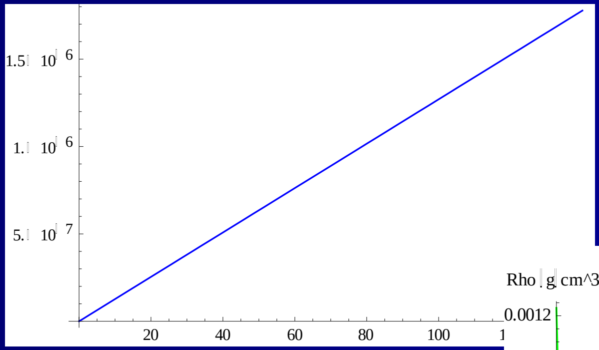
deling
Sub-theme 2
Mathematical
modeling on
symbiotic systems
of organisms
Test of
hypotheses
biodiversity

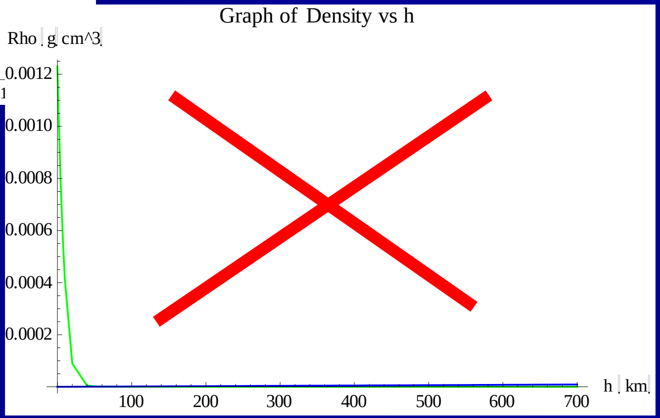
Modeling Part …

Modeling …..

```
f = Fit[data, {h, 1}, h]
(* To find a suitable polynomial or relationship *)
g2 = Plot[f, {h, hgh[[1]], hgh[[Length[hgh]]]},
  PlotStyle → {RGBColor[0, 0, 1], PointSize[0.02]}]
(* To plot the predicted model *)
Show[{g1, g2}]
```





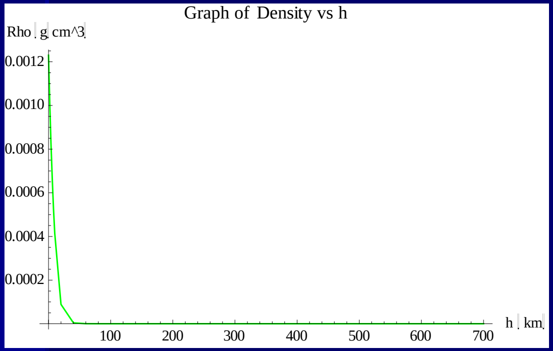


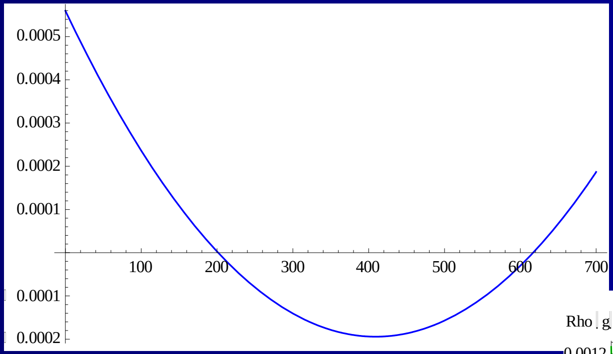
Modeling Part …

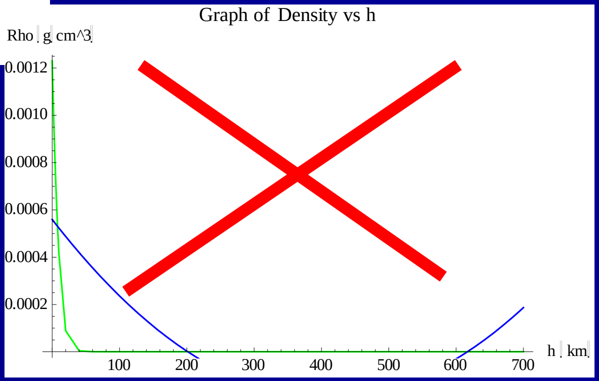
```
f = Fit[data, {h, 1}, h]
(* To find a suitable polynomial or relationship *)
g2 = Plot[f, {h, hgh[[1]], hgh[[Length[hgh]]]},
  PlotStyle → {RGBColor[0, 0, 1], PointSize[0.02]}]
(* To plot the predicted model *)
Show[{g1, g2}]
```

Modeling …..

{h^2, h, 1}



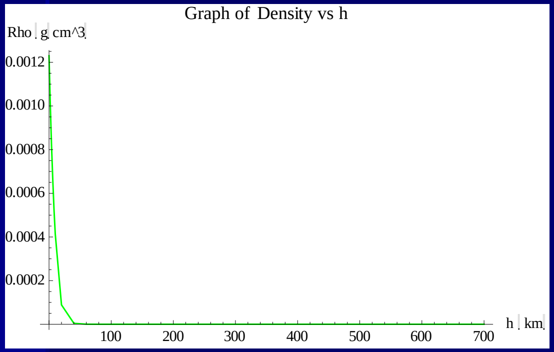




Modeling Part …

```
f = Fit[data, {h, 1}, h]
(* To find a suitable polynomial or relationship *)
g2 = Plot[f, {h, hgh[[1]], hgh[[Length[hgh]]]},
  PlotStyle → {RGBColor[0, 0, 1], PointSize[0.02]}]
(* To plot the predicted model *)
Show[{g1, g2}]
```

Modeling …..



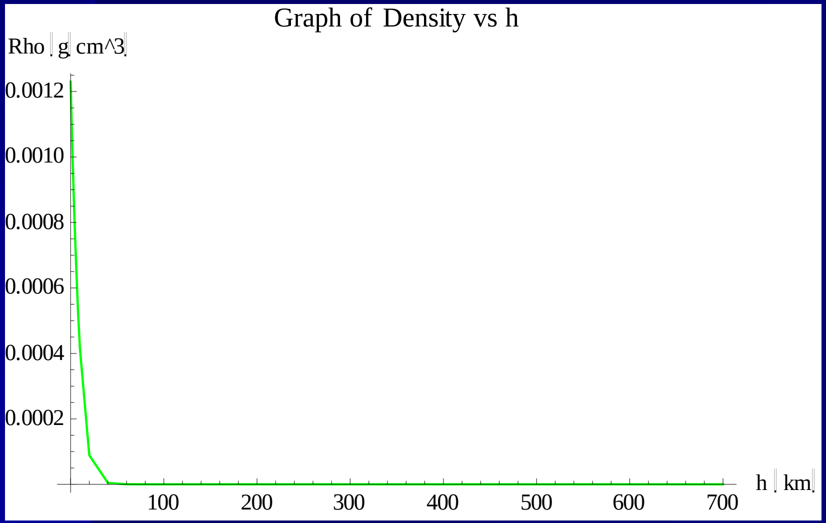
{h^2, h, 1}

Sin[x]	Cos[x]	Tan[x]
x^2	x^3	…….
1/x	1/x^2	1/x^3
Exp[x]	Exp[-x]	Exp[- a x]

Modeling Part …

Modeling …..

```
f = Fit[data, {h, 1}, h]
(* To find a suitable polynomial or relationship *)
g2 = Plot[f, {h, hgh[[1]], hgh[[Length[hgh]]]},
  PlotStyle → {RGBColor[0, 0, 1], PointSize[0.02]}]
(* To plot the predicted model *)
Show[{g1, g2}]
```



Exp[- a x]

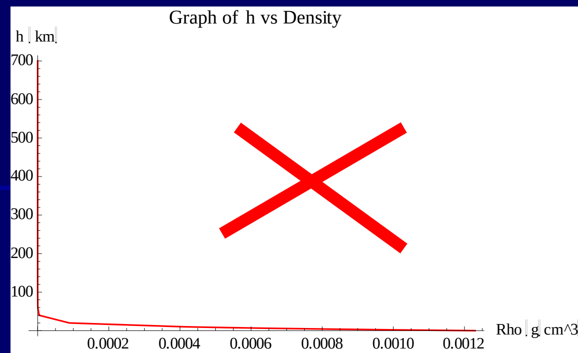
a = 1, 2, 3, …..

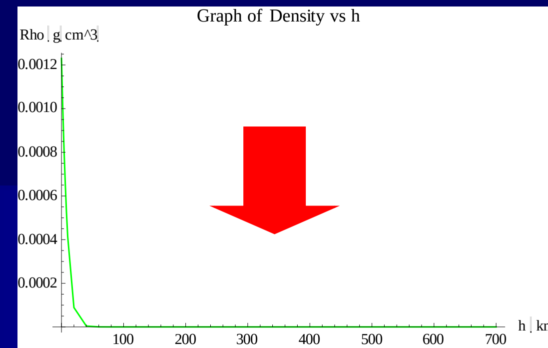
a = ½, 1/3, …

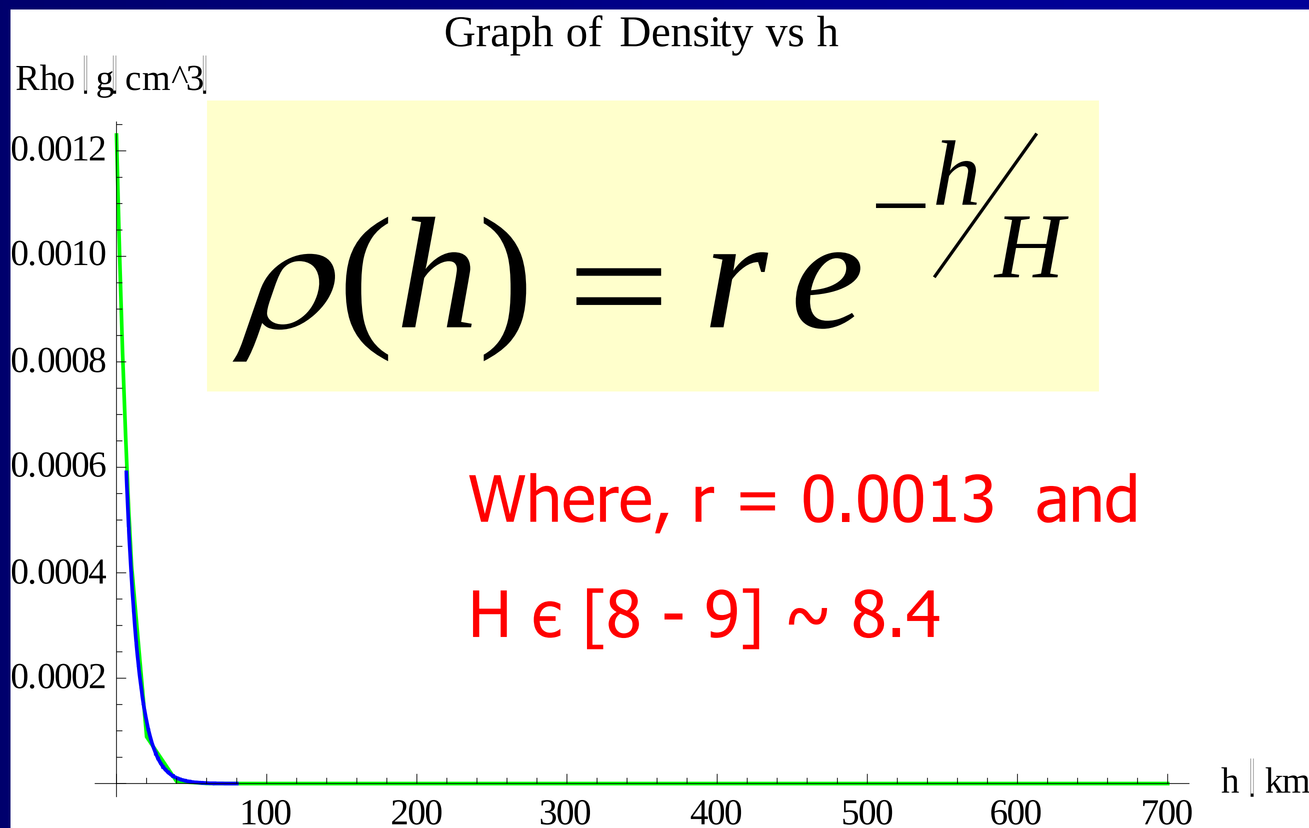
a = 1/8 - 1/9

The graph of h (in km) vs Density (in gr cm^(-3))

Altitude in km	Density in gr/cm^{-3}
0	1.23×10^{-3}
2	1.01×10^{-3}
4	8.19×10^{-4}
6	6.60×10^{-4}
8	5.26×10^{-4}
10	4.14×10^{-4}
20	8.89×10^{-5}
40	4.00×10^{-6}
60	3.06×10^{-7}
80	2.00×10^{-8}
100	4.97×10^{-10}
140	3.39×10^{-12}
180	5.86×10^{-13}
220	1.99×10^{-13}
260	8.04×10^{-14}
300	3.59×10^{-14}
400	6.50×10^{-15}
500	1.58×10^{-15}
600	4.64×10^{-16}
700	1.54×10^{-16}







$$\rho(h) = r\, e^{-h/H}$$

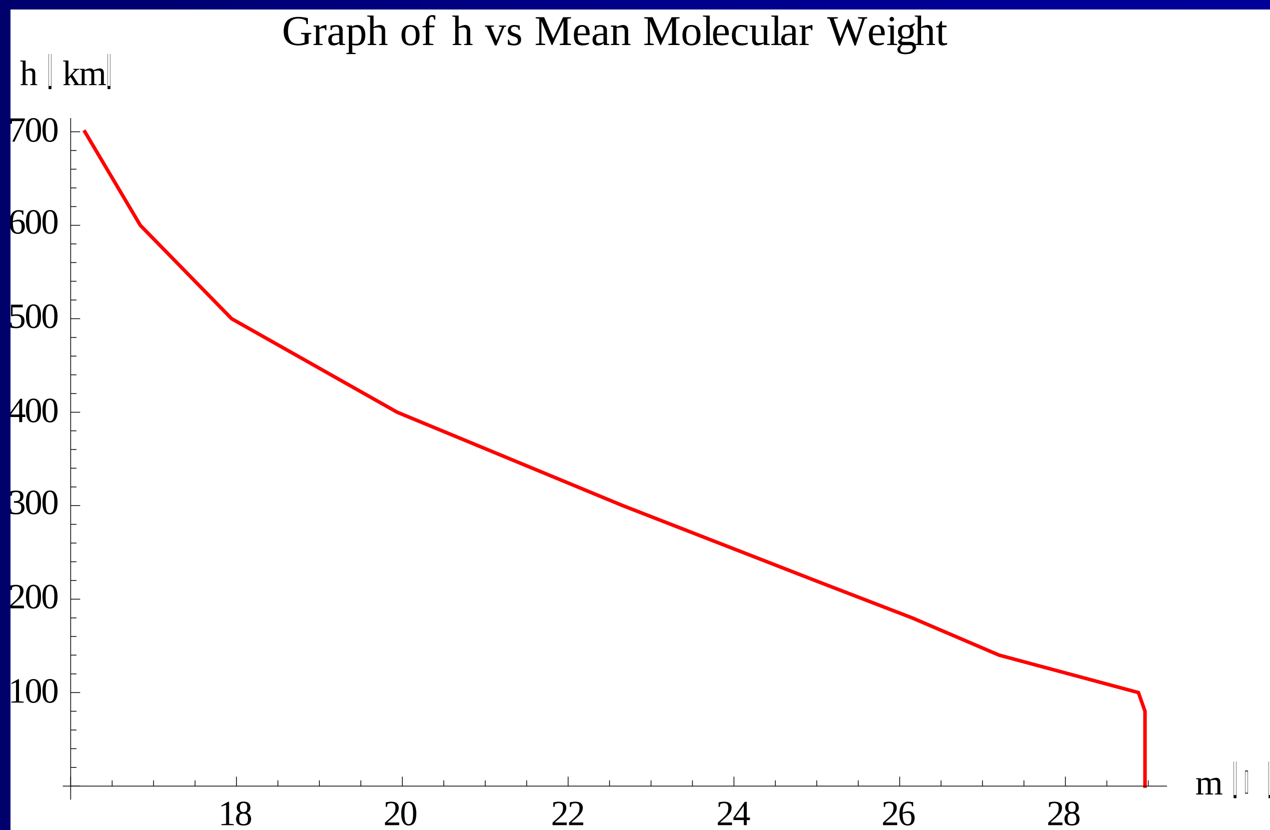
Where, r = 0.0013 and

H ϵ [8 - 9] ~ 8.4

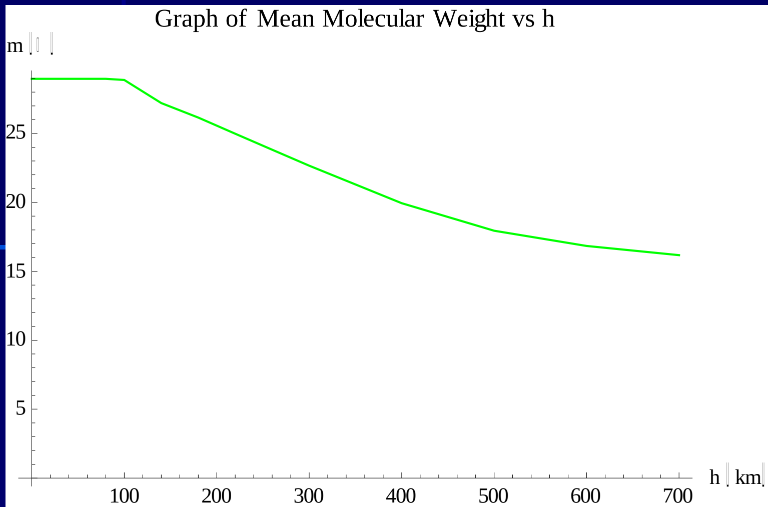
The graph of h (in km) vs Mean Molecular Weight

Altitude in km	Mean Mol. Weight
0	28.96
2	28.96
4	28.96
6	28.96
8	28.96
10	28.96
20	28.96
40	28.96
60	28.96
80	28.96
100	28.88
140	27.20
180	26.15
220	24.98
260	23.82
300	22.66
400	19.94
500	17.94
600	16.84
700	16.17

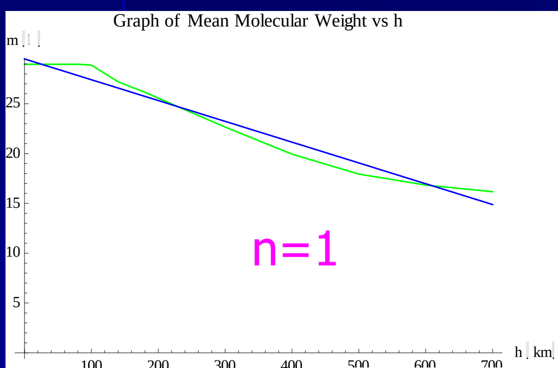
Enter the data as two 1D arrays....

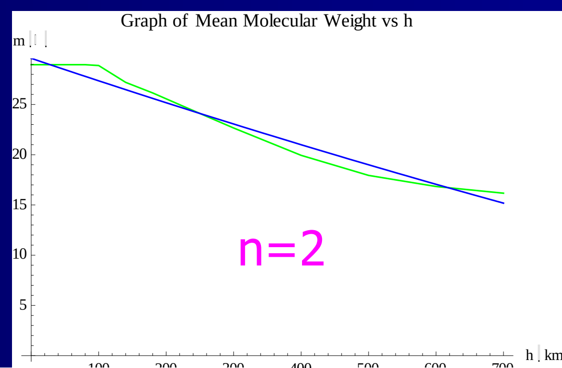


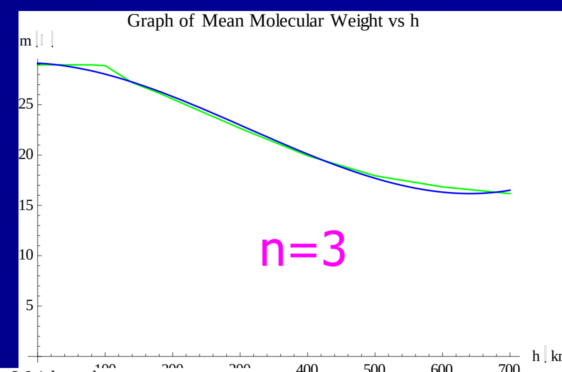
To model the data set …

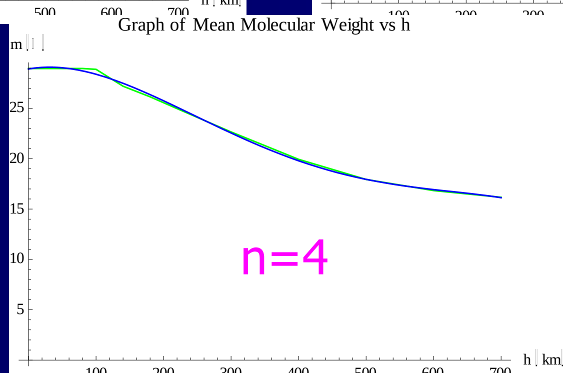


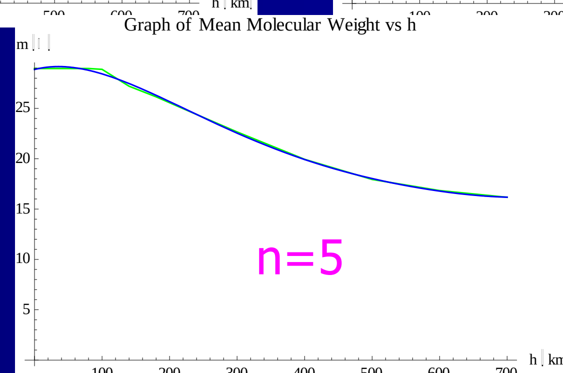
To Model …..





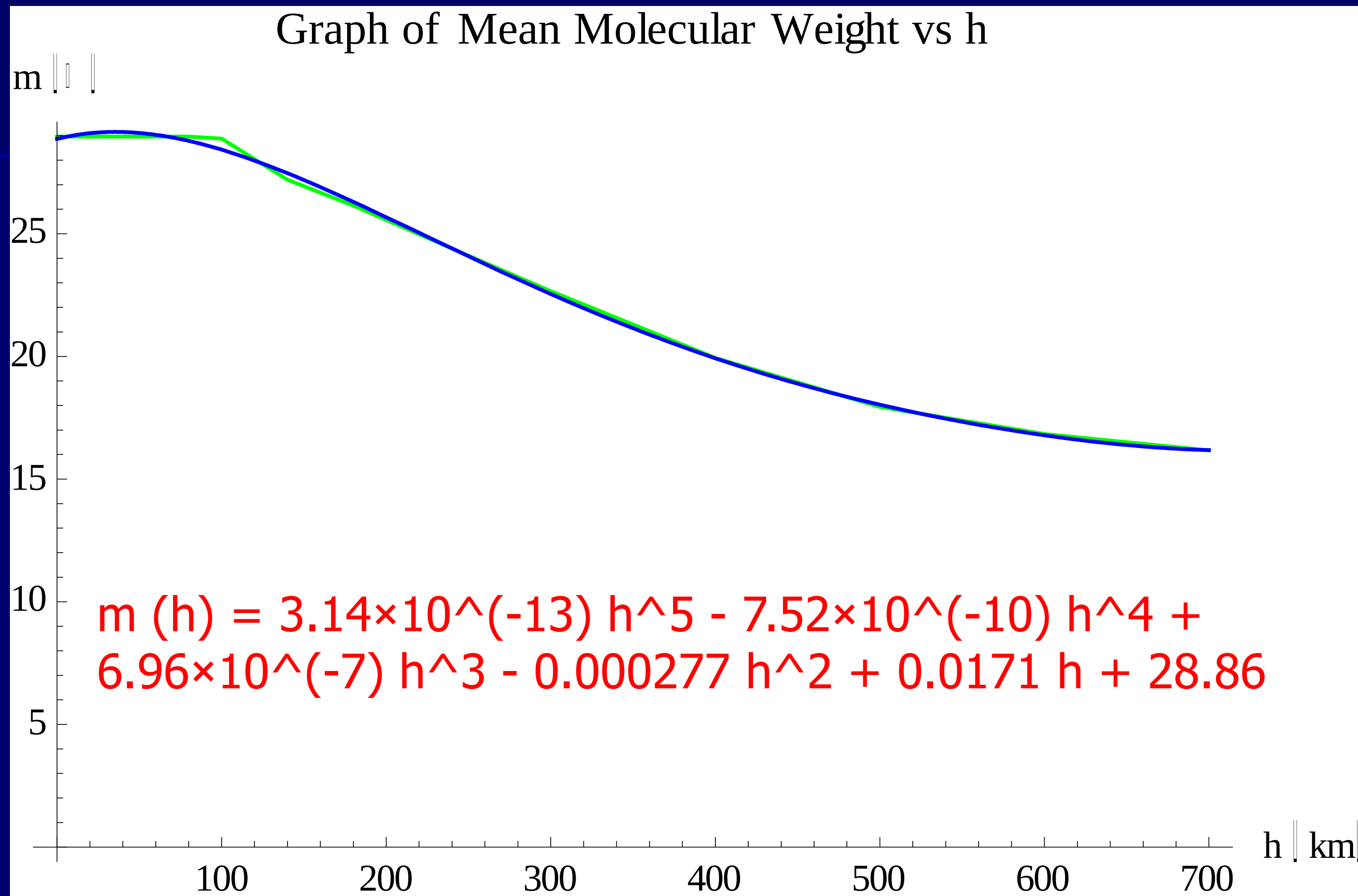






The graph of h (in km) vs m

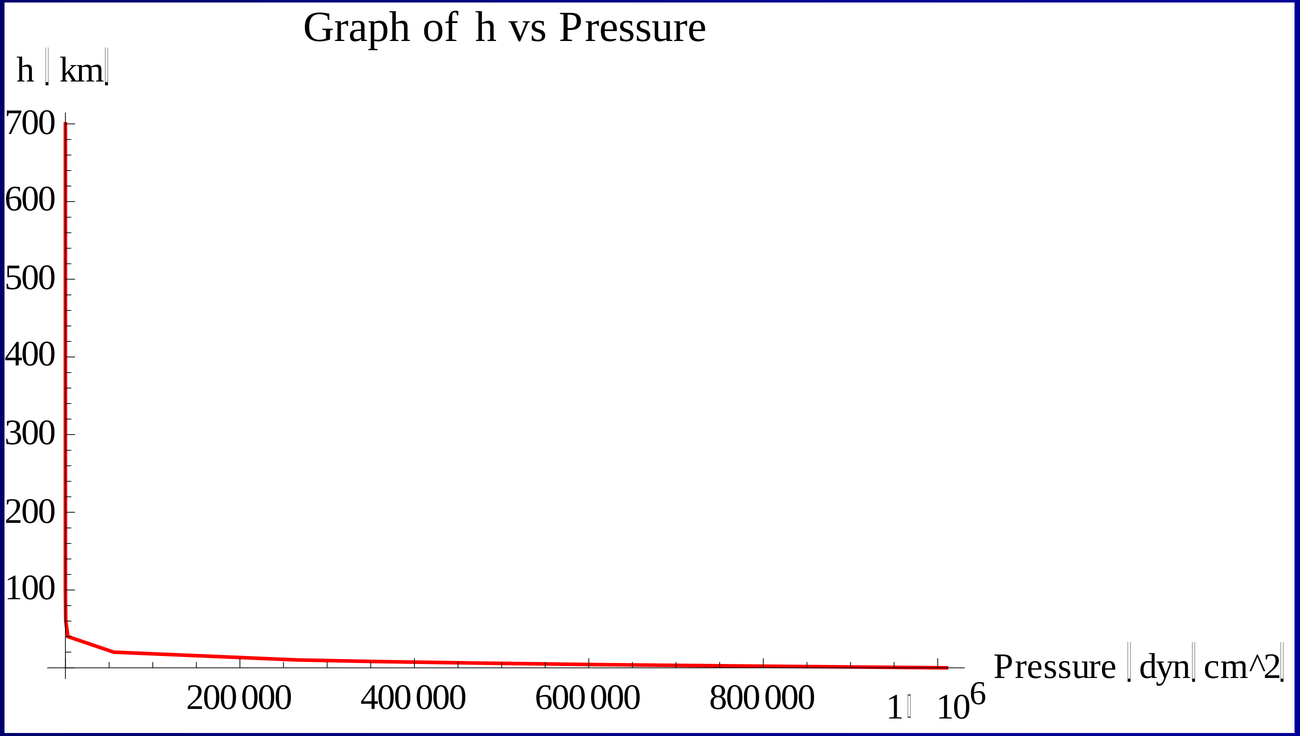
Altitude in km	Mean Mol. Weight
0	28.96
2	28.96
4	28.96
6	28.96
8	28.96
10	28.96
20	28.96
40	28.96
60	28.96
80	28.96
100	28.88
140	27.20
180	26.15
220	24.98
260	23.82
300	22.66
400	19.94
500	17.94
600	16.84
700	16.17



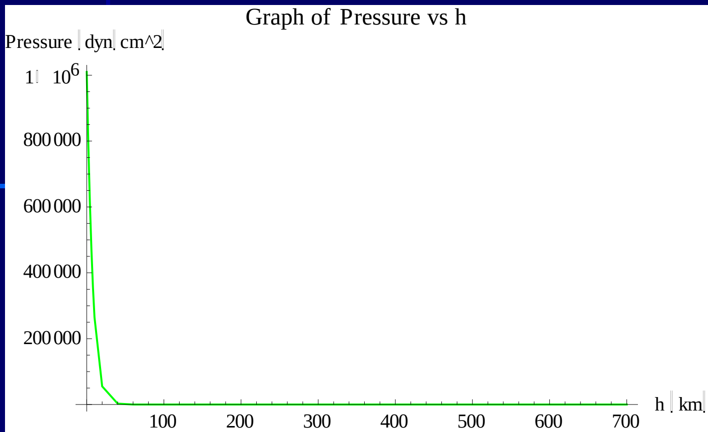
The graph of h (in km) vs Pressure (in dyn/cm^2)

Altitude in km	Pressure in dyn/cm^2
0	1.01×10^6
2	7.95×10^5
4	6.17×10^5
6	4.72×10^5
8	3.57×10^5
10	2.65×10^5
20	5.53×10^4
40	2.87×10^3
60	2.25×10^2
80	1.04×10
100	3.01×10^{-1}
140	7.41×10^{-3}
180	2.15×10^{-3}
220	8.58×10^{-4}
260	3.86×10^{-4}
300	1.88×10^{-4}
400	4.03×10^{-5}
500	1.10×10^{-5}
600	3.45×10^{-6}
700	1.19×10^{-6}

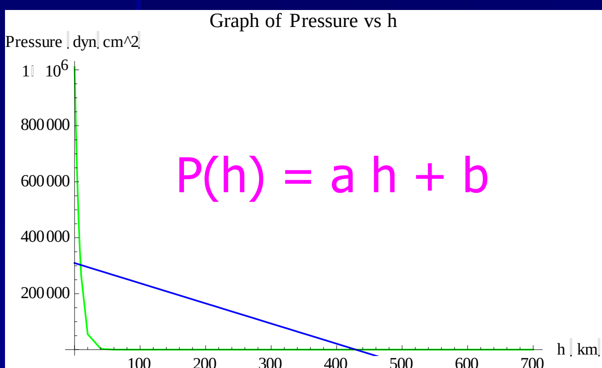
Enter the data as two 1D arrays….

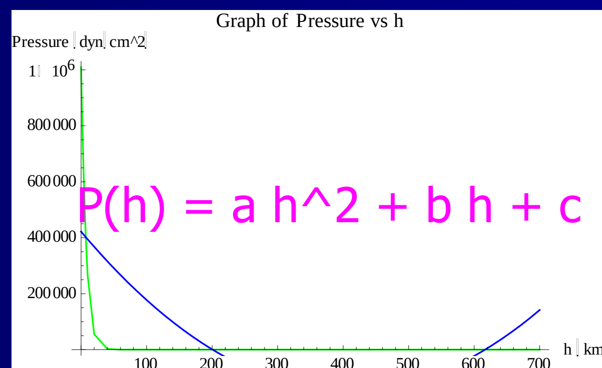


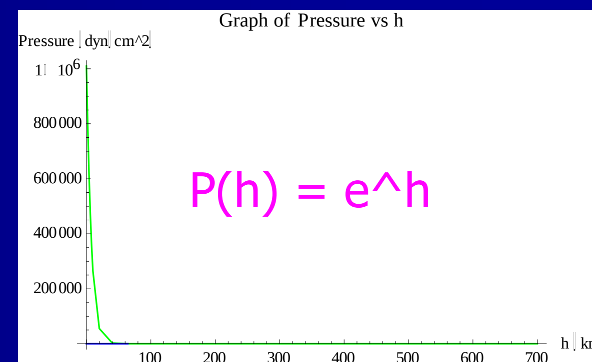
To model the data set …

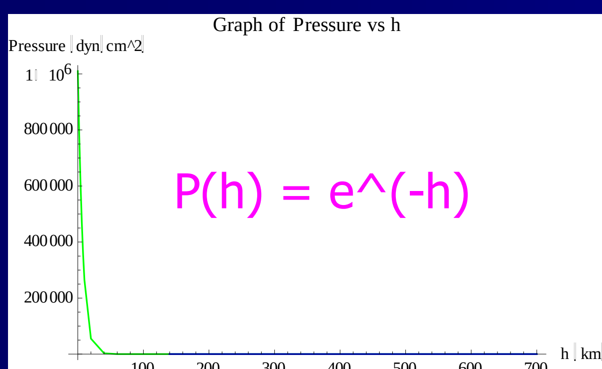


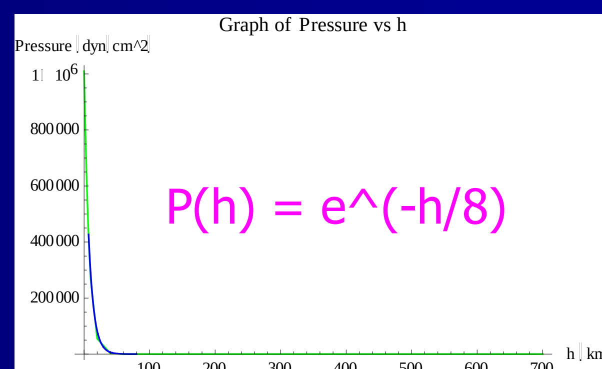
To Model …..





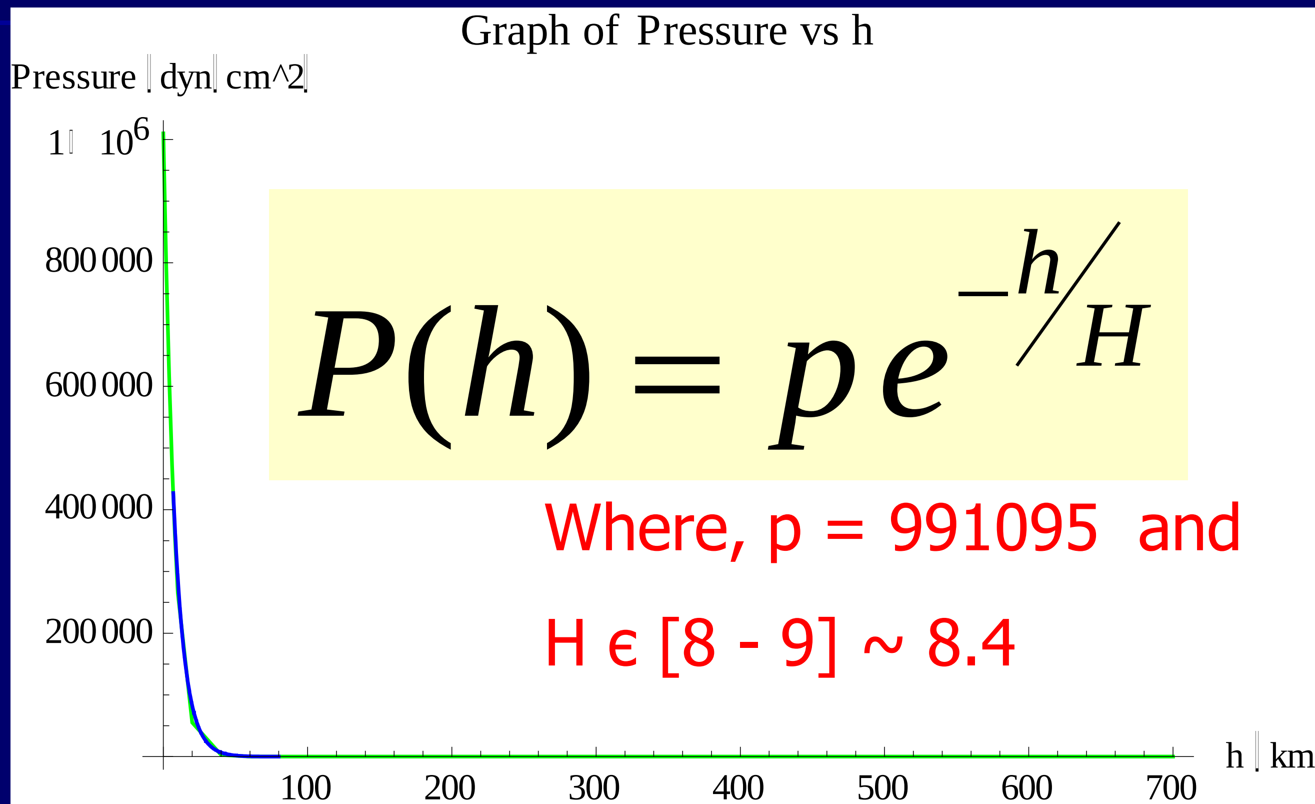






The graph of h (in km) vs Pressure (in dyn/cm^2)

Altitude in km	Pressure in dyn/cm^2
0	1.01×10^6
2	7.95×10^5
4	6.17×10^5
6	4.72×10^5
8	3.57×10^5
10	2.65×10^5
20	5.53×10^4
40	2.87×10^3
60	2.25×10^2
80	1.04×10
100	3.01×10^{-1}
140	7.41×10^{-3}
180	2.15×10^{-3}
220	8.58×10^{-4}
260	3.86×10^{-4}
300	1.88×10^{-4}
400	4.03×10^{-5}
500	1.10×10^{-5}
600	3.45×10^{-6}
700	1.19×10^{-6}



$$P(h) = p\, e^{-h/H}$$

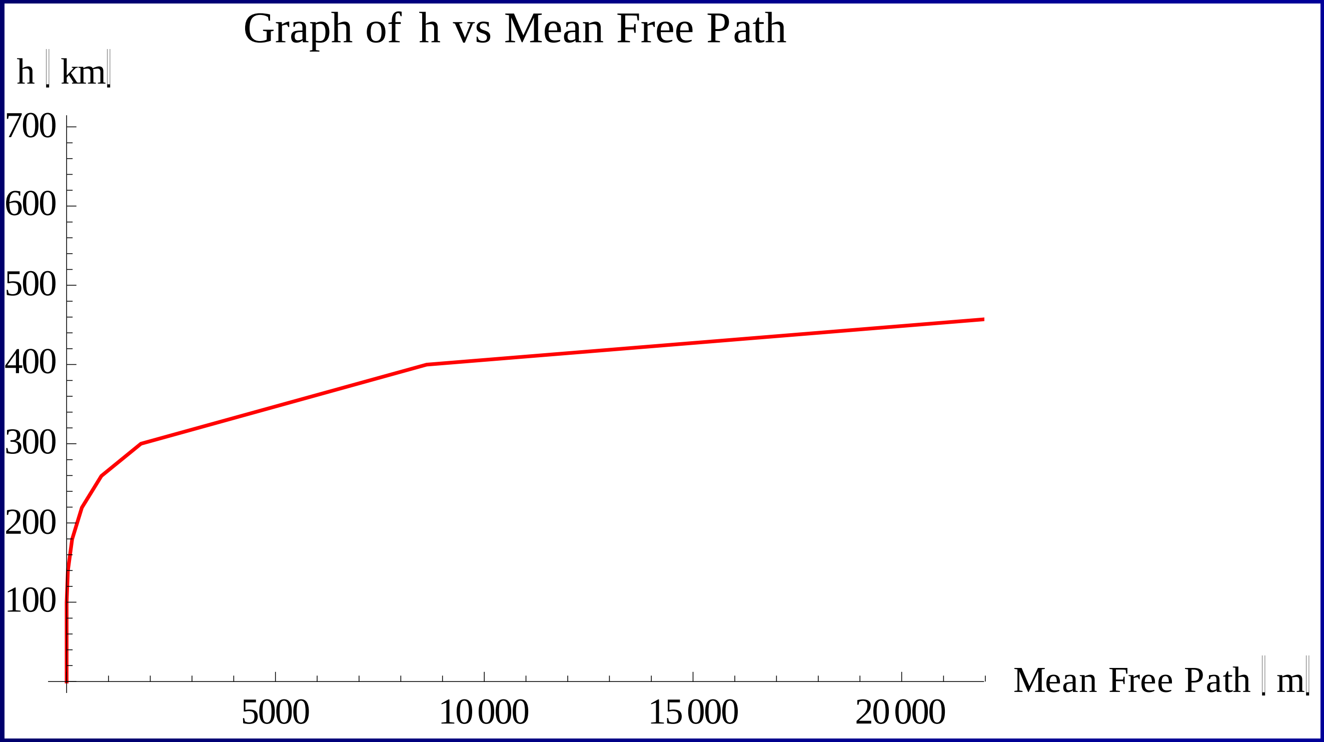
Where, p = 991095 and

H є [8 - 9] ~ 8.4

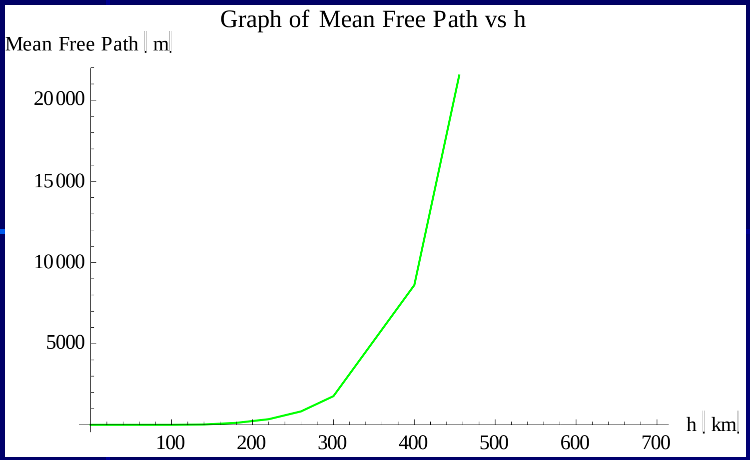
The graph of h (in km) vs Mean Free Path (in m)

Altitude in km	Mean Free Path in m
0	6.63×10^{-8}
2	8.07×10^{-8}
4	9.92×10^{-8}
6	1.23×10^{-7}
8	1.55×10^{-7}
10	1.96×10^{-7}
20	9.14×10^{-7}
40	2.03×10^{-5}
60	2.66×10^{-4}
80	4.07×10^{-3}
100	1.63×10^{-1}
140	2.25×10
180	1.25×10^{2}
220	3.52×10^{2}
260	8.31×10^{2}
300	1.77×10^{3}
400	8.61×10^{3}
500	3.19×10^{4}
600	1.02×10^{5}
700	2.95×10^{5}

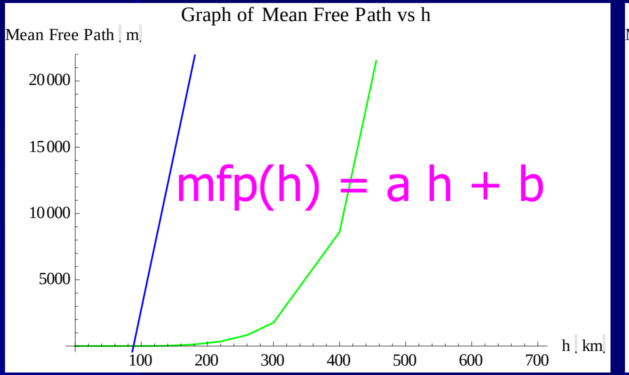
Enter the data as two 1D arrays....

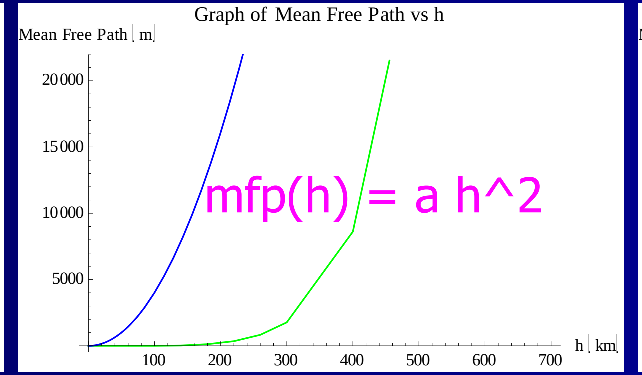


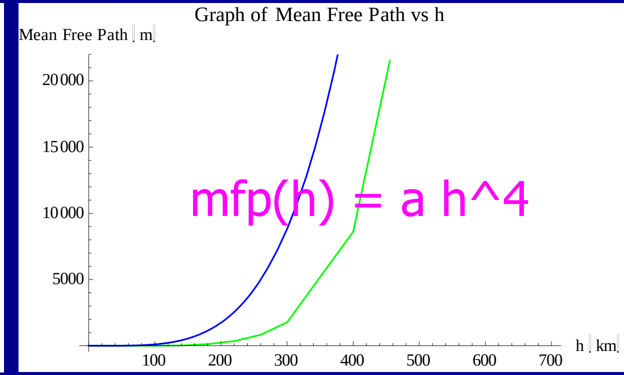
To model the data set …

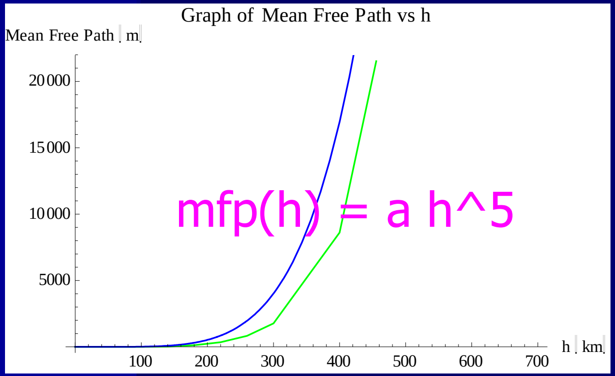


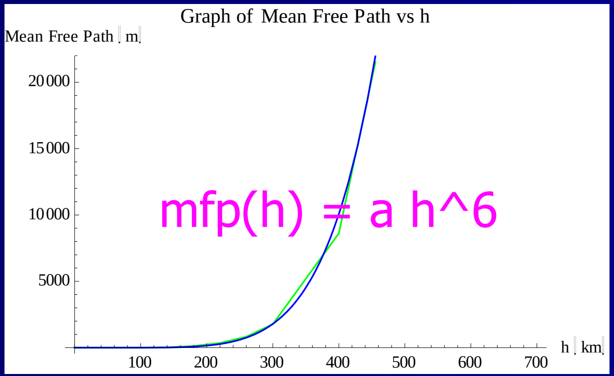
To Model …..

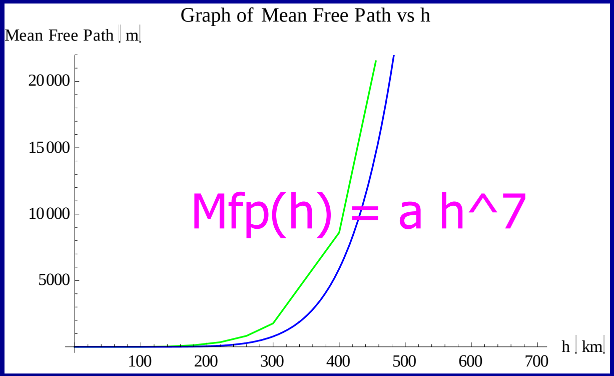






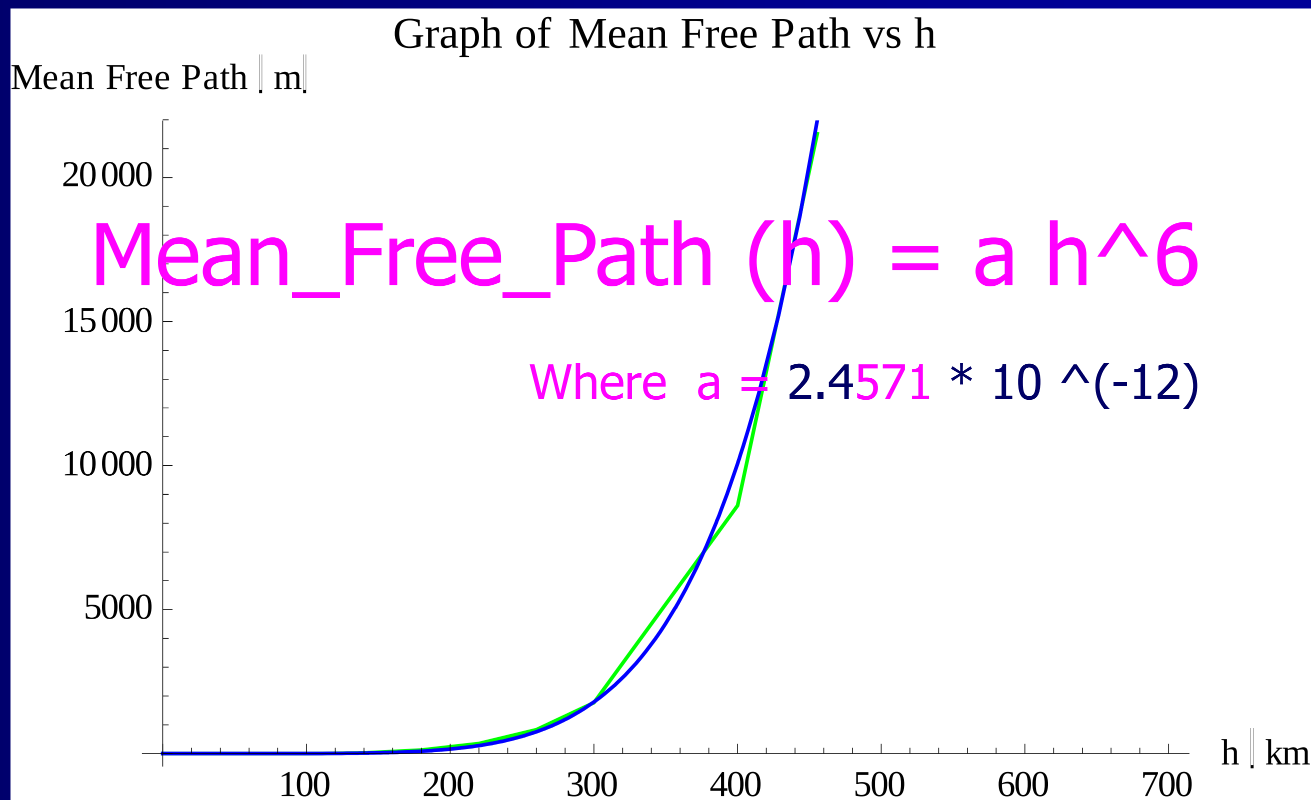






The graph of h (in km) vs Mean Free Path (in m)

Altitude in km	Mean Free Path in m
0	6.63×10^{-8}
2	8.07×10^{-8}
4	9.92×10^{-8}
6	1.23×10^{-7}
8	1.55×10^{-7}
10	1.96×10^{-7}
20	9.14×10^{-7}
40	2.03×10^{-5}
60	2.66×10^{-4}
80	4.07×10^{-3}
100	1.63×10^{-1}
140	2.25×10
180	1.25×10^{2}
220	3.52×10^{2}
260	8.31×10^{2}
300	1.77×10^{3}
400	8.61×10^{3}
500	3.19×10^{4}
600	1.02×10^{5}
700	2.95×10^{5}



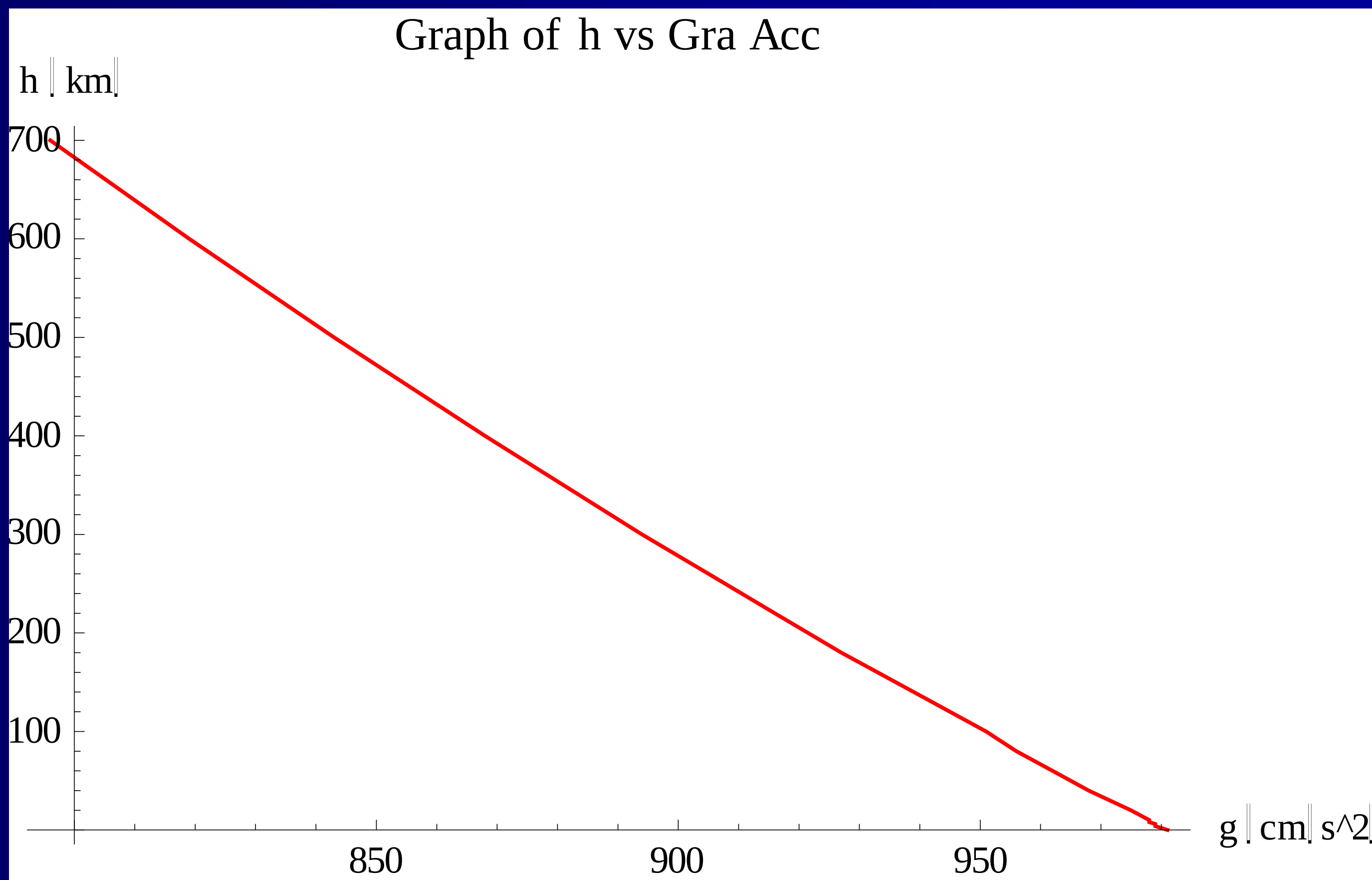
Mean_Free_Path (h) = a h^6

Where a = 2.4571 * 10 ^(-12)

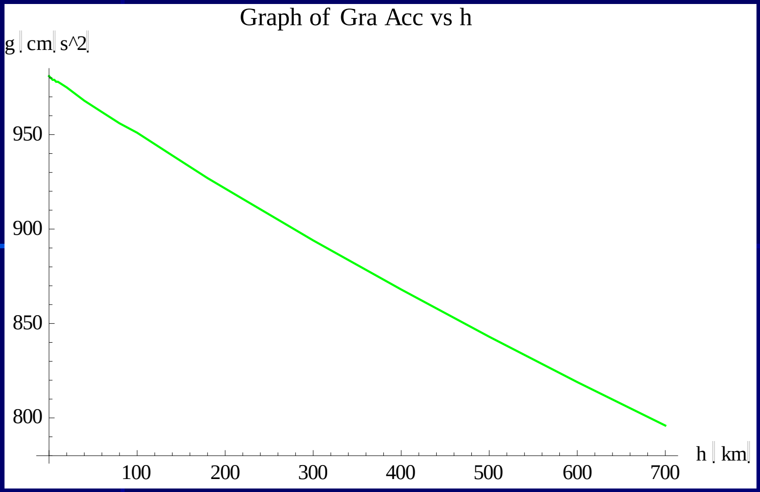
The graph of h (in km) vs Gravitational Acceleration (in cm/s^2)

Altitude in km	Accel. Grav. in cm/s²
0	981
2	980
4	979
6	979
8	978
10	978
20	975
40	968
60	962
80	956
100	951
140	939
180	927
220	916
260	905
300	894
400	868
500	843
600	819
700	796

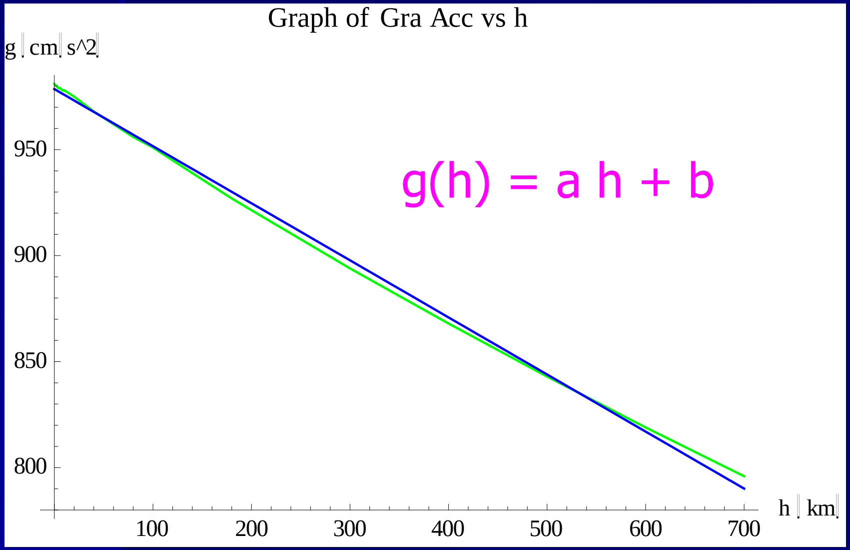
Enter the data as two 1D arrays....

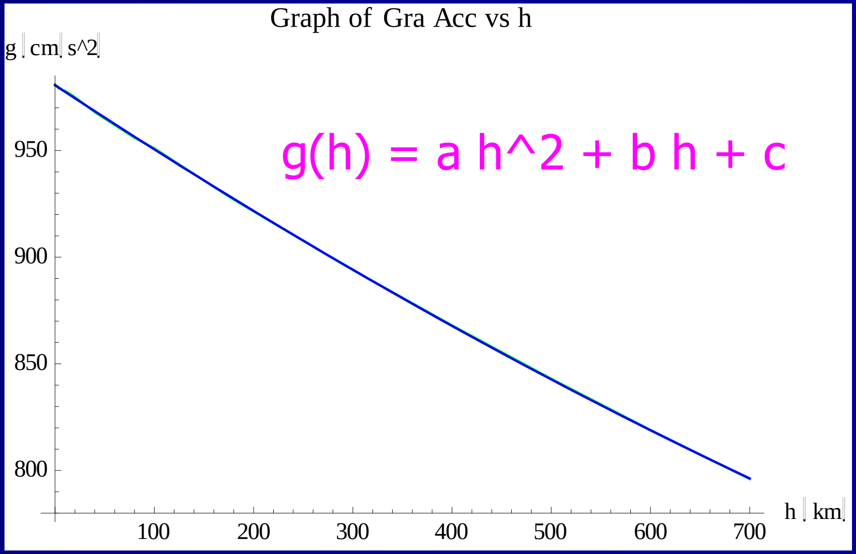


To model the data set …



To Model …..





The graph of h (in km) vs Gravitational Acceleration (in cm/s^2)

Altitude in km	Accel. Grav. in cm/s^2
0	981
2	980
4	979
6	979
8	978
10	978
20	975
40	968
60	962
80	956
100	951
140	939
180	927
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260	905
300	894
400	868
500	843
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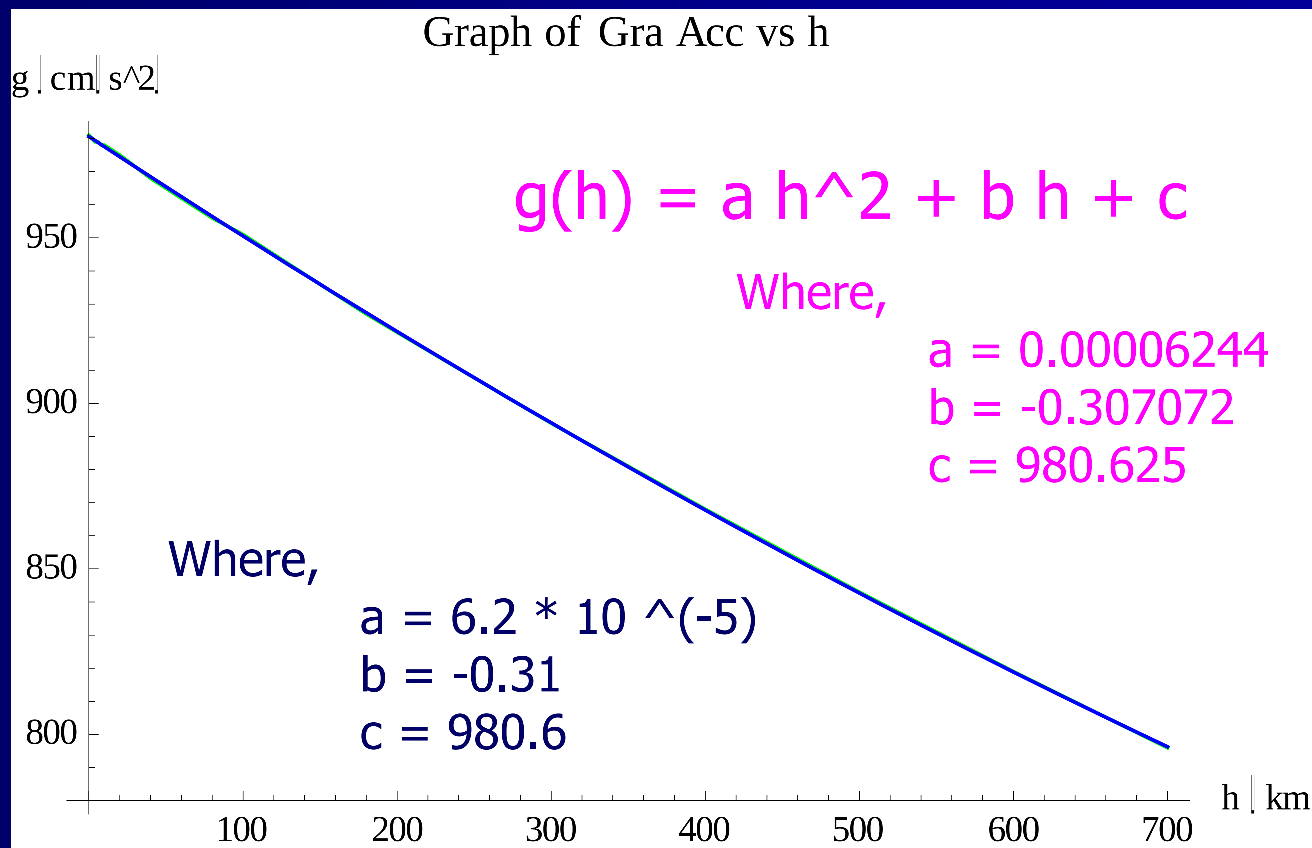
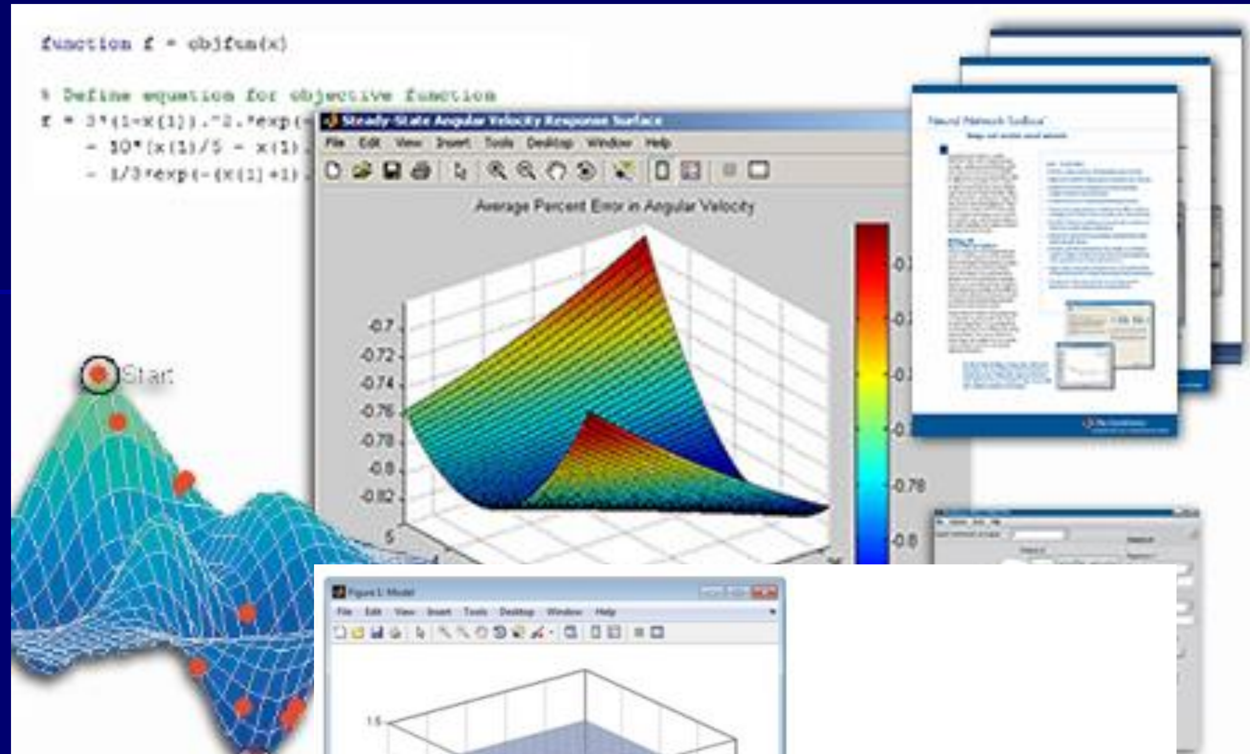
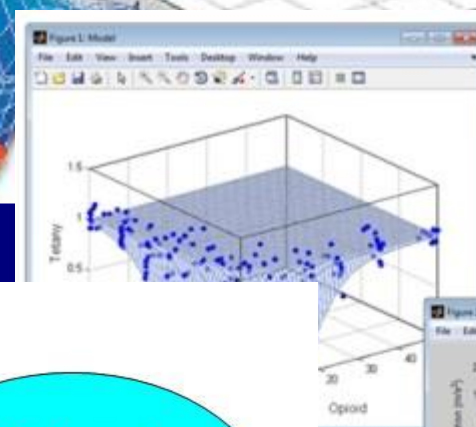
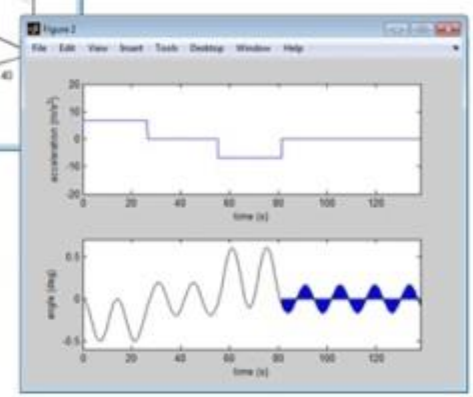
700	796

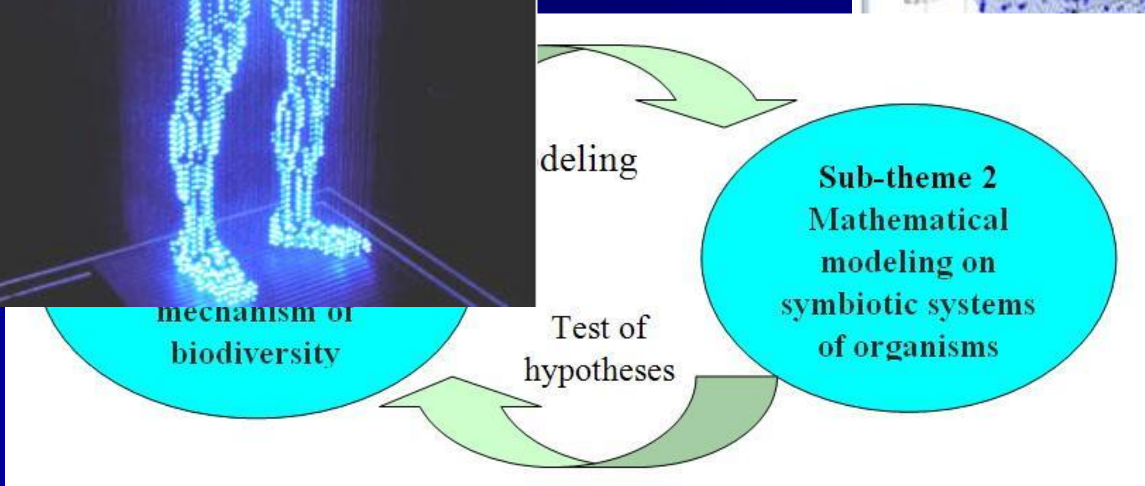


Part by Part Modeling…









The graph of h (in km) vs T (in K)

Altitude in km	Temperature in °K
0	288
2	275
4	262
6	249
8	236
10	223
20	217
40	250
60	256
80	181
100	210
140	714
180	1156
220	1294
260	1374
300	1432
400	1487
500	1499
600	1506
700	1508

```
(* This program is to plot the Earth's parameters w.r.t altitude from the Earth Surface *)

hgh = {0, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100, 140, 180, 220, 260, 300, 400, 500, 600, 700} ; (* Altitude *)
tem = {288, 275, 262, 249, 236, 223, 217, 250, 256, 181, 210, 715, 1156, 1294, 1374, 1432, 1487, 1499, 1506, 1508} ;(* Temperature *)
d1 = Transpose[{tem, hgh}]; (* to get the h vs T data set *)
ListPlot[d1, PlotJoined → True, PlotStyle → {RGBColor[1, 0, 0], PointSize[0.02]}, PlotLabel → "Graph of h vs Temperature", AxesLabel → {"T (K)", "h (km)"}] ;
(* To Plot the h vs T graph *)

(* To model the data set in T vs h format *)
data = Transpose[{hgh, tem}];
g1 = ListPlot[data, PlotStyle → {RGBColor[0, 1, 0], PointSize[0.02]}, PlotLabel → "Graph of Temperature vs h", AxesLabel → {"h (km)", "T (K)"}] ;
nnn = 11; (* Division point of the height *)
tt1 = Take[data, nnn];
tt2 = Take[data, - (nnn - 1)];
gtt1 = ListPlot[tt1, PlotStyle → {RGBColor[0, 1, 0], PointSize[0.02]}, PlotLabel → "Graph of Temperature vs h", AxesLabel → {"h (km)", "T (K)"}]
gtt2 = ListPlot[tt2, PlotStyle → {RGBColor[0, 1, 1], PointSize[0.02]}, PlotLabel → "Graph of Temperature vs h", AxesLabel → {"h (km)", "T (K)"}]
s1 = Show[{gtt1, gtt2}];
(* g1=ListPlot[data,PlotJoined→True,PlotStyle→ {RGBColor[0,1,0], PointSize[0.02]}, PlotLabel→ "Graph of Temperature vs h",AxesLabel→ {"h (km)","T (K)"}] *)
(* To Plot the T vs h graph *)

n1 = 5; (* Order of the polynomai *)
ph1 = Table[h^i, {i, 0, n1}] ; (* to create coifficient of polynomial *)
Print["Suitable First Polynomail Function is : "]
f1 = Fit[tt1, ph1, h] (* To find a suitable polynomial or relationship *)
gf1 = Plot[f1, {h, hgh[[1]], hgh[[nnn]]}, PlotStyle → {RGBColor[0, 0, 1], PointSize[0.02]}, PlotRange → Full] ; (* To plot the predicted model *)
s1 = Show[{gtt1, gf1}]

n2 = 5; (* Order of the polynomai *)
ph2 = Table[h^i, {i, 0, n2}] ; (* to create coifficient of polynomial *)
Print["Suitable Second Polynomail Function is : "]
f2 = Fit[tt2, ph2, h] (* To find a suitable polynomial or relationship *)
gf2 = Plot[f2, {h, hgh[[nnn]], hgh[[Length[hgh]]]}, PlotStyle → {RGBColor[1, 0, 1], PointSize[0.02]}, PlotRange → Full] ; (* To plot the predicted model *)
s2 = Show[{gtt2, gf2}]
mm1 = f1 /. h -> Take[hgh, nnn - 1];
mm2 = f1 /. h -> Take[hgh, - (nnn - 1)];
tempm = Join[mm1, mm2];
datam = Transpose[{hgh, tempm}];
gtm = ListPlot[datam, PlotStyle → {RGBColor[0, 1, 1], PointSize[0.02]}, PlotLabel → "Graph of Temperature vs h", AxesLabel → {"h (km)", "T (K)"}] ;
Show[{g1, gtm}]
```

The graph of h (in km) vs T (in K)

Altitude in km	Temperature in °K
0	288
2	275
4	262
6	249
8	236
10	223
20	217
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180	1156
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260	1374
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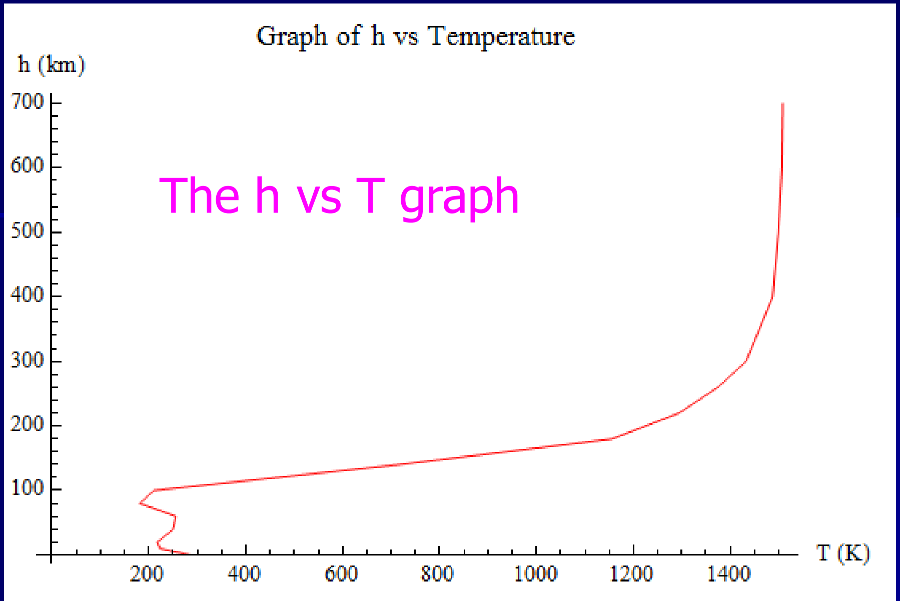

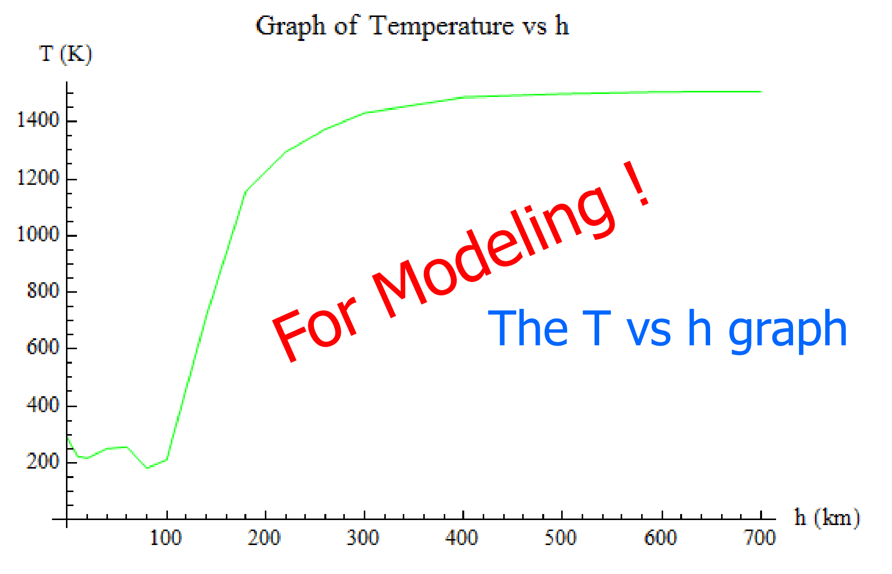

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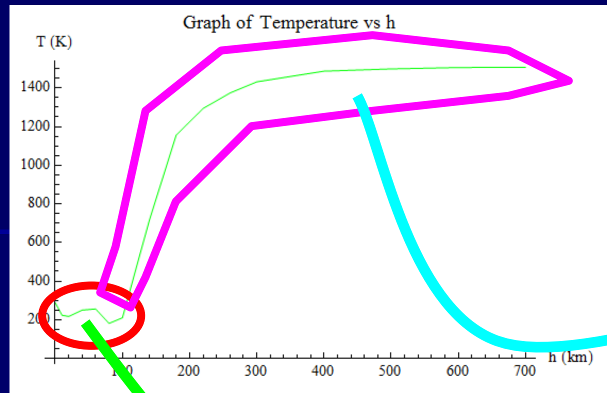

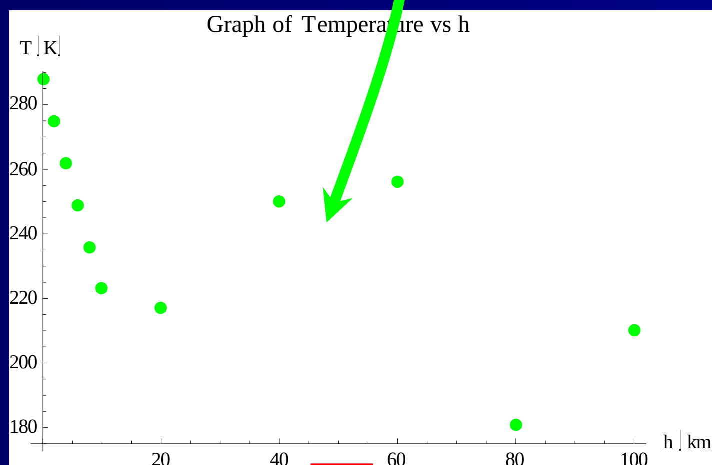

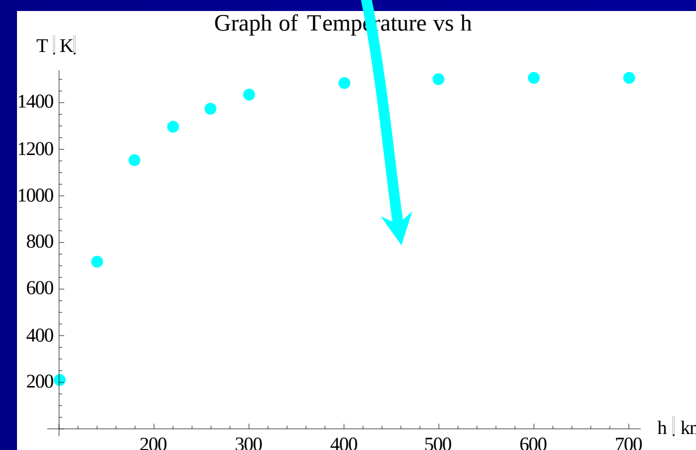

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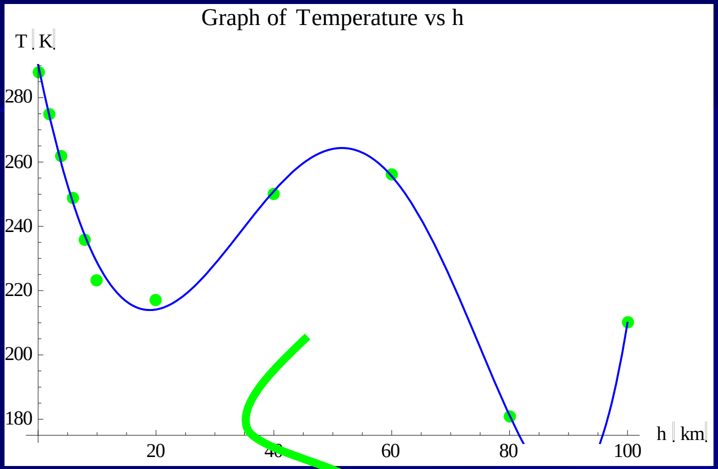

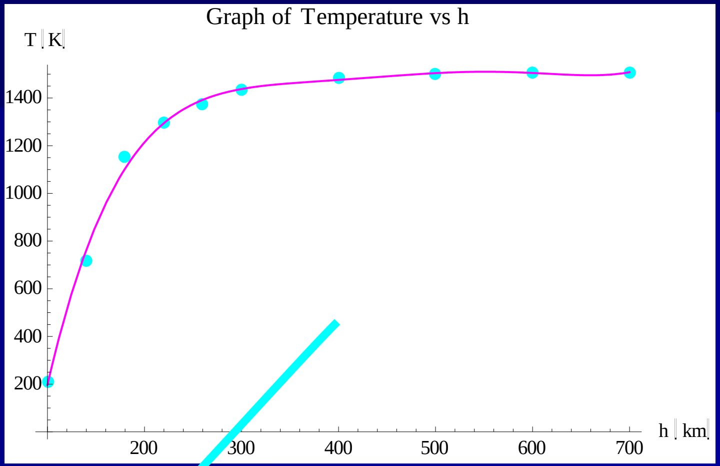

T1(h) = 3.40×10^(-7) h^5 - 0.000042 h^4 - 0.0013 h^3 + 0.299 h^2 - 8.98185 h + 290.294

T2(h) = 2.0×10^(-10) h^5 - 4.81×10^(-7) h^4 + 0.00045 h^3 - 0.2089 h^2 + 47.7 h - 2895.7

Altitude in km	Temperature in °K
0	288
2	275
4	262
6	249
8	236
10	223
20	217
40	250
60	256
80	181
100	210
140	714
180	1156
220	1294
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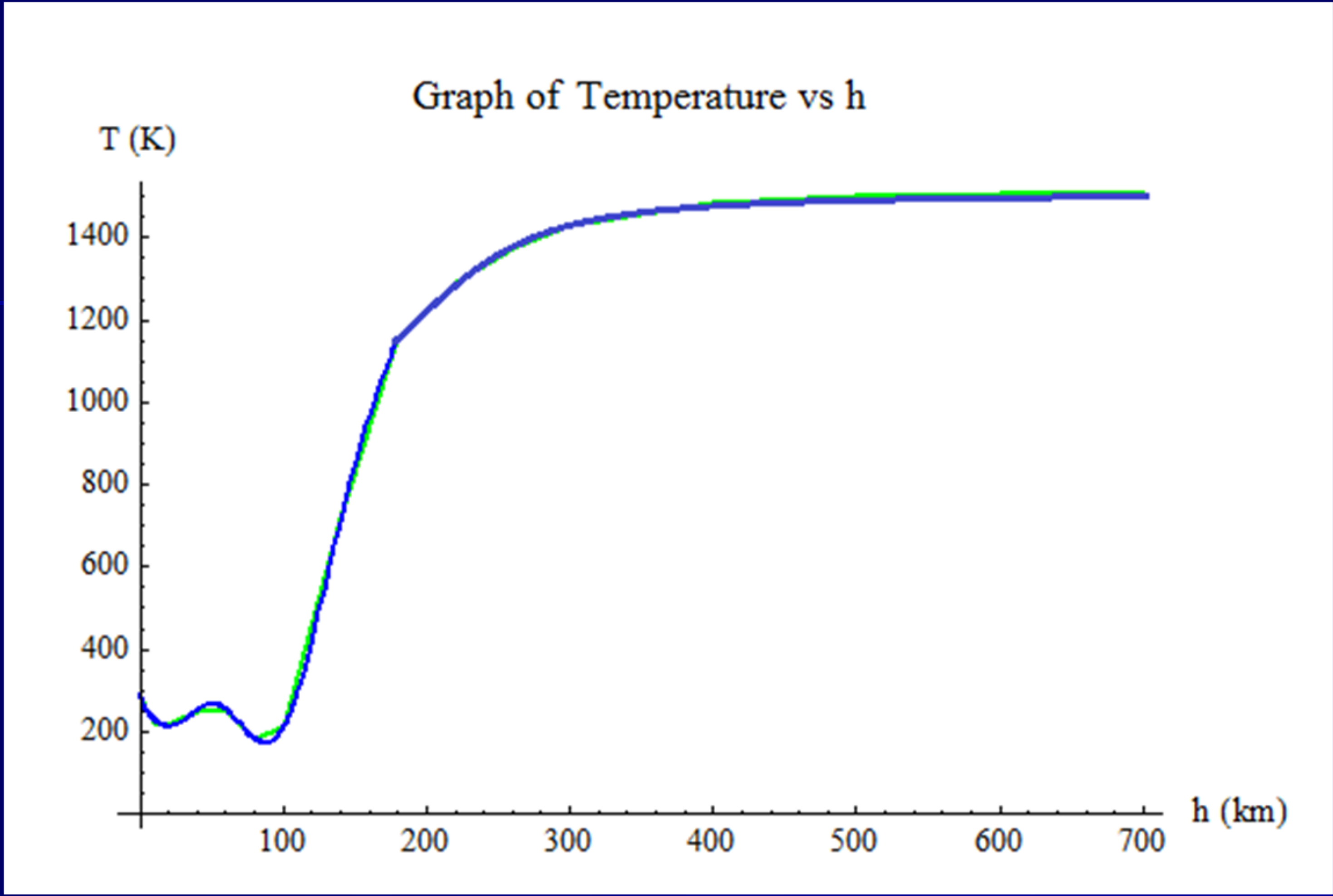


$$T(h) = \begin{cases} 3.4\times10^{-7}h^5 - 4.2\times10^{-5}h^4 - 1.3\times10^{-3}h^3 + 2.9\times10^{-1}h^2 - 8.98h + 290.3 & 0km \le h \le 100km \\ 2.0\times10^{-10}h^5 - 4.8\times10^{-7}h^4 + 4.5\times10^{-4}h^3 - 2.1\times10^{-1}h^2 + 47.7h - 2895.7 & 100km < h \le 700km \end{cases}$$

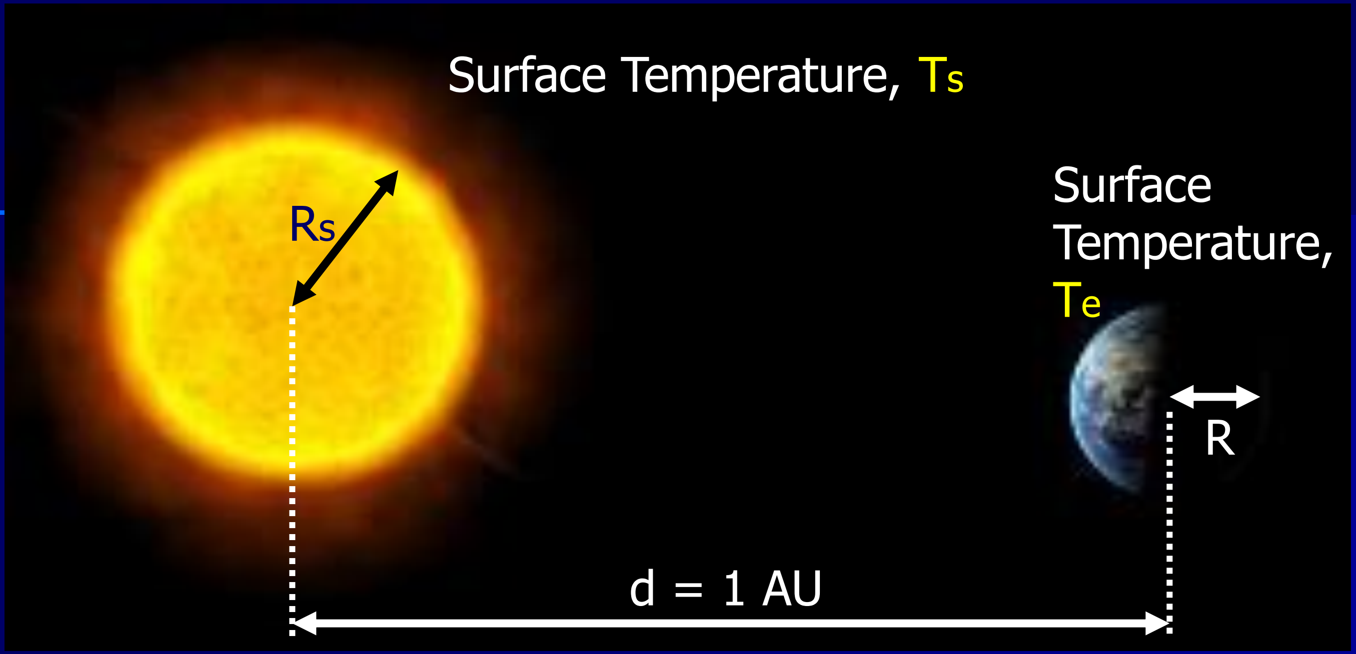
Thank You !

Planetary Atmospheres

Planetary Atmospheres

- Formation and Evolution of Planetary Atmospheres
- The Structure of the Terrestrial Atmosphere
- **The Temperature of the Neutral Atmosphere**
- The Escape of the Atmospheric Gases
- The Atmospheres of the Planets

The Temperature of the Neutral Atmosphere



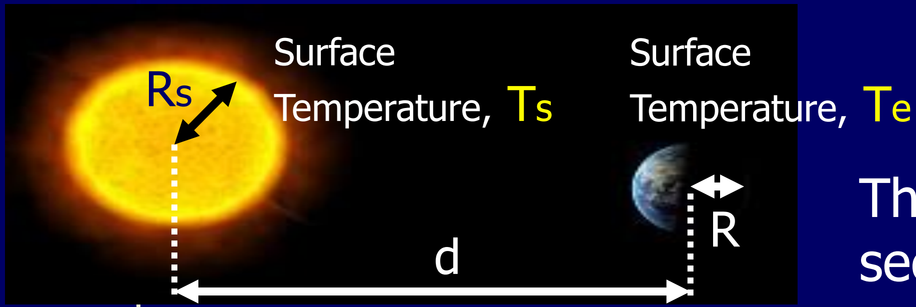
AL Method

$$\sigma = 5.67 \times 10^{-8}\ Js^{-1}m^{-2}K^{-4}$$

$$T_s = 5778K \ \ (\sim 6000K)$$

$$R_s = 695500km \ (\sim 7 \times 10^5\ km)$$

$$d = 149598000km \ (1AU)$$



Using Stephan's Law;

The **Energy** emitted per unit area, per second by the Sun = $E = \sigma T_S^4$

The **Total Energy** emitted per second by the Sun = $\sigma T_S^4 \times 4\pi R_S^2$

The Energy Density per second (Energy per unit area) at our orbit = $\dfrac{\sigma T_S^4 \times 4\pi R_S^2}{4\pi d^2}$

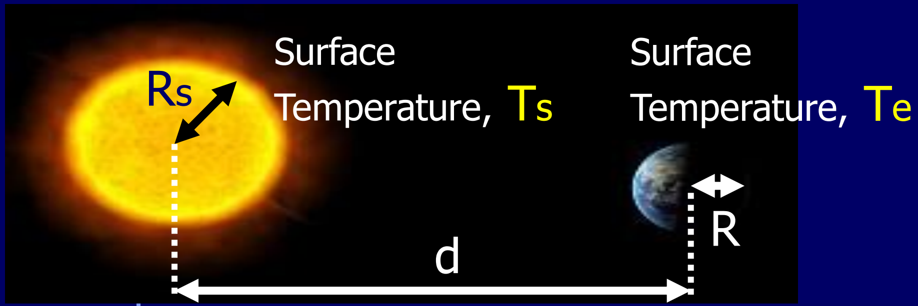
Where this ***d*** is distance from the Sun to the Earth's Orbit (1.0 AU)

The Total Energy to the Earth = $\dfrac{\sigma T_S^4 \times 4\pi R_S^2}{4\pi d^2} \times \pi R^2$ → 1

The Energy absorbed by the Earth = $e\sigma T_e^4 \times 2\pi R^2$ → 2

Using Stephan's Law

Where this ***e*** is emissivity of the Earth (Factor per BB, for BB ; e=1)



Connect equation 1 & 2 ;

$$\Rightarrow \quad \frac{\sigma T_S^{\,4} \times 4\pi R_S^{\,2}}{4\pi d^2} \times \pi R^2 = e\,\sigma T_e^{\,4} \times 2\pi R^2$$

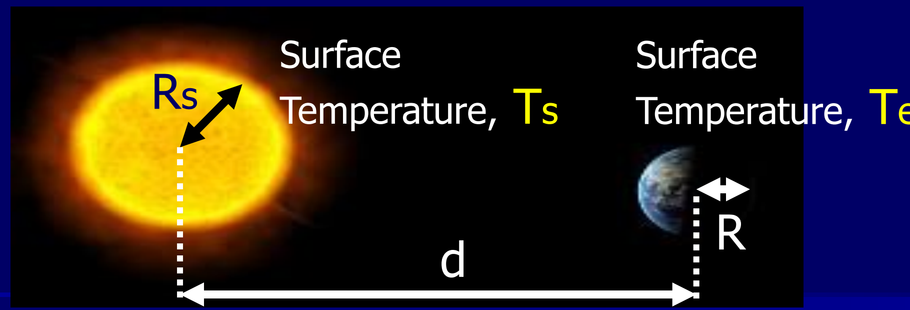
Where *e* should be [0 – 1]

$$\Rightarrow \quad T_e^{\,4} = \frac{\sigma T_S^{\,4} \times 4\pi R_S^{\,2} \times \pi R^2}{4\pi d^2 \times e\ \sigma \times 2\pi R^2} \quad \Rightarrow \quad T_e = \left(\frac{T_S^{\,4} \times R_S^{\,2}}{e\ d^2 \times 2} \right)^{1/4}$$

$\Rightarrow$

$$T_e = \{1047.62, 589.123, 393.97, 340.13, 332.121, 331, 288\}\ C$$

???



Space Physics 01.nb * - Wolfram Mathematica 10.0

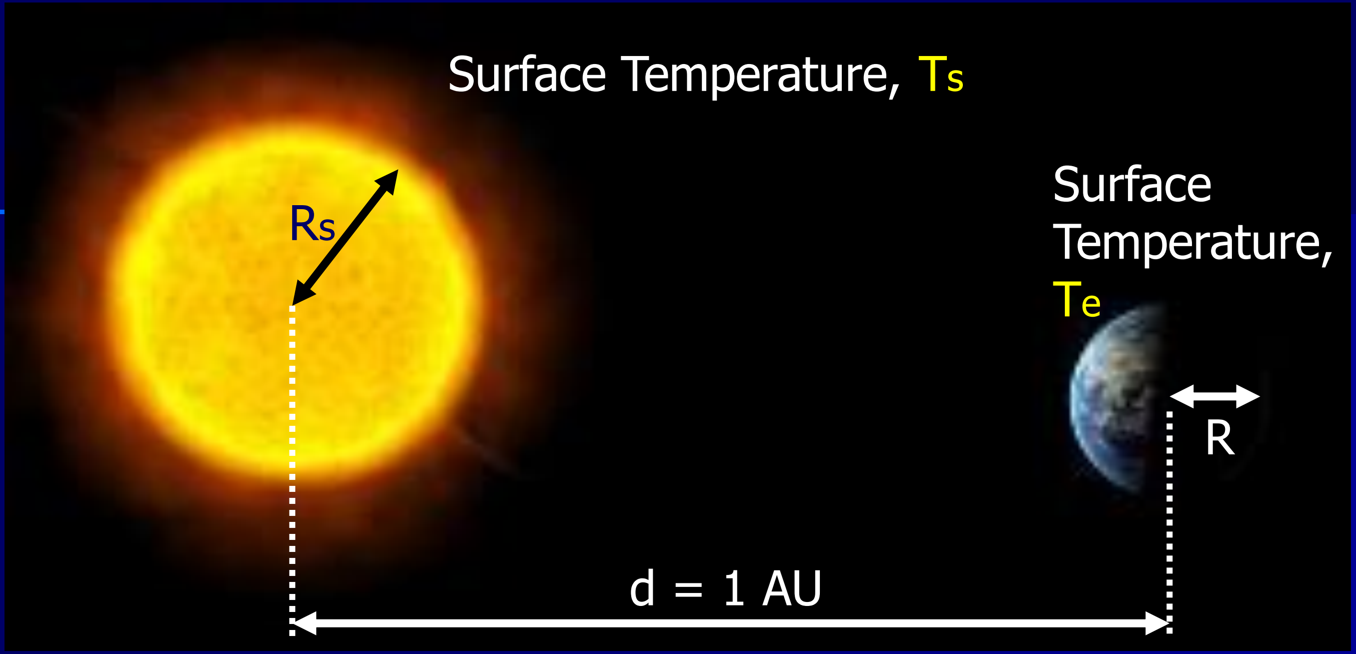
File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

```
In[61]:= sig = 5.67 * (10 ^ (-8)); (* in J/s m^2 K^4 *)
ts = 5778; (* in K *)
rs = 695500 * 1000; (* in m *)
re = 6400 * 1000; (* in m *)
d = 149598000 * 1000; (* in m *)
e = {0.01, 0.1, 0.5, 0.9, 0.99, 1.0} ; (* e in between 0 and 1 *)

te = (  ((ts ^ 4) * (rs ^ 2)) / (e * (d ^ 2) * 2)  ) ^ (1 / 4);
Print["Temperature on the Earth is : ", te - 273, " C"]

Temperature on the Earth is : {774.625, 316.123, 120.97, 67.1301, 59.1214, 58.288} C
```

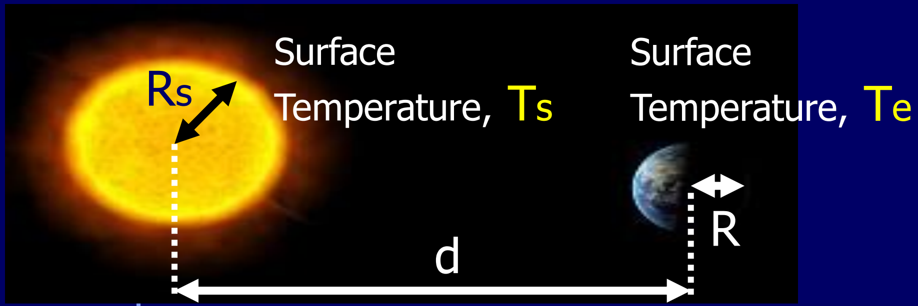
The Temperature of the Neutral Atmosphere



Let us first consider the Earth as a rapidly rotating solid sphere of radius *R*. Let the reflectivity of this sphere be such that it reflects a fraction *A* (Albedo) and absorbs the remaining fraction (*1 – A*) of the incoming solar radiation. Let the sphere also radiate like a black body at an effective temperature T_e.

The Energy absorbed by the Earth = $\sigma T_e^{4} \times 4\pi R^{2}$

Using Stephan's Law



The Energy emitted by the Earth = $(1-A)\times S_o \times \pi R^2$

Where S_o is the Solar Flux at 1AU.

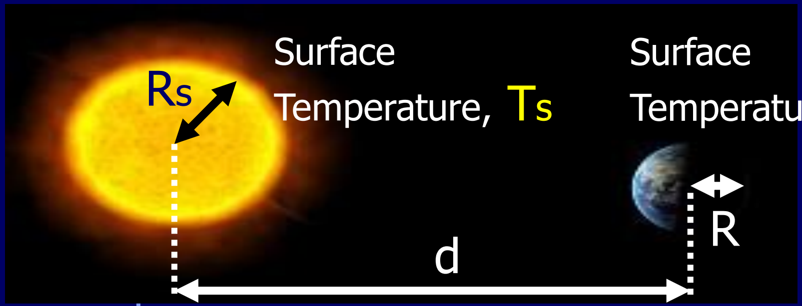
Under condition of thermal equilibrium;

The Energy absorbed by the Earth = The Energy emitted by the Earth

$$\sigma T_e^{\,4} \times 4\pi R^2 = (1-A)\times S_o \times \pi R^2$$

$$4\sigma T_e^{\,4} = (1-A) S_o \longrightarrow 1$$

The Total Energy emitted per second by the Sun = $\sigma T_S^{\,4} \times 4\pi R_S^{\,2}$



The Total Energy emitted per second by the Sun = $\sigma T_S^{\,4} \times 4\pi R_S^{\,2}$

The Energy Density (Energy per unit area) at our orbit = $\dfrac{\sigma T_S^{\,4} \times 4\pi R_S^{\,2}}{4\pi d^2}$

This is called Solar Flux, *So* at 1AU (at *d* or at our orbit)

$$\therefore \quad S_o = \sigma T_S^{\,4} \left(\frac{R_S}{d} \right)^2 \longrightarrow 2$$

Connect equation 1 & 2 ;

$$4\sigma T_e^{\,4} = (1-A) S_o$$

$$4\sigma T_e^{\,4} = (1-A)\ \sigma T_S^{\,4} \left(\frac{R_S}{d} \right)^2$$

$$T_e^{\,4} = \frac{(1-A)}{4} \left(\frac{R_S}{d} \right)^2 T_S^{\,4}$$

$$T_e = \left(\frac{R_S}{d} \right)^{1/2} \left(\frac{1-A}{4} \right)^{1/4} T_S$$

Solar Flux at 1 AU

$$S_o = \sigma T_S^{\,4}\left(\frac{R_S}{d}\right)^2$$

Where,

$$\sigma = 5.67\times10^{-8}\ Js^{-1}m^{-2}K^{-4}$$

$$T_S = 5778K\ \ (\sim 6000K)$$

$$R_S = 695500\,km\ (\sim 7\times10^5\ km)$$

$$d = 149598000\,km\ (1AU)$$

$$\Rightarrow\quad S_o = 1365.95\ \ Jm^{-2}s^{-1}$$

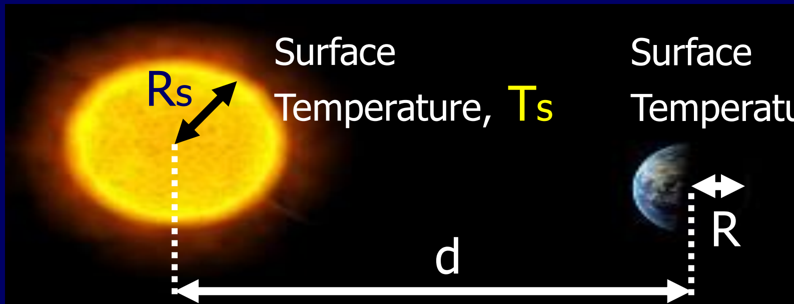
in our orbit...

The value of the effective temperature of the Earth,

$$T_e = \left(\frac{R_S}{d}\right)^{1/2}\left(\frac{1-A}{4}\right)^{1/4} T_S$$

Where, $A = 0.4$ Albedo of the Earth...

$$\Rightarrow\quad T_e = 245.181K$$



The value of the effective temperature of the Earth,

$$T_e = 245.181K$$

The value of Te is ~ 245 K. It is approximately 45 K lower than the average ground temperature, Tg (Tg = 290 K) of the Earth.
The difference is due to the Green House Effect of the terrestrial atmosphere which act as follows.

The incident Solar radiation has its maximum intensity in the visible portion of the spectrum and passes with practically no attenuation through the transparent atmosphere of the Earth. Thus, the (1 – A) fraction of the Solar radiation that is not reflect back, is absorbed by the ground and heats it up. The Earth radiates as a black body at a temperature Tg = 290 K , which is the average temperature of its surface. At Tg = 290 K most of the emitted energy is in the infra-red region.

The maximum intensity, according to Wien's Law, occures at a wavelength λ_m = 10^(-5) m.

$$\lambda_m T = hc/6k \approx 0.003mK$$

The maximum intensity, according to Wien's Law, occures at a wavelength λ_m = 10^(-5) m.

$$\lambda_m T = \frac{hc}{6k} \approx 0.003 mK$$

$$\Rightarrow \lambda_m = \frac{0.003}{T}$$

$$\Rightarrow \lambda_m = \frac{0.003}{290}$$

$$\Rightarrow \lambda_m = 10^{-5}\, m$$

The infra-red spectrum is strongly absorbed by the tri-atomic molecules of the atmosphere, namely CO_2, H_2O and O_3. The energy absorbed by these molecules is re-emitted in part toward the outer space and in part toward the ground, thus providing an additional heating source for the surface of the Earth.

Upward flux from the ground = Downward flux from the tri-atomic molecules of the atmosphere + flux of Solar radiation absorbed by the Earth

$$4\pi R^2 \sigma T_g^4 = 4\pi R^2 F_d + 4\pi R^2 \sigma T_e^4$$

Upward flux from the ground = Downward flux from the tri-atomic molecules of the atmosphere + flux of Solar radiation absorbed by the Earth

$$\sigma T_g^4 = F_d + \sigma T_e^4$$

→ 4

The Equation of Radiative Transfer

Using Eddington Approximation $F_d = \pi I_d$ (See Appendix I)

and the intensity of the downward flowing radiation (I_d)

$$I_d = \frac{F}{\pi}\left(\frac{3\tau}{4}\right)$$

$$\therefore \quad F_d = \pi I_d \quad \Rightarrow \quad F_d = \pi\left\{\frac{F}{\pi}\left(\frac{3\tau}{4}\right)\right\}$$

$$\Rightarrow \quad F_d = F\frac{3\tau}{4}$$

Where, $F = \sigma T_e^4$ and $\tau = \tau_o$

The opacity in the infra-red of the terrestrial atmosphere

$F = \sigma T_e^4$ and $\tau = \tau_o$ ⇒ $F_d = F \dfrac{3\tau}{4}$ ∴ $F_d = \sigma T_e^4 \dfrac{3\tau_o}{4}$

Using Eq 4:

$$\sigma T_g^4 = F_d + \sigma T_e^4$$

$$\sigma T_g^4 = \sigma T_e^4 \frac{3\tau_o}{4} + \sigma T_e^4$$

$$\sigma T_g^4 = \sigma T_e^4 \left(1 + \frac{3\tau_o}{4}\right)$$

$$T_g = T_e \left(1 + \frac{3\tau_o}{4}\right)^{1/4}$$

It has been observed that approximately 85% of the infra-red radiation is absorbed in the atmosphere and only 15% of the ground intensity (I_g) makes it through the Earth's atmosphere.

How to find τ_o :

$$I = I_g\, e^{-\tau_o} \Rightarrow \tau_o = -\ln\left(\frac{I}{I_g}\right)$$

Where, $$\frac{I}{I_g} = \frac{15}{100} = 0.15$$

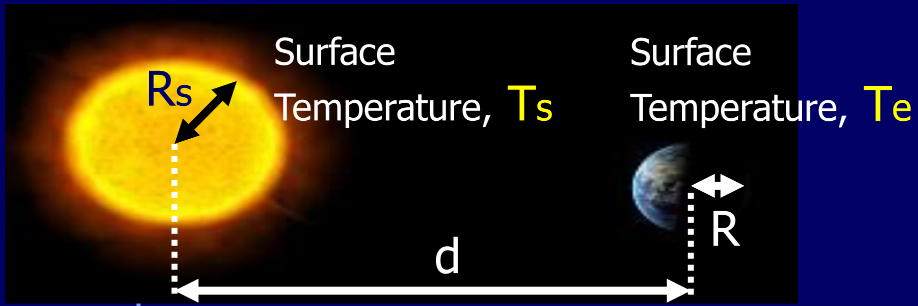
$$\Rightarrow \tau_o = -\ln(0.15)$$

$$\Rightarrow \tau_o = -(-1.89712)$$

$$\Rightarrow \tau_o = 1.89712 \approx 1.9$$

The value of ground temperature of the Earth,

$$T_g = T_e\left(1 + \frac{3\tau_o}{4}\right)^{1/4} \Rightarrow T_g = 245\left(1 + \frac{3(1.9)}{4}\right)^{1/4} \Rightarrow T_g \approx 305\,K$$



The value of ground temperature of the Earth,

$$T_e = 305K$$

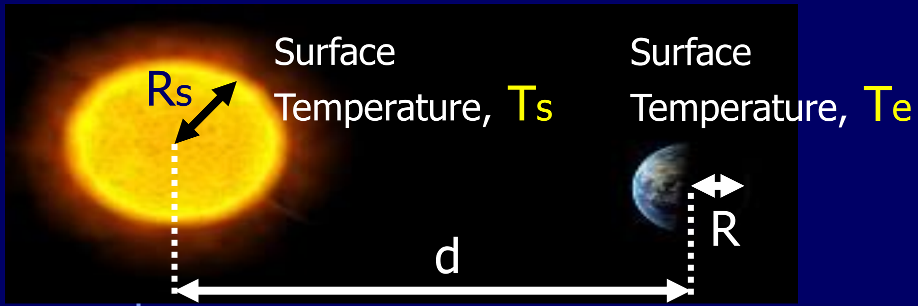
The temperature obtained in the above equation is somewhat higher than the average temperature on the surface on the Earth, but still it describes to a good approximation the Green House Effect. The small excess we have found in T_g occurs in part because we have neglected the convective transport of heat in the lower atmosphere, which would tend to cool down the surface of the Earth.

Note that the temperature of the air T_a near the ground is given by (A-30, appendix I), which yields a value for T_a lower than T_g.

$$T_a = T_e \left(\frac{1}{2} + \frac{3\tau_o}{4} \right)^{1/4}$$

$$T_a = 245 \left(\frac{1}{2} + \frac{3\,(1.9)}{4} \right)^{1/4}$$

$$T_a = 288\,K$$

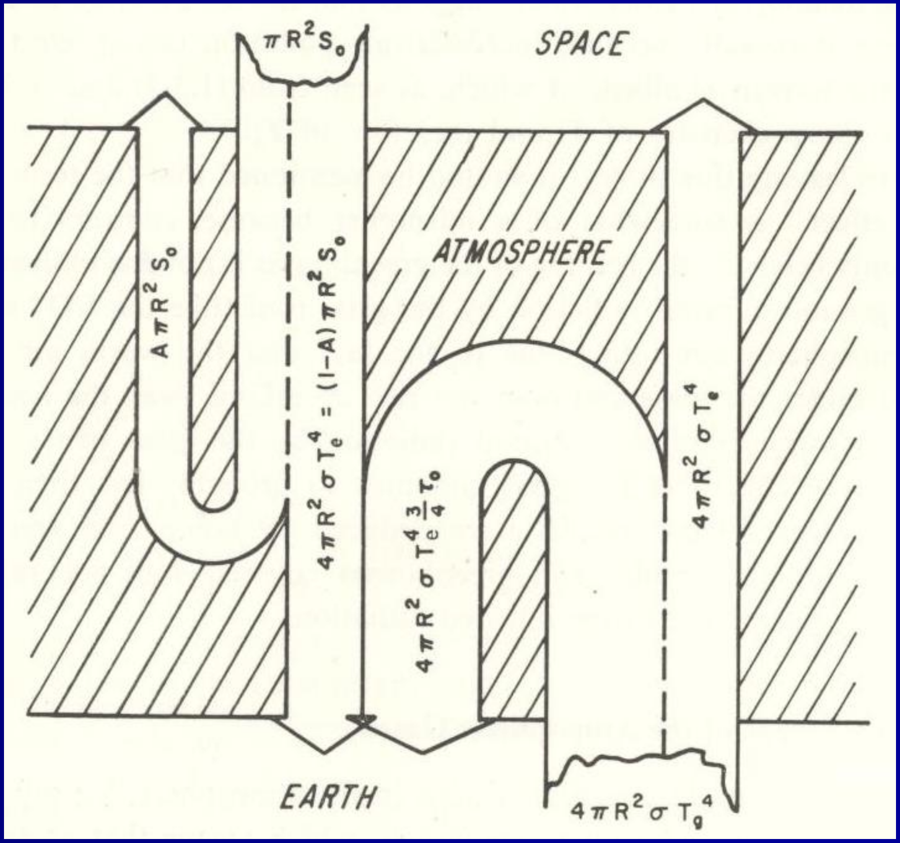


The temperature of the air T_a near the ground,

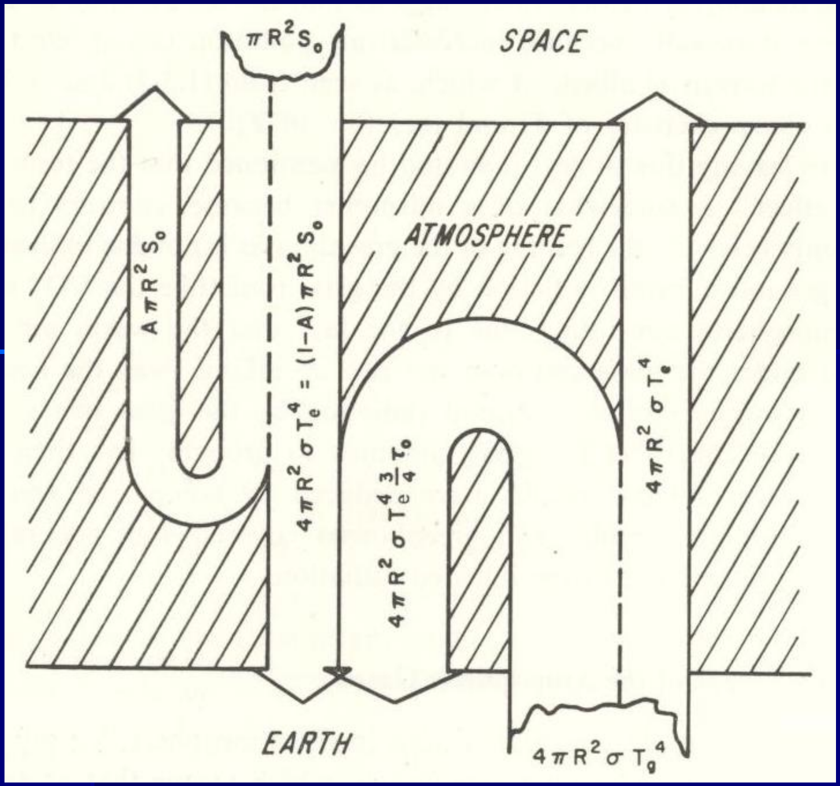
$$T_a = 288K$$

The discontinuity between T_g and T_a is in practice removed through conduction and convection and tends to lower the value of T_g obtained above.

This figure describes the balance between the radiation received and the radiation emitted by the Earth, including the green house effect.

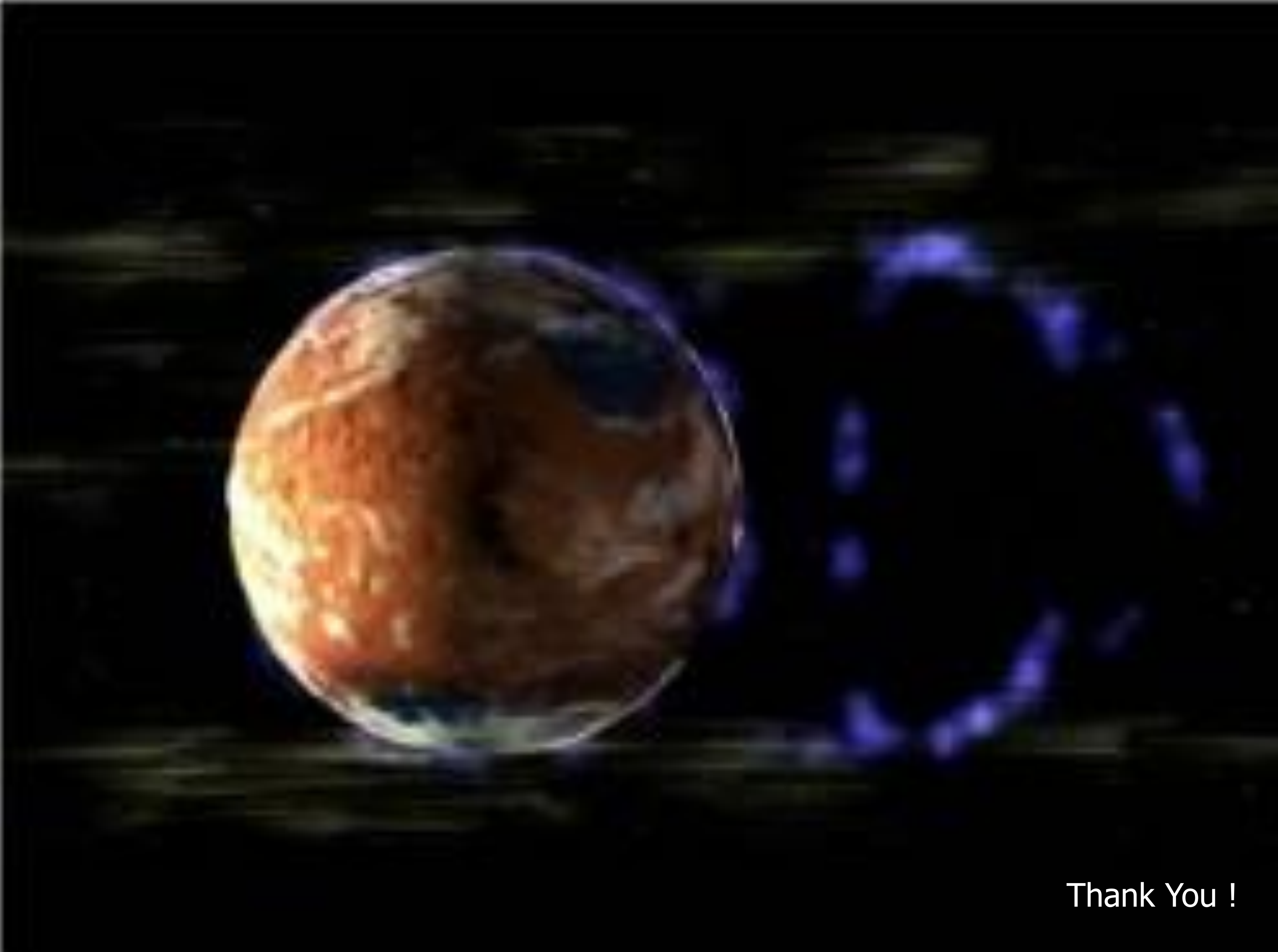


A diagram showing the balance of heat, including the G.H.E. in the atmosphere of the Earth.



A diagram showing the balance of heat, including the G.H.E. in the atmosphere of the Earth.

It is significant to note that if the Earth did not have an atmosphere, or if the terrestrial atmosphere did not have any absorbing molecules such as CO_2, H_2O and O_3, we would have $\tau_o=0$ and $T_g = T_e = 245$ K = -28 °C. This shows the importance of the green house effect, i.e. the trapping of the infra-red radiation emitted from the ground by the tri-atomic molecules of the atmosphere, and emphasizes the critical role of the minor atmospheric constituents.



Thank You !