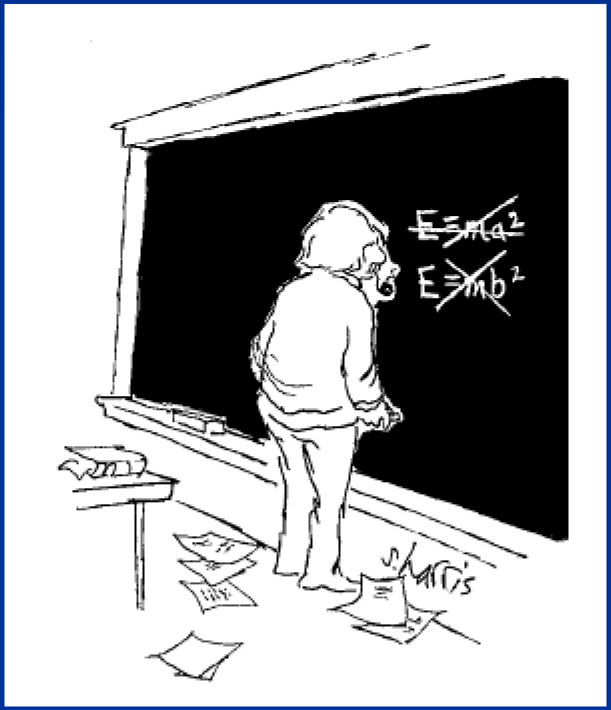
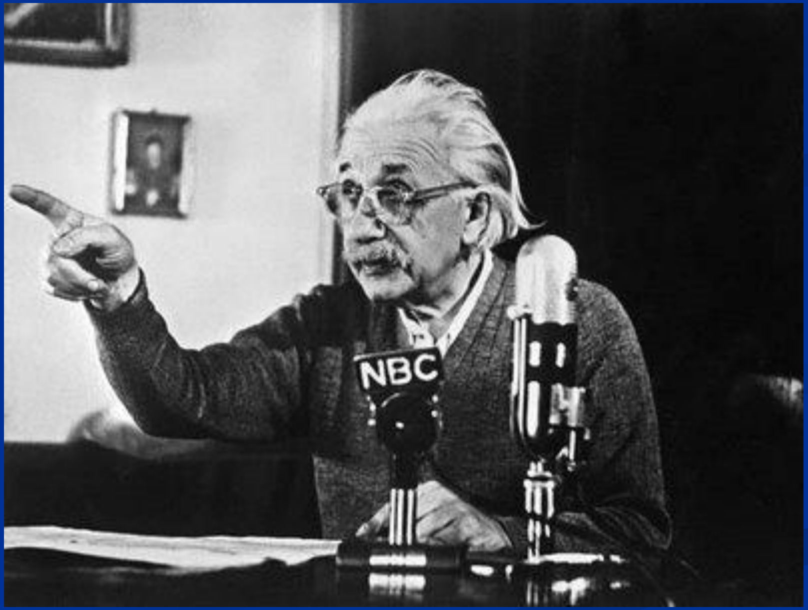
Special Theory of Relativity





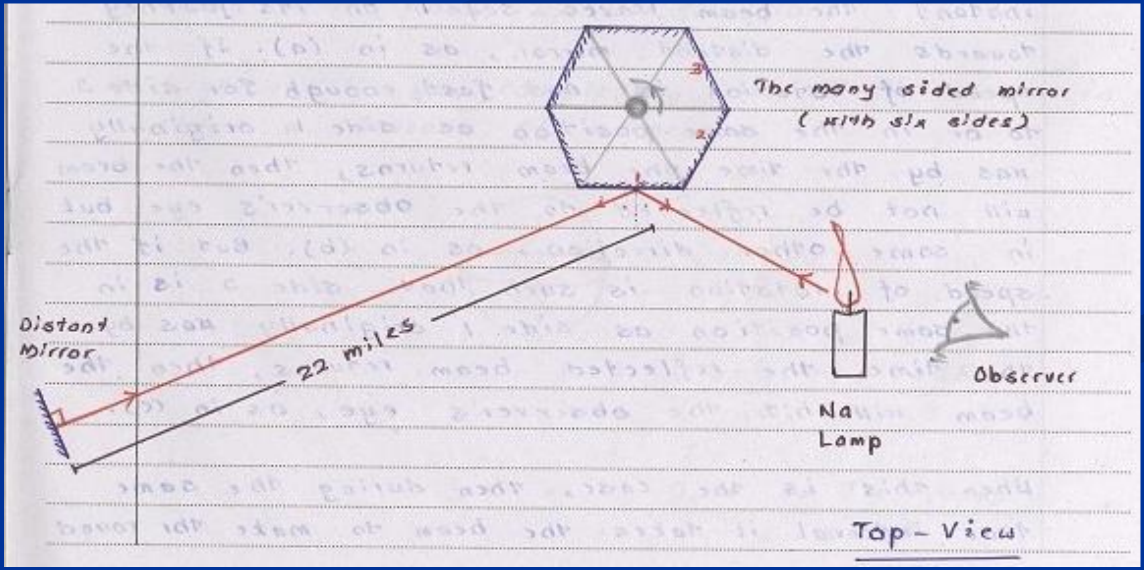
2nd Lecture

Michelson's Precise Method

The most widely known measurement of the velocity of light is that performed by Michelson (1852 - 1931) in 1879.

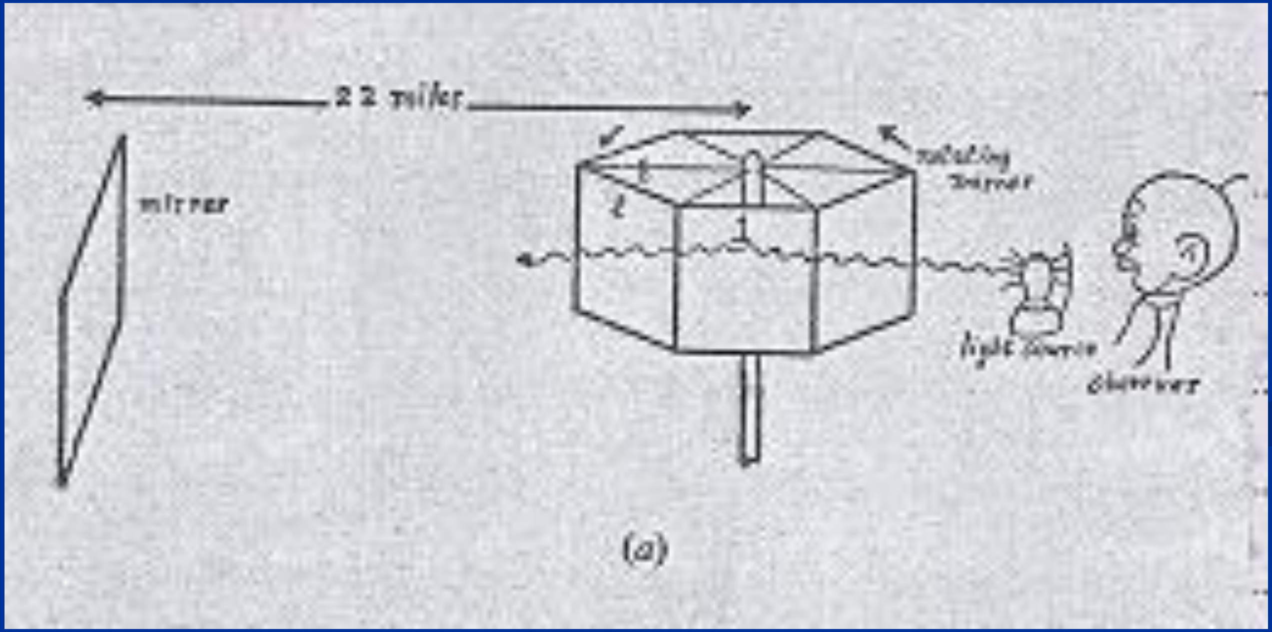
Michelson perfected the revolving mirror method. This is some what similar to Fizeau's cog-wheel but uses a rotating, many- sided mirror to chop up the original light wave into individual beams which, like Fizeau's are sent to a distant mirror (here 22 miles away!) and back again (see following figure).

The many-sided mirror is represented by one with six sides and can be rotated at any desired speed with an electric motor.



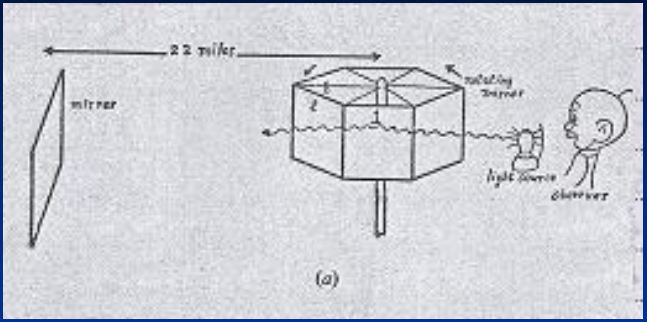
Michelson's Precise Method

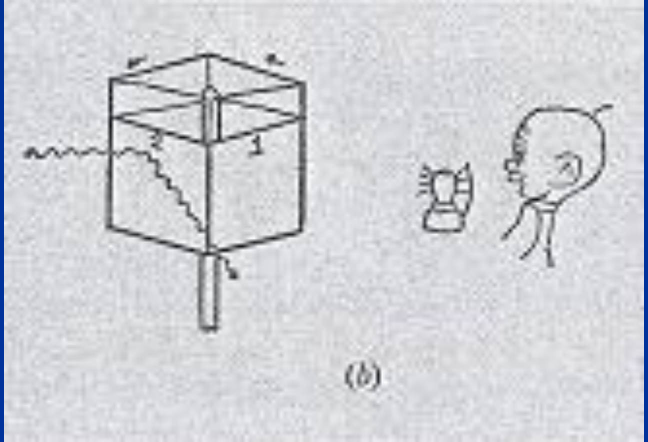
We first consider that would happen if the mirror was not rotating and set at position (a). Light would leave the light source and hit side 1, from which it would be reflected to the distant mirror. Upon reflection there, it would return via the same general path, hit side 1 again, and would be seen by the eye near the lamp as it returned towards the general direction of the lamp.

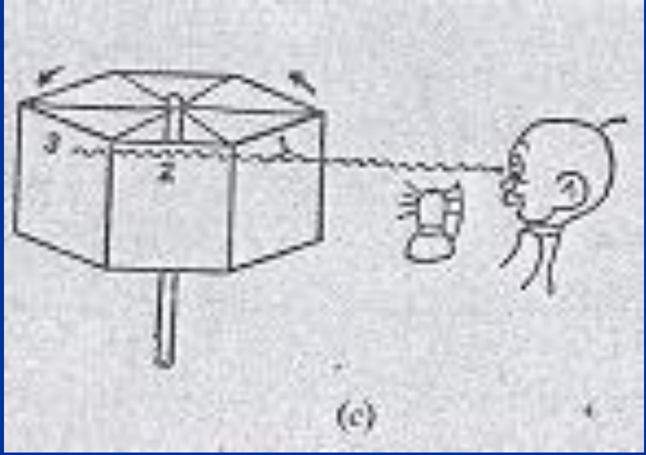


(a)

Michelson's Precise Method







But now suppose, as happens in the actual experiment, that the mirror is rotated at the instant the beam leaves side 1 on its journey towards the distant mirror, as in (a). If the speed of rotation is not fast enough for side 2 to be in the same position as side 1 originally was by the time the beam returns, then the will not be reflected to the observer's eye but in the some other direction, as in (b). But if the speed of rotation is such that side 2 is in the same position as side 1 originally was by the time reflected beam returns, then the beam will hit the observer's eye as in (c).

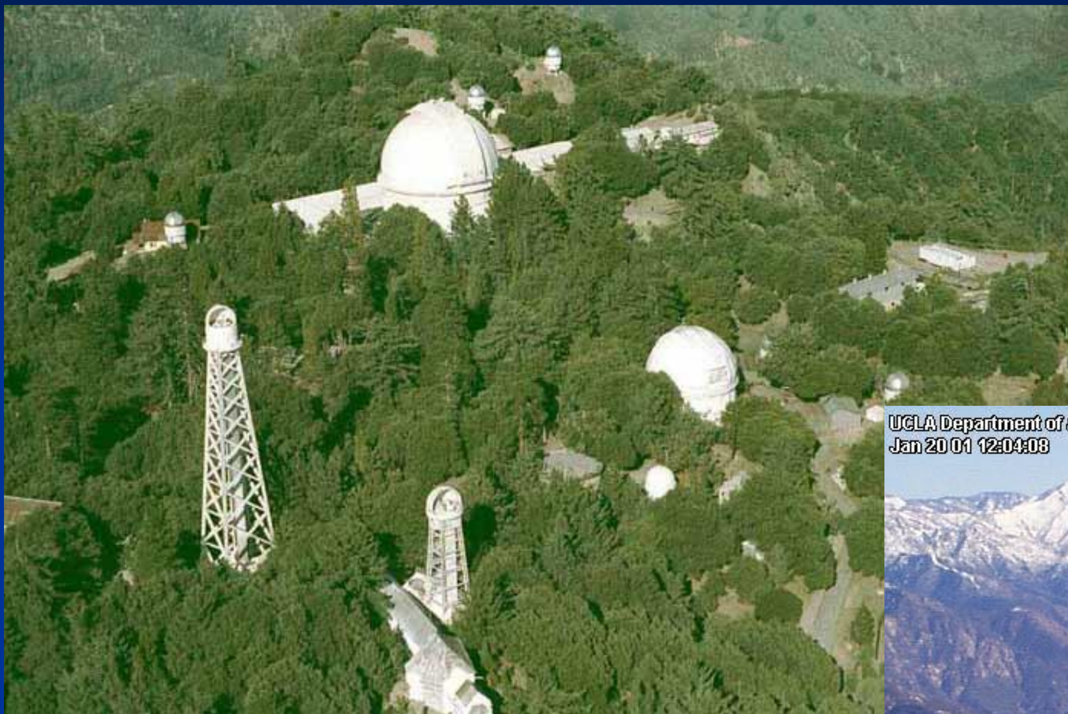
Michelson's Precise Method

When this is the case, then during the same time interval it takes the beam to make the round trip to the distant mirror and back, the rotating rotates by a sixth of a rotation. Further, since the speed of rotation of the mirror is known, the time for one revolution is known, and one sixth of this is the time it takes for the light beam to make its round-trip. Dividing the round trip distance by this time interval gives the velocity of light.

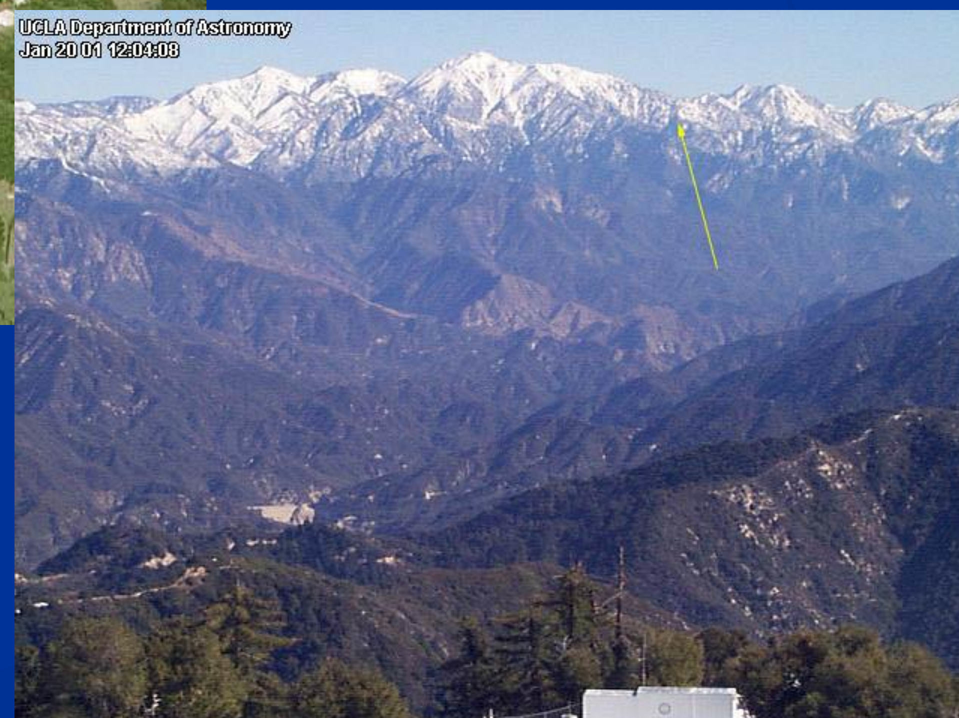
In Michelson's actual experiment various rotating mirrors of 8, 12 and 16 sides were used. This apparatus was set up on Mount Wilson in California. The mirror system comprising the distant mirror was set up on Mount San Antonio, approximately 22 miles away.

As a result of this and Michelson's subsequent experiments, the velocity of light is approximately **186,000 miles a second**.

Michelson's Precise Method



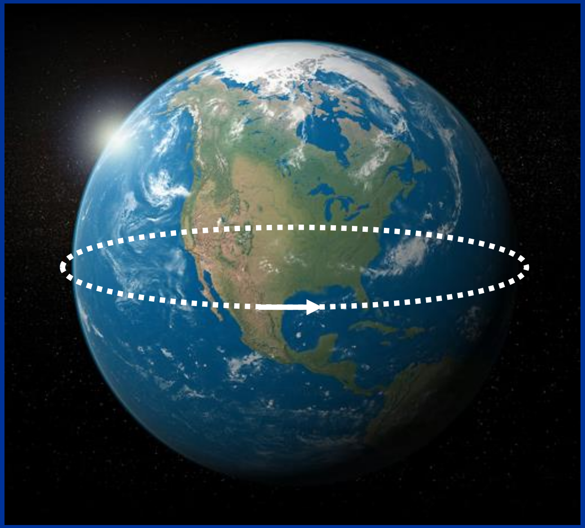
Mount Wilson in California



Mount San Antonio

Properties of Light Waves :

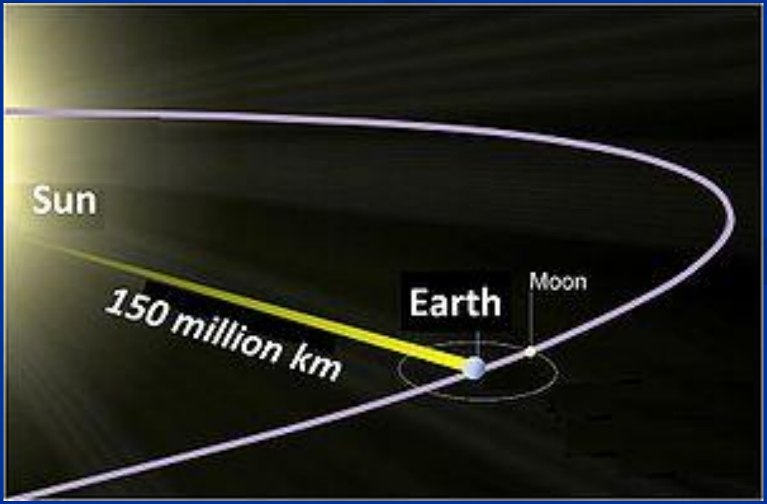
186,000 miles a second [297 600 000 m/s] may seem like too high speed to imagine, but it can be put into ideas with which we are familiar.



Light will go completely around the Earth in about a seventh of a second.

$$\sim \frac{1}{7} s$$

It takes about 08 minutes and 30 seconds for sunlight to travel the 93 million miles from the Sun to the Earth.

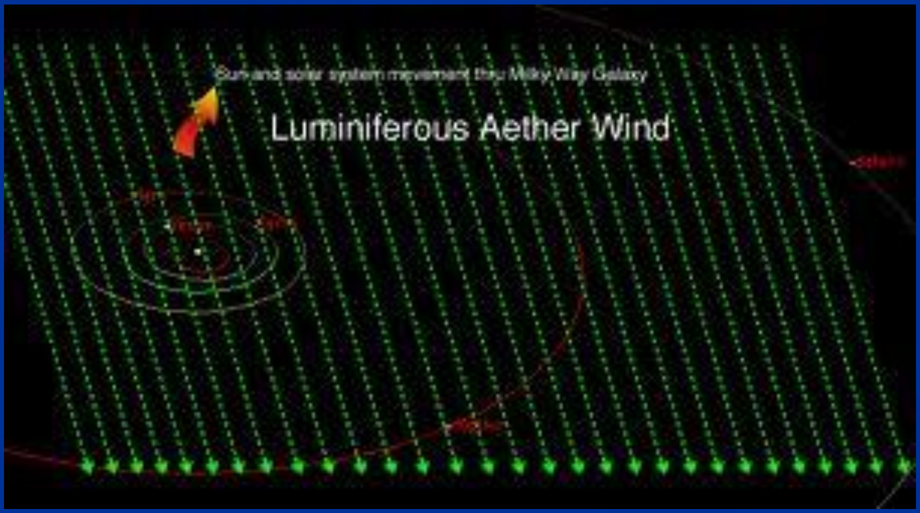


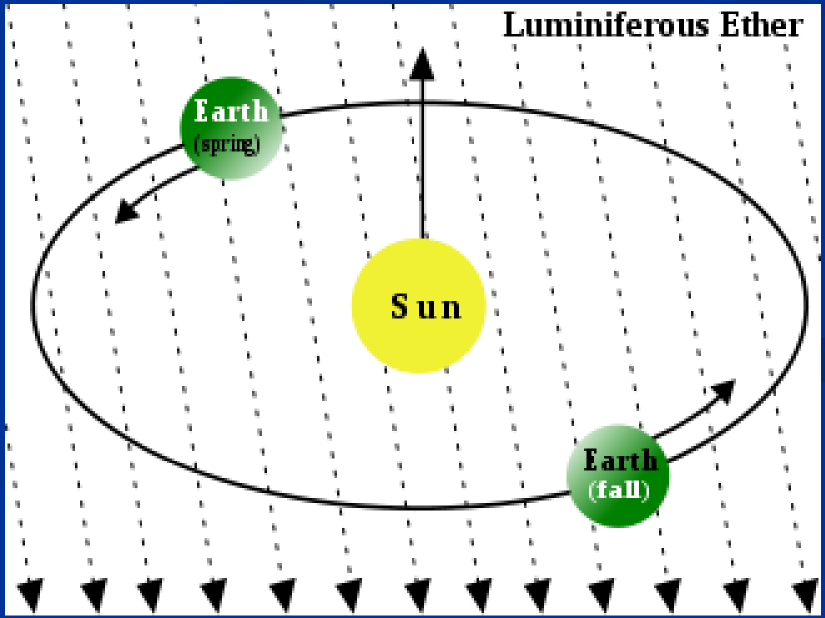
Propagation Media of Light

The concept of the *Ether* [luminiferous ether]

With proof afforded by many excellent experiments that light travels with a finite velocity of about 186000 miles a second, scientists next turned their attention to the consideration of the medium which carried or propagated the light waves.

Scientists created a special word for the hypothetical carrier of light waves. They called it the *lumeniferous ether*, or just plain *ether*.



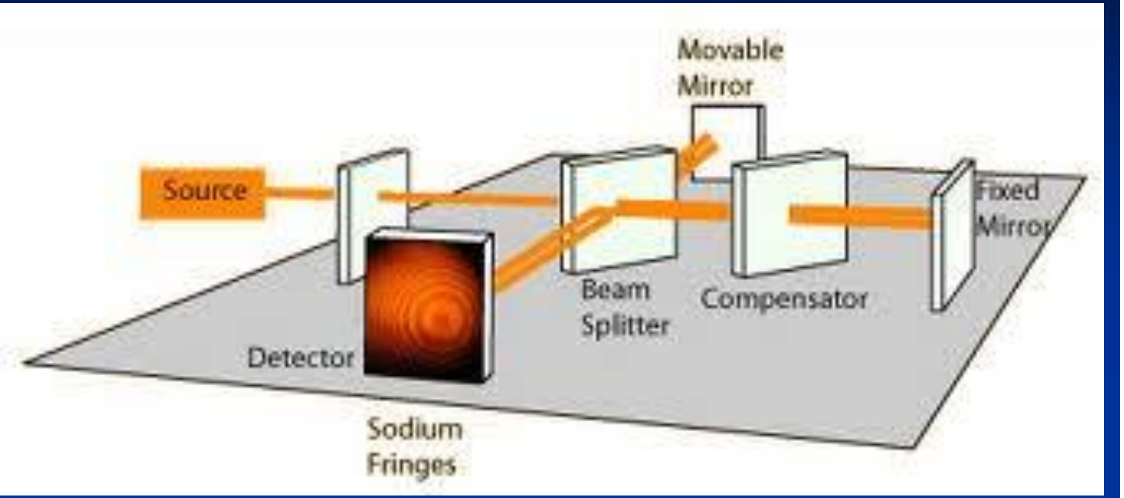


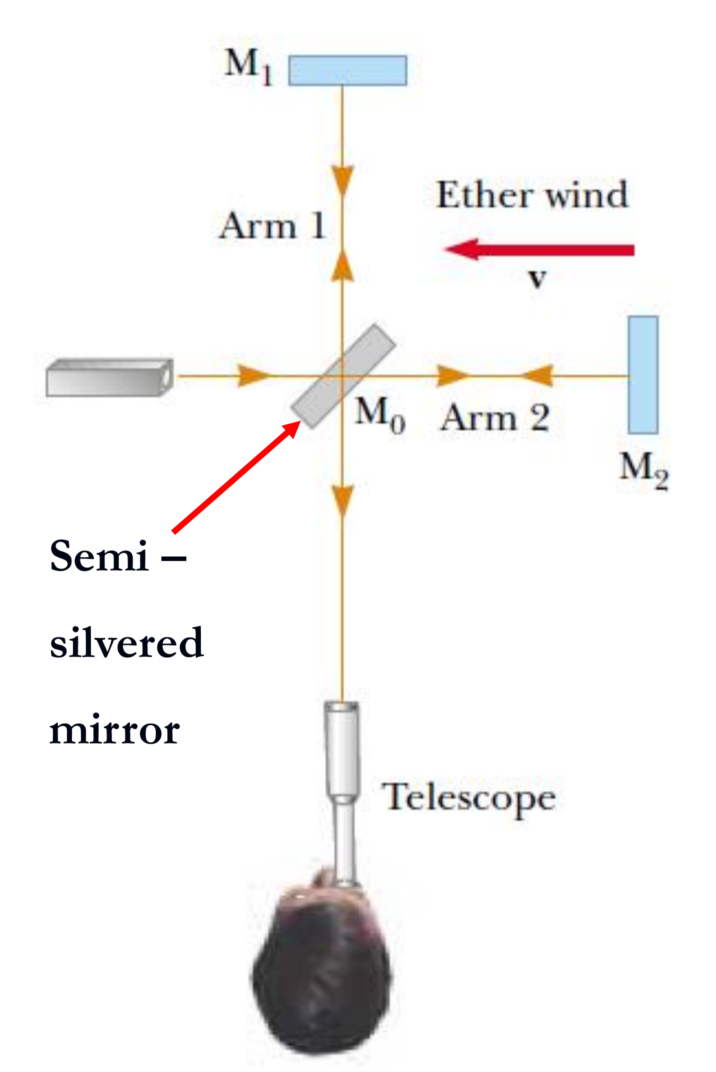
The Michelson – Morley Experiment

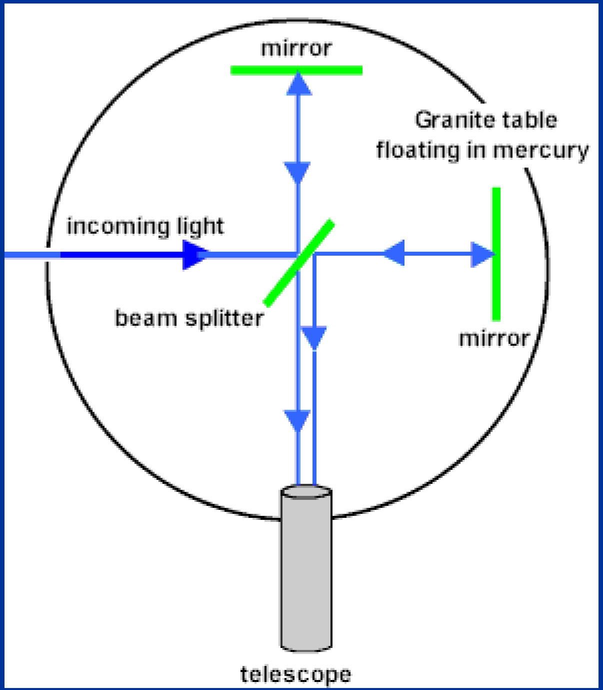
The Michelson – Morley experiment was performed in **1887** by **Albert Michelson** and **Edward Morley** at what is now **Case Western Reserve University**. Its results are generally considered to be the first strong evidence against the theory of a **luminiferous ether**.

The experiment was designed to determine the velocity of the Earth relative to that of the hypothetical ether.

The Michelson – Morley Experiment

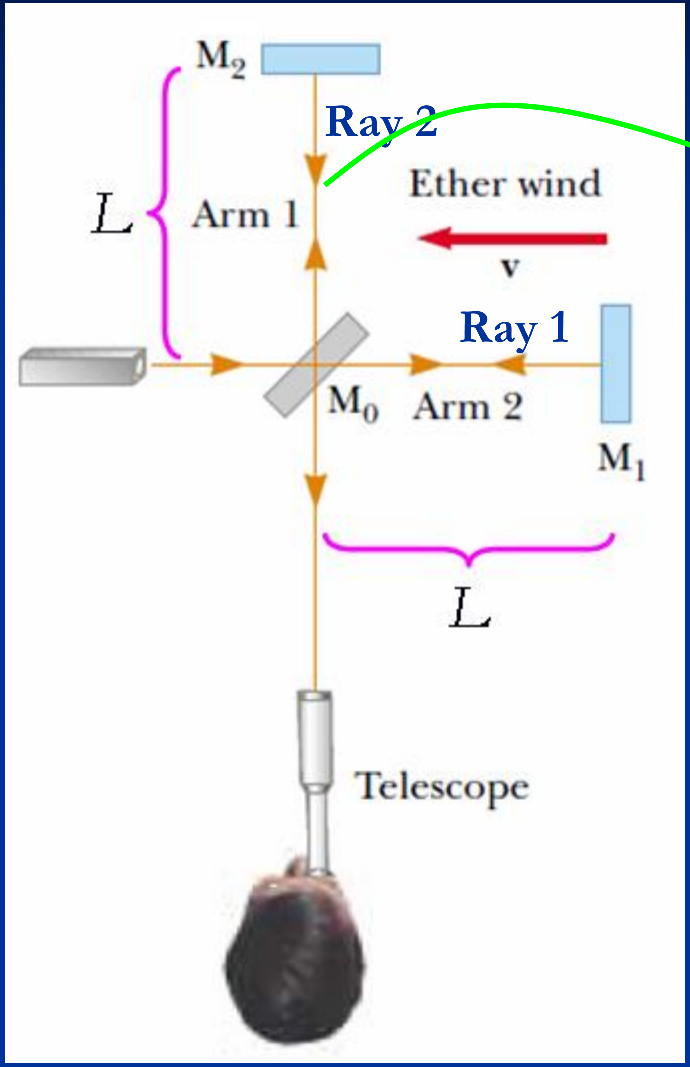




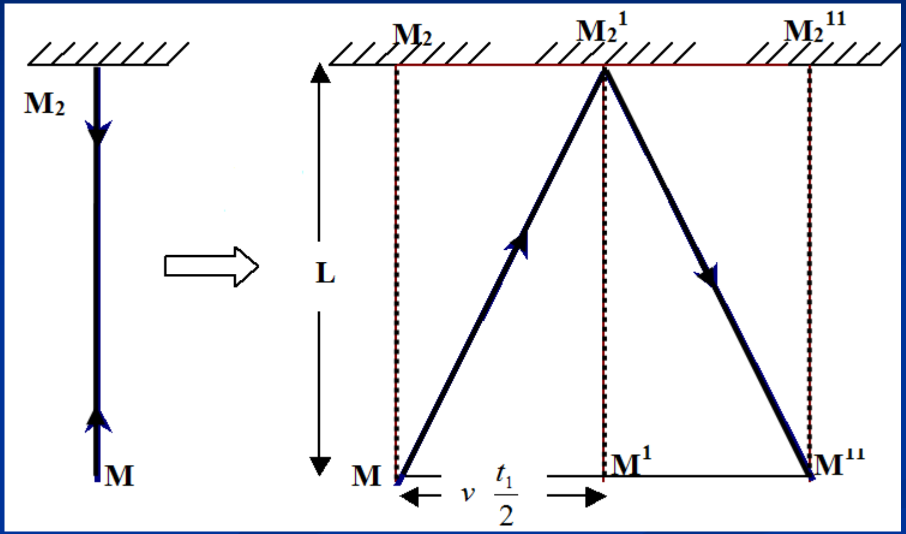


The experimental tool used was the Michelson interferometer is shown in the above Figures.

Details of the Michelson–Morley Experiment



For Ray 2:

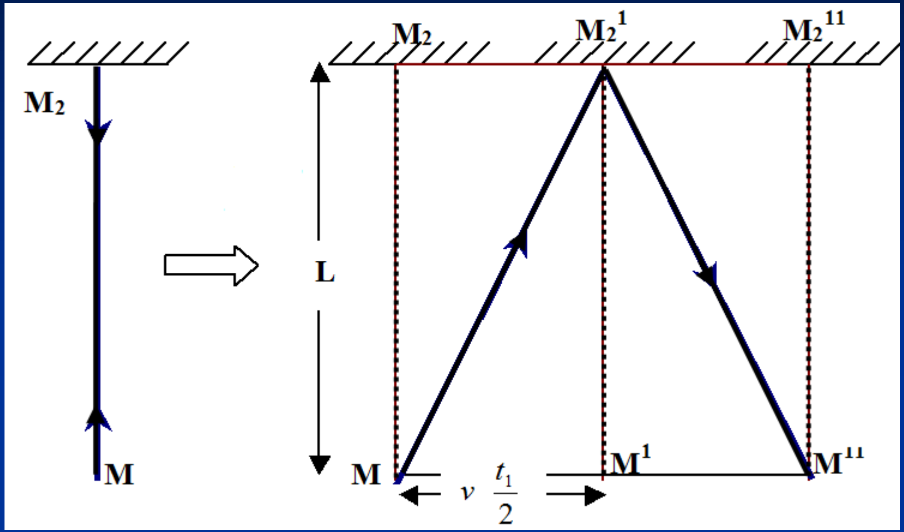


The distance of travel for each half of the trip is,

$$M\ M_2^1 = \sqrt{L^2 + \left(v\frac{t_1}{2}\right)^2}$$

Details of the Michelson–Morley Experiment

For Ray 2:



The total distance of travel for the round trip is,

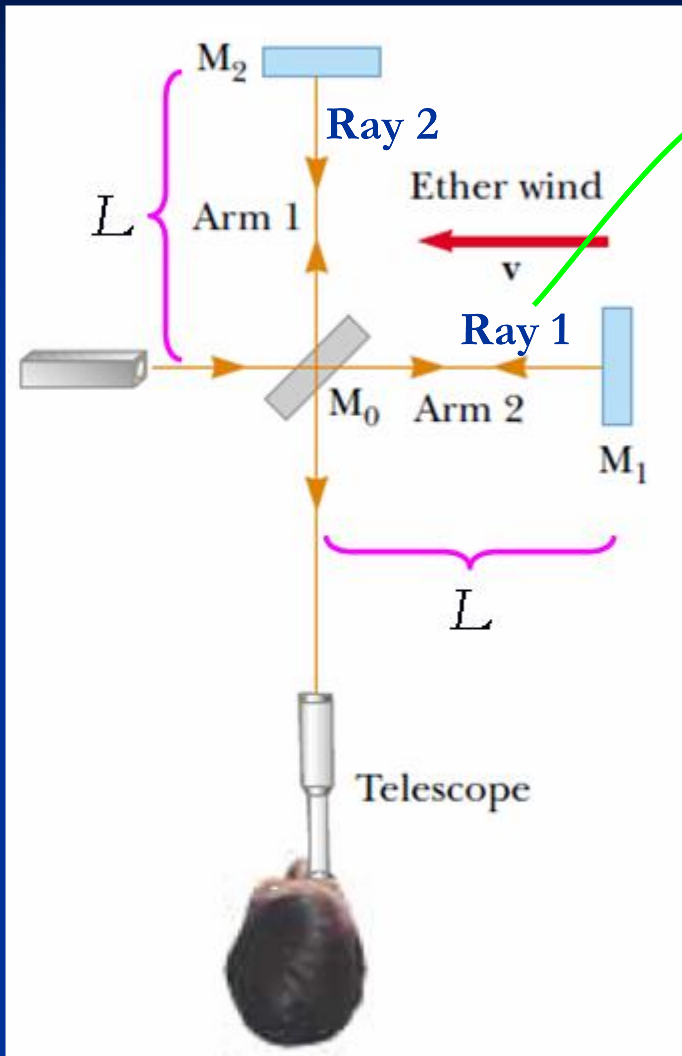
$$M\ M_2^1\ M^{11} = 2\sqrt{L^2 + \left(v\frac{t_1}{2}\right)^2}$$

:. The total time of travel for the round trip is,

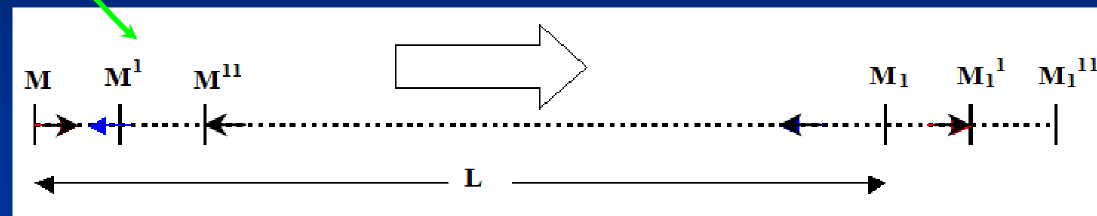
$$t_1 = \frac{2\left[L^2 + \left(v\frac{t_1}{2}\right)^2\right]^{1/2}}{c}$$

$$t_1 = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

Details of the Michelson–Morley Experiment



For Ray 1:

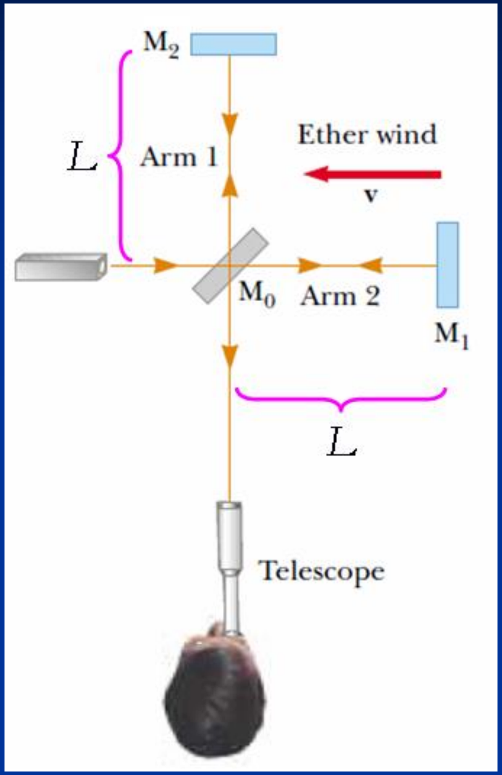


The time of travel to the right is $L/(c - v)$, and the time of travel to the left is $L/(c + v)$.
The total time needed for the round trip is,

$$t_2 = \frac{L}{c+v} + \frac{L}{c-v}$$

$$t_2 = \frac{2L}{c} \frac{1}{\left[1 - v^2/c^2\right]}$$

Details of the Michelson–Morley Experiment



Thus, the time difference between the horizontal round trip and the vertical round trip is,

$$t_2 - t_1 = \frac{2L}{c}\frac{1}{\left(1-\frac{v^2}{c^2}\right)} - \frac{2L}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

If the ether wind exists, this time difference is not zero !

This time difference between the two instants at which the reflected beams arrive at the viewing telescope gives rise to a phase difference between the beams, producing an interference pattern when they combine at the position of the telescope.

Details of the Michelson–Morley Experiment

The instrument used by Michelson and Morley could detect shifts as small as 0.01 fringe. However, it detected no shift whatsoever in the fringe pattern. Since then, the experiment has been repeated many times by different scientists under a wide variety of conditions, and no fringe shift has ever been detected. Thus, it was concluded that the motion of the Earth with respect to the postulated ether cannot be detected.

The stage was set for Einstein, who solved the problem in 1905 with his special theory of relativity.

Special Theory of Relativity

In the previous section we noted the impossibility of measuring the speed of the ether with respect to the Earth. Einstein proposed a theory that boldly removed these difficulties and at the same time completely altered our notion of space and time. **He based his special theory of relativity on two postulates:**

What is a Postulate ???

1. Scientific assumptions / presuppositions for making a theory.
2. The verb "to postulate" means to assert a claim as true, without proof.
3. An accepted statement without proof.

Eg: Your parents,
The statement " She loves me ", ….

Special Theory of Relativity

Einstein's Two Postulates in STR

Postulate 01 : The Principle of Relativity:

The laws of physics must be the same in all inertial reference frames.

The laws of Physics are the same for all observers in uniform motion relative to one another.

Postulate 02 : The constancy of the speed of light :

The speed of light in vacuum has the same value, $c = 3 \times 10^8$ m/s in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

Measurement of Time in STR

What is Time ???

Time is the indefinite continued progress of existence and events that occur in apparently **irreversible** succession from the **past** through the **present** to the **future**!

Time is a component quantity of many measurements used to sequence events, to compare the durations of events and the intervals between them, and to quantify rates of change of quantities in material reality or in the conscious experience.

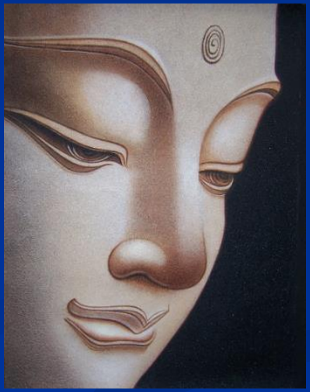
The fundamental unit of time in the International System of Units (SI) is the second (symbolized s or sec).

One second elapses during the occurrence of exactly 9,192,631,770 (9.192631770 x 10^9) cycles of the radiation produced by the transition between two levels of the cesium 133 atom.

Measurement of Time in STR

What is Time ???

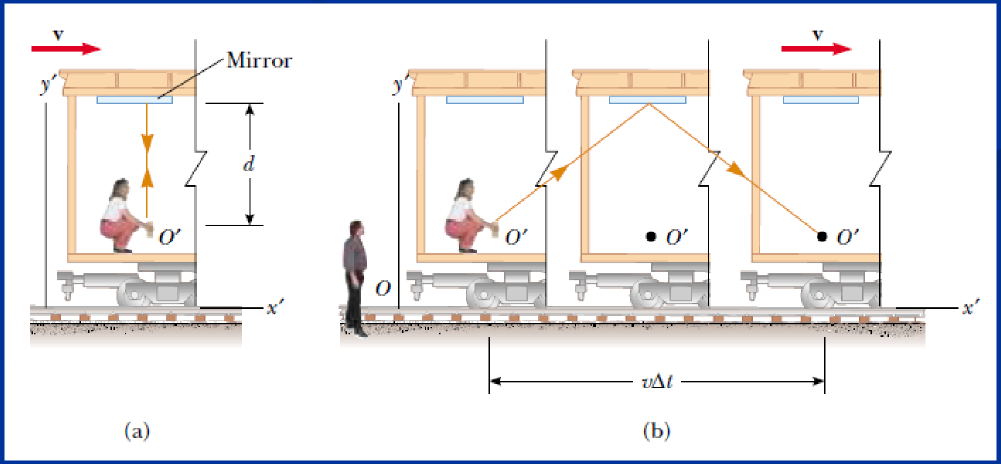
The Gap between two events is called Time (difference) !
– Lord Buddha!



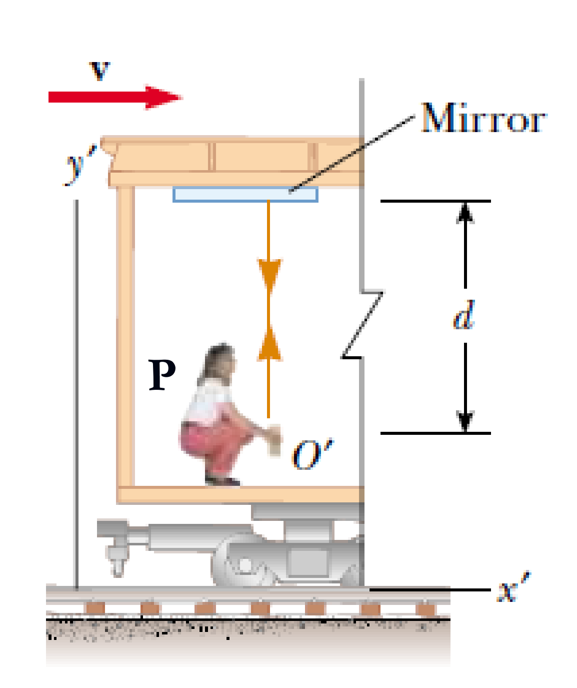
There is no belief in a personal god. Buddhists believe that nothing is fixed or permanent and that change is always possible. The path to Enlightenment is through the practice and development of morality, meditation and wisdom. Buddhists believe that life is both endless and subject to impermanence, suffering and uncertainty. These states are called the *tilakhana*, or the three signs of existence. Existence is endless because individuals are reincarnated over and over again, experiencing suffering throughout many lives.

Measurement of Time in STR

Now consider the same pair of events as viewed by observer O in a second frame, as shown in Figure b. According to this observer, the mirror and laser are moving to the right with a speed v, and as a result the sequence of events appears entirely different. By the time the light from the laser reaches the mirror, the mirror has moved to the right a distance $v\,(\Delta to/2)$, where Δto is the time it takes the light to travel from O' to the mirror and back to O' as measured by O. In other words, O concludes that, because of the motion of the vehicle, if the light is to hit the mirror, it must leave the laser at an angle with respect to the vertical direction. Comparing Figure a and Figure b, we see that the light must travel farther in (b) than in (a).



For the Observer, P on the vehicle :

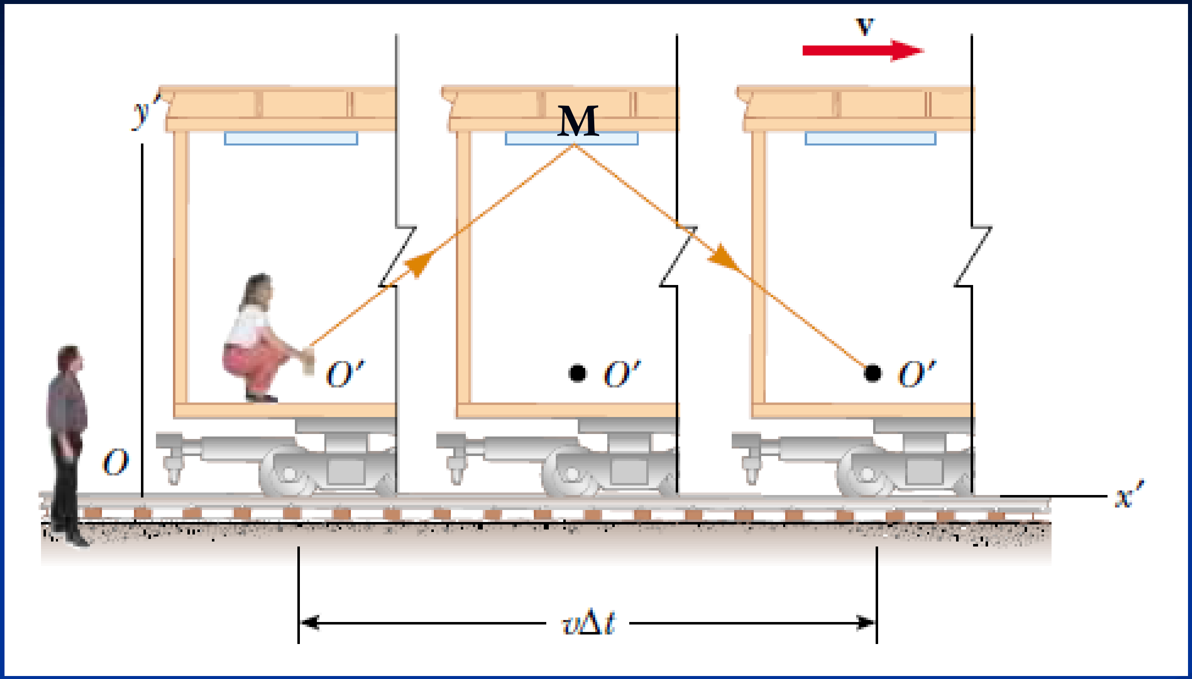


In the frame where the clock is moving, the light pulse traces out a path of length $2d$ and the period of the clock is $2d$ divided by the speed of light, c .

$$\Delta t_P = \frac{2d}{c}$$

where Δt_P is the time it takes the light to travel from O' to the mirror and back to O' as measured by P.

For the Observer, O on the ground :



Total time for the light pulse to trace its path with respect to O.

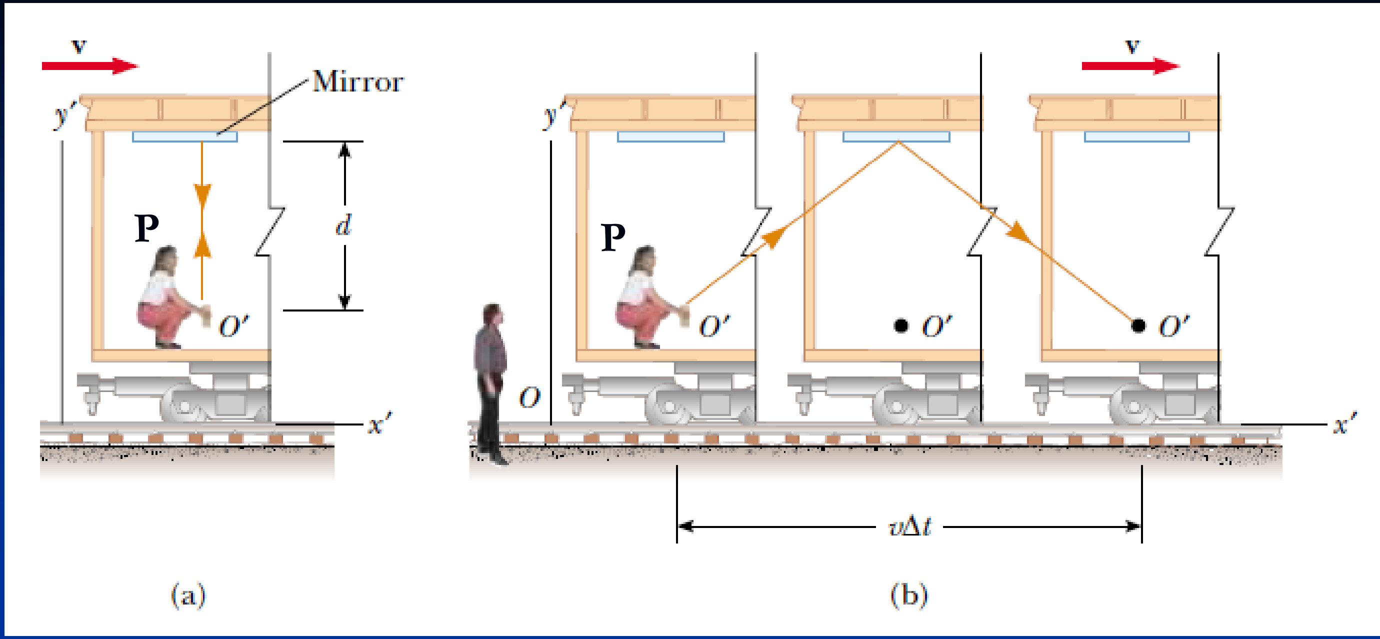
$$\Delta t_O = \frac{O^1MO^1}{velocity}$$

(Using postulate 01)

$$\Delta t_O = \frac{2\left(d^2 + \left[v\frac{\Delta t_O}{2}\right]^2\right)^{1/2}}{c}$$

(Using postulate 02)

$$\Delta t_O = \frac{2d}{c}\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$



$$\Delta t_O = \Delta t_P \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where, v is the Relative Speed of the Two Frames

This equation express the fact that for the moving observer the period of the clock is shorter than in the frame of the ground observer itself !

$$\Delta t_O \rangle \Delta t_P$$

Time interval w. r. t the stationary frame

Time interval w. r. t the moving frame

Time Dilation

Suppose,

$$t_2 = t_1 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

This equation is called relativistic time equation!

If v > 0 ➡ $\frac{v}{c} < 1$ ➡ $\frac{v^2}{c^2} < 1$ ➡ $1-\frac{v^2}{c^2} < 1$

➡ $\sqrt{1-\frac{v^2}{c^2}} < 1$ ➡ $t_2 = t_1 \frac{1}{(<1)}$

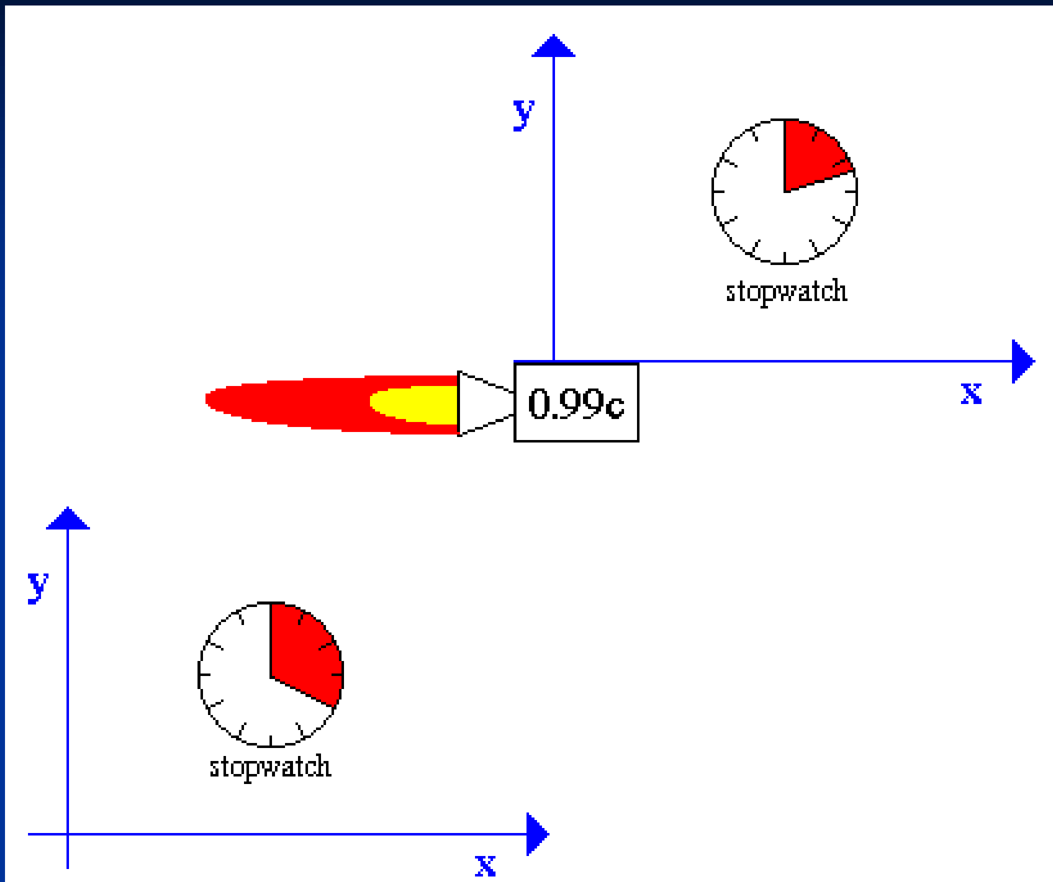
$$t_2 > t_1$$

Time interval w. r. t the stationary frame (t_2)

Time interval w. r. t the moving frame (t_1)

This is called Time Dilation !

Time Dilation



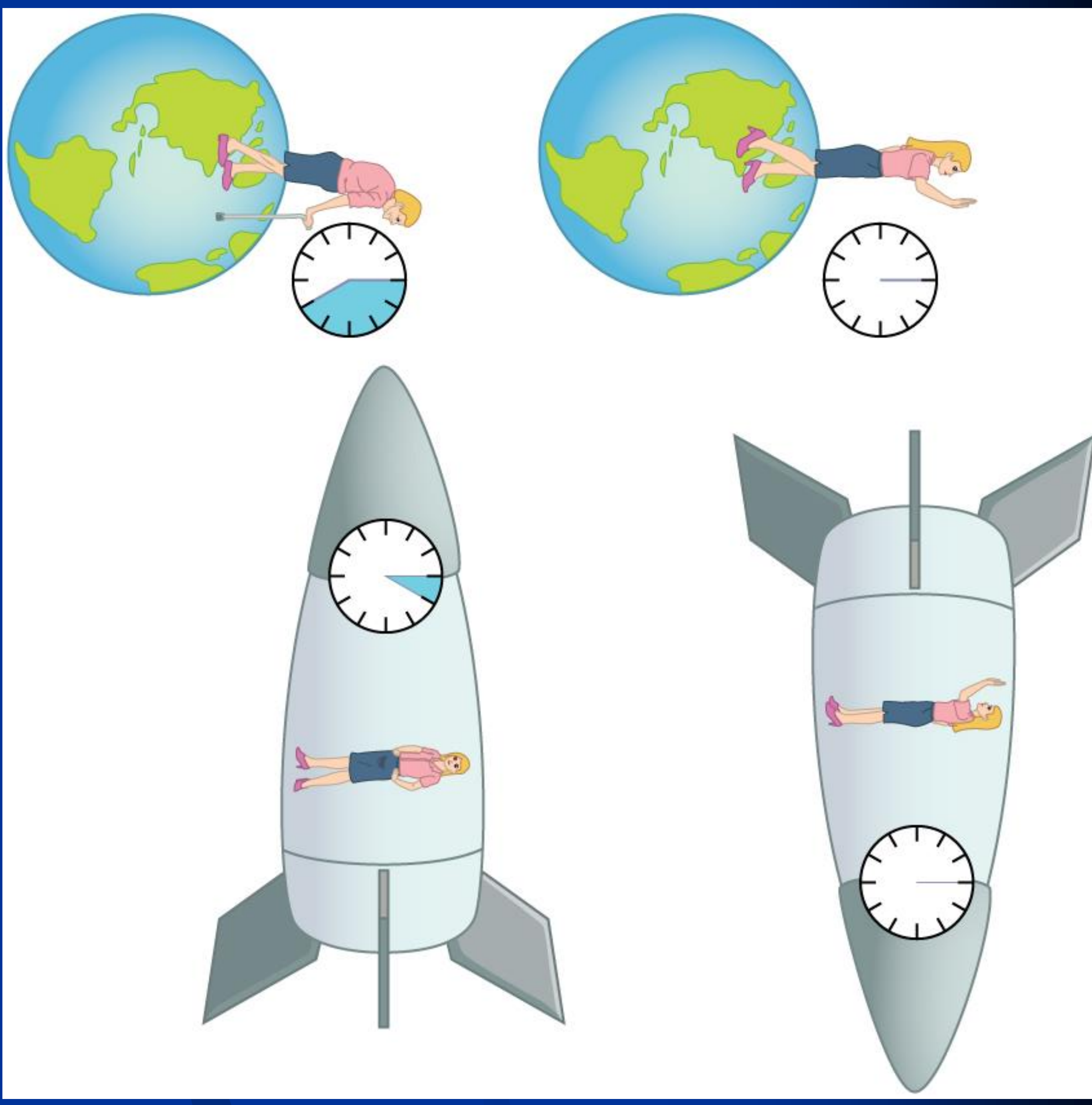
$$t_2 > t_1$$

Time interval w. r. t the stationary frame

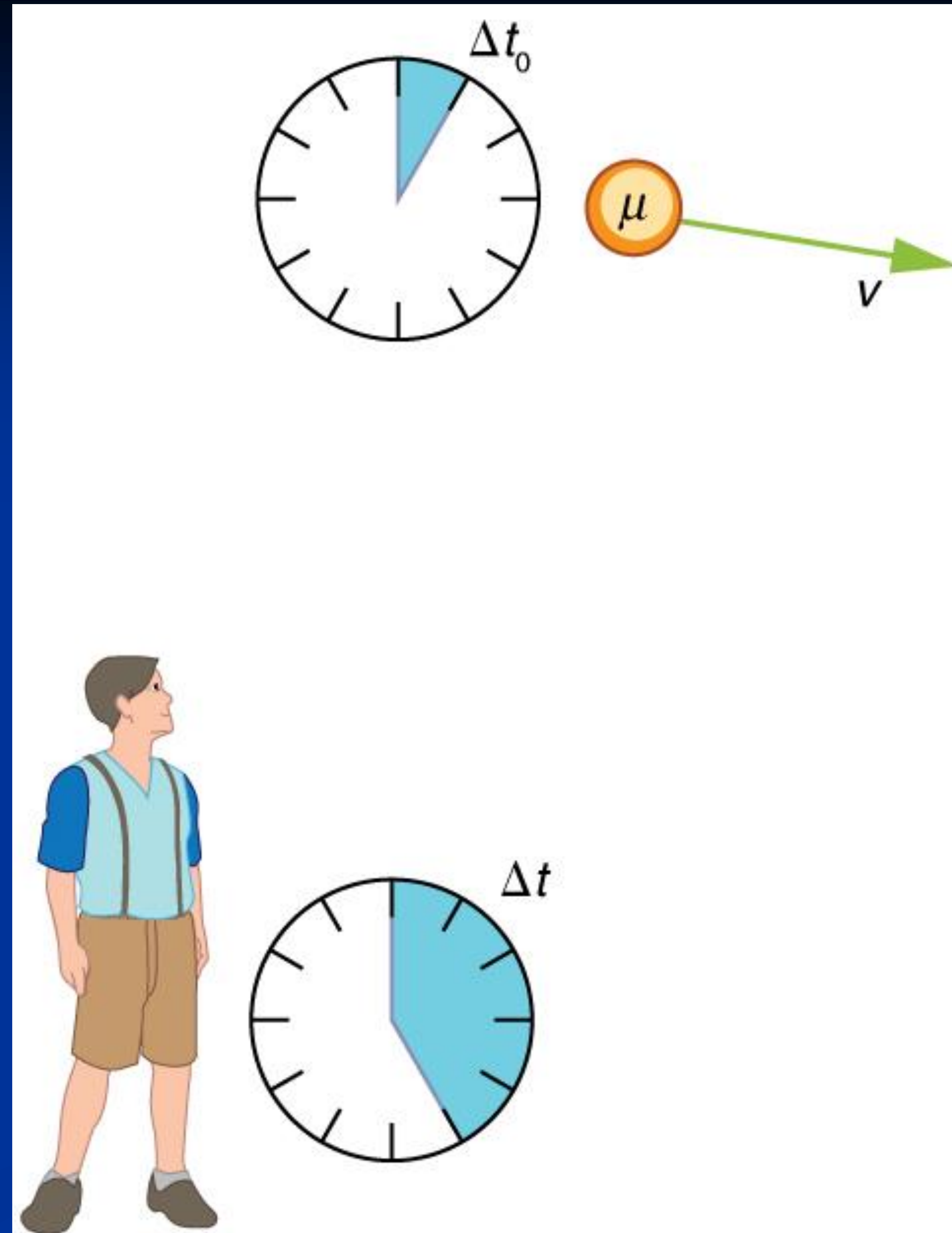
Time interval w. r. t the moving frame

Time dilation is an actual difference of elapsed time between two events as measured by observers either moving relative to each other.

Time Dilation



Time Dilation



Thank You!