RELATIVITY

3rd year (semester II) - Department of Physics, USJP.

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1. Describe the Michelson–Morley experiment. With what aim in view was the experiment performed? Discuss the results obtained.

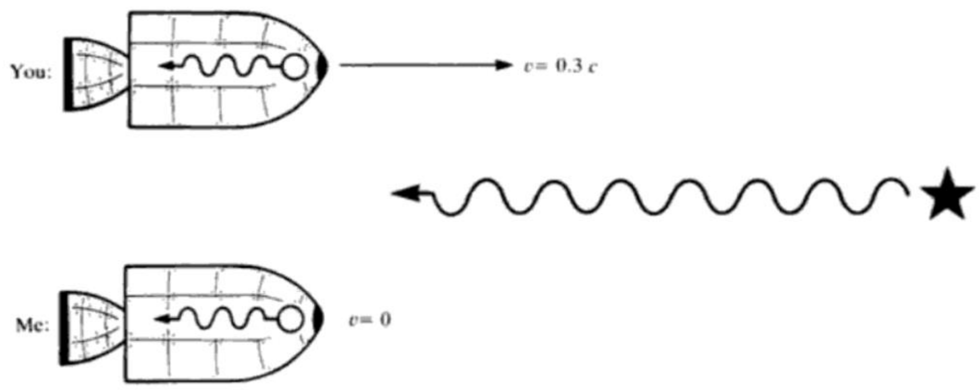
2. 

Figure 1

Figure 1 shows a distant star and two rocket ships, my ship at rest with respect to the star and your ship moving towards it at a of *0.3c* Do you and I measure the same speed for the light from the star?

3. Charlie has just caught a lake trout 20 inches long. Zipping by in her motor bike, the game warden sees the fish as 12 inches long. Uh-oh! the minimum length legal length is 16inches. How fast the game warden going?

4. What will be the distance a pion travels before decaying, If its speed is $v = 0.99c$ and its average life time is $2.6 \times 10^{-8} s$? How much will be the distance traveled classically?

5. Obtain following time equation, from starting Einstein's Postulates.

$$t = T\frac{1}{\sqrt{1-v^2/c^2}}$$ (Symbols have their usual meanings)

Particle X, which is created in a particle accelerator, travels a total distance of 100.0 m between two detectors in 410 ns as measured in the laboratory frame before decaying into other particles. What is the lifetime of the particle X as measured in its own frame.

6. A particle has a velocity of *0.998 c* and it travels a distance of *300 km* before decaying. Find the average life time of the particle.

7. A rod of length *2l* is held at an angle of 45^o to the horizontal. It s now projected with a velocity of *0.9c* along the horizontal such that the rod always keeps the same angle of 45^o during the motion. What will be the length of the rod as seen by,
   (a) An observer stationary on the ground.
   (b) An observer moving with the rod.

8. (I) Consider an isosceles triangle *ABC*, with *BC* as the base, and each side of length *a*. The triangle is now projected with velocity *0.5c* along the *BC* direction such that its plane always remains vertical. Find the area of the triangle seen by

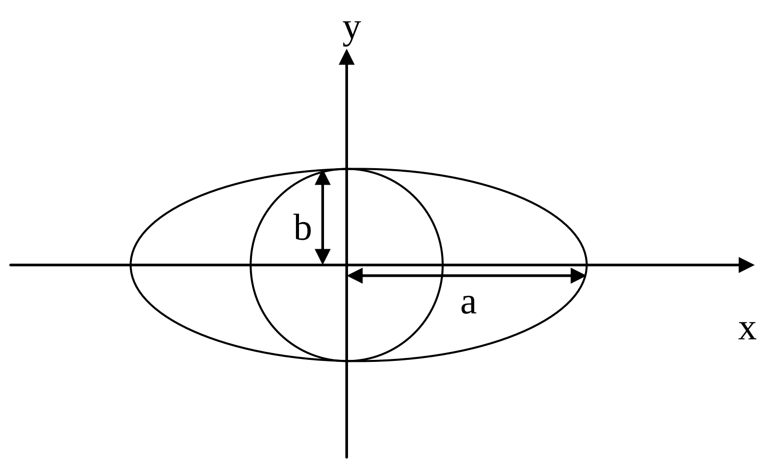
   (a) A stationary observer.

   (b) An observer moving with the triangle.

   (II) With what speed an ellipse whose area is πab be projected along the direction of the major axis such that it is seen as an article of radius πb^2?

9. Write down the mathematical relationship between **proper time** and **improper time.**

   Drive an expression for "**Length Contraction**" starting above relationship.



An ellipse having an area πab is project with a certain velocity. It was observed that the ellipse appears as a circle of area πb^2. Determine the velocity of projection of the ellipse.(Where ,a > b).

10. A cube of each side *a* is projected with a velocity *v* along one of its sides. What will be its volume its volume seen by a stationary observer?

11. A star known as Alfa Centauri is about *4.0 light years* (*1ly* = $9.46x10^{15}m$) distant from us. If suppose a rocket from earth is to reach it in one day how fast would it have to go?

12. A beam of pions travels with a speed of *90%* of light. What is the half life of these particles in

    (a) Frame in which pions are stationary.

    (b) In a laboratory frame?

13. The first human trip to the moon took about *three days* (approximately $3x10^5$ *seconds*) each way. The distance from the earth to the moon is roughly $4\ x10^8$ *m*. When they returned, how much younger were the astronauts than their twin brother who remained on earth?

14. A certain spectral line in the spectrum of a double star ordinary has a frequency of *0.5*x*10^15^ cycles/sec*. If the star is approaching the earth at *200km/sec*, approximately by what fraction $\Delta f / f$ will the frequency be changed?

15. What is meant by the **Doppler effect** in relativity for a moving light source?

    You are giving the following mathematical equation for the Doppler effect,

    $$f_0 = \frac{f_s}{\gamma(1-\beta\cos\theta)}$$

    Where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = \frac{v}{c}$ and other symbols have their usual meanings.

    A yellow coloured vehicle appears as a green coloured vehicle to a stationary observer due to its speed. Find the velocity of a vehicle. (Wavelengths of yellow and green light are 550 nm and 500nm respectively).

    Is the above incident practically possible? Briefly explain your answer.

16. Derive an expression for length contraction starting, ($t = T\frac{1}{\sqrt{1-v^2/c^2}}$) time relationship.

    Length of a vehicle is L. When the vehicle is moving with a certain velocity, it was observed that the length of the vehicle is l (L > l). Determine the velocity of the vehicle.

    Now, this vehicle is taken into a garage of length l, and its door was closed.
    Explain whether this can be done practically.

17. (a) Derive an expression for variation of mass according to relativistic mechanics.
    (b) The rest mass of the electron is $9\text{x}10^{-31}kg$. What will be its mass if it were moving with velocity *0.8c* ?

18. (a) Derive the relativistic expression for kinetic energy of a moving body.
    (b) Show that for small speeds it reduces to the classical form.
    (c) Calculate the energy in electron volts released when $10^{-3}kg$ of matter is converted into energy. Given $c= 3\text{x}10^8\ m/sec$.

19. Write short notes on,
    (a) Michelson Morley Experiment and
    (b) Twin Paradox
    in relativity.

20. Prove that the velocity of a partical having the rest energy E_0 and kinetic energy K is given by, $v = \frac{c\sqrt{K(K+2E_0)}}{K+E_0}$

21. For a particle of the rest mass m_0, relativistic mass m, rest energy E_0 and total relativistic energy E prove the following relations

    (a) $v = \frac{c\sqrt{(m+m_0)(m-m_0)}}{m}$

    (b) $v = \frac{c\sqrt{(E+E_0)(E-E_0)}}{E}$

22. For what value of v/c will the relativistic mass of a particle exceed its rest mass by a given fraction f ?

23. If *1.0 grm* of matter could be converted entirely into energy, what would be the energy in *kWh* (kilo-watt-hour).

24. Figure 2 shows two reference frames in relative motion. *Alice*'s frame moves with speed $v = \frac{3}{5}c$ relative to *Bob*'s frame. When the origin of *Alice*'s frame passes the origin of *Bob*'s frame, clocks at both origins read *zero*. In this frame, *Bob* notes the following coordinates for events *1* and *2*.

    Event 1 : x_1 =*5 meters*; t_1=2x*10*$^{-8}$ *seconds*.

    Event 2 : x_2 =*10 meters*; t_2=*3*x*10*$^{-8}$ *seconds*.

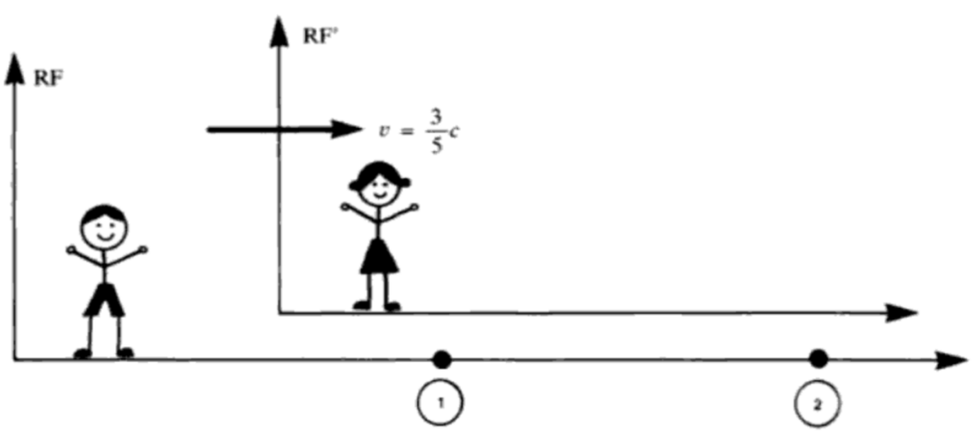


Figure 2 The vertical offset is for clarity only.

Use the Lorentz transformations (where appropriate) to answer the following questions.

(a) Do you find that $1/\sqrt{1-(v^2/c^2)}$ equals $\frac{5}{4}$?

(b)What are the coordinates of event *1* as measured in *Alice*'s frame?

(c) Similarly, what are coordinates of event *2*?

(d) I find $x_2' - x_2' = 4$ meters. Do you concur?

(e) Does *Alice* note the two events to be simultaneous?

(f) If *Alice* were moving (relative to *Bob*) *10%* faster than specified here, would she perceive the two events to be simultaneous? What if she were moving *10%* slower?

25. **Identify** the symbols E, p and m_0 in the equation ,

$$E^2 - p^2c^2 = m_o^2c^4$$

Derive the equation for **mass variation** in relativistic dynamics.

Rest mass of an electron is accelerated to a velocity 0.8 c by using a particle accelerator. Calculate the mass of the moving electron.

Briefly explain reasons for changing the moving plane of planets, which are revolving about the sun. **(Mathematical explanations not needed).**

26. (a) State the **basic postulates** of the *special theory of relativity*.
(b) Show that the transformation equations

$$x' = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \left(1 - \frac{v^2}{c}\right)^{-1/2}\left(t - \frac{vx}{c^2}\right)$$

may be obtained from the above postulates.
(c) Discuss simultaneity of events in two interval frames of reference.

27. A *muon* decays with a mean life-time T_a when it is at rest. *Muons* in motion are observed to decay with a mean life-time $6T_a$. Determine their velocity.

28. A train *200m* long passes through a tunnel *100m* long. (a) How fast the train should move in order that it is just contained in the tunnel according to an observer on the ground? (b) What is the length of the tunnel according to an observer in the train?
[Hint: Use the equation $L = L_0\sqrt{1 - v^2/c^2}$ **from which** $v = \frac{c}{L_0}\sqrt{(L_0 + L)(L_0 - L)}$ **]**

29. A vector in the primed frame is represented by $12\vec{i}' + 6\vec{j}'$. Find its representation in the unprimed frame if the primed frame moves with $\vec{v} = 0.75\ c\ \vec{i}$ with respect to the unprimed frame.

30. If
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma[t - \frac{vx}{c^2}]$$
Show that
$$x = \gamma(x' + vt)$$
$$y = y'$$
$$\gamma = \gamma'$$
$$t = \gamma[t' + \frac{vx'}{c^2}]$$
.

31. State the postulates of the special theory of relativity. Derive the *Lorentz* transformation equations.

$$x' = \frac{x - vt}{\sqrt{1 - (v/c)^2}}, \quad y' = y, \quad z' = z \ \text{ and } \ t' = \frac{t - \frac{vx}{c}}{\sqrt{1 - (v/c)^2}}.$$

Where the symbols have their usual meanings.
Hence deduce the *Lorentz* relativistic *length contraction* formula.

A spacecraft is moving directly away from the earth with a uniform velocity of *0.8c* relative to the earth. An observer on the earth measures the time taken by the spacecraft to travel from the earth to the moon and finds it to be *1.6 s*. According to the astronauts in the space craft

(a) How much time is required for this trip?
(b) What is the distance from the earth to the moon?

32. What is meant by "**Twin Paradox"** in relativity?

Let A be the twin on the earth and B be the twin in the ship in the twin paradox episode. Comment on the following statement using your Knowledge of special of relativity.

"The twin B can go to the future, but can not go to the past"

33. Consistency of Special Theory of Relativity with the concept of 'Causality" lead to its acceptance at the initial stages. The two different from of Causality as follows,

(a) Causality in Eastern Philosophy
(b) Causality in Western Philosophy

What is idea of causality in Eastern Philosophy?
How it differs from causality in Western Philosophy?

Explain how the Special Theory of Relativity was strengthened by the concept of Causality.

Note bane:

You may use following formula.

$$x^1 = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad x = \frac{x^1 + vt^1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y^1 = y \qquad y = y^1$$

$$z^1 = z \qquad z = z^1$$

$$t^1 = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t^1 + \frac{vx^1}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma(u^1) = \gamma(u)\gamma(v)\left[1 - \frac{vu_x}{c^2}\right],$$

Where $\gamma(w) = \sqrt{1 - \frac{w^2}{c^2}}$ (Above symbols have their usual meanings)

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